1 Examples

Example 1.7 (biological example: synchronization of two fireflies) Consider a biological example of the synchronization of two fireflies flashing. The fireflies can be modeled mathematically as periodic oscillators which tend to synchronize their flashing until they are flashing in phase with each other. A state value of $\tau_i = 1$ corresponds to a flash, and after each flash, the firefly automatically resets its internal timer (periodic cycle) to $\tau_i = 0$. The synchronization of the fireflies can be modeled as an interconnection of two hybrid systems because every time one firefly flashes, the other firefly notices and jumps ahead in its internal timer τ by $(1+\varepsilon)\tau$, where ε is a biologically determined coefficient. This happens until eventually both fireflies synchronize their internal timers and are flashing simultaneously. Each firefly can be modeled as a hybrid

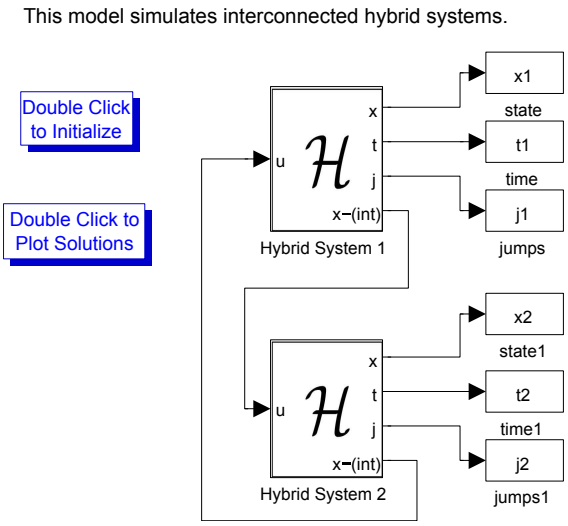


Figure 1: Interconnection Diagram for Example 1.7

system given by

$$f_i(\tau_i, u_i) := 1, \tag{1}$$

$$C_i := \{(\tau_i, u_i) \in \mathbb{R}^2 \mid 0 \le \tau_i \le 1\} \cap \{(\tau_i, u_i) \in \mathbb{R}^2 \mid 0 \le u_i \le 1\} \tag{2}$$

$$g_i(\tau_i, u_i) := \begin{cases} (1+\varepsilon)\tau_i & (1+\varepsilon)\tau_i < 1 \\ 0 & (1+\varepsilon)\tau_i \ge 1 \end{cases} \tag{3}$$

$$D_i := \{(\tau_i, u_i) \in \mathbb{R}^2 \mid \tau_i = 1\} \cup \{(\tau_i, u_i) \in \mathbb{R}^2 \mid u_i = 1\}. \tag{4}$$

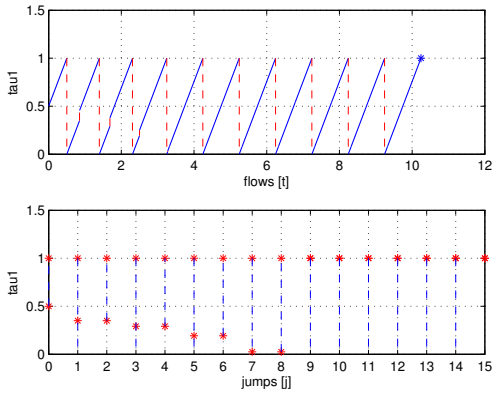
The interconnection diagram for this example is simpler than in the previous example because now no external inputs are being considered. The only event that affects the flashing of a firefly is the flashing of the other firefly. The interconnection diagram can be seen in Figure 1.

For hybrid system $\mathcal{H}_i$, $i = 1, 2$:

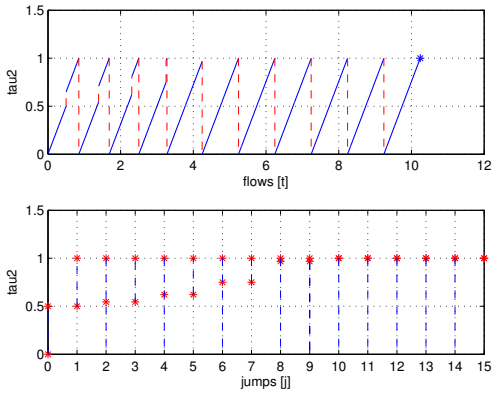
Flow map

```
function taudot = f(tau, u)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Matlab Function  Author: Ricardo Sanfelice
%
% Project: Simulation of a hybrid system (interconnection)
%
% Name: f.m
%
% Description: Flow map
```



(a) Solution for system $\mathcal{H}_1$



(b) Solution for system $\mathcal{H}_2$

Figure 2: Solution of Example 1.7

```
%
% Version: 1.0
% Required files: -
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% flow map
taudot = 1;
```

Flow set

```
function v  = C(tau, u)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Matlab Function  Author: Ricardo Sanfelice
%
% Project: Simulation of a hybrid system (interconnection)
%
% Name: C.m
%
% Description: Flow set
%
% Version: 1.0
% Required files: -
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%flow set
if ((tau > 0) && (tau < 1)) || ((u > 0) && (u <= 1))  % flow condition
    v = 1;  % report flow
else
    v = 0;  % do not report flow
end
```

Jump map

```
function tauplus = g(tau, u, e)

```

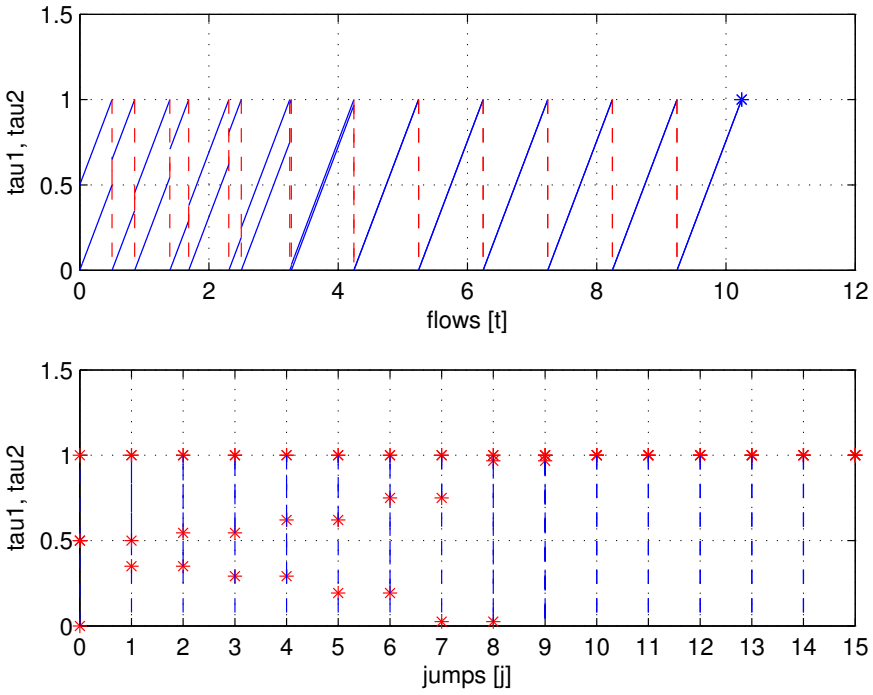


Figure 3: Solution of Example 1.7 for interconnection of $\mathcal{H}_1$ and $\mathcal{H}_2$

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Matlab Function  Author: Ricardo Sanfelice
%
% Project: Simulation of a hybrid system (interconnection)
%
% Name: g.m
%
% Description: Jump map
%
% Version: 1.0
% Required files: -
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% jump map
if (1+e)*tau < 1
    tauplus = (1+e)*tau;
elseif (1+e)*tau >= 1
    tauplus = 0;
else
    tauplus = tau;
end
```

Jump set

```
function v  = D(tau, u)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
% Matlab Function  Author: Ricardo Sanfelice
%
% Project: Simulation of a hybrid system (interconnection)
%
% Name: D.m
%
% Description: Jump set
%
% Version: 1.0
% Required files: -
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

if (u >= 1) || (tau >= 1) % jump condition
    v = 1;  % report jump
else
    v = 0;  % do not report jump
end
```

A solution to the interconnection of hybrid systems $\mathcal{H}_1$ and $\mathcal{H}_2$ with $T = 15, J = 15$, $rule = 1$, $\varepsilon = 0.3$ is depicted in Figure 3. Both the projection onto t and j are shown. A solution to the hybrid system $\mathcal{H}_1$ is depicted in Figure 2(a). A solution to the hybrid system $\mathcal{H}_2$ is depicted in Figure 2(b).

These simulations reflect the expected behavior of the interconnected hybrid systems. The fireflies initially flash out of phase with one another and then synchronize to flash in the same phase.

For MATLAB files of this example, see Examples/Example_1.7.

□