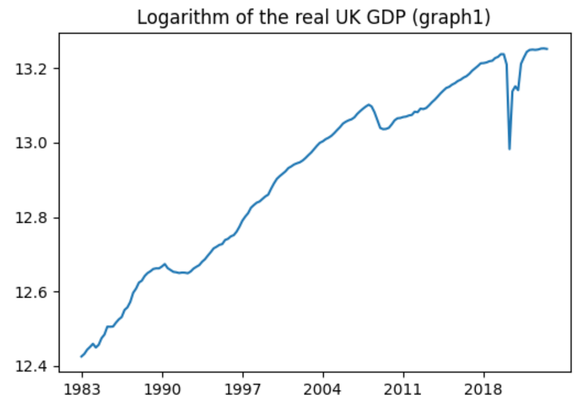
26/30 P

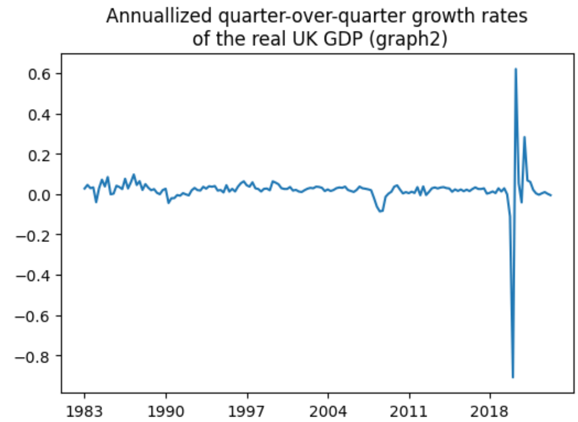
Take home exam 1358810

1.

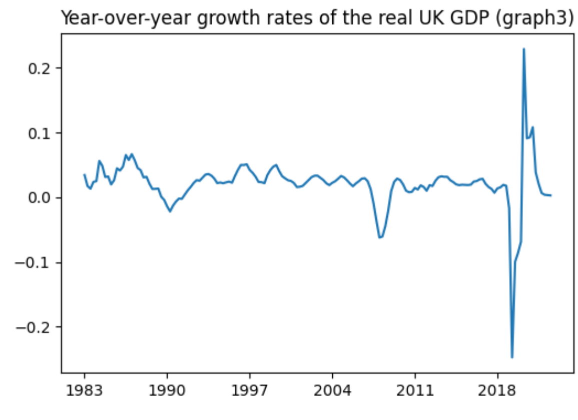
a)



Main feature of graph1:
Throughout the graph, we can see that real GDP has been growing over time; we can also see that the financial crisis of 2008 and 2009 caused an economic recession, and that there was a sharp economic decline in 2020 due to the impact of the COVID-19 pandemic.

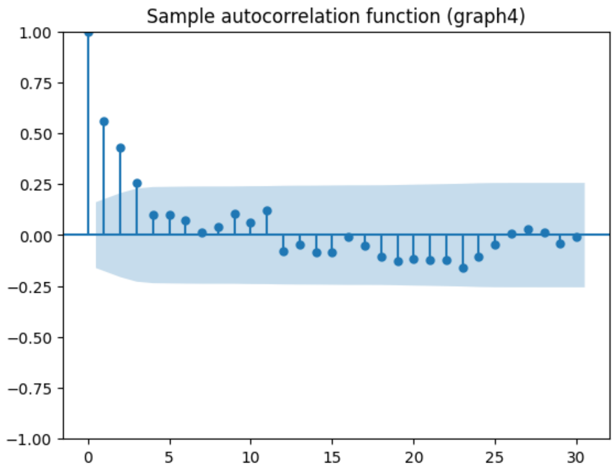


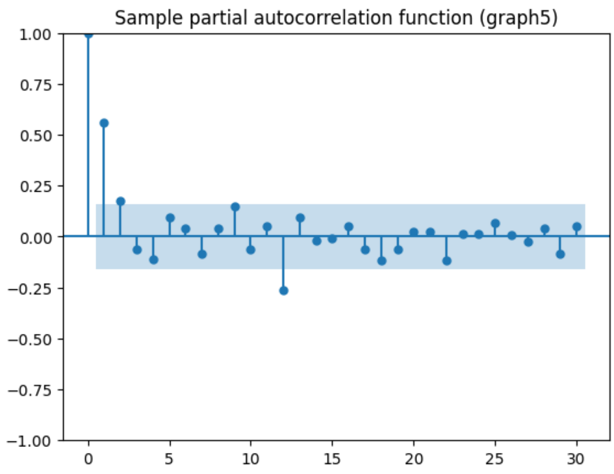
Short-term quarterly fluctuations are evident, showing positive and negative growth rates. There were a relatively stable growth fluctuations in the late 1990s and 2000s and very large negative growth in 2020 due to the COVID-19 pandemic, followed by a sharp recovery followed by a rapid recovery.



Main feature of graph3:
The graph of annual growth rates shows a greater range of fluctuation than Graph 2. Compared to the 1980s, there is a trend toward a gradual decline in the peak growth rate. This is thought to indicate that the speed of growth is slowing down as the economy matures. As in Graph 2, there are also fluctuations due to the impact of the pandemic.

b)





According to the graph 4, which is the plot of the sample autocorrelation function, the autocorrelation from lags 1 to 3 exceeds the bounds, so I suggest the MA(3) model.

c)

AIC
table 1

	0
0	-664.959104
1	-718.081821
2	-720.540362
3	-719.104524
4	-718.980181
5	-718.313981
6	-716.546047
7	-715.548349
8	-713.777688

Since AIC value for lag 2 is -720.54, the suggested AIC is lag2.

HQIC
table 2

	0
0	-662.529013
1	-714.436685
2	-715.680181
3	-713.029298
4	-711.689909
5	-709.808664
6	-706.825685
7	-704.612943
8	-701.627236

Since HQIC value for lag 2 is -715.68, the suggested HQIC is lag2.

BIC
tabe3

	0
0	-662.529013
1	-714.436685
2	-715.680181
3	-713.029298
4	-711.689909
5	-709.808664
6	-706.825685
7	-704.612943
8	-701.627236

(√) −0.5P

Since BIC value for lag 1 is -709.11, the suggested BIC is lag2

d)

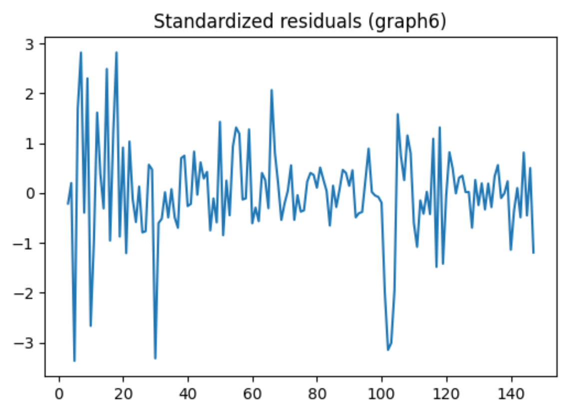
table4

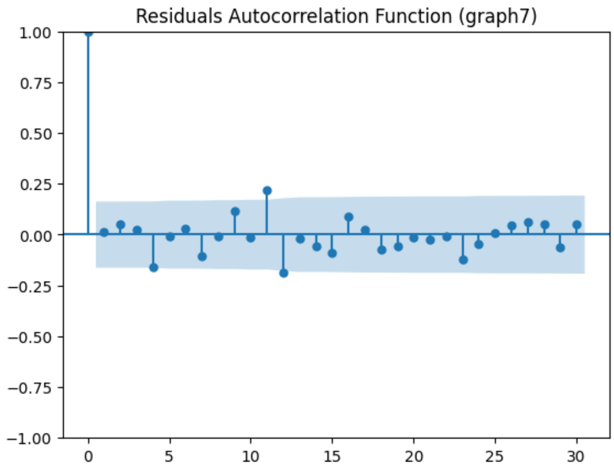
```
                            OLS Regression Results
==============================================================================
Dep. Variable:           NGDPRSAXDCGBQ   R-squared:                       0.336
Model:                             OLS   Adj. R-squared:                  0.326
Method:                  Least Squares   F-statistic:                     35.88
Date:                 Sun, 23 Jun 2024   Prob (F-statistic):           2.44e-13
Time:                         21:58:45   Log-Likelihood:                 359.07
No. Observations:                  145   AIC:                            -712.1
Df Residuals:                      142   BIC:                            -703.2
Df Model:                            2
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0077      0.002      3.197      0.002       0.003       0.013
x1             0.4626      0.083      5.585      0.000       0.299       0.626
x2             0.1745      0.083      2.107      0.037       0.011       0.338
==============================================================================
Omnibus:                       18.404   Durbin-Watson:                   1.965
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               43.972
Skew:                          -0.474   Prob(JB):                     2.83e-10
Kurtosis:                       5.525   Cond. No.                         60.7
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

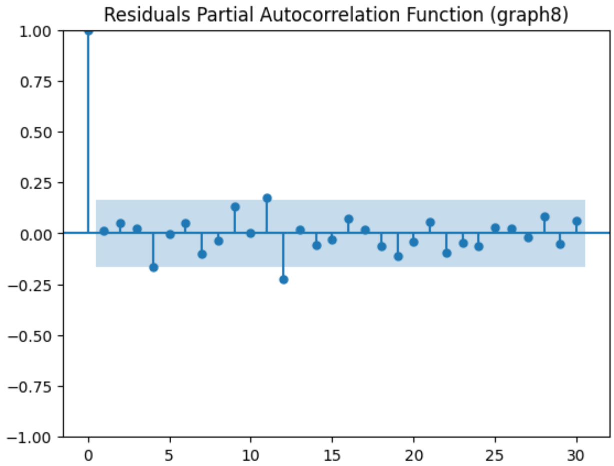
parameters are:
const 0.007739
x1 0.462613
x2 0.174506
dtype: float64

asymptotic standard errors are:
const 0.002421
x1 0.082828
x2 0.082836
dtype: float64

e)







- The modified Portmanteau test:

The null hypothesis is that the autocorrelation coefficients of the residual series are zero for all lags from lag 1 to lag h.

$$\rho_{\epsilon,1} = \rho_{\epsilon,2} = \cdots = \rho_{\epsilon,h} = 0$$

, where

$$\rho_{\epsilon,i} = \mathrm{Corr}(\epsilon_t, \epsilon_{t-i})$$

The alternative hypothesis is that the autocorrelation coefficient of the residual series is non-zero for at least one lag from lag 1 to lag h.

$$\rho_{\epsilon,i} \neq 0 \text{ for at least one } i = 1, \ldots, h$$

Under the null the appropriate distribution of the test statistic is a chi-squared distribution with (h - p - q) degrees of freedom, in this case degree of freedom is h - 2

We can reject the null at 5% significant level when the lag is from 12 to 16.

The test statistics and the p values are:

table5

	lb_stat	lb_pvalue
1	0.022412	NaN
2	0.433621	NaN
3	0.536285	0.463976
4	4.379414	0.111950
5	4.385201	0.222762
6	4.544627	0.337289
7	6.327009	0.275687
8	6.336595	0.386559
9	8.368572	0.301218
10	8.405694	0.394876
11	15.881195	0.069404
12	21.575952	0.017417
13	21.641021	0.027307
14	22.122678	0.036168
15	23.438652	0.036704
16	24.829985	0.036283
17	24.942003	0.050728

	lb_stat	lb_pvalue
18	25.842148	0.056290
19	26.358118	0.068178
20	26.390506	0.091159

√

Comment on the adequacy of the model: While a larger p-value may be natural up to lag 8 for autocorrelation, a larger p-value at 9-12lags may require consideration of seasonality since the null hypothesis of no autocorrelation in the residuals cannot be rejected.

- Breusch-Godfrey LM-test:

Considering the AR(h) model for errors

$$\epsilon_t = \beta_1 \epsilon_{t-1} + \cdots + \beta_h \epsilon_{t-h} + v_t$$

The null hypothesis is that there is no autocorrelation

$$\beta_1 = \cdots = \beta_h = 0$$

The alternative hypothesis is that at least one of the coefficients is not zero.

$$\beta_1 \neq 0 \text{ or } \cdots \text{ or } \beta_h \neq 0$$

Under the null the appropriate distribution of the test statistic is a chi-squared distribution with h degrees of freedom

The test statistics and the p values are:
LM Statistic: 22.0443
LM p-value: 0.1418
Thus, we cannot reject the null at 10% significance level.

Comment on the adequacy of the model:
Based on the test results, the null hypothesis ("there is no autocorrelation in the residuals") cannot be rejected at the 10% significance level, so it is concluded that there is no significant autocorrelation in the residuals of the model. This suggests that the model likely captures the structure of the data well.

- The result of LMF-test is as follows: h?
  The null and alternative hypothesis are the same as the LM-test.

Under the null the appropriate distribution of the test statistic is a F distribution with F(h, T-p-h-1) degrees of freedom. T is the number of the observations in the dataset used for the regression analysis. p is the number of parameters estimated in the model, excluding the intercept.
In this case, the degree of freedom is F(h, 145-2-h-1)=F(h, 142-h)

The test statistics and the p values are: h? -0.5P (√)
F Statistic: 1.4119
F p-value: 0.1463
Thus, we cannot reject the null at 10% significance level.

Comment on the adequacy of the model:
similarly, we cannot reject the null, meaning that the model likely fit the time series data.

- The result of a non normality test (Jarque-Bera) is as follows:

The null hypothesis is that there are no skewness and no kurtosis, which means that third and fourth moments of

$$\epsilon_t^s$$

are as in standard normal distribution

$$E[(\epsilon_t^s)^3] = 0 \quad \text{and} \quad E[(\epsilon_t^s)^4] = 3$$

The alternative hypothesis is that there is a skewness or kurtosis, which means that either third or fourth moments of the epsilon are not as in standard normal distribution.

$$E[(\epsilon_t^s)^3] \neq 0 \quad \text{and} \quad E[(\epsilon_t^s)^4] = 3$$

Under the null the appropriate distribution of the test statistic is a chi-squared distribution with (2) degrees of freedom

The test statistics and the p values are:
JB Statistic: 43.9719
JB p-value: 0.0000
thus, we can reject the null at 1% significance level. √

Comment on the adequacy of the model:
The fact that we could reject the null indicates that the residuals of the model cannot normally distributed, and the model does not fit well.

f)

'coefficients': {
'c': 0.008275022982464035,
'alpha1': 0.46059682514988554, √
'alpha2': 0.1700016053151856,
'sigma2': 0.0004204558496704455
}

'std_errors': {
'c': 1.198407600676266,
'alpha1': 30.19927370754045, × -0.5P The L-BFGS-B doesn't give you the correct Hessian Matrix
'alpha2': 22.989317817023696,
'sigma2': 0.01643904490632986
}

'max_log_likelihood': 362.6035008759334} √

Checking the result by comparing the built-in function ARIMA

table6

```
                               SARIMAX Results
==============================================================================
Dep. Variable:                      y   No. Observations:                  147
Model:                 ARIMA(2, 0, 0)   Log Likelihood                 362.617
Date:                Mon, 24 Jun 2024   AIC                           -717.234
Time:                        01:02:05   BIC                           -705.272
Sample:                             0   HQIC                          -712.374
                                - 147
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0224      0.005      4.757      0.000       0.013       0.032
ar.L1          0.4507      0.054      8.310      0.000       0.344       0.557
ar.L2          0.1811      0.065      2.794      0.005       0.054       0.308
sigma2         0.0004   3.43e-05     12.240      0.000       0.000       0.000
===================================================================================
Ljung-Box (L1) (Q):                   0.02   Jarque-Bera (JB):                41.07
Prob(Q):                              0.88   Prob(JB):                         0.00
Heteroskedasticity (H):               0.58   Skew:                            -0.39
Prob(H) (two-sided):                  0.06   Kurtosis:                         5.47
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
```

Do you get the same parameter estimates as from Python's statsmodel ARIMA function? (✓)

-> As for the constant, I got a different value, but in terms of the coefficients of the alphas and sigma square, I got the values with the deviation of less than one standard error.

↳ the SARIMAX estimates μ ...

What is the estimated mean based on the AR(2)?

->According to the function I created, the estimated mean based on the AR(2) is 1.1984, while the value is 0.0224 calculated by the built-in function. -0.5P

$$\mu = \frac{c}{1-\alpha_1-\alpha_2}$$ ✗

g)

Estimating the AR(2) model over the sample period ending 2023Q3

table 7

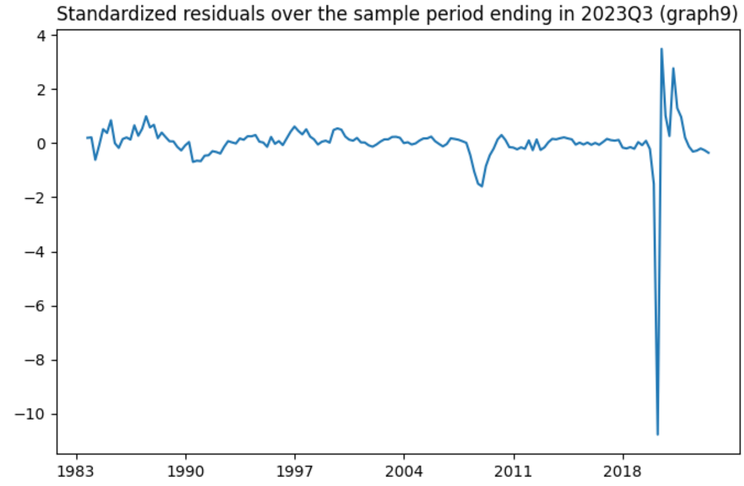
```
                            OLS Regression Results
==============================================================================
Dep. Variable:          NGDPRSAXDCGBQ   R-squared:                       0.081
Model:                            OLS   Adj. R-squared:                  0.070
Method:                 Least Squares   F-statistic:                     6.952
Date:                Mon, 24 Jun 2024   Prob (F-statistic):            0.00128
Time:                        01:03:22   Log-Likelihood:                 158.24
No. Observations:                 160   AIC:                            -310.5
Df Residuals:                     157   BIC:                            -301.3
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0288      0.008      3.766      0.000       0.014       0.044
x1            -0.2906      0.079     -3.670      0.000      -0.447      -0.134
x2            -0.1257      0.079     -1.587      0.115      -0.282       0.031
==============================================================================
Omnibus:                      267.214   Durbin-Watson:                   1.971
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            47048.979
Skew:                          -7.460   Prob(JB):                         0.00
Kurtosis:                      85.673   Cond. No.                         12.4
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

✓

parameters are:
const 0.028755
x1 -0.290561
x2 -0.125667
dtype: float64 ✓

asymptotic standard errors are:
const 0.007635
x1 0.079182
x2 0.079190
dtype: float64

Standardized residuals over the sample period ending in 2023Q3 (graph9)

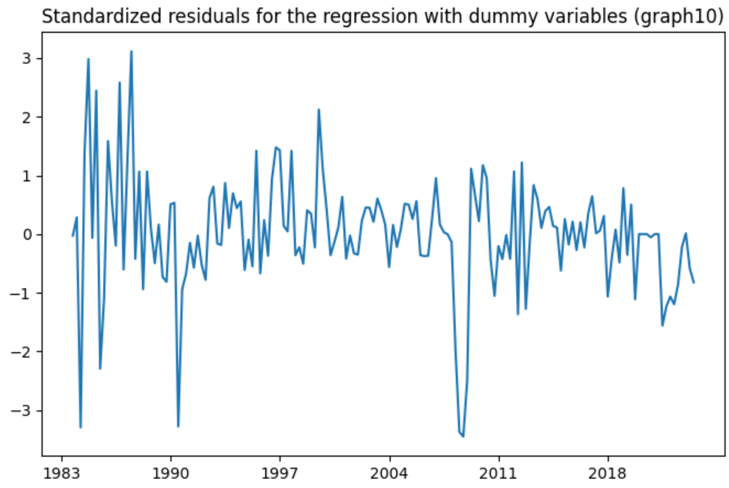


With dummy variables
table8

```
                            OLS Regression Results
==============================================================================
Dep. Variable:           NGDPRSAXDCGBQ   R-squared:                       0.954
Model:                             OLS   Adj. R-squared:                  0.952
Method:                  Least Squares   F-statistic:                     452.5
Date:                 Mon, 24 Jun 2024   Prob (F-statistic):           2.35e-98
Time:                         01:06:23   Log-Likelihood:                 398.15
No. Observations:                  160   AIC:                            -780.3
Df Residuals:                      152   BIC:                            -755.7
Df Model:                            7
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0090      0.002      4.055      0.000       0.005       0.013
x1             0.3468      0.040      8.758      0.000       0.269       0.425
x2             0.1861      0.028      6.636      0.000       0.131       0.242
x3            -0.1245      0.021     -6.012      0.000      -0.165      -0.084
x4            -0.8793      0.021    -40.919      0.000      -0.922      -0.837
x5             0.9470      0.045     21.181      0.000       0.859       1.035
x6            -0.1843      0.027     -6.781      0.000      -0.238      -0.131
x7             0.2781      0.021     13.376      0.000       0.237       0.319
==============================================================================
Omnibus:                       21.777   Durbin-Watson:                   1.659
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               62.595
Skew:                          -0.469   Prob(JB):                     2.56e-14
Kurtosis:                       5.917   Cond. No.                         39.1
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

parameters are:
const 0.009001
x1 0.346809
x2 0.186094
x3 -0.124460
x4 -0.879289
x5 0.947018
x6 -0.184317
x7 0.278121
dtype: float64

asymptotic standard errors are:
const 0.002220
x1 0.039600
x2 0.028044
x3 0.020703
x4 0.021489
x5 0.044712
x6 0.027181
x7 0.020793
dtype: float64



Interpret the sign of the estimated coefficients on the dummy variables.
-> The coefficients for 2020Q1, 2020Q2, and 2021Q1 are negative, indicating that annualized quarter-over-quarter growth rates of the real GPD are fairly lower in these three quarters than in other years. Conversely, the coefficients for 2020Q3 and 2021Q2 are positive, indicating quite higher growth rates in those years compared to other years.

How does the inclusion of the dummy variables affect the estimates of the AR coefficients?
->
constant:
The constant term has been significantly reduced. This is due to the fact that the dummy variable corrected for anomalous values during the pandemic period more accurately reflects the average growth rate during normal periods of economic activity.

the coefficients of AR(1), x1:
The negative value before the addition of the dummy variable has changed to a positive value. This indicates that the short-term autoregressive relationship has been modified in a more stable and positive direction as a result of the removal of the anomalous shocks during the pandemic period.

the coefficients of AR(2), x2:
Similarly, the values have changed from negative to positive. This also indicates that, as a result of removing the effects of the pandemic, the autoregressive relationship over the two periods now reflects normal economic conditions.

2.

a)

i.

The null hypothesis is that the process does not have a unit root, which means that the process is stationary.
When we assume the data generating process is:

$$y_t = x_t + z_t$$

, where

$$x_t = x_{t-1} + v_t, \quad v_t \sim \text{i.i.d. } (0, \sigma_v^2)$$

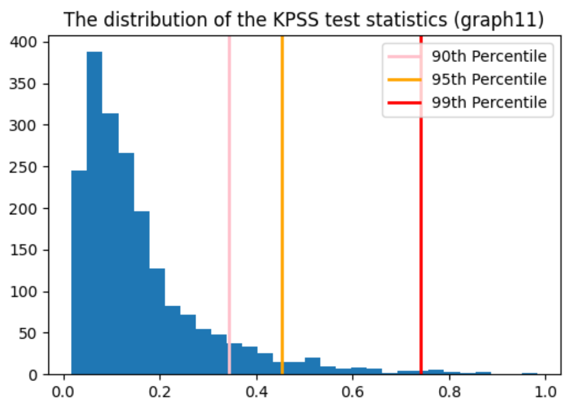
and z is a stationary process, under the null,

$$\sigma_v^2 = 0$$

y equals a constant plus a stationary process.

ii.

0.90 quantile: 0.3386386043212102
0.95 quantile: 0.4502681281940961
0.99 quantile: 0.710575009129688



iii.

Reported quantiles were very similar to the critical values of the KPPS test.
These values are the critical values of the KPSS test.

10%: 0.347
5%: 0.463
1%: 0.739

d)

KPSS test:
The null hypothesis is that the process does not have a unit root, which means that the process is stationary.

The alternative hypothesis is that the process has a unit root.

Under the null the appropriate distribution of the test statistic can be like graph11, which is not normal distribution.

We can reject the null at 5% significant level if the test statistic is larger than 0.463.

The rejection frequencies are summarized in g and h.

e), f)

Augmented Dickey Fuller test with the constant and one lagged difference:
When we assume that data generating process is

$$y_t = (1-\alpha)\mu_0 + \alpha y_{t-1} + \epsilon_t$$

The null hypothesis is that the process is that the process has a unit root.

$$\alpha = 1$$

The alternative hypothesis is that the process does not have a unit root.

Under the null the appropriate distribution of the test statistic is a non-standard limiting distribution.

The test regression is

$$\Delta y_t = c + \phi y_{t-1} + \epsilon_t$$

The test statistics is:

$$t_{DF,c} = \frac{\hat{\phi}}{\hat{\sigma}_{\hat{\phi}}}$$

We can reject the null at 5% significance level if the test statistic is smaller than -2.86

g)

table 9

	alpha1	alpha2	T	KPSS 1	KPSS 2	ADF 1	ADF 2
0	0.4	0.2	50	0.1280	0.0180	0.5915	0.8445
1	0.4	0.2	100	0.1385	0.0430	0.9885	0.9830
2	0.4	0.2	200	0.1550	0.0720	1.0000	1.0000
3	0.4	0.2	500	0.1305	0.0635	1.0000	1.0000
4	0.5	0.2	50	0.2075	0.0195	0.4045	0.7330
5	0.5	0.2	100	0.2045	0.0595	0.9195	0.9310
6	0.5	0.2	200	0.2010	0.0735	1.0000	1.0000
7	0.5	0.2	500	0.1680	0.0660	1.0000	1.0000
8	0.6	0.2	50	0.2655	0.0270	0.2120	0.5055
9	0.6	0.2	100	0.2950	0.0920	0.6395	0.7700
10	0.6	0.2	200	0.2950	0.1030	0.9975	0.9965
11	0.6	0.2	500	0.2485	0.0920	1.0000	1.0000
12	0.7	0.2	50	0.4085	0.0570	0.1120	0.2725
13	0.7	0.2	100	0.4675	0.1545	0.2275	0.4185
14	0.7	0.2	200	0.5140	0.1975	0.6815	0.7410
15	0.7	0.2	500	0.5075	0.1995	1.0000	1.0000
16	0.8	0.2	50	0.6875	0.3380	0.0560	0.0945
17	0.8	0.2	100	0.8375	0.5850	0.0565	0.0930
18	0.8	0.2	200	0.9385	0.7260	0.0530	0.0760
19	0.8	0.2	500	0.9895	0.8890	0.0495	0.0510

✓

-1P

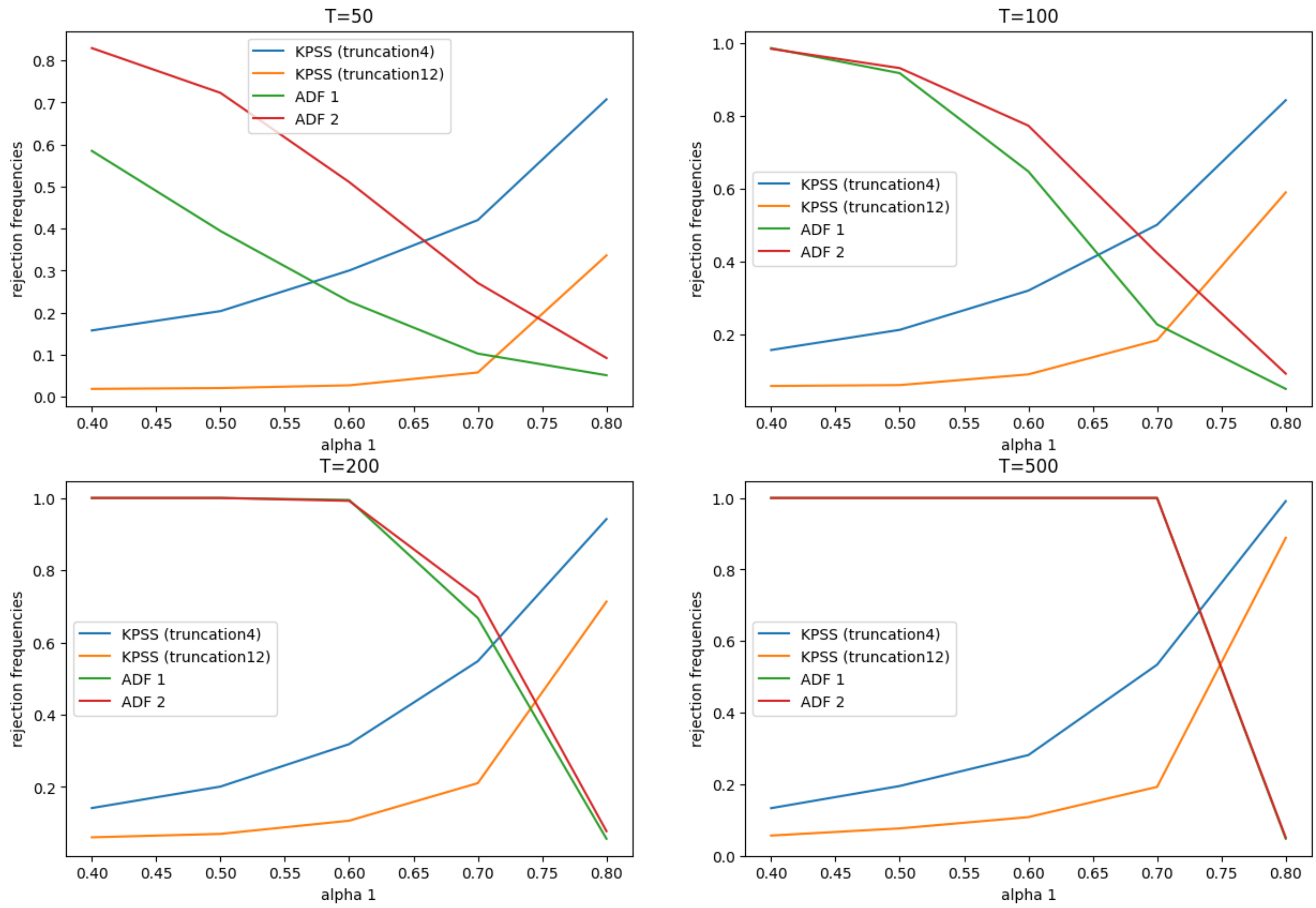
the size and power is flipped ← For $\alpha_1 = 0.8$ we have a unit root

- Which of the rejection frequencies correspond to the empirical size and power of the test?
  -> For the KPSS test, the null hypothesis is that the series is stationary. Therefore, the rejection frequencies when the series is stationary represent the empirical size of the test.
  For the ADF test, the null hypothesis is that the series has a unit root (non-stationary). Thus, when the series is stationary, the rejection frequencies represent the empirical power of the test, as they indicate how often the test correctly rejects the false null hypothesis of a unit root."
- What size values would you expect in an ideal test?
  -> In an ideal test, the size value would match the significance level of the test. Common significance levels are 5%, also meaning the null hypothesis should be incorrectly rejected about 5% of the time (✓)
- How does the empirical size values compare against this?
  -> To compare the empirical size values against the ideal values, we need to examine the rejection frequencies when the null hypothesis is true. In an ideal scenario, the empirical size should be close to the significance level, 5%.
  The null hypothesis for the KPSS test is that the series is stationary. The empirical size is reflected in the rejection frequencies for stationary series, which are the cases

with lower α1 values. For example, when α1=0.4 and T=50, the KPSS rejection frequency is 0.1280.
The null hypothesis for the ADF test is that the series has a unit root (non-stationary). The empirical size is reflected in the rejection frequencies for non-stationary series, which are the cases with higher α1 values. For example, when α1=0.8 and T=50, the ADF rejection frequency is 0.0560. (✓)
The empirical size values are higher than the ideal 0.05. This suggests that the KPSS test is more likely to incorrectly reject the null hypothesis of stationarity, indicating a higher rate of Type I errors compared to the ideal scenario. The empirical size values are close to the ideal 0.05, suggesting that the ADF test is well-calibrated in terms of Type I errors.

h)

Graph 12



✓

- Which of the KPSS test variants is doing better?
  -> For KPSS, the variant with truncation lag 4 generally shows better performance, especially for larger sample sizes (T=200, T=500). It consistently has higher power (higher rejection rates for larger α1) while maintaining a reasonable empirical size. The size of q=4 is worse! -0.5P
- Which variant of the ADF test is doing better?
  -> For ADF, the variant using the BIC criterion (ADF 2) generally performs better. It has higher power for larger α1 values while maintaining an empirical size close to the nominal level of 5%. ✓
- Briefly describe any differences between the ADF test and the KPSS test in terms of correct unit root detection.
  -> The KPSS test is designed to test the null hypothesis of stationarity against the alternative of a unit root. Therefore, a rejection in the KPSS test suggests the presence of a unit root (non-stationary).
  The ADF test is designed to test the null hypothesis of a unit root against the alternative of stationarity. A rejection in the ADF test suggests that the series is stationary.

↳ compare the plots -0.5P