Kurczak Michał #21

Numerical Methods (ENUME 2019) – Project
Assignment C: Solving ordinary differential equations

1. Develop a program for solving the following differential equation:

   $9y'' + 6y' + 10y = 0$ for $t \in [0,10]$, $y(0) = 0$ and $y'(0) = 2$

   by means of the implicit Lobatto IIID order 4 method defined by the following Butcher table:

$$\begin{array}{c|ccc} 0 & \frac{1}{6} & 0 & -\frac{1}{6} \\ \frac{1}{2} & \frac{1}{12} & \frac{5}{12} & 0 \\ 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ \hline & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{array}$$

   Compare the solution, obtained by means of this program for the constant integration step $h = 0.01$, with the solution, obtained by means of the MATLAB operator ***ode45***. Optimise the parameters ***RelTol*** and ***AbsTol*** of the latter in such a way as to get the most accurate solution $\dot{\mathbf{y}}(t;h)$ (to be used as a reference in the following sections).

2. Carry out a systematic investigation of the dependence of the accuracy of the solution $\hat{\mathbf{y}}(t;h)$ on the integration step h. Use the following accuracy indicators for this purpose:

$$\delta_2(h) = \frac{\left\|\hat{\mathbf{y}}(t;h) - \dot{\mathbf{y}}(t,h)\right\|_2}{\left\|\dot{\mathbf{y}}(t,h)\right\|_2} \quad \text{(the root-mean-square error)}$$

$$\delta_\infty(h) = \frac{\left\|\hat{\mathbf{y}}(t;h) - \dot{\mathbf{y}}(t,h)\right\|_\infty}{\left\|\dot{\mathbf{y}}(t,h)\right\|_\infty} \quad \text{(the maximum error)}$$

   Make the graphs $\delta_2(h)$ and $\delta_\infty(h)$.

3. Repeat the systematic investigation, defined in Section 3, for the implicit Euler method. Add the curves representative of $\delta_2(h)$ and $\delta_\infty(h)$ to the graph obtained in Section 2.