Relativistic Spacetime and Non-Causal Game Theory

1 Introduction

This document explores the incorporation of relativistic spacetime intervals into game theory, with a focus on scenarios where causality is challenged, leading to potential reverse influence of outcomes on decision-making processes.

2 Spacetime Geometry and Relativity Basics

In special relativity, the position and time of an event are represented by a four-vector:

$$x^\mu = (ct, x, y, z), \tag{1}$$

where c is the speed of light. The spacetime interval s^2 is defined as:

$$s^2 = (ct)^2 - x^2 - y^2 - z^2. \tag{2}$$

Depending on the sign of s^2, the interval is classified as:

- $s^2 > 0$: Time-like interval (causal relationship possible).
- $s^2 = 0$: Light-like interval (events lie on the light cone).
- $s^2 < 0$: Space-like interval (no causal relationship).

In space-like intervals, events may appear simultaneous or even reversed in causality in certain reference frames.

3 Game Theory in Non-Causal Settings

Consider a simple two-player game with players A and B, where:

- Player A makes a decision a at event E_A.
- Player B makes a decision b at event E_B.

The events E_A and E_B are separated by a space-like interval, implying no clear causal order between them.

The payoff functions for the players are:

$$U_A(a, b), \quad U_B(a, b), \tag{3}$$

representing their respective utilities based on actions a and b. In a non-causal setting, decisions may depend on outcomes that are, in principle, influenced retrocausally.

3.1 Information Flow and Strategy Dependence

Under relativistic constraints, we assume:

1. **Action dependence on outcomes**: Player A's decision a depends not only on local information I_A but also on b, which is determined at E_B.
2. **Strategy representation**: Let $a = f_A(I_A, b)$ and $b = f_B(I_B, a)$.
3. **Reverse causality**: Player B's decision b can also be influenced by a, forming a closed-loop dependence.

4 Fixed-Point Analysis

In the non-causal framework, equilibrium strategies can be analyzed through fixed-point equations. Suppose:

$$a = f_A(b), \quad b = f_B(a), \tag{4}$$

where f_A and f_B are strategy functions. Solving for the fixed points gives:

$$a^* = f_A(b^*), \quad b^* = f_B(a^*). \tag{5}$$

4.1 Example

Let the payoff functions be:

$$U_A(a, b) = -a^2 + 2ab, \quad U_B(a, b) = -b^2 + 2ab. \tag{6}$$

Assume linear strategies:

$$a = \alpha b, \quad b = \beta a. \tag{7}$$

Substituting, we have:

$$a = \alpha(\beta a) \implies a(1 - \alpha\beta) = 0. \tag{8}$$

The solutions are:

$$a = 0 \quad \text{or} \quad \alpha\beta = 1. \tag{9}$$

The corresponding b values are $b = \beta a$. This demonstrates the calculation of fixed points under non-causal influences.

5 Interpretation and Physical Implications

In a relativistic framework:

- Fixed points represent equilibria where decisions and payoffs are balanced.
- Under space-like intervals, actions may mutually influence each other despite the absence of a causal order.
- This analysis parallels retrocausal effects observed in quantum mechanics, such as time-symmetric interpretations.

6 Conclusion

Integrating relativistic spacetime into game theory introduces intriguing possibilities for decision-making under non-causal conditions.