# 5 Examples

The examples below illustrate the use of the Simulink implementation above.

**Example 5.10** (Sample-and-hold Feedback Control) Consider a physical process modeled as in Example 5.7 and a state feedback interconnection. The algorithm (static gain) uses measurements of its output and controls the input of the physical process with the goal of steering its state to zero. Suppose the sampling device is ideal and that the signals are connected to the plant via a DAC modeled as follows. The digital signals in the cyber components need to be converted to analog signals for their use in the physical world. Digital-to-analog converters (DACs) perform such a task by converging digital signals into analog equivalents. One of the most common models for a DAC is the zero-order hold model (ZOH). In simple terms, a ZOH converts a digital signal at its input into an analog signal at its output. Its output is updated at discrete time instants, typically periodically, and held constant in between updates, until new information is available at the next sampling time. We will model DACs as ZOH devices with similar dynamics. Let $\tau_h \in \mathbb{R}_{\geq 0}$ be the timer state, $m_h \in \mathbb{R}^{r_C}$ be the sample state (note that the value of $h$ indicates the number of DACs in the interface), and $v_h \in \mathbb{R}^{r_C}$ be the inputs of the DAC. Its operation is as follows. When $\tau_h \leq 0$, the timer state is reset to $\tau_r$ and the sample state is updated with $v_h$ (usually the output of the embedded computer), where $\tau_r \in [T^{\min}, T^{\max}]$ is a random variable that models the time in-between communication instants and $T^{\min} \leq T^{\max}$. A model that captures this mechanism is given by

$$\dot{\tau}_h = -1, \quad \dot{m}_h = 0 \qquad \text{when} \quad \tau_h \in [T^{\min}, T^{\max}] \tag{1}$$

$$\tau_h^+ = \tau_r, \quad m_h^+ = v_h \qquad \text{when} \quad \tau_h \leq T^{\min} \tag{2}$$

The interconnection between the models of the physical process, the sampling device, the finite state machine, and the DAC has the feedback topology shown in Figure 1. In particular, the output of the DAC is connected to the input $u$ of the physical process by a matrix gain $K$, while the input $v$ of the finite state machine is equal to the output $y$ of the physical process at every sampling instant.

- Physical process:

$$f_P(x,u) := Ax + Bu, \; C_P := \{(x,u) \in \mathbb{R}^2 \times \mathbb{R}^2\}, \tag{3}$$

$$G_P(x,u) := x, \; D_P := \emptyset, \; y = h(x) := x \tag{4}$$

  where $A = \begin{bmatrix} 0 & 1 \\ 0 & -b/m \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1/m \end{bmatrix}$, and $x = (x_1, x_2) \in \mathbb{R}^2$.

- Analog-to-Digital Converter (ADC):

$$f(x,u) := \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \; C := \{(x,u) \mid \tau_s \in [0, T_s^*]\}, \tag{5}$$

$$g(x,u) := \begin{bmatrix} u \\ 0 \end{bmatrix}, \; D := \{(x,u) \mid \tau_s \geq T_s^*\}, \; y = h(x) := x \tag{6}$$

  where $x = (m_s, \tau_s) \in \mathbb{R} \times \mathbb{R}^2$, and $u \in \mathbb{R}^2$.

- Zero-Order Hold (ZOH):

$$f(x,u) := \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \; C := \{(x,u) \mid \tau_s \in [0, T_s^*]\}, \tag{7}$$

$$g(x,u) := \begin{bmatrix} u \\ 0 \end{bmatrix}, \; D := \{(x,u) \mid \tau_s \geq T_s^*\}, \; y = h(x) := x \tag{8}$$

  where $x = (m_s, \tau_s) \in \mathbb{R} \times \mathbb{R}^2$, and $u \in \mathbb{R}^2$.

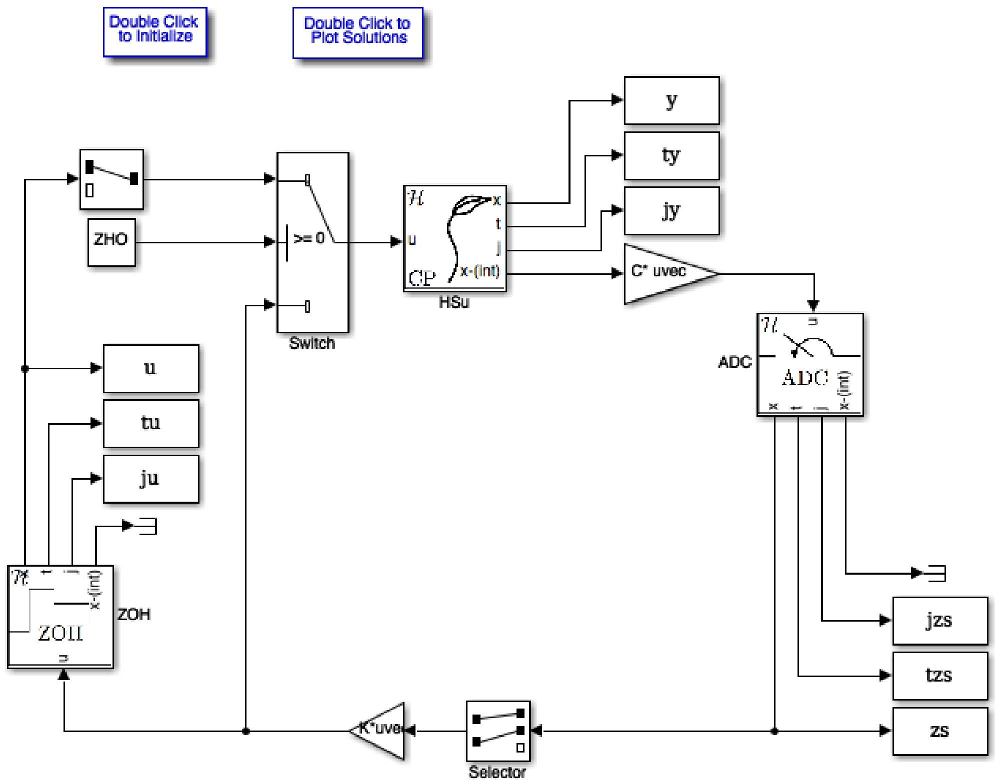


Figure 1: Sample and hold feedback interconnection for Example 5.10

The feedback gain is given by $K = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}$.

For each hybrid system in Figure 1 (HSu, ADC, and ZOH) we have the following Matlab embedded functions that describe the sets $C$ and $D$ and functions $f$ and $g$. Figure 2 presents a numerical simulation of the interconnected system.

- Continuous process:

  Flow map

```
function xdot = f(x, u, ctes)
%--------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
%--------------------------------------------------------------------------
% Project: Simulation of a hybrid system (plant with constraints in the
% state and the input)
% Description: Flow map
%--------------------------------------------------------------------------
%--------------------------------------------------------------------------
%   See also PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC, PLOTHARCCOLOR,
%   PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00
%--------------------------------------------------------------------------
% flow map: xdot=f(x,u);
```

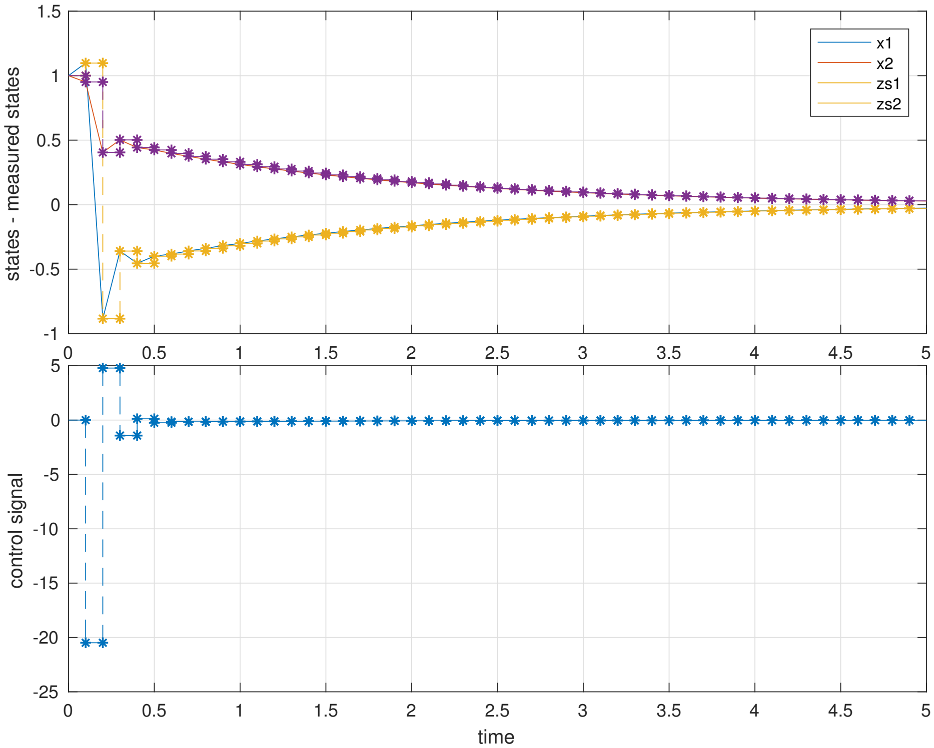


Figure 2: Measuread states and inputs.

```

% ctes = [A,B,C,K'];

A = ctes(:,1:2);
B = ctes(:,3);
C = ctes(:,4:5);
K = ctes(:,6)';

xdot = A*x + B*u;
```

Flow set

```
function v  = C(x, u)
%--------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
%--------------------------------------------------------------------------
% Project: Simulation of a hybrid system
% Description: Flow set
%--------------------------------------------------------------------------
%--------------------------------------------------------------------------
%   See also PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC, PLOTHARCCOLOR,
%   PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00
```

```
%
% Check on flow conditions
% E.g.,
% if (x(1) >= u(1))  % flow condition
%     v = 1;  % report flow
% else
%     v = 0;   % do not report flow
% end


v = 1; % report flow

```

Jump map

```
function xplus = g(x, u)
%--------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
%--------------------------------------------------------------------------
% Project: Simulation of a hybrid system
% Description: Jump map
%--------------------------------------------------------------------------
%--------------------------------------------------------------------------
%   See also PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC, PLOTHARCCOLOR,
%   PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00
%--------------------------------------------------------------------------
% jump map: xplus = g(x,u);

xplus = x;


```

Jump set

```
function v  = D(x, u)
%--------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
%--------------------------------------------------------------------------
% Project: Simulation of a hybrid system
% Description: Jump set
%--------------------------------------------------------------------------
%--------------------------------------------------------------------------
%   See also PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC, PLOTHARCCOLOR,
%   PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00
%
% Check on jump conditions
% % E.g.,
```

```
% if (x(1) <= u(1)) && (x(2) <= 0)  % jump condition
%     v = 1;  % report jump
% else
%     v = 0;    % do not report jump
% end

v = false; % do not report jump
```

- ADC and ZOH:

  Flow map

```
function xdot = f(x,vs)
%--------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
%--------------------------------------------------------------------------
% Project: Simulation of a hybrid system Analog-to-Digital converter (ADC)
% Description: Flow map
%--------------------------------------------------------------------------
%--------------------------------------------------------------------------
%   See also HYEQSOLVER, PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC,
%   PLOTHARCCOLOR, PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00

n = length(vs); % measured input size

msdot = zeros(n,1); % measured continuous dynamics
tau_sdot = 1; % Timer tau_s
xdot = [msdot;tau_sdot];

```

  Flow set

```
function v  = C(x, vs, Ts)
%--------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
%--------------------------------------------------------------------------
% Project: Simulation of a hybrid system Analog-to-Digital converter (ADC)
% Description: Flow set
%--------------------------------------------------------------------------
%--------------------------------------------------------------------------
%   See also HYEQSOLVER, PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC,
%   PLOTHARCCOLOR, PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00
%
% Check on flow conditions
% E.g.,
% if (x(1) >= u(1))  % flow condition
%     v = 1;  % report flow
```

```
% else
%      v = 0;    % do not report flow
% end

n = length(vs); % measured input size

tau_s = x(end); % timer state

if tau_s>=0 && tau_s<= Ts
    v = 1; % report flow
elseif tau_s> Ts
    v = 0; % do not report flow
else
    v = 0;
end
```

Jump map

```
function xplus = g(x, vs)
%--------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
%--------------------------------------------------------------------------
% Project: Simulation of a hybrid system Analog-to-Digital converter (ADC)
% Description: Jump map
%--------------------------------------------------------------------------
%--------------------------------------------------------------------------
%   See also HYEQSOLVER, PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC,
%   PLOTHARCCOLOR, PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00


n = length(vs) % measured input size

msplus = vs; % output = measured input
tau_splus = 0; % Timer tau_s
xplus = [msplus;tau_splus];



```

Jump set

```
function v  = D(x, vs, Ts)
%--------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
%--------------------------------------------------------------------------
% Project: Simulation of a hybrid system Analog-to-Digital converter (ADC)
% Description: Jump set
%--------------------------------------------------------------------------
%--------------------------------------------------------------------------
```

```
%   See also HYEQSOLVER, PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC,
%   PLOTHARCCOLOR, PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00
%
% Check on jump conditions
% % E.g.,
% if (x(1) <= u(1)) && (x(2) <= 0)  % jump condition
%      v = 1;  % report jump
% else
%      v = 0;   % do not report jump
% end

n = length(vs); % measured input size

tau_s = x(end); % timer state

if tau_s>=0 && tau_s<= Ts
    v = 0; % do not report jump
elseif tau_s> Ts
    v = 1; % report jump
else
    v = 0;
end
```

There are numerous practical examples of systems that can be modeled within the general model for sample-and-hold feedback control defined above. For example, one "classical" example is the control of the temperature of a room by turning on and off a heater so as to keep the temperature within a desired range. Another widely known example is the control of the level of a water tank.

□