1 Examples

The examples below illustrate the use of the Simulink implementation above.

Example 1.5 (vehicle following a track with boundaries) Consider a vehicle modeled by a Dubins vehicle model traveling along a given track with state vector $x = [\xi_1, \xi_2, \xi_3]^\top$ with dynamics given by $\dot{\xi}_1 = u\cos\xi_3$, $\dot{\xi}_2 = u\sin\xi_3$, and $\dot{\xi}_3 = -\xi_3 + r(q)$. The input u is the tangential velocity of the vehicle, ξ_1 and ξ_2 describe the vehicle's position on the plane, and ξ_3 is the vehicle's orientation angle. Also consider a switching controller attempting to keep the vehicle inside the boundaries of a track given by $\{(\xi_1, \xi_2) : -1 \leq \xi_1 \leq 1\}$. A state $q \in \{1, 2\}$ is used to define the modes of operation of the controller. When $q = 1$, the vehicle is traveling to the left, and when $q = 2$, the vehicle is traveling to the right. A logic variable r is defined in order to steer the vehicle back inside the boundary. The state of the closed-loop system is given by $x := [\xi^\top \; q]^\top$. A model of such a closed-loop system is given by

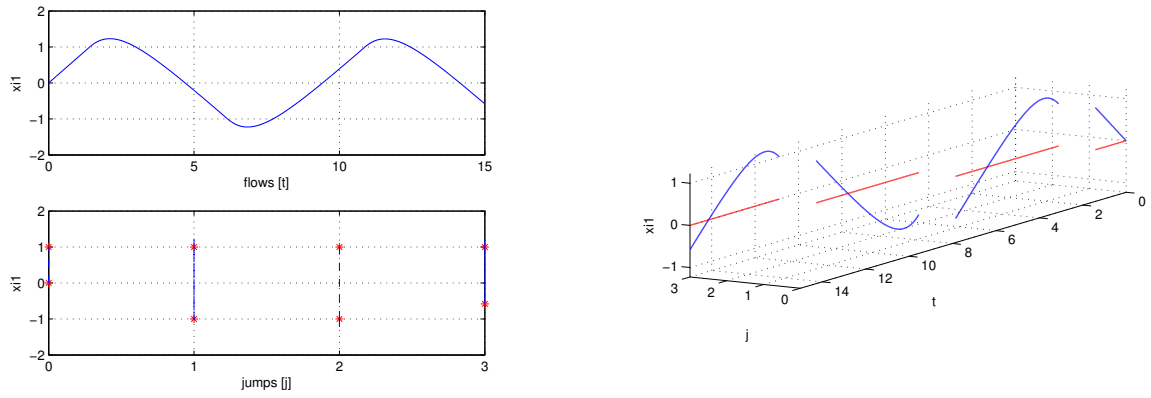
$$f(x,u) := \begin{bmatrix} \begin{bmatrix} u\cos(\xi_3) \\ u\sin(\xi_3) \\ -\xi_3 + r(q) \end{bmatrix} \\ u \end{bmatrix}, \quad r(q) := \begin{cases} \frac{3\pi}{4} & \text{if} \quad q = 1 \\ \frac{\pi}{4} & \text{if} \quad q = 2 \end{cases} \tag{1}$$

$$C := \left\{(\xi, u) \in \mathbb{R}^3 \times \{1,2\} \times \mathbb{R} \mid (\xi_1 \leq 1, q = 2) \text{ or } (\xi_1 \geq -1, q = 1)\right\}, \tag{2}$$

$$g(\xi,u) := \begin{cases} \begin{bmatrix} \xi \\ 2 \end{bmatrix} & \text{if} \quad \xi_1 \leq -1, \quad q = 1 \\ \begin{bmatrix} \xi \\ 1 \end{bmatrix} & \text{if} \quad \xi_1 \geq 1, \quad q = 2 \end{cases}, \tag{3}$$

$$D := \left\{(\xi, u) \in \mathbb{R}^3 \times \{1,2\} \times \mathbb{R} \mid (\xi_1 \geq 1, q = 2) \text{ or } (\xi_1 \leq -1, q = 1)\right\} \tag{4}$$

The MATLAB scripts in each of the function blocks of the implementation above are given as follows. The tangential velocity of the vehicle is chosen to be $u = 1$, the initial position on the plane is chosen to be $(\xi_1, \xi_2) = (0, 0)$, and the initial orientation angle is chosen to be $\xi_3 = \frac{\pi}{4}$ radians.



(a) Trajectory

(b) Hybrid arc

Figure 1: Solution of Example 1.5

Flow map

```
function xdot = f(x, u)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Matlab Function  Author: Ricardo Sanfelice
%
```

```
% Project: Simulation of a hybrid system (Vehicle Traveling on a Track)
%
% Name: f.m
%
% Description: Flow map
%
% Version: 1.0
% Required files: -
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

xtemp = zeros(4,1);
xtemp = x;
x = xtemp;

% state
xi1 = x(1);       %x-position
xi2 = x(2);       %y-position
xi3 = x(3);       %orientation angle
q = x(4);

% q = 1 --> going left
% q = 2 --> going right

if q == 1
    r = 3*pi/4;
elseif q == 2
    r = pi/4;
else
    r = 0;
end

% flow map: xidot=f(xi,u);
xi1dot = u*cos(xi3);  %tangential velocity in x-direction
xi2dot = u*sin(xi3);  %tangential velocity in y-direction
xi3dot = -xi3 + r;      %angular velocity
qdot = 0;

xdot = [xi1dot;xi2dot;xi3dot;qdot];
```

Flow set

```
function v  = C(x, u)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Matlab Function  Author: Ricardo Sanfelice
%
% Project: Simulation of a hybrid system (Vehicle Traveling on a Track)
%
% Name: C.m
%
% Description: Flow set
%
% Version: 1.0
% Required files: -
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
15
16  xtemp = zeros(4,1);
17  xtemp = x;
18  x = xtemp;
19
20  % state
21  xi1 = x(1);       %x-position
22  xi2 = x(2);       %y-position
23  xi3 = x(3);       %orientation angle
24  q = x(4);
25
26  % q = 1 --> going left
27  % q = 2 --> going right
28
29  if ((xi1 < 1) && (q == 2)) || ((xi1 > -1) && (q == 1))  % flow condition
30      v = 1;  % report flow
31  else
32      v = 0;   % do not report flow
33  end
```

Jump map

```
1   function xplus = g(x, u)
2
3   %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
4   % Matlab Function  Author: Ricardo Sanfelice
5   %
6   % Project: Simulation of a hybrid system (Vehicle Traveling on a Track)
7   %
8   % Name: g.m
9   %
10  % Description: Jump map
11  %
12  % Version: 1.0
13  % Required files: -
14  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
15
16  xtemp = zeros(4,1);
17  xtemp = x;
18  x = xtemp;
19  % state
20  xi1 = x(1);       %x-position
21  xi2 = x(2);       %y-position
22  xi3 = x(3);       %orientation angle
23  q = x(4);
24
25  % q = 1 --> going left
26  % q = 2 --> going right
27  xi1plus=xi1;
28  xi2plus=xi2;
29  xi3plus=xi3;
30  qplus=q;
31
32  % jump map
33  if ((xi1 >= 1) && (q == 2)) || ((xi1 <= -1) && (q == 1))
```

```
    qplus = 3-q;
else
    qplus = q;
end

xplus = [xi1plus;xi2plus;xi3plus;qplus];
```

Jump set

```
function v  = D(x, u)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Matlab Function  Author: Ricardo Sanfelice
%
% Project: Simulation of a hybrid system (Vehicle Traveling on a Track)
%
% Name: D.m
%
% Description: Jump set
%
% Version: 1.0
% Required files: -
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

xtemp = zeros(4,1);
xtemp = x;
x = xtemp;

% state
xi1 = x(1);       %x-position
xi2 = x(2);       %y-position
xi3 = x(3);       %orientation angle
q = x(4);

% q = 1 --> going left
% q = 2 --> going right

 if ((xi1 >= 1) && (q == 2)) || ((xi1 <= -1) && (q == 1))  % jump condition
     v = 1;  % report jump
 else
     v = 0; % do not report jump
 end
```

A solution to the system of a vehicle following a track in $\{(\xi_1, \xi_2) : -1 \leq \xi_1 \leq 1\}$, and with $T = 15, J = 10$, $rule = 1$, is depicted in Figure 1(a) (trajectory). Both the projection onto t and j are shown. Figure 1(b) depicts the corresponding hybrid arc.

For MATLAB files of this example, see Examples/Example_1.5.

□