1 Examples

The example below shows how to use the HyEQ solver to simulate a bouncing ball.

Example 1.2 (bouncing ball with Lite HyEQ Solver) Consider the hybrid system model for the bouncing ball with data given in Example 1.1 in the instructions file.

For this example, we consider the ball to be bouncing on a floor at zero height. The constants for the bouncing ball system are $\gamma = 9.81$ and $\lambda = 0.8$. The following procedure is used to simulate this example in the Lite HyEQ Solver:

- Inside the MATLAB script `run.m`, initial conditions, simulation horizons, a rule for jumps, ode solver options, and a step size coefficient are defined. The function `HyEQsolver.m` is called in order to run the simulation, and a script for plotting solutions is included.
- Then the MATLAB functions `f.m, C.m, g.m, D.m` are edited according to the data given above.
- Finally, the simulation is run by clicking the run button in `run.m` or by calling `run.m` in the MATLAB command window.

Example code for each of the MATLAB files `run.m, f.m, C.m, g.m,` and `D.m` is given below.

```
%-------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
% Filename: run_ex1_2a.m
%-------------------------------------------------------------------------
% Project: Simulation of a hybrid system (bouncing ball)
%-------------------------------------------------------------------------
%-------------------------------------------------------------------------
%   See also HYEQSOLVER, PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC,
%   PLOTHARCCOLOR, PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00

function run_ex1_2a
% initial conditions
x1_0 = 1;
x2_0 = 0;
x0 = [x1_0;x2_0];

% physical variables
global gamma lambda
gamma = -9.81;  % gravity constant
lambda = 0.8;   % restitution coefficent

% simulation horizon
TSPAN=[0 10];
JSPAN = [0 20];

% rule for jumps
% rule = 1 -> priority for jumps
% rule = 2 -> priority for flows
rule = 1;

options = odeset('RelTol',1e-6,'MaxStep',.1);
```

```
36
37 % simulate
38 [t j x] = HyEQsolver( @f_ex1_2a,@g_ex1_2a,@C_ex1_2a,@D_ex1_2a,...
39     x0,TSPAN,JSPAN,rule,options);
40
41 % plot solution
42 figure(1) % position
43 clf
44 subplot(2,1,1), plotHarc(t,j,x(:,1));
45 grid on
46 ylabel('x_1 position')
47 subplot(2,1,2), plotHarc(t,j,x(:,2));
48 grid on
49 ylabel('x_2 velocity')
50
51 % plot phase plane
52 figure(2) % position
53 clf
54 plotHarcColor(x(:,1),j,x(:,2),t);
55 xlabel('x_1')
56 ylabel('x_2')
57 grid on
58
59 % plot hybrid arc
60 figure(3)
61 plotHybridArc(t,j,x)
62 xlabel('j')
63 ylabel('t')
64 zlabel('x1')
65
```

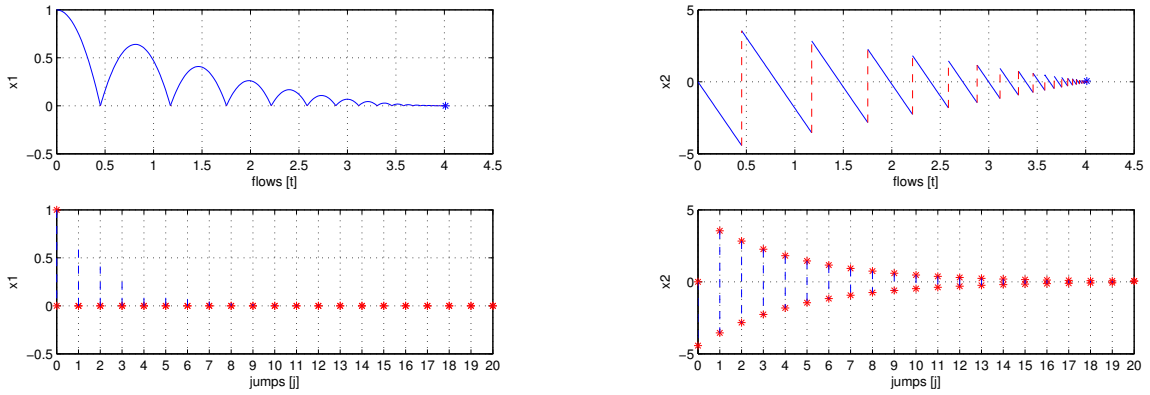
```
1  function xdot = f_ex1_2a(x)
2  %-------------------------------------------------------------------------
3  % Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
4  % https://hybrid.soe.ucsc.edu/software
5  % http://hybridsimulator.wordpress.com/
6  % Filename: f_ex1_2a.m
7  %-------------------------------------------------------------------------
8  % Project: Simulation of a hybrid system (bouncing ball)
9  % Description: Flow map
10 %-------------------------------------------------------------------------
11 %-------------------------------------------------------------------------
12 %   See also HYEQSOLVER, PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC,
13 %   PLOTHARCCOLOR, PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
14 %   Copyright @ Hybrid Systems Laboratory (HSL),
15 %   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00
16
17 % state
18 x1 = x(1);
19 x2 = x(2);
20
21 global gamma
22
23 % differential equations
```

```
xdot = [x2 ; gamma];
end
```

```
function [value] = C_ex1_2a(x)
%-------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
% Filename: C_ex1_2a.m
%-------------------------------------------------------------------------
% Description: Flow set
% Return 0 if outside of C, and 1 if inside C
%-------------------------------------------------------------------------
%-------------------------------------------------------------------------
%   See also HYEQSOLVER, PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC,
%   PLOTHARCCOLOR, PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00


x1 = x(1);

if x1 >= 0
    value = 1;
else
    value = 0;
end
end
```

```
function xplus = g_ex1_2a(x)
%-------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
% Filename: g_ex1_2a.m
%-------------------------------------------------------------------------
% Project: Simulation of a hybrid system (bouncing ball)
% Description: Jump map
%-------------------------------------------------------------------------
%-------------------------------------------------------------------------
%   See also HYEQSOLVER, PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC,
%   PLOTHARCCOLOR, PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00

% state
x1 = x(1);
x2 = x(2);

global lambda

xplus = [-x1 ; -lambda*x2];
end
```

```
function inside = D_ex1_2a(x)
%--------------------------------------------------------------------------
% Matlab M-file Project: HyEQ Toolbox @  Hybrid Systems Laboratory (HSL),
% https://hybrid.soe.ucsc.edu/software
% http://hybridsimulator.wordpress.com/
% Filename: D_ex1_2a.m
%--------------------------------------------------------------------------
% Description: Jump set
% Return 0 if outside of D, and 1 if inside D
%--------------------------------------------------------------------------
%--------------------------------------------------------------------------
%   See also HYEQSOLVER, PLOTARC, PLOTARC3, PLOTFLOWS, PLOTHARC,
%   PLOTHARCCOLOR, PLOTHARCCOLOR3D, PLOTHYBRIDARC, PLOTJUMPS.
%   Copyright @ Hybrid Systems Laboratory (HSL),
%   Revision: 0.0.0.3 Date: 05/20/2015 3:42:00

x1 = x(1);
x2 = x(2);
if (x1 <= 0 && x2 <= 0)
    inside = 1;
else
    inside = 0;
end
end
```



(a) Height

(b) Velocity

Figure 1: Solution of Example 1.2

A solution to the bouncing ball system from $x(0,0) = [1,0]^\top$ and with $TSPAN = [0\ \ 10], JSPAN = [0\ \ 20]$, $rule = 1$, is depicted in Figure 1(a) (height) and Figure 1(b) (velocity). Both the projection onto t and j are shown. Figure 2 depicts the corresponding hybrid arc for the position state.

For MATLAB files of this example, see Examples/Example_1.2.

□

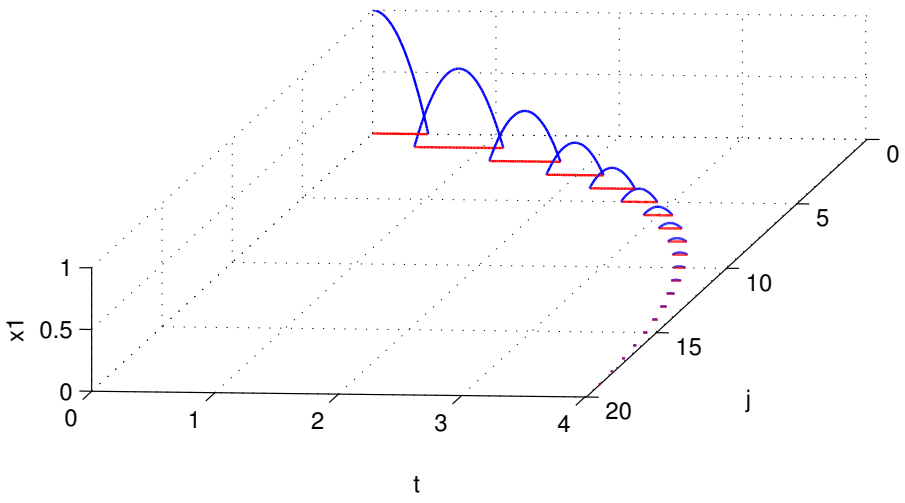


Figure 2: Hybrid arc corresponding to a solution of Example 1.2: height