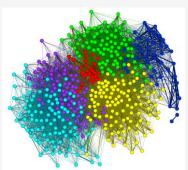
Data Analytics

Lecture 6



Outline

- Modeling and Machine Learning part 1
- Decision Trees
- Instance Based Learning

http://wwwusers.cs.umn.edu/~kumar/dmbook/ch4.pdf

> Adapted from Introduction to Data Mining Tan, Steibeck, Kumar

What is a model?

- An attempt to represent reality through a particular lens.
- An artificial construct that does not contain unnecessary detail and makes a set of assumptions.

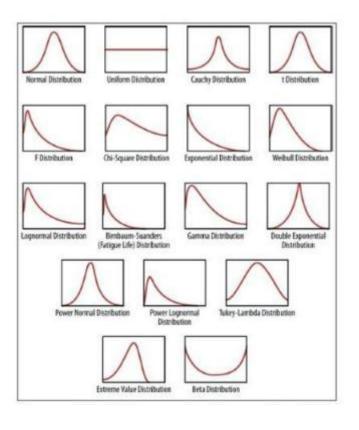
Statistical Modeling

- A way to express a model using mathematics
- The model designer makes an assumption about the **generative process** of the data
- The goal is to estimate the parameters of the model given a particular data set
- A level of confidence is always given for the model, e.g. confidence intervals

Statistical modeling questions

- What is the process that generated the data?
- What happened first?
- What influences what?
- What causes what?
- o How can I test these?

Common Distributions



- Basis for statistical models
- Natural processes generate "shapes" of distributions that can often be approximated by a mathematical function, given a few parameters that are estimated using the data.
- Not all processes generate data that looks like a named distribution.

From book: Doing Data Science

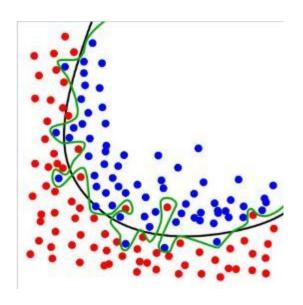
Deciding on a model to use

- Conduct exploratory analysis
- Develop a hypothesis to test
 - Try a linear function first why?
 - Write down assumptions
 - Does this make sense?
 - If necessary, begin looking at more sophisticated models
 - Write assumptions
 - Does this make sense?

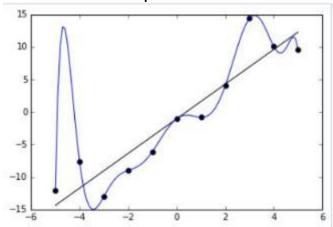
Fitting a Model

- When you fit a model, you estimate its parameters using real world collected data (samples).
- Fitting a model often requires optimization techniques and algorithms.
- Over fitting is a common problem that needs to be avoided.
 - Can end up describing random error or noise rather than the underlying distribution.
 - Can occur when a model is too complex.

Avoid testing and training using the same or overlapping data.



Visual Examples of over fitting



Learning Styles

Supervised Learning

 Labeled input data exist to train a model. The model is then used to predict the class on unseen data.

Unsupervised Learning

 Input data are not labeled and the result is not known.

Semi-supervised Learning

• Input data is a **mix** of labeled and unlabeled examples.

Reinforcement Learning

- A model that interacts with and learns from its environment.
- Feedback is provided as punishments and rewards in the environment.

Supervised Examples:

Regression, Decision Tree, Random Forest, KNN, Logistic Regression, Naive Bayes, Support Vector Machines, Neural Networks

Unsupervised Examples:

kmeans clustering, Association Rules

Reinforcement Learning Examples:

Q-Learning, Temporal Difference (TD), Deep Adversarial Networks

Interesting References:

https://machinelearningmastery.com/a-tour-of-machine-learning-algorithms/

What is Classification

Given: a collection of records/vectors (training dataset) Each record contains a set of attributes (variable values), one of the attributes must be the class.

Goal: Find a *model* (some function of the variable values) to identify the **class of a new vector/record**.

Table 4.1. The vertebrate data set.

Name	Body Temperature	Skin Cover	Gives Birth	Aquatic Creature	Aerial Creature	Has Legs	Hiber- nates	Class Label	→ CLASS
human	warm-blooded	hair	Ves	no	no	yes	no	mammal	
python	cold-blooded	scales	no	no	no	no	yes	reptile	
salmon	cold-blooded	scales	no	ves	no	no	no	fish	
whale	warm-blooded	hair	yes	yes	no	no	no	mammal	
frog	cold-blooded	none	no	semi	no	yes	yes	amphibian	
komodo dragon	cold-blooded	scales	no	no	no	yes	no	reptile	
bat	warm-blooded	hair	yes	no	yes	yes	yes	mammal	
pigeon	warm-blooded	feathers	no	no	Ves	yes	no	bird	
cat	warm-blooded	fur	yes	no	no	yes	no	mammal	
leopard shark	cold-blooded	scales	yes	yes	no	no	no	fish	
turtle	cold-blooded	scales	no	semi	no	yes	no	reptile	
penguin	warm-blooded	feathers	no	semi	no	yes	no	bird	
porcupine	warm-blooded	quills	yes	no	no	yes	yes	mammal	
eel	cold-blooded	scales	no	ves	no	no	no	fish	
salamander	cold-blooded	none	no	semi	no	ves	ves	amphibian	

Cross-validation

- A test set is used to determine the accuracy of the model.
- Usually, the given data set is divided into training and test sets, with training sets used to build the model and the test set used to validate it.
- Training sets and testing sets should not overlap in values.
- Cross-validation (leave-one-out) is often used.

Concepts for ML Classification

Input data: collection of records (also called an instance or example).

- For example: tuple(x, y), where x is the set (vector) of known attributes (variable values) and y is the class label (called the target).

Classification: learning a target/class **function** f that maps any vector of attributes, **x** to a predefined class y.

 $f: \mathbf{x} \rightarrow y$ f is a classification model

- Descriptive Modeling: Classification model that can distinguish between objects of different classes.
- Predictive Modeling: Using a classification model to predict a label/class given a vector/record x

Example: Feature Table

Table 4.1. The vertebrate data set.

Name	Body Temperature	Skin Cover	Gives Birth	Aquatic Creature	Aerial Creature	Has Legs	Hiber- nates	Class Label
human	warm-blooded	hair	yes	no	no	yes	no	mammal
python	cold-blooded	scales	no	no	no	no	yes	reptile
salmon	cold-blooded	scales	no	yes	no	no	no	fish
whale	warm-blooded	hair	yes	yes	no	no	no	mammal
frog	cold-blooded	none	no	semi	no	yes	yes	amphibian
komodo	cold-blooded	scales	no	no	no	yes	no	reptile
dragon		2012/03/05/05	23000	3430	2009903	M. Care	322500	2594222000
bat	warm-blooded	hair	yes	no	yes	yes	yes	mammal
pigeon	warm-blooded	feathers	no	no	yes	yes	no	bird
cat	warm-blooded	fur	yes	no	no	yes	no	mammal
leopard	cold-blooded	scales	yes	yes	no	no	no	fish
shark		0.0000000000000000000000000000000000000	- W	(W (2.755))	413 40400	017070031	950500	95054454
turtle	cold-blooded	scales	no	semi	no	yes	no	reptile
penguin	warm-blooded	feathers	no	semi	no	yes	no	bird
porcupine	warm-blooded	quills	yes	no	no	ves	ves	mammal
eel	cold-blooded	scales	no	yes	no	no	no	fish
salamander	cold-blooded	none	no	semi	no	ves	yes	amphibian

http://www-users.cs.umn.edu/~kumar/dmbook/ch4.pdf, page 147

- 1. What are the classes, y?
- 2. What are the records, x?

Answer:

- The first record x₁ is:
 (human, warm-blooded, hair, yes, no, no, yes, no)
 The label or class y is mammal
- The second record **x**₂ is: (python, cold-blooded, scales, no, no, no, no, yes) The label or class y is reptile.

Given a new vector x_n (grib, warm-blooded, hair, yes, no, no, yes, no) Predict the class?

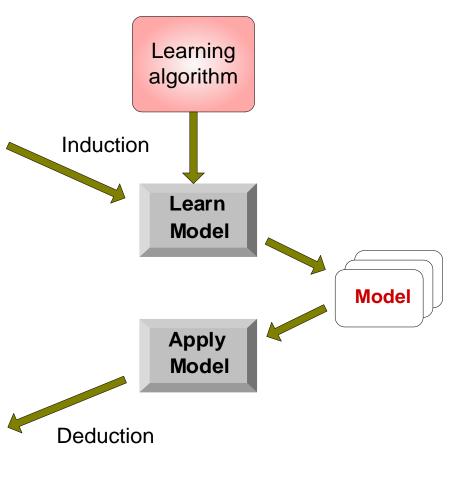
Illustrating A Classification Task

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	No
2	No	Medium	100K	No
3	No	Small	70K	No
4	Yes	Medium	120K	No
5	No	Large	95K	Yes
6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
9	No	Medium	75K	No
10	No	Small	90K	Yes

Training Set

Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	?
12	Yes	Medium	80K	?
13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

Test Set



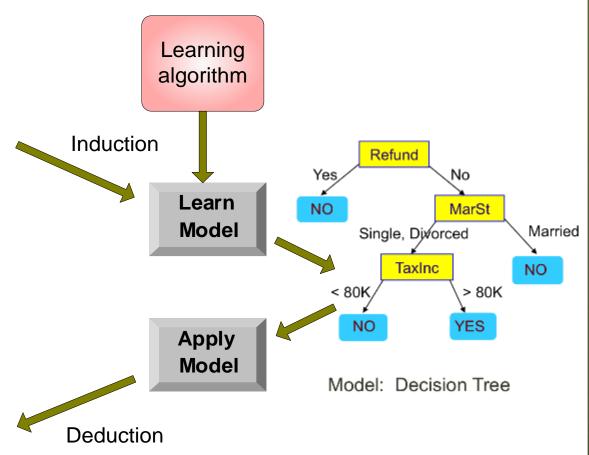
EXAMPLE: Is Someone Cheating on Their Taxes?

Tid	Attrib1	Attrib2	Attrib3	Class
110	Attiol	Attribe	Attribo	Olass
1	Yes	Large	125K	No
2	No	Medium	100K	No
3	No	Small	70K	No
4	Yes	Medium	120K	No
5	No	Large	95K	Yes
6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
9	No	Medium	75K	No
10	No	Small	90K	Yes

Training Set

Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	?
12	Yes	Medium	80K	?
13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

Test Set



Example of Training Data and Decision Tree Model

categorical continuous

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Yes No

NO

MarSt

Single, Divorced

TaxInc

< 80K

NO

YES

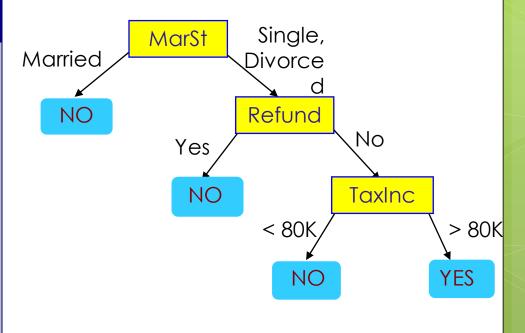
Model: Decision Tree

Training Data

Another Example of Decision Tree – there are infinite tree options

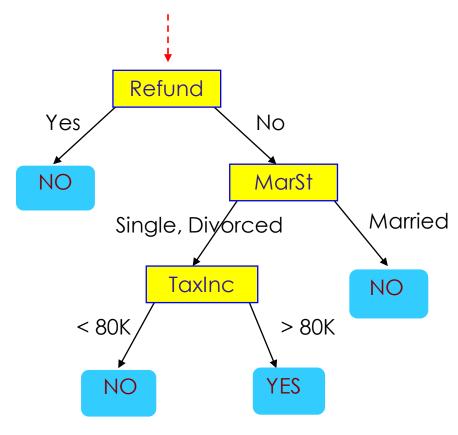
categorical continuous

Tid	Refund	Marital Status	Taxable Income	Cheat
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2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



Apply Model to Test Data

Start from the root of tree.



Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?

Performance Evaluation

Confusion matrix for a 2-class problem.

		Predicted Class		
		Class = 1	Class = 0	
Actual	Class = 1	f_{11}	f_{10}	
Class	Class = 0	f_{01}	f_{00}	

The performance of a classification model can be based on **counts** of test records **correctly** or **incorrectly** predicted.

f11: Record was class 1 and was predicted as class 1 correctly

fo1: Record was class 0 and incorrectly predicted as Class 1

Accuracy =
$$\frac{\text{Number of correct predictions}}{\text{Total number of predictions}} = \frac{f_{11} + f_{00}}{f_{11} + f_{10} + f_{01} + f_{00}}$$

Equivalently, the performance of a model can be expressed in terms of its error rate, which is given by the following equation:

Error rate =
$$\frac{\text{Number of wrong predictions}}{\text{Total number of predictions}} = \frac{f_{10} + f_{01}}{f_{11} + f_{10} + f_{01} + f_{00}}$$

Metrics for Performance Evaluation: Confusion Matrix

Confusion Matrix:

Accuracy =
$$\frac{a+d}{a+b+c+d} = \frac{TP+TN}{TP+TN+FP+FN}$$

	PREDICTED CLASS			
		Class=Yes	Class=No	
ACTUAL CLASS	Class=Yes	True Positive	False Negative	
	Class=No	False Positive	True Negative	

Is accuracy always a good measure?

Can you think of an example when it is not?

	PREDICTED CLASS				
		Class=Yes	Class=No		
ACTUAL	Class=Yes	a (TP)	b (FN)		
CLASS	Class=No	c (FP)	d (TN)		

Accuracy =
$$\frac{a+d}{a+b+c+d} = \frac{TP+TN}{TP+TN+FP+FN}$$

Example when Accuracy is not a good measure:

Consider a 2-class problem

- Number of Class 0 examples = 9990
- Number of Class 1 examples = 10

If the model predicts everything to be in class 0, accuracy is 9990/10000 = 99.9 %

 Accuracy is misleading because model does not detect any class 1 examples.

Using a Cost Matrix

	PREDICTED CLASS		
	C(i j)	Class=Yes	Class=No
ACTUAL CLASS	Class=Yes	C(Yes Yes)	C(No Yes)
	Class=No	C(Yes No)	C(No No)

C(i | j): Cost of misclassifying class j, as class i

EXAMPLE: Computing Cost of Classification

This the actual prediction from the model

Model M ₁	PREDICTED CLASS		
ACTUAL CLASS		+	-
	+	150	40
	-	60	250

This is the Cost Matrix

Cost Matrix	PREDICTED CLASS		
ACTUAL CLASS	C(i j)	+	-
	+	-1	100
	-	1	0

Accuracy

$$= (150 + 250) / (150 + 40 + 60 + 250) = 80\%$$

Cost

$$= (150)(-1) + (40)(100) + (60)(1) + (250)(0) = 3910$$

Cost vs Accuracy

Count	PREDICTED CLASS		
ACTUAL CLASS		Class=Yes	Class=No
	Class=Yes	а	b
	Class=No	С	d

Cost	PREDICTED CLASS		
		Class=Yes	Class=No
ACTUAL CLASS	Class=Yes	р	q
	Class=No	q	р

Accuracy is proportional to cost if

Proof:

Classification Techniques

- ODecision Tree Methods
- Instance-based Methods
- Bayesian algorithms (Naïve Bayes)
- Support Vector Machines
- Ensembles (Random Forest)

Decision Trees

ML Topic 1

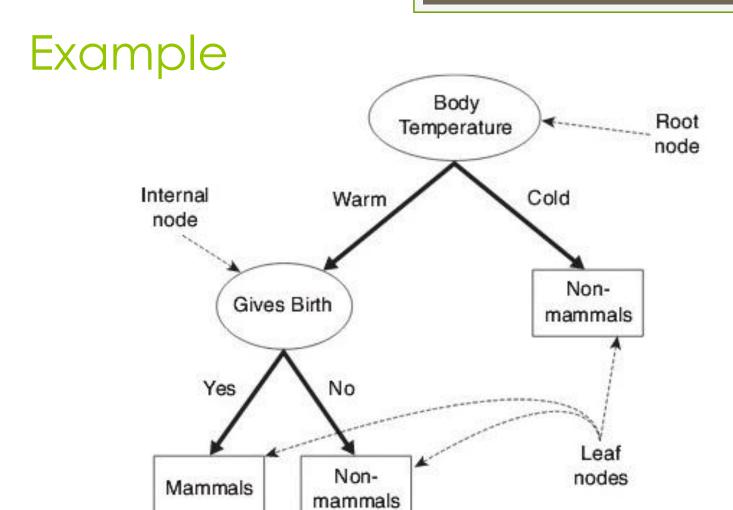
Decision Tree Overview

Build a classifier that is a directional tree structure.

The tree has

- Root Node: no incoming edges and zero or more outgoing edges. (contains attribute test condition(s))
- Internal Nodes: Exactly ONE incoming edge and TWO or more outgoing. (contains attribute test condition(s))
- Leaf/terminal Nodes: ONE incoming, no outgoing.

Each leaf node is assigned a class label.



Building a Decision Tree

- There are an **infinite** number of possible decision trees that can be constructed from a set of attributes.
- Finding the optimal tree is an intractable problem as the search space is exponential.
- Algorithms can find "good" decision trees using the Greedy approach – they make a series of locally optimal decisions.

Example: Hunt's Algorithm

o Hunt is the basis of ID3, C4.5, and CART

Hunt's Algorithm: Decision Tree

Assumptions:

- Let Dt be the set of **training records**, associated with node t in the tree.
- Let **xi** be record i such that yi is the class label.
- All training records are: $\{x1, x2, ..., xn\}$ with associated class labels $\{y1, y2, ..., yn\}$

Method: The tree is created in a **recursive** fashion by continuing to partition the training records (**x**) into purer subsets.

Steps:

- If all records in Dt belong to the same class yt, then t is a leaf node labeled as yt.
- 2) If Dt contains records that belong to MORE THAN one class, an **attribute test condition** is selected to partition into smaller subsets.
- 3) The above is recursively repeated

Practice: Predict whether a Loan Applicant will repay their loan:

Class label 1

Defaulted = No

Class label 2

Defaulted = Yes

Next, examine a known **training** set.

What are the attributes of each record?

What is the label of each?

Build a Decision Tree...

Sinary

categorical continuous

1855

Tid	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

http://www-users.cs.umn.edu/~kumar/dmbook/ch4.pdf

Step 1

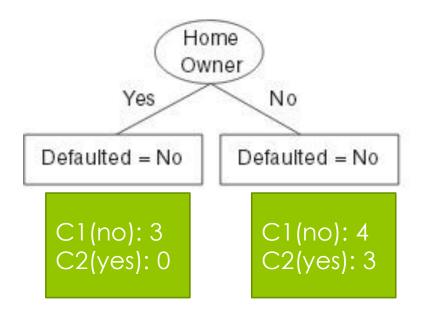
Defaulted = No

C1(no): 7 C2 (yes): 3

The initial tree is a single node that represents the fact that most borrows did pay and so the majority is default=no.

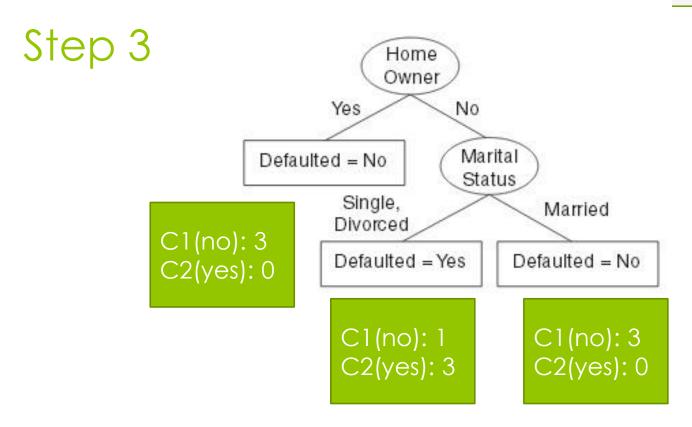
However, this current node contains records from both classes and so must be further refined (split).

Step 2



Using the attribute condition test of "Home Owner" still results one mixed class, with the majority of each labeled as Default=NO.

This must be further refined until the node is **pure**.



All "Home Owners" are class: Default=NO. Therefore, that node is pure and does not require further partitioning.

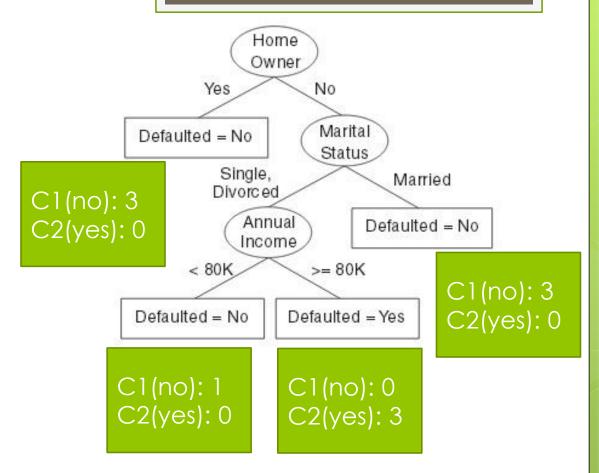
If the borrower is not a home owner, they are further partitioned by Marital Status.

The only node that is still not pure here is "Single/Divorces" AND "not home owner". Another attribute condition must be added.

Step 4

"Annual Income" is used as an attribute condition with <80K, or >80K.

Now, all nodes are pure and the leaf nodes contain the classification.



Test it!

Suppose a non-married person with 75K per year who does not own a home gets a loan – will they pay it back?

About Hunt

- Hunt works well if the training set contains every combination of all possible attributes.
- What happens if it does not?

Design Issues with Decision Trees

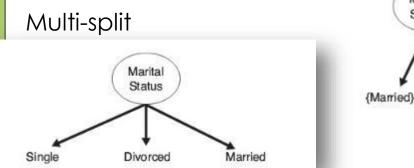
- How should training records be split?
 - How can the "best" attribute test condition be selected?
- O How should the splitting stop?
 - One option is to expand a node until all records are in the same class or have identical attribute values.

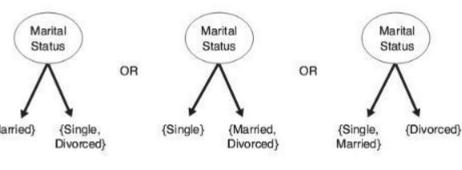
Expressing Attribute Test Conditions:

 Binary attributes: two possible outcomes such as married or not married.

Nominal Attributes:

- Multi-split one node for each attribute name
- binary split (CART does this) determined by investigating the best of the $2^k 1$ options for splitting. (k is the number of attributes)



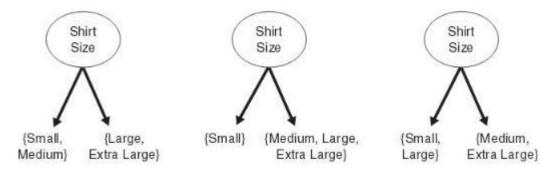


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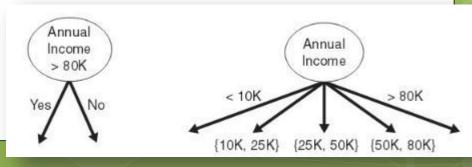
Binary split options for 3 attributes

Expressing Attribute Test Conditions:

- Ordinal attributes: can also be split using binary or multi.
 - o Why is the last option here not as good?



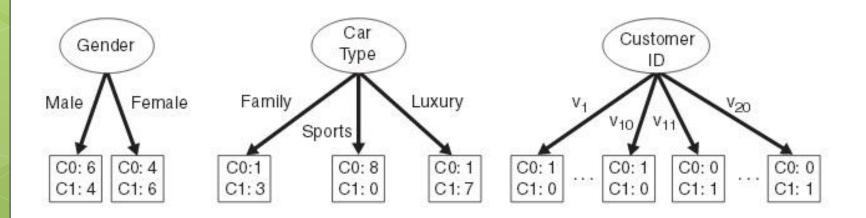
- Continuous Attributes:
 - Can use comparison set: (A < x) OR (A >= x)
 - Can use a range of options (for mult):



Comparing Splits

 Note: C0:6 means that there are 6 records of class "0" in the partition.

Which split created purer classes?



Methods for Measuring "Best" Split

$$\operatorname{Entropy}(t) = -\sum_{i=0}^{c-1} p(i|t) \log_2 p(i|t),$$

$$\operatorname{Gini}(t) = 1 - \sum_{i=0}^{c-1} [p(i|t)]^2,$$

$$\operatorname{Classification error}(t) = 1 - \max_i [p(i|t)],$$

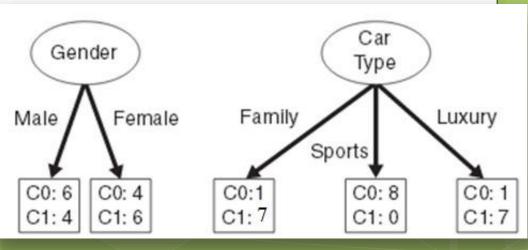
Step 1: Class Node Probability

Let p(i | t) be the fraction of records belonging to class i at a given node t.

For the Gender Partition: p(C0 | Male) = 6/10 p(C1 | Female) = 6/10

For the Car Type partition

p(C0 | Family) = 1/8



Calculating Entropy

Entropy(t) =
$$-\sum_{i=0}^{c-1} p(i|t) \log_2 p(i|t),$$

Node N_2	Count
Class=0	1
Class=1	5

$$p(i|t) = p(class 0 | node N2) = 1/6$$
 (recall that i is the class)
 $p(i|t) = p(class 1 | node N2) = 5/6$

Negative sum over all classes:

entropy =
$$-(1/6)\log 2(1/6) - (5/6)\log 2(5/6) = .65$$

Entropy ranges from 0 to 1, where 0 is pure and 1 is the worst case.

Comparison

Node N_1	Count
Class=0	0
Class=1	6

Gini =
$$1 - (0/6)^2 - (6/6)^2 = 0$$

Entropy = $-(0/6) \log_2(0/6) - (6/6) \log_2(6/6) = 0$
Error = $1 - \max[0/6, 6/6] = 0$

Node N_2	Count
Class=0	1
Class=1	5

$$\begin{aligned} & \text{Gini} = 1 - (1/6)^2 - (5/6)^2 = 0.278 \\ & \text{Entropy} = -(1/6)\log_2(1/6) - (5/6)\log_2(5/6) = 0.650 \\ & \text{Error} = 1 - \max[1/6, 5/6] = 0.167 \end{aligned}$$

Node N_3	Count
Class=0	3
Class=1	3

$$\begin{aligned} & \text{Gini} = 1 - (3/6)^2 - (3/6)^2 = 0.5 \\ & \text{Entropy} = -(3/6)\log_2(3/6) - (3/6)\log_2(3/6) = 1 \\ & \text{Error} = 1 - \max[3/6, 3/6] = 0.5 \end{aligned}$$

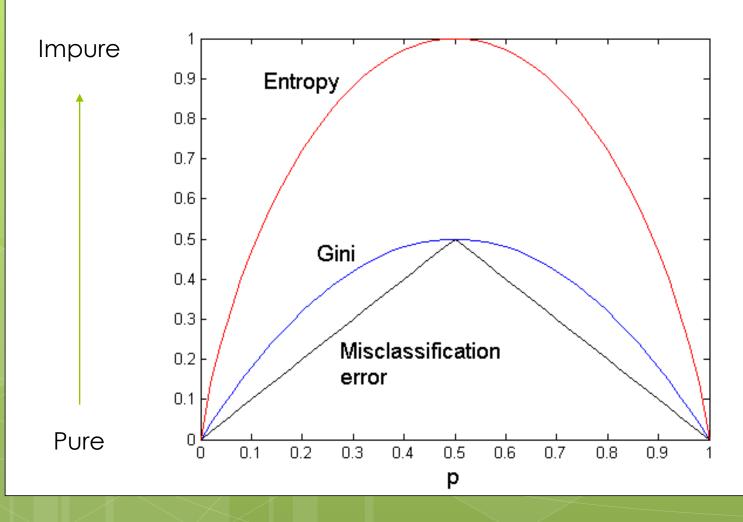
- 1) which has lowest impurity?
- 2) Which has highest impurity?

Intuition:

- If a partition results in p(i | t) = .5, it is very poor. Why?
- If a partition results in p(i | t) = 0, it is pure. Why?
- The lower the impurity of the partition (so the more pure it is), the more skewed the class distribution. Why?
- A node with class distribution of C1:0, C2:10 is skewed and pure.
- A node distribution with C1:5 and C2:5 is has the highest impurity an no skew.

Comparison among Splitting Criteria

For a 2-class problem:



Information Gain

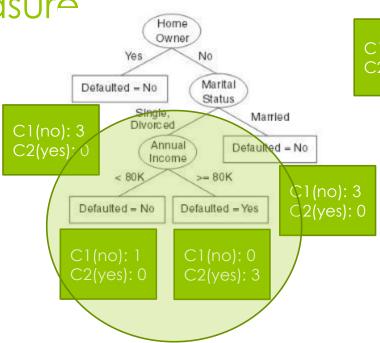
- To determine the **strength of a partition** compare purity of parent node (before split) to child nodes (after split).
- The greater the difference the better the partition condition.
- The Gain (Δ) is a measure for goodness of split.
- I is the impurity measure of a node, N is the number of records at parent node, k is the number of attribute values. N(vj) is the number of records in child vj.
- If entropy is used as the impurity measure, the difference in entropy is the information gain.
- This method is used in ID3

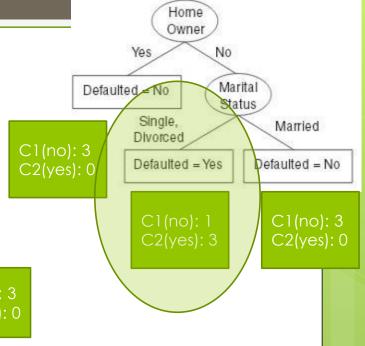
$$\Delta = I(\text{parent}) - \sum_{j=1}^{k} \frac{N(v_j)}{N} I(v_j)$$

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Example: Information

Gain using Entropy as the purity measure





$$\Delta = I(\text{parent}) - \sum_{j=1}^{k} \frac{N(v_j)}{N} I(v_j),$$

Calculations for Information Gain Using Entropy

Entropy for Parent =

$$-(1/4)\log(1/4) - (3/4)\log(3/4) =$$

 $-(1/4)(-2) - (1/4)(-.415) = .604$

Entropy for left node =

$$-(1/1)\log(1/1) - (0/1)\log(0/1) = 0 - 0 = 0$$

Entropy for right node =

$$-(0/3)\log(0/3) - (3/3)\log(3/3) = 0 - 0 = 0$$

Information GAIN:

```
I(Parent) - sum over all children N(v)/N * I(v) = .604 - (1)/(4) * 0 - (3)/(4)*0 = .604
This is the max possible difference and so is the best partition.
```

N is the num records at parent k is the num attribute values (ours has two possible values) N(v) is the num of records in child I in this case is the entropy

The greater the difference between I(Parent) and children – the better the partition condition.

Splitting Based on Information Gain

• Gain Ratio:

$$GainRATIO_{split} = \frac{GAIN_{split}}{SplitINFO}$$

$$SplitINFO = -\sum_{i=1}^{k} \frac{n_i}{n} \log \frac{n_i}{n}$$

Parent Node, p is split into k partitions n_i is the number of records in partition i

• Used in C4.5

Decision Tree Based Classification

• Advantages:

- Inexpensive to construct
- Extremely fast at classifying unknown records
- Easy to interpret for small-sized trees
- Robust for missing values
- Redundant attributes do not adversely affect accuracy of prediction
- Accuracy is comparable to other classification techniques for many simple data sets

Decision tree issues

- Choosing Splitting Attributes
- Ordering of Splitting Attributes
- Tree Structure
- Stopping Criteria
- Training Data
- Pruning

Occam's Razor

- Given two models of similar generalization errors, one should prefer the simpler model over the more complex model
- For complex models, there is a greater chance that it was fitted accidentally by errors in data
- Therefore, one should include model complexity when evaluating a model

Classification Techniques

- Decision Tree Methods
- Instance-based Methods
- Bayesian algorithms (Naïve Bayes)
- Support Vector Machines
- Ensembles (Random Forest)

Instance-based Learning

Part 2

Training with Labeled Records

Set of Stored Cases

Atr1	 AtrN	Class
		A
		В
		В
		С
		A
		С
		В

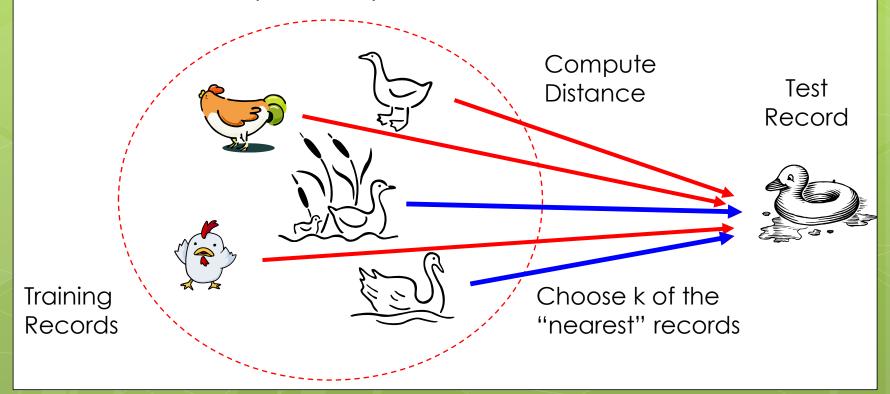
- Store the training records
- Use training records to train a predictor.
- Predict the class label of unseen cases

Instance Based Classifiers

- Examples:
 - Rote-learner
 - Memorizes entire training data and performs classification only if attributes of record match one of the training examples exactly
 - Nearest neighbor
 - Uses k "closest" points (nearest neighbors) for performing classification

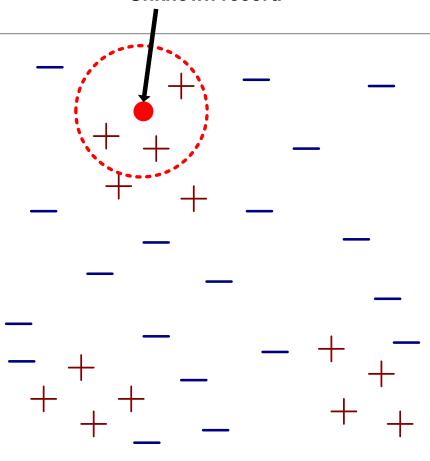
Nearest Neighbor Classifiers

- Basic idea:
 - If it walks like a duck, quacks like a duck, then it's probably a duck



Nearest-Neighbor Classifiers

Unknown record



Requires three things

- The set of stored records
- Distance Metric to compute distance between records
- The value of k, the number of nearest neighbors to retrieve
- To classify an unknown record:
 - Compute distance to other training records
 - Identify k nearest neighbors
 - Use class labels of nearest neighbors to determine the class label of unknown record (e.g., by taking majority vote)

Nearest Neighbor Distance

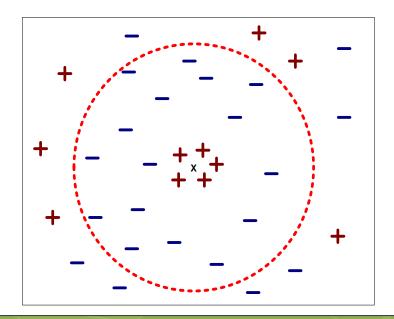
- Compute distance between two points:
 - Euclidean distance

$$d(p,q) = \sqrt{\sum_{i} (p_{i} - q_{i})^{2}}$$

- Determine the class from nearest neighbor list
 - take the majority vote of class labels among the k-nearest neighbors
 - Weigh the vote according to distance
 - weight factor, $w = 1/d^2$

Nearest Neighbor – Determine k

- Choosing the value of k:
 - If k is too small, sensitive to noise points
 - If k is too large, neighborhood may include points from other classes



Nearest Neighbor Issues

Scaling issues

- Attributes may have to be scaled to prevent distance measures from being dominated by one of the attributes
- Example:
 - height of a person may vary from 1.5m to 1.8m
 - weight of a person may vary from 90lb to 300lb
 - o income of a person may vary from \$10K to \$1M

Nearest neighbor summary

- k-NN classifier
 - It does not build models explicitly
 - Unlike eager learners such as decision tree induction and rule-based systems (lazy learner)
 - Classifying unknown records are relatively expensive