Programming assignment No. 3

General hints and rules:

- Although the submission of this assignment is not mandatory, you are advised and encouraged to submit your solution, as you can earn bonus points (to be added to your final exam score).
- The solutions can be submitted until Friday, June 7 2024, 11.30 am to tilmann.haertl@uni-konstanz.de. Late submissions will not be considered.
- We recommend to use the programming language Python* for the following programming problems. The solutions should be handed in form of a code file. Answers to questions requiring text (e.g. for interpretations) should be given as comments in the code.
- Name your code file with the assignment number and your student number (not your name). Example: For programming assignment 3 (PA3) and if your student number is 007007, call the file PA3_007007.py
- Policy with regard to academic dishonesty: Students who wish to work together on assignment material may do so. Please indicate the student numbers of all fellow students that have worked in your group. **Each student must submit her or his individual code/solution file**.
- Write your code general enough such that it can be easily reused. Document your code.
- Avoid loops as much as possible and use matrix language techniques whenever possible.

1) Write a function that computes the Modified Portmanteau test statistic. Your function should take a $T \times 1$ vector of residuals, the lag orders p and q of the fitted ARMA(p, q) process, the number of residual autocorrelations that are tested and the significance level of the tests. The function should return both Portmanteau test statistics and the critical value.
2) Simulate a time series with length $T = 1000$ from an AR(2) process with $c = 0.1$, $\alpha_1 = 0.6$, $\alpha_2 = 0.2$ and ε_t iid $N(0, \sigma^2)$, $\sigma = 1$. Fit an AR(1) and an AR(2) to the resulting series and conduct the Modified Portmanteau test procedure on the estimated residuals. Test for the joint significance of the first $h = 16$ residual autocorrelations with a significance level α of 5%. Interpret your results.
3) Program a function that estimates an AR(1) with intercept by exact ML. The function should take the time series as an input and should return the MLEs of c, α_1 and σ^2 together with asymptotic standard errors and the value of the maximized log-likelihood function. *The log-likelihood function can be found on the lecture slides. Basics of numerical optimization in Python are added to the python introduction file. Hint: Use an optimizer that also returns the Hessian matrix.*
4) Apply your function from 3) to estimate an AR(1) with intercept on quarterly European GDP **growth rates** (see lecture, raw GDP data in `awm19up18.csv` and column `YER`). Compare your results with those of the `ARIMA` function of Python's `statsmodel` module.

*A download link for Python can be found on ILIAS. There is also a link to a Python documentation.