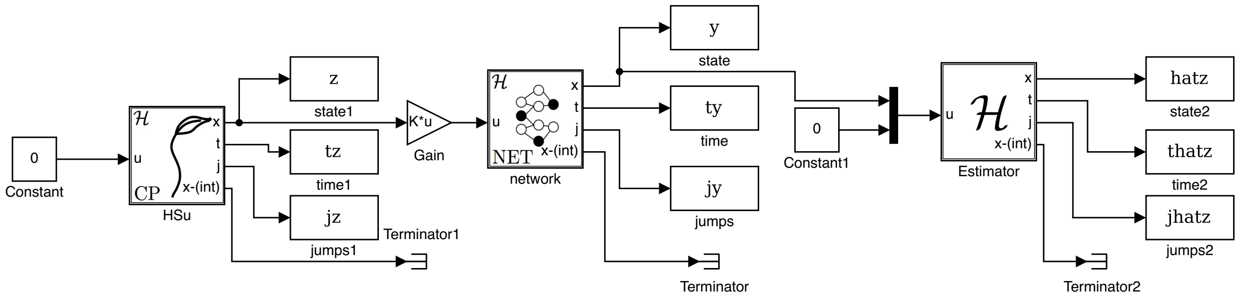

Figure 1: Estimation over a network scheme

5 Examples

The examples below illustrate the use of the Simulink implementation above.

Example 5.9 (Estimation Over a Network) Consider a physical process given in terms of the state-space model

$$\dot{x} = Ax, \qquad y = Mx, \qquad x \in \mathbb{R}^{n_P}, y \in \mathbb{R}^{r_P} \tag{1}$$

where x is the state and y is the measured output. The output is digitally transmitted through a network. At the other end of the network, a computer receives the information and runs an algorithm that takes the measurements of y to estimate the state x of the physical process. In Figure 1 the scheme of the simulation is presented. We consider an estimation algorithm with a state variable $\hat{x} \in \mathbb{R}^{n_P}$, which denotes the estimate of x, that is appropriately reset to a new value involving the information received. More precisely, denoting the transmission times by t_i and L a constant matrix to be designed, the estimation algorithm updates its state as follows

$$\hat{x}^+ = \hat{x} + L(y - M\hat{x}) \tag{2}$$

at every instant information is received. In between such events, the algorithm updates its state continuously so as to match the evolution of the state of the physical process, that is, via

$$\dot{\hat{x}} = A\hat{x} \tag{3}$$

Modeling the network as a hybrid system, which, in particular, assumes zero transmission delay, the state variables of the entire system are j_N, $\tau_N \in \mathbb{R}$, $m_N \in \mathbb{R}^{r_P}$, and $x,\ \hat{x} \in \mathbb{R}^{n_P}$. Then, transmissions occur when $\tau_N \leq 0$, at which events the state of the network is updated via

$$\tau_N^+ \in [T_N^{*\min}, T_N^{*\max}], \quad j_N^+ = j_N + 1, \qquad m_N^+ = y$$

and the state of the algorithm is updated via (2). Note that since the state of the physical process does not change at such events, we can use the following trivial difference equation to update it at the network events:

$$x^+ = x$$

In between events, the state of the network is updated as

$$\dot{\tau}_N = -1, \qquad \dot{j}_N, \qquad \dot{m}_N = 0$$

the state of the algorithm changes continuously according to (3), and the state of the physical process changes according to (1). Considering the equations above, we arbitrarily pick the following data for each of the subsystems in Figure 1:

- Physical process:

$$f_P(x,u) := Ax + Bu,\ C_P := \{(x,u) \in \mathbb{R}^{n_p} \times \mathbb{R}^1\} \tag{4}$$
$$G_P(x,u) := x,\ D_P := \emptyset,\ y = h(x) := Mx \tag{5}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ -2 & 1 & -1 & 0 \\ 2 & -2 & 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad M = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, \tag{6}$$

$n_p = 4$, $r_p = 1$, $x \in \mathbb{R}^{n_p}$, $y \in \mathbb{R}^{r_p}$, and $u \in \mathbb{R}^1$.

- Network:

$$f(x_N, u_N) := \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}, \qquad C := \{(x_N, u_N) \mid \tau_N \geq 0\}, \tag{7}$$

$$g(x_N, u_N) := \begin{bmatrix} u_N \\ j_N + 1 \\ \tau_r \end{bmatrix}, \qquad D := \{(x_N, u_N) \mid \tau_N \leq 0\}, \quad y_N = h(x_N) := x_N \tag{8}$$

where $\tau_r \in [T_N^{*\min}, T_N^{*\max}]$ is a random variable that models the time in-between communication instances. Also, the sate and the input are given by $x_N = (m_N, j_N, \tau_N) \in \mathbb{R}\times\mathbb{R}\times\mathbb{R}$, and $u_N = y \in \mathbb{R}^{r_p}$, respectively.

- Estimator:

$$f(x_E, u_E) := \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} x_E + \begin{bmatrix} B \\ 0 \end{bmatrix} v, \qquad C := \{(x_E, u_E) \mid j_E = j_N\}, \tag{9}$$

$$g((\hat{x}, j_E), u_E) := \begin{bmatrix} \hat{x} + L(m_N - M\hat{x}) \\ j_N \end{bmatrix}, \qquad D := \{(x, u) \mid j_E \neq j_N\}, \tag{10}$$

where L, which is designed as in [1], is given by

$$L := \begin{bmatrix} 1 \\ -0.9433 \\ -0.6773 \\ 1.6274 \end{bmatrix}, \tag{11}$$

the input and the state are given by $u_E = (x_N, v) = ((m_N, j_N, \tau_N), v) \in \mathbb{R}\times\mathbb{R}\times\mathbb{R}\times\mathbb{R}$, and $x_E = (\hat{x}, j_E) \in \mathbb{R}^4 \times \mathbb{R}$, respectively. Notice that the estimator block (Estimator) in Figure 1 is implemented using a regular hybrid system block with embedded functions.

For each hybrid system in Figure 1 (HSu, network, and Estimator) we have the following Matlab embedded functions that describe the sets C and D and functions f and g. Figure 2 depicts the corresponding hybrid arc for the position state.

- Continuous process:

  Flow map

```
function zdot = f(z, u, ctes)
%--------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
%--------------------------------------------------------------------------
% Project: Simulation of a hybrid system (plant with constraints in the
% state and the input)
% Description: Flow map
```

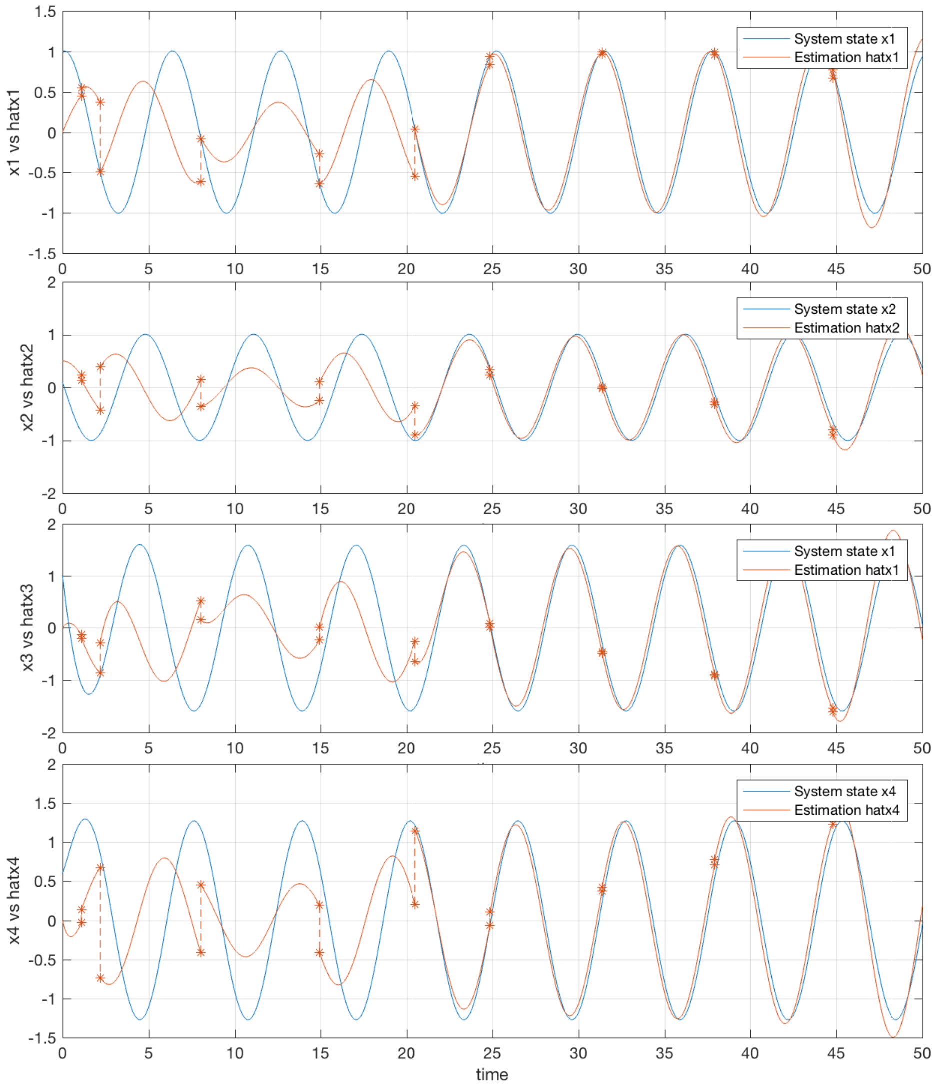

Figure 2: Estimated states Vs. continuous plant states

```
%-------------------------------------------------------------------------
%-------------------------------------------------------------------------
%   See also HYEQSOLVER, PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC,
%   PLOTHARCCOLOR, PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00
%-------------------------------------------------------------------------
% flow map: xdot=f(x,u);
% ctes = [A,B,M',L,P];

A = ctes(:,1:4);
B = ctes(:,5);
% M = ctes(:,6)';
% L = ctes(:,7);

```

```
zdot = A*z + B*u;
```

Flow set

```
function v  = C(x, u)
%--------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
%--------------------------------------------------------------------------
% Project: Simulation of a hybrid system
% Description: Flow set
%--------------------------------------------------------------------------
%--------------------------------------------------------------------------
%   See also HYEQSOLVER, PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC,
%   PLOTHARCCOLOR, PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00
%
% Check on flow conditions
% E.g.,
% if (x(1) >= u(1))  % flow condition
%     v = 1;  % report flow
% else
%     v = 0;   % do not report flow
% end


v = 1; % report flow

```

Jump map

```
function xplus = g(x, u)
%--------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
%--------------------------------------------------------------------------
% Project: Simulation of a hybrid system
% Description: Jump map
%--------------------------------------------------------------------------
%--------------------------------------------------------------------------
%   See also HYEQSOLVER, PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC,
%   PLOTHARCCOLOR, PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00
%--------------------------------------------------------------------------
% jump map: xplus = g(x,u);

xplus = zeros(4,1);
xplus = x;

```

Jump set

```
function v  = D(x, u)
%--------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
%--------------------------------------------------------------------------
% Project: Simulation of a hybrid system
% Description: Jump set
%--------------------------------------------------------------------------
%--------------------------------------------------------------------------
%   See also HYEQSOLVER, PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC,
%   PLOTHARCCOLOR, PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00
%
% Check on jump conditions
% % E.g.,
% if (x(1) <= u(1)) && (x(2) <= 0)  % jump condition
%     v = 1;  % report jump
% else
%     v = 0;   % do not report jump
% end

v = 0; % do not report jump
```

- Network:

  Flow map

```
function xdot = f(x,vs)
%--------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
%--------------------------------------------------------------------------
% Project: Simulation of a hybrid system Digital network (net)
% Description: Flow map
%--------------------------------------------------------------------------
%--------------------------------------------------------------------------
%   See also HYEQSOLVER, PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC,
%   PLOTHARCCOLOR, PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00

n = length(vs); % measured input size

msdot = zeros(n,1); % measured continuous dynamics
jdot = 0;
tau_sdot = -1; % Timer tau_s
xdot = [msdot;jdot;tau_sdot];

```

Flow set

```matlab
function v  = C(x, vs, Tnmax)
%--------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
%--------------------------------------------------------------------------
% Project: Simulation of a hybrid system Digital network (net)
% Description: Flow set
%--------------------------------------------------------------------------
%--------------------------------------------------------------------------
%   See also HYEQSOLVER, PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC,
%   PLOTHARCCOLOR, PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00
%
% Check on flow conditions
% E.g.,
% if (x(1) >= u(1))  % flow condition
%      v = 1;  % report flow
% else
%      v = 0;   % do not report flow
% end

tau_s = x(end); % timer state

if tau_s>=0 && tau_s<= Tnmax
    v = 1; % report flow
elseif tau_s> Tnmax
    v = 0; % do not report flow
else
    v = 0;
end
```

Jump map

```matlab
function xplus = g(x, vs, Tnmax, Tnmin, tk)
%--------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
%--------------------------------------------------------------------------
% Project: Simulation of a hybrid system Analog-to-Digital converter (ADC)
% Description: Jump map
%--------------------------------------------------------------------------
%--------------------------------------------------------------------------
%   See also HYEQSOLVER, PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC,
%   PLOTHARCCOLOR, PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00

n = length(vs); % measured input size
xtemp = zeros(n+2,1);
xtemp = x;
x = xtemp;
```

```
j = x(n+1);
msplus = vs; % output = measured input
% The value tau_s is updated as a function of vs, e.g.,
tau_splus = tk(j+1); % Timer tau_s
j_plus = j+1;
xplus = [msplus;j_plus;tau_splus];
```

Jump set

```
function v  = D(x, vs)
%--------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
%--------------------------------------------------------------------------
% Project: Simulation of a hybrid system Analog-to-Digital converter (ADC)
% Description: Jump set
%--------------------------------------------------------------------------
%--------------------------------------------------------------------------
%   See also HYEQSOLVER, PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC,
%   PLOTHARCCOLOR, PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00
%
% Check on jump conditions
% % E.g.,
% if (x(1) <= u(1)) && (x(2) <= 0)  % jump condition
%     v = 1;  % report jump
% else
%     v = 0;   % do not report jump
% end


tau_s = x(end); % timer state

if tau_s>=0
    v = 0; % do not report jump
elseif tau_s<= 0
    v = 1; % report jump
else
    v = 0;
end
```

- Estimator:

  Flow map

```
function xdot = f(x, v,ctes)
%--------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
```

```
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
%--------------------------------------------------------------------------
% Project: Simulation of a hybrid system
% Description: Flow map
%--------------------------------------------------------------------------
%--------------------------------------------------------------------------
%   See also HYEQSOLVER, PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC,
%   PLOTHARCCOLOR, PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00
%--------------------------------------------------------------------------
% flow map: xdot=f(x,u);
% ctes = [A,B,M',L,P];

A = ctes(:,1:4);
B = ctes(:,5);
% M = ctes(:,6)';
% L = ctes(:,7);
n = length(A);
u = v(end);
z = x(1:n);
jdot = 0;
zdot = A*z + B*u;
xdot = [zdot;jdot];
```

Flow set

```
function out  = C(x, v)
%--------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
%--------------------------------------------------------------------------
% Project: Simulation of a hybrid system
% Description: Flow set
%--------------------------------------------------------------------------
%--------------------------------------------------------------------------
%   See also HYEQSOLVER, PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC,
%   PLOTHARCCOLOR, PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00
%
% Check on flow conditions
% E.g.,
% if (x(1) >= u(1))  % flow condition
%     v = 1;  % report flow
% else
%     v = 0;   % do not report flow
% end

j = x(end); % internal communication memory event
jnet = v(end-2); % communication event

```

```
if j == jnet
    out = 1; % report flow
else
    out = 0;
end
```

Jump map

```
function xplus = g(x, v, ctes)
%--------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
%--------------------------------------------------------------------------
% Project: Simulation of a hybrid system
% Description: Jump map
%--------------------------------------------------------------------------
%--------------------------------------------------------------------------
%   See also HYEQSOLVER, PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC,
%   PLOTHARCCOLOR, PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00
%--------------------------------------------------------------------------
% jump map: xplus = g(x,u);

A = ctes(:,1:4);
M = ctes(:,6)';
L = ctes(:,7);

n = 4;
z = zeros(4,1);
zplus = zeros(4,1);
xplus = zeros(5,1);
z = x(1:4);
jnet = v(2); % communication event
y = v(1);

jplus = jnet;
zplus = z + L*(y-M*z);
xplus = [zplus;jplus];
```

Jump set

```
function out  = D(x, v)
%--------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
%--------------------------------------------------------------------------
% Project: Simulation of a hybrid system
% Description: Jump set
%--------------------------------------------------------------------------
%--------------------------------------------------------------------------
%   See also HYEQSOLVER, PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC,
%   PLOTHARCCOLOR, PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
```

```
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00
%
% Check on jump conditions
% % E.g.,
% if (x(1) <= u(1)) && (x(2) <= 0)  % jump condition
%     v = 1;  % report jump
% else
%     v = 0;   % do not report jump
% end


n = length(x)-1; % measured input size
j = x(end); % internal communication memory event
jnet = v(end-2); % communication event

if j ~= jnet
    out = 1; % report jump
else
    out = 0;
end

```

□

References

[1] F. Ferrante, F. Gouaisbaut, R. G. Sanfelice, and S. Tarbouriech. State estimation of linear systems in the presence of sporadic measurements. *Automatica*, 73:101–109, November 2016.