Real-time Optimal Path Planning of UAVs in an Obstacle-filled Environment using Vector Fields

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***Abstract*— In this paper, the optimal path is developed for a fixed-wing Unmanned Air Vehicle (UAV) constrained by a minimum turn radius, to converge to a circular path starting from any initial state using the strategy based on Dubins path. Then the UAV is guided through the generated optimal path using a new guidance law which is a combination of two different vector field methods formulated using Lyapunov method of stability. A strategy for real-time obstacle avoidance using the vector fields is also proposed to generate the global optimal path in an obstacle-filled environment. Simulation results are included to show the robustness of the algorithm in different environmental scenarios, which include obstacles and steady wind.**

I. Introduction

Path planning in an obstacle-filled environment for UAVs have been active research fields for last few years and obstacle avoidance following an optimal path will enhance the performance of an UAV considerably. As fixed-wing UAVs have limitation on lateral acceleration, the minimum turn radius of the vehicle is bounded.

The shortest path problem was first addressed by Dubins [1] for a car moving forward. In [1], a mathematical proof was shown to prove that shortest path for a constrained turn radius vehicle consists of line segments and arcs of circles minimum turn radius. So the shortest path is either CCC, CSC or a subset of them. Where C implies circular arc of minimum turn radius and S implies line segments. So the Dubins set consists of six paths D = (LSL, LSR, RSL, RSR, RLR, LRL). Where R and L are the left and right turns respectively. Dubins path when backward motion is allowed is shown in [2]. Pontryagin's minimum principle has been is used in [3] to obtain the same results as in [1]. The lengths of CLC paths for given initial and final poses are calculated in [4]. Dubins problem where terminal direction is not prescribed is solved in [5]. Time optimal path to converge to a straight line is discussed in [6]. The necessary conditions needed to reach tangentially to a circle in minimum-length paths have been deduced in [7]. One of the primary challenges with an UAV is flight in windy conditions. To address this problem we propose a guidance law which will allow the UAV to be stable even in presence of wind or any other disturbances.

After path planning the natural next step is path following. In [8], survey of different path following algorithms has been presented. A guidance law for straight line and circular paths has been presented in [9]. A problem of waypoint tracking in presence of wind is solved in [10]. In [11], a vector field path following method is proposed for both straight line and curved paths. It has been seen that vector field guidance laws have very low cross track error compared to other guidance laws. For this reason vector field guidance laws have been adopted in our work. A Lyapunov vector field method is proposed in [12], which is globally stable, easier to implement and takes less computation time compared to [11]. In [13] tangent plus Lyapunov vector field guidance law is proposed which works only in the case of a tangent being available from the initial circle to the final circle. In [14], the same method has been extended for 3D path planning.

The third and most important problem that is being addressed in this paper is obstacle avoidance. Many different methods have been developed over the years to address this issue. Potential Field Method [15], Artificial Potential Field methods are frequently used techniques, which use repulsive forces to avoid collision with obstacles. Sample based motion planning, e.g. probabilistic road map (PRM), rapidly-exploring random trees (RRT) [16] are useful in complex problems. Heuristic approaches have got a significant importance in recent times, e.g. A* algorithm [17] and Neural Networks [18], Fuzzy logic [19]. These algorithms are useful in real time motion planning. Vector Field method for obstacle avoidance has been presented in [13] and [14] but the optimality and UAV's turn rate constraint haven't been considered there. In our work, we will be presenting a strategy for real time obstacle avoidance using vector fields to reach the target following the global optimal path.

The remainder of the paper is structured as follows: Problem statement for optimal path generation is given in section 2. Section 3 consists of computation of optimal path. A vector field path following algorithm has been proposed in section 4. Section 5 generates the global optimal path for obstacle avoidances. Simulation results have been presented in the respective sections. Summary of the work is mentioned in section 6.

II. Problem Description

In the first part of the problem, A fixed-wing UAV is required to converge to a circular path in minimum time

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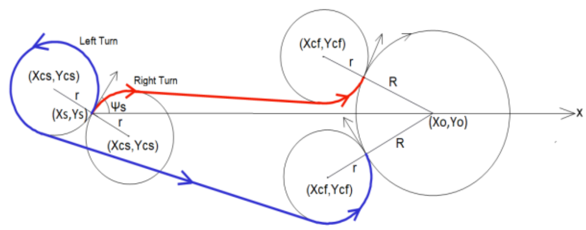


Figure 1: Possible Path Geometry (LSL and RSL)

starting from any initial orientation and position which sufficiently far away from the circular path to be followed. For this purpose, kinetic model of a UAV in 2D is as follows:

$$\dot{x} = V_a cos\Psi \tag{1}$$

$$\dot{y} = V_a sin\Psi \tag{2}$$

$$\dot{\Psi} = \omega \tag{3}$$

Where,

V_a= Air Speed of UAV

Ψ = Orientation angle of UAV air speed measured anti-clockwise w.r.t X-axis,

ω = angular speed of the UAV.

The air speed (V_a) is considered as constant. The control input, ω, is constrained due to UAV's minimum turn radius.

$$|\omega| \leq \frac{V_a}{r} \tag{4}$$

Where, r is the minimum turn radius of the UAV.

Following initial conditions are given while planning the path: initial location P =(x_s, y_s), Initial pose = Ψ_s measured anti-clockwise with x-axis. Location of center of circular path to be converged is (x_o, y_o). Here, it is set to converge the circular path in clock-wise direction.

III. PATH PLANNING

Paths possible for an UAV converging clock-wise to a circular path sufficiently far away are RSL and LSL as shown in Fig. 1.

A. *Optimal path generation*

The partial result which provides a set of possible optimal paths for this problem is presented in [7]. Here several improvements have been made to choose the optimal path from the set for a given initial configurations with the given circular path to be followed.

Now to derive the property of the shortest LSL, let us consider the geometry shown in Fig. 2, where center of the first circular turn is (x_{cs}, y_{cs}) and the center of second circular turn is (x_{cf}, y_{cf}). From the Fig. 2, we have,

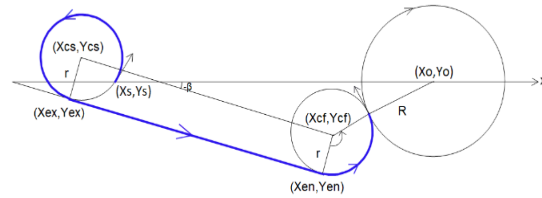


Figure 2: LSL Path Geometry

$$x_{cf} = (r + R)sin\theta_f + x_o \tag{5}$$

$$y_{cf} = (r + R)cos\theta_f + y_o \tag{6}$$

And

$$x_{cs} = x_s - rsin\Psi_s$$

$$y_{cs} = y_s + rcos\Psi_s$$

From Fig. 2, β is the angle made by the line passing through (x_{cs}, y_{cs}) and (x_{cf}, y_{cf}) with positive x-axis. The angle made by the external tangent of the initial and the final circular arcs with positive x-axis is also β.

$$tan\beta = \frac{y_{cf} - y_{cs}}{x_{cf} - x_{cs}}$$

Exit point of UAV,(x_{ex}, y_{ex}) from the initial circle is given by,

$$x_{ex} = x_{cs} + rsin\beta$$

$$y_{ex} = y_{cs} - rcos\beta$$

Entry point of UAV,(x_{en}, y_{en}) on the final circle is given by,

$$x_{en} = x_{cf} + rsin\beta$$

$$y_{en} = y_{cf} - rcos\beta$$

Length of circular paths covered l_c is,

$$l_c = r(\frac{3\pi}{2} + \theta_f - \Psi_s)$$

Length of tangent l_t is,

$$l_t = \sqrt{(x_{cs} - x_{cf})^2 + (y_{cs} - y_{cf})^2}$$

Total path length P is calculated to be,

$$P = r\left(\frac{3\pi}{2} + \theta_f - \Psi_s\right) + l_t$$

For optimizing path length,

$$\frac{dP}{d\theta_f} = 0$$

$$r + \frac{(x_{cs} - x_{cf})(y_{cf} - y_o) - (y_{cs} - y_{cf})(x_{cf} - x_o)}{l_t} = 0$$

This later simplifies to,

$$\frac{y_{en} - y_{ex}}{x_{en} - x_{ex}} = \frac{y_o - y_{ex}}{x_o - x_{ex}}$$

This implies that the tangent line is passing through center of the final circle to become the optimal LSL path. The same approach can be adopted for obtaining the optimal RSL path. Using this result following lemma can be written.

Lemma 1: The straight line segment of any optimal CSC path passes through the center of the final circle to be converged.

Using Lemma 1, the optimal CSC can be easily generated. As the straight line segment is tangent to the initial circular arc and passes through the center of the circular path to be followed, slope m of the line segment can be deduced as the solution of the following equation,

$$m^4 x_{cf}^2 - 2m^3 x_{cf} y_{cf} + m^2\left(y_{cf}^2 + x_{cs}^2 - r^2\right) = 2x_{cs} y_{cs} m + y_{cs}^2 - r^2$$

The larger of the two real roots obtained is considered as the slope of the line segment for LSL path and the smaller root is considered for the slope of the line segment for RSL path.

We know for the LSL path,

$$tan\beta = \frac{y_{en} - y_o}{x_{en} - x_o}$$

From this equation and using (5), (6) following results can be deduced,

$$x_{cf} = -rsin\beta \pm cos\beta(R(R+2r))^{\frac{1}{2}} + x_o$$

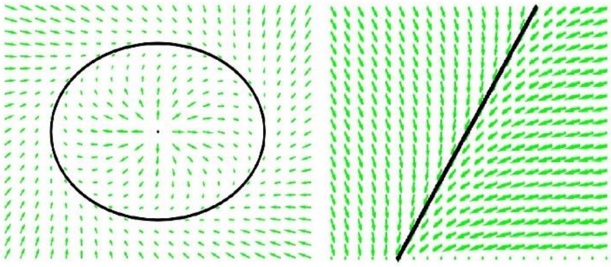
$$y_{cf} = rcos\beta \pm sin\beta(R(R+2r))^{\frac{1}{2}} + y_o$$

x_{cf} takes smaller value if initially the UAV is left to the final circular path and takes larger value if the UAV is initially right to the final circular path and the same is applicable for y_{cf}. From this, θ_f can be calculated and hence the optimal LSL path can be generated. A similar approach can be adopted for computation of the optimal RSL path.

The other important results which have been obtained are summarized in the following Lemma without showing the details proof for brevity.

Lemma 2 The time-optimal path for converging to a circular path in the absence of wind is found to be RSL for initial orientation of vehicle's airspeed in $(0, \pi]$ and LSL for initial orientation of vehicle's airspeed in $[\pi, 2\pi)$ measured anticlockwise from the line joining the initial location of the vehicle and the center of the circular path.

Using Lemma 1 and 2 the particular optimal path can easily be generated for a given initial configuration of UAV.



3(a) 3(b)

Figure 3: Generated Vector Fields

IV. Vector Field Guidance Law

Here we propose a Path following algorithm which is a combination of Lyapunov Vector Field Guidance Law [12] and Vector Field Path Following [11]. The basic idea is to generate a stable vector field attractor to make the UAV follow the above generated optimal paths.

A. Lyapunov Vector Field Guidance

The idea is to build a limit cycle which is a circular attractor and globally stable. For this, the vector field is the addition of two parts, one which directs the vehicle towards the center of the desired circle and the other which allows the UAV to rotate in the circle.

Let the vector field be $\dot{\bar{r_d}} = v(\bar{r}, \theta)$

Where $\bar{r}$ is vector directed from current UAV position (x, y) towards the center of the desired circle (x_o, y_o) and $\dot{\bar{r_d}}$ is the desired direction in which vehicle is desired to move. For the vector field to be globally stable a function $V_F(\bar{r})$ should exist such that it is positive definite, $\dot{V}_F(\bar{r})$ is negative definite and $V_F(\bar{r}) \to \infty$ as $x \to \infty$.

So the following V_F is chosen,

$$V_F(\bar{r}) = \tfrac{1}{2}(\bar{r} - \rho)^2$$

$$\frac{dV_F}{d\bar{r}} = (|\bar{r}| - \rho)\hat{r}$$

Where ρ is the the radius of stand-off circle and $\hat{r}$ is the a unit vector in the the direction of $\bar{r}$. Vector field is chosen as,

$$\dot{\bar{r_d}} = v(\bar{r}, \theta) = \frac{1}{N(\bar{r})}\left(-\left[\frac{dV_F}{d\bar{r}}\right]^T \pm \beta\bar{r_n}\right) \tag{7}$$

Where, β is a positive constant and $N(\bar{r})$ is normalization factor since the UAV is moving with constant velocity i.e, $|\dot{\bar{r}}| = v$

$$N(\bar{r}) = \frac{1}{\mathrm{v}}((\bar{r} - \rho)^2 + \rho^2\beta^2)$$

We know,

$$\dot{V}_F(\bar{r}) = \frac{dV_F}{d\bar{r}}\dot{\bar{r}}$$

From these results all the three above conditions for stability are satisfied and the chosen vector field is proved to be globally stable.

From (7) the desired horizontal and vertical velocities of UAV can be deduced as,

$$\dot{x} = \frac{1}{N(\bar{r})}\left(1 - \frac{\rho}{|\bar{r}|}\right)x \pm \gamma y$$

$$\dot{y} = \frac{1}{N(\bar{r})}\left(1 - \frac{\rho}{|\bar{r}|}\right)y \mp \gamma x$$

For clockwise rotation positive in $\dot{x}$ and negative in $\dot{y}$ is chosen and for anti-clockwise rotation opposite signs are chosen. Fig. 3(a) shows the generated vector field and it

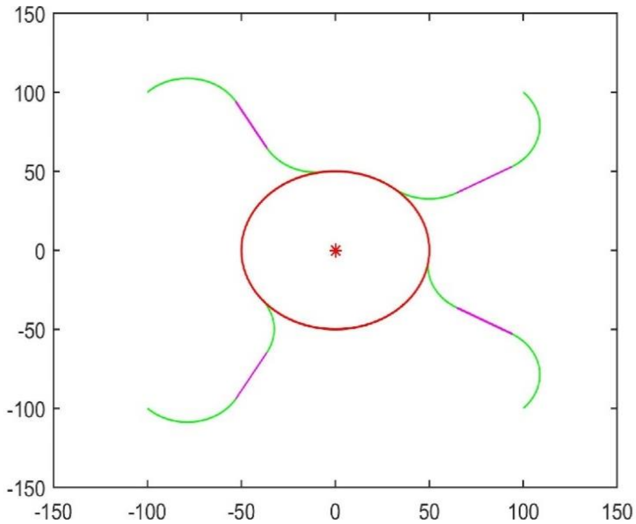


Figure 4. Trajectories of UAV following optimal path

Can be implied that UAV starting at any point will be attracted to circle and this method is used for loiter path.

Following control law is used to navigate the UAV through generated vector field,

$$\ddot{r} = -\chi(\dot{r} - \dot{r_d}) + \ddot{r_d} \tag{8}$$

B. Vector Field Guidance

Using this method the UAV is directed to a straight line path from any initial location by construction vector fields heading towards the straight line. Let Ψ be the current heading angle Ψ_c be the desired heading angle. $\Psi_{d,}$ difference between Ψ_c and Ψ is eventually desired to become zero from any initial heading angle. So the following vector field is built for a straight line with slope m and y-intercept c,

$$\Psi_d = \pm\Psi_\infty \frac{2}{\pi} \tan^{-1}(k(y - mx) - c) \tag{9}$$

Positive is considered when y is supposed to decrease and negative is used otherwise. Ψ_∞ and k are constants to be chosen. The value of k should not be too large or too small. The optimal range is between 0.01 and 0.05.

The UAV is directed in the course of the vector field using the control law given in [11],

$$\Psi_c = \Psi - \frac{2\Psi_\infty k V_a}{\alpha\pi(1 + (ky)^2)} \sin\Psi - \frac{\kappa}{\alpha} sat(\frac{\Psi - \Psi_d}{\gamma}) \tag{10}$$

Where κ, α, γ are positive constants and V_a is the airspeed of the UAV. In the Fig. 3(b), we can see that UAV starting at any initial point will be attracted to the straight line desired and this vector field can be asymptotically stable using Lyapunov stability criteria.

C. Simulation of Optimal paths using proposed Guidance law

As the optimal path generated in the previous section is a combination of circular arcs and line segments we will be

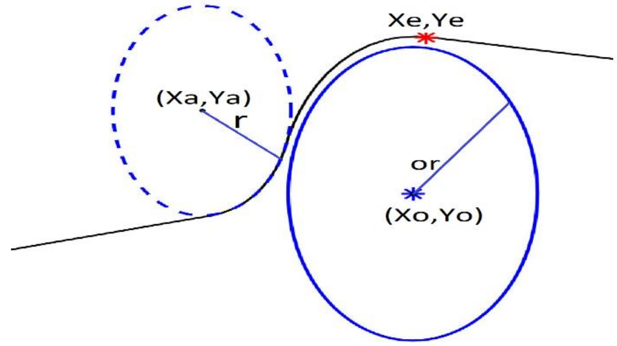


Figure 5. Illustration of obstacle avoidance

using LVF method for loiter guidance and VF method for straight line guidance.

Table 1. Initial conditions of the UAV and corresponding path

Case	x(m)	y(m)	Heading angle(degrees)	Path
1	-100	-100	315	LSL
2	-100	100	45	RSL
3	100	100	315	LSL
4	100	-100	45	RSL

The simulations are made for a UAV with airspeed $V_a = 10m/s$, turn radius $r = 20m$, wind speed $V_W = 3m/s$, wind direction $\alpha = 45^o$ the radius of stand-off circle $R = 50m$. The positive constants are taken as β=0.2, χ=5, k=0.02, κ=π/2, ϵ=0.1, α=1.65. For the initial conditions given in Table 1. As UAV is guided using the proposed vector field guidance law, even in the presence of wind it will travel through the same path The wind case can be generated by making following changes to the velocity vectors,

$$\dot{x} = V_a cos\Psi + W_x$$

$$\dot{y} = V_a sin\Psi + W_y$$

The trajectories of UAV for above given initial conditions have been shown in Fig. 5. We can see that there is a very little cross-track which proves the robustness of the guidance law.

V. OBSTACLE AVOIDANCE

In this paper, we will be solving the problem of real-time obstacle avoidance using the above-obtained results. We assume that information about the circle circumscribing any obstacle is obtained at a sufficient distance from the obstacle. Using the established fact that UAV will reach the destination in optimal distance if it is directed towards the center of the target circle, we propose a strategy here that UAV is supposed to move towards the center unless it meets an obstacle. If an obstacle is present in its way then it escapes the collision as shown in Fig. 5, where r is the minimum turn radius and or is the radius of circumscribing circle of an obstacle. (x_a, y_a) is obtained by the following analysis,

It is the center of the circle with radius r and adjacent to circle with center (x_o, y_o) and radius $R = or + 1$, as the

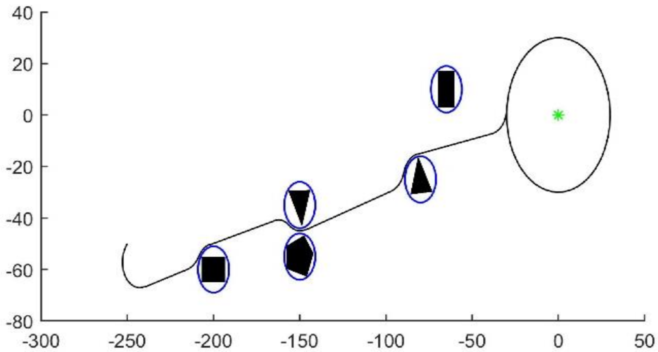


Figure 6: Real Time obstacle avoidance with r=10m, or= 15m, R=30m and initial coordinates (-250,-50)

wing-span of UAV is considered to be less than 1m. The circle is also tangent to a line with equation $y = mx + c$ so

$$\text{Let } k = c \pm r\sqrt{1+m^2} - y_o$$

x_a is the solution of the following equation,

$$x_a^2(1+m^2) + x_a(2mk - 2x_o) + x_o^2 + k^2 - (r+R)^2 = 0$$

Smaller value of x_a is considered if UAV is approaching from the left and larger value for the opposite.

$$y_a = mx_a + r\sqrt{1+m^2} \pm c$$

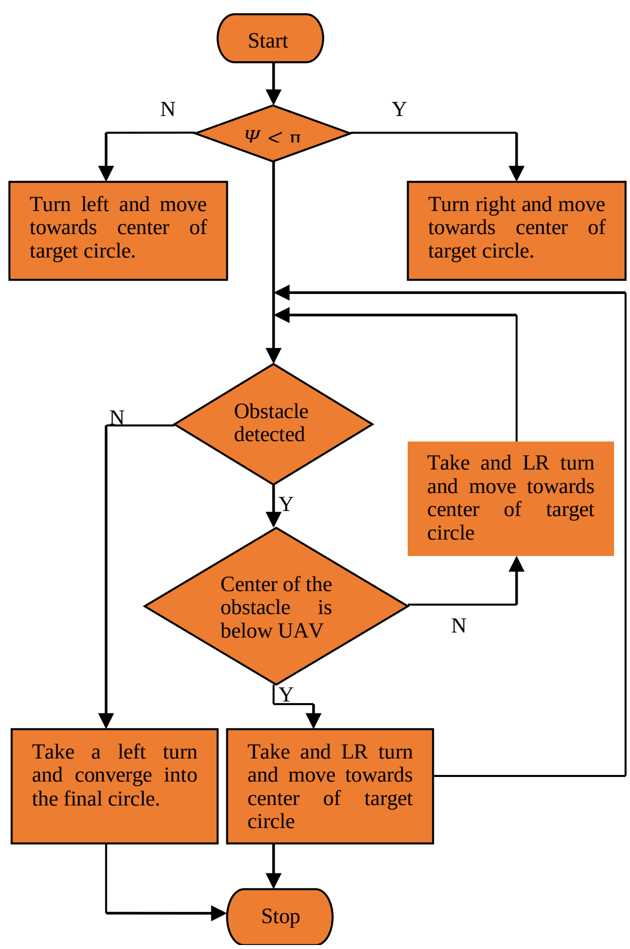


Chart 1. Flow chart showing the algorithm to achieve optimal path

Positive for LR turn and negative for RL turn. LR turn is chosen if the line $y = mx + c$ is below (x_o, y_o) and RL turn is taken if the line is above(x_o, y_o). (x_e, y_e) is the point where the UAV leaves the circular arc.

A detailed set of commands are mentioned in Chart 1 so that a UAV starting at any point in any orientation can follow these commands to reach and converge a target circle in the optimal distance in presence of wind and obstacles.

In Fig. 6 we can see that the UAV is following all the commands of Chart 1. It is avoiding the obstacles in its way by making the turns greater than or equal to the minimum turn radius to satisfy the constraint and always directed towards the center of the destination circle to achieve global optimal path. We can also see that it is making LR and RL turns based on above-given conditions.

VI. Conclusion And Future Work

In this paper, we have proposed optimal path planning in presence and absence of wind and a corresponding vector field guidance law. The main advantage of the proposed method is that the path generated takes dynamic constraints of the UAV into consideration while generating the optimal path. The guidance law is also proved to be globally stable which is a huge advantage during an autonomous flight in hazardous conditions. The obstacle avoidance strategy proposed is also concentrated on developing an optimal which was not discussed in [13]. The simulation results show robustness and efficiency of the algorithm in many different scenarios.

This work can be extended for the case of UAV being closer to the final circle. Three-dimensional analysis can be done using the same strategies. It can also be extended to moving targets and time-varying winds.

References

[1] Dubins, L.E. (1957) On Curves of Minimal Length with a Constraint on Average Curvature, and with Prescribed Initial and Terminal Positions and Tangents. American Journal of Mathematics, 79, 497-516.

[2] Shepp, L. A. “Optimal Paths for a Car That Goes Both Forwards and Backwards.” Pacific Journal of Mathematics, Pacific Journal of Mathematics.

[3] J-D Boissonnat, André Cérézo, and Juliette Leblond. Shortest paths of bounded curvature in the plane. In Robotics and Automation, 1992. Proceedings, 1992 IEEE International Conference on, pages 2315–2320. IEEE, 1992.

[4] Andrei M Shkel and Vladimir Lumelsky. Classification of the dubins set. Robotics and Autonomous Systems, 34(4):179–202, 2001.

[5] Boris Thomaschewski. Dubins’ problem for the free terminal direction. Preprint, pages 1–14, 2001.

[6] Sikha Hota and Debasish Ghose. A modified dubins method for optimal path planning of a miniature air vehicle converging to a straight line path. In American Control Conference, 2009. ACC’09., pages 2397–2402. IEEE, 2009.

[7] A Balluchi and P Soueres. Optimal feedback control of dubins’ car tracking circular reference paths. In Decision and Control, 1996, Proceedings of the 35th IEEE Conference on, volume 3, pages 3558–3563. IEEE, 1996.

[8] Sujit, P. B., Saripalli, S., & Sousa, J. B. (2014). Unmanned aerial vehicle path following: A survey and analysis of algorithms for fixed-wing unmanned aerial vehicless. IEEE Control Systems, 34(1), 42-59.

[9] Mangal Kothari, Ian Postlethwaite, and Da-Wei Gu. Uav path following in windy urban environments. Journal of Intelligent & Robotic Systems, 74(3-4):1013, 2014.

[10] John Osborne and Rolf Rysdyk. Waypoint guidance for small uavs in wind. In Infotech@ Aerospace, page 6951. 2005.

[11] Derek R Nelson, D Blake Barber, Timothy W McLain, and Randal W Beard. Vector field path following for miniature air vehicles. IEEE Transactions on Robotics, 23(3):519–529, 2007.

[12] D. A. Lawrence, E. W. Frew, and W. J. Pisano, “Lyapunov vector fields for autonomous UAV flight control,” presented at the AIAA Guidance, Navigation Control Conf. Exhibit, Hilton Head, SC, Aug. 2007, Paper AIAA 2007-6317.

[13] H. Chen, K. C. Chang, and C. S. Agate, “Tracking with UAV using tangent-plus-Lyapunov vector field guidance,” in Proc. 12th Int. Conf. Information Fusion, 2009, pp. 363–372.

[14] P. Yao H. Wang Z. Su, “Real-Time Path Planning of Unmanned Aerial Vehicle for Target Tracking and Obstacle Avoidance in Complex Dynamic Environment.” Aerospace Science and Technology, Elsevier Masson, 2015.

[15] F.Arambula, Cosío, M.A.Padilla Castañeda “Autonomous Robot Navigation Using Adaptive Potential Fields.” Mathematical and Computer Modelling, Pergamon, 2005.

[16] J Lee, O Kwon, L Zhang, S.E.Yoon, “A Selective Retraction-Based RRT Planner for Various Environments”, IEEE Trans. Robot, 2014.

[17] Hong Yang and Yiyuan Zhao. "Trajectory Planning for Autonomous Aerospace Vehicles amid Known Obstacles and Conflicts", Journal of Guidance, Control, and Dynamics, Vol. 27, No. 6 (2004), pp. 997-1008.

[18] Zou AM., Hou ZG., Fu SY., Tan M. (2006) Neural Networks for Mobile Robot Navigation: A Survey. In: Wang J., Yi Z., Zurada J.M., Lu BL., Yin H. (eds) Advances in Neural Networks - ISNN 2006. ISNN 2006. Lecture Notes in Computer Science, vol 3972. Springer, Berlin, Heidelberg

[19] Zhu, Anmin, and Simon X. Yang. "A fuzzy logic approach to reactive navigation of behavior-based mobile robots." Robotics and Automation, 2004. Proceedings. ICRA'04. 2004 IEEE International Conference on. Vol. 5. IEEE, 2004.