5 Examples

The examples below illustrate the use of the Simulink implementation above.

Example 5.7 (Linear time-invariant plant)
Following the model of a physical component

$$\dot{x} = f(x,u), \qquad C := \{x \mid x \in \mathbb{R}^n\}, \tag{1a}$$
$$x^+ = g(x) = \emptyset, \qquad D := \emptyset, \tag{1b}$$
$$y = h(x,u) \tag{1c}$$

a linear time-invariant model of the physical component is defined by

$$f(x,u) = f_P(x,u) = A_P x + B_P u, \qquad h(x,u) = h(x,u) = M_P x + N_P u$$

where A_P, B_P, M_P, and N_P are matrices of appropriate dimensions. State and input constraints can directly be embedded into the set C_P. For example, the constraint that x has all of its components nonnegative and that u has its components with norm less or equal than one is captured by

$$\begin{aligned} C_P \;=\; & \{(x,u) \in \mathbb{R}^{n_P} \times \mathbb{R}^{m_P} \mid x_i \geq 0 \ \forall i \in \{1,2,\ldots,n_P\}\} \\ & \cap \{(x,u) \in \mathbb{R}^{n_P} \times \mathbb{R}^{m_P} \mid |u_i| \leq 1 \ \forall i \in \{1,2,\ldots,m_P\}\} \end{aligned}$$

For example, the evolution of the temperature of a room with a heater can be modeled by a linear-time invariant system with state x denoting the temperature of the room and with input $u = (u_1, u_2)$, where u_1 denotes whether the heater is turned on ($u_1 = 1$) or turned off ($u_1 = 0$) while u_2 denotes the temperature outside the room. The evolution of the temperature is given by

$$\dot{x} = -x + \begin{bmatrix} z_\Delta & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad \text{when } (x,u) \in C_P = \left\{(x,u) \in \mathbb{R} \times \mathbb{R}^2 \mid u_1 \in \{0,1\}\right\} \tag{2}$$

where z_Δ is a constant representing the heater capacity. ◻