

# 算法与数据结构体系课程

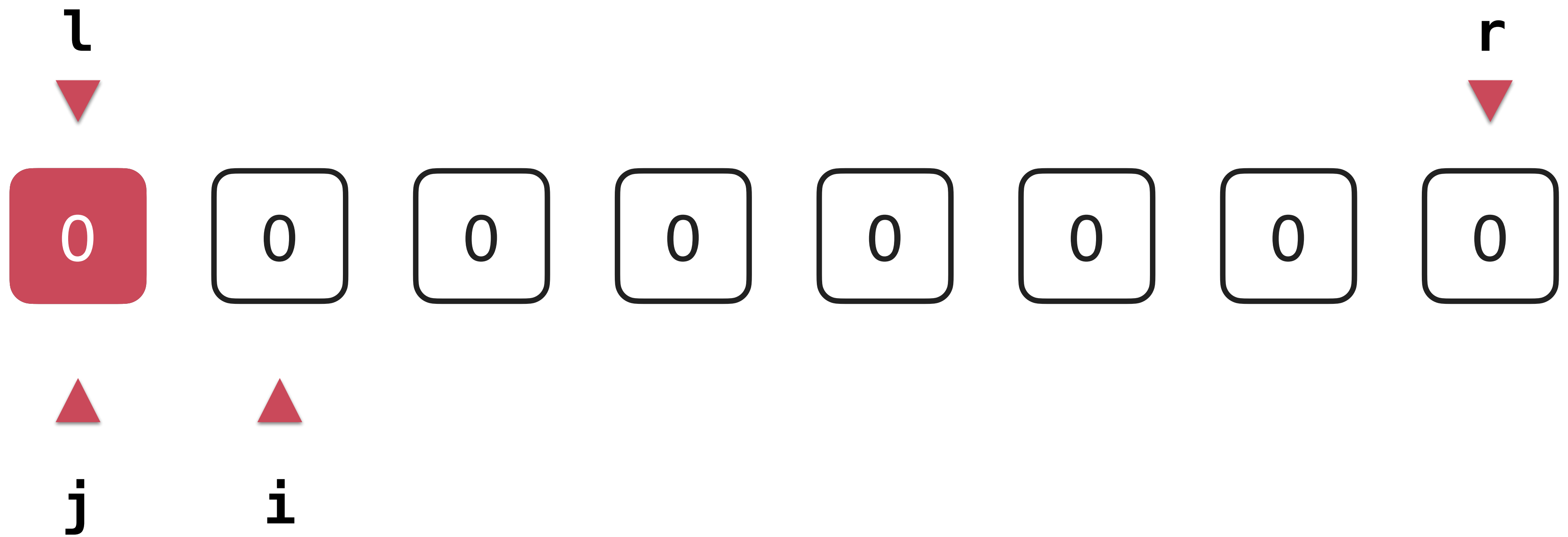
liuyubobobo

更多关于快速排序法

快速排序法还有问题

实践：展示快速排序法的问题

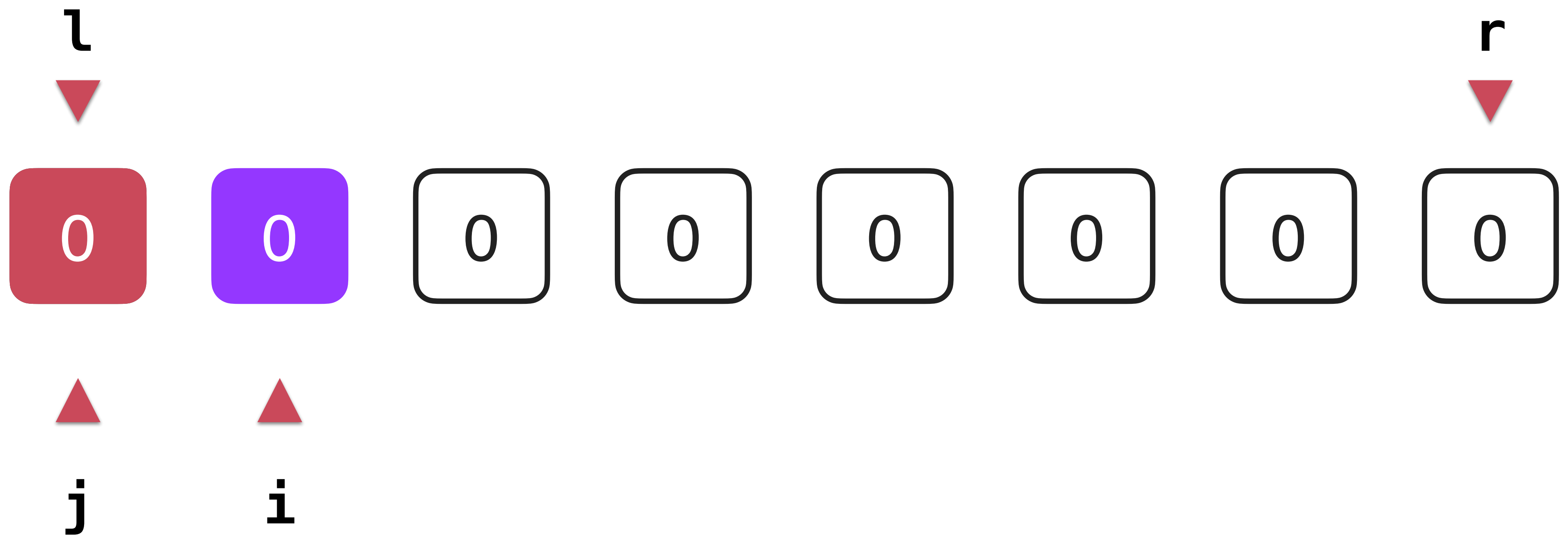
# 快速排序法还有问题



$\text{arr}[\text{l}+1 \dots \text{j}] < v$

$\text{arr}[\text{j}+1 \dots \text{i}-1] \geq v$

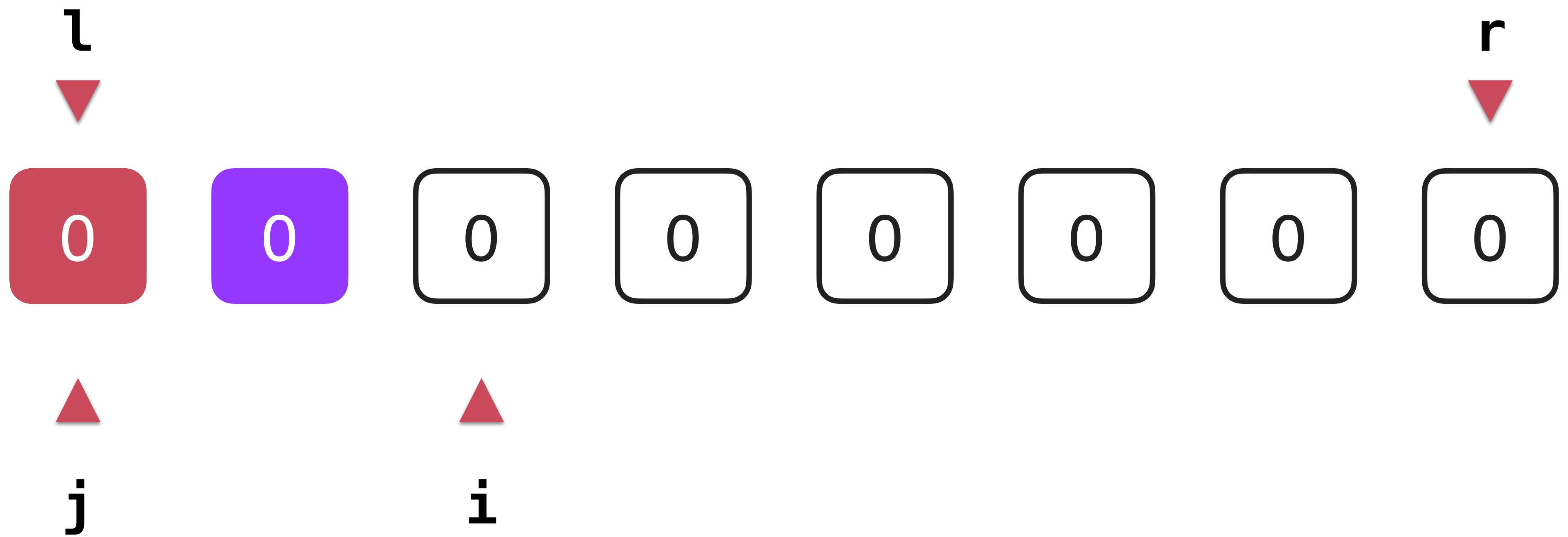
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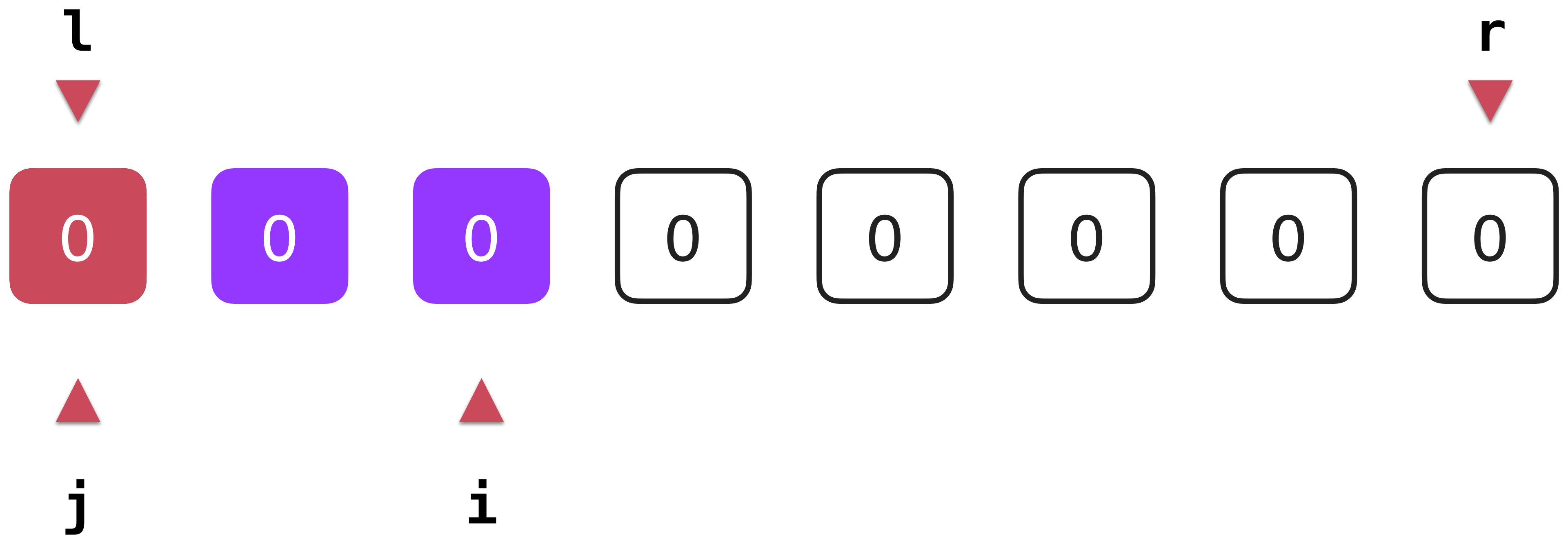
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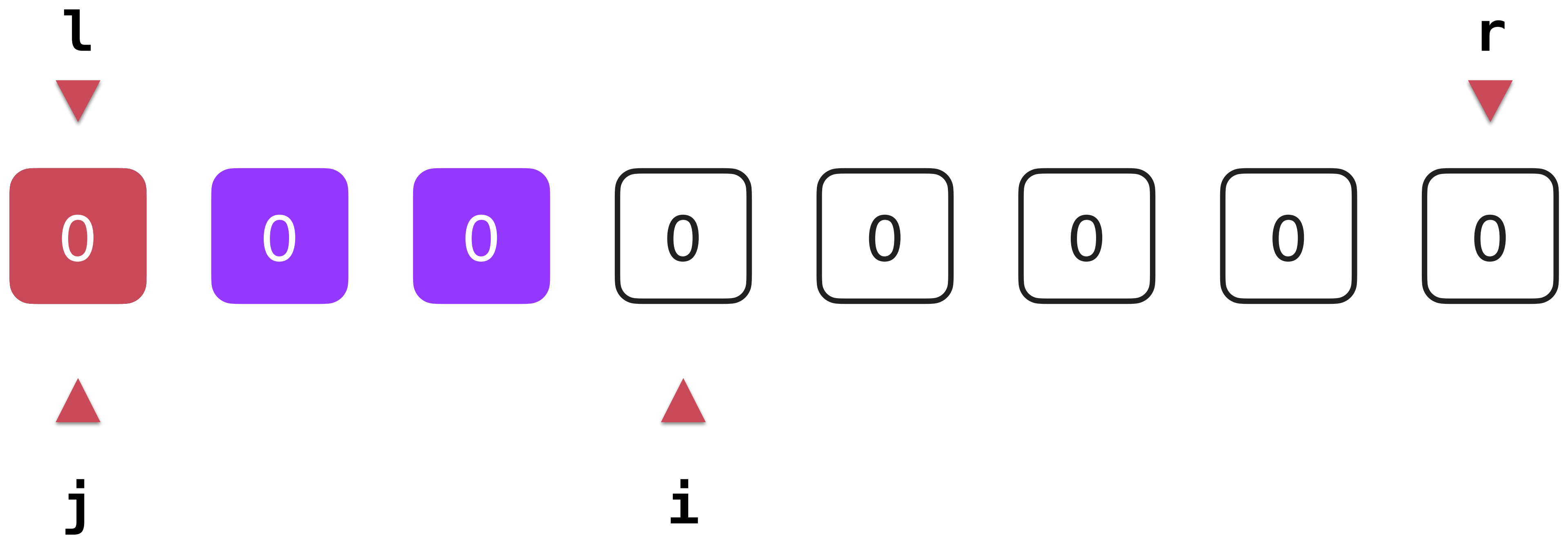


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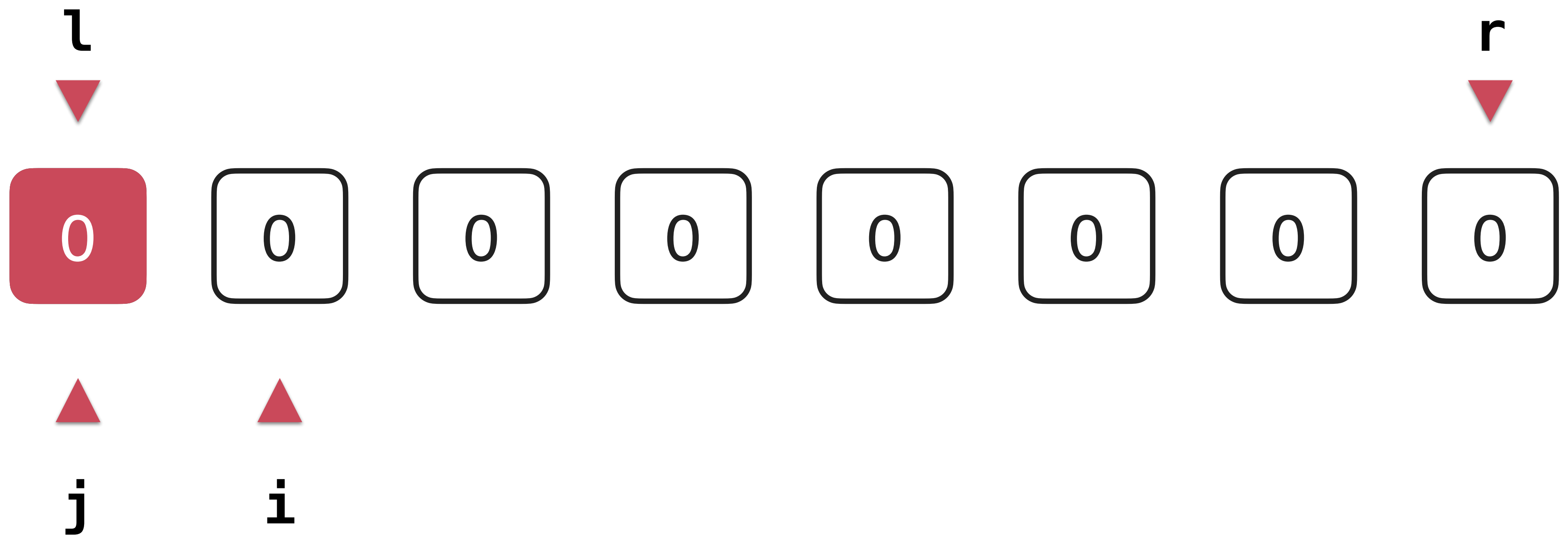
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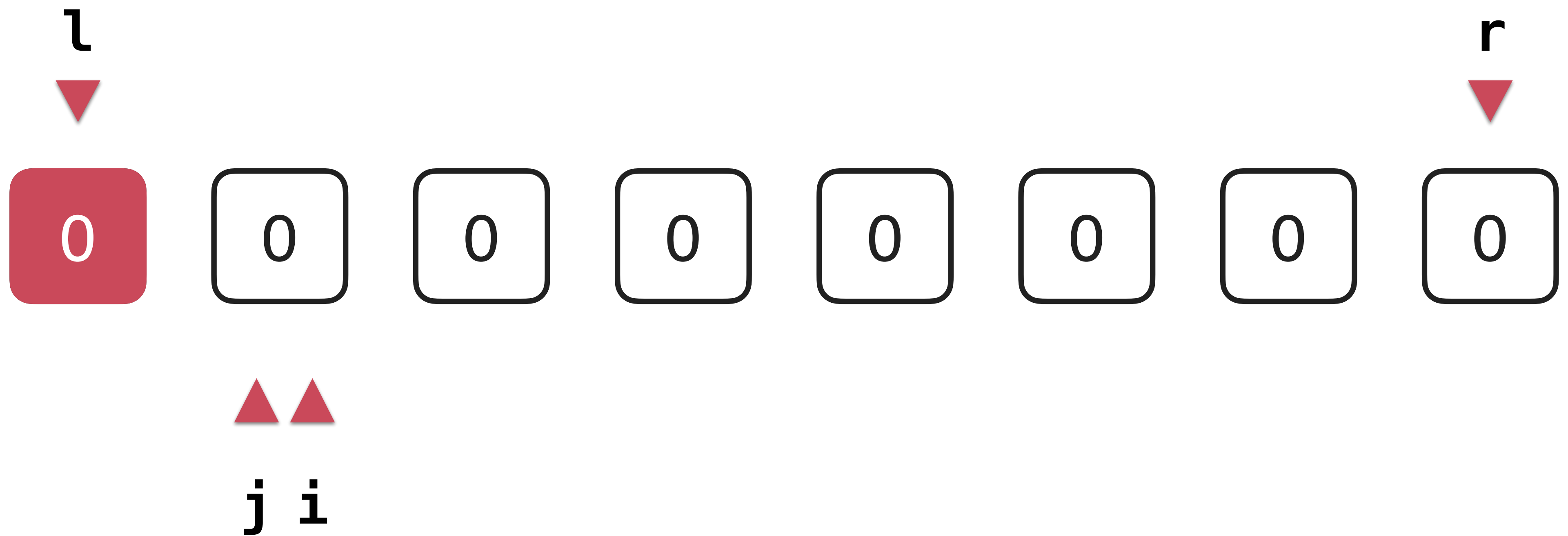
# 快速排序法还有问题



$\text{arr}[l+1 \dots j] \leq v$

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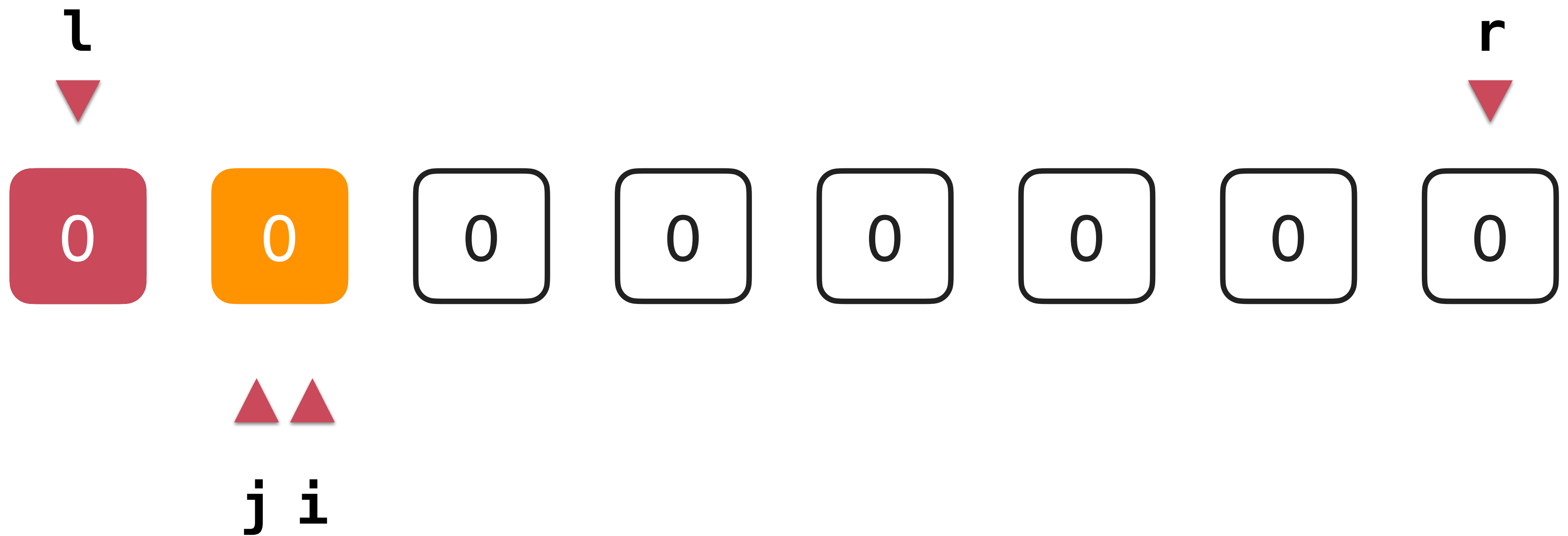
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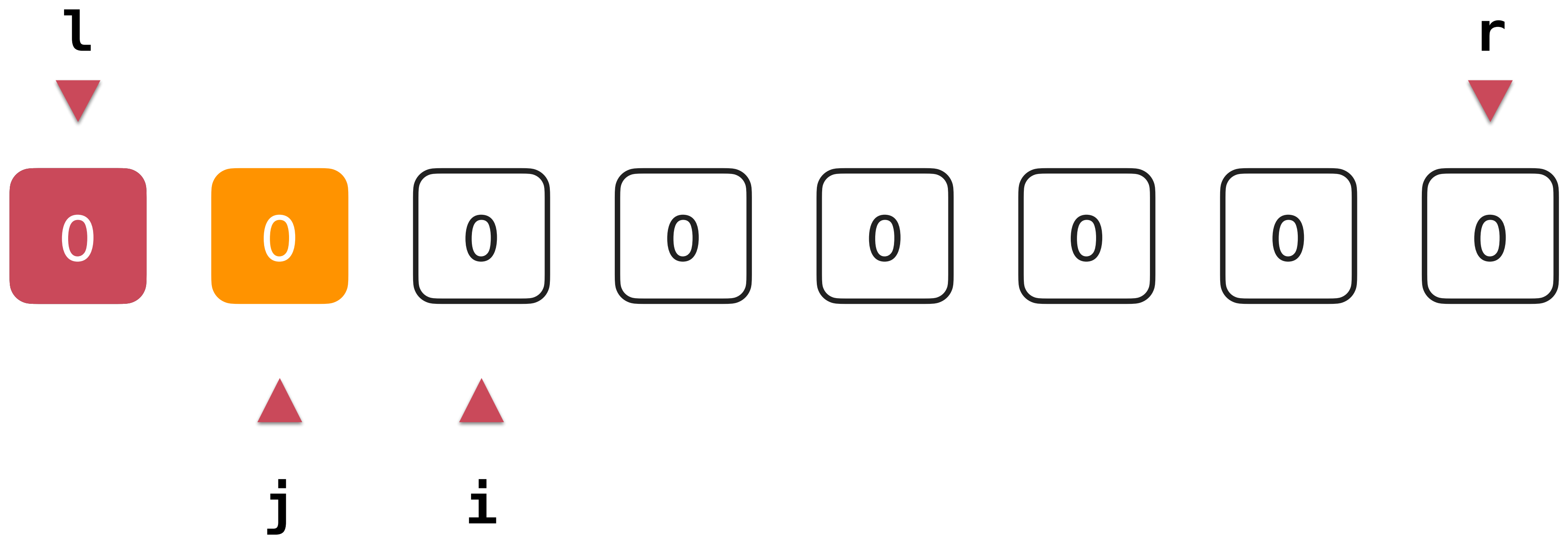
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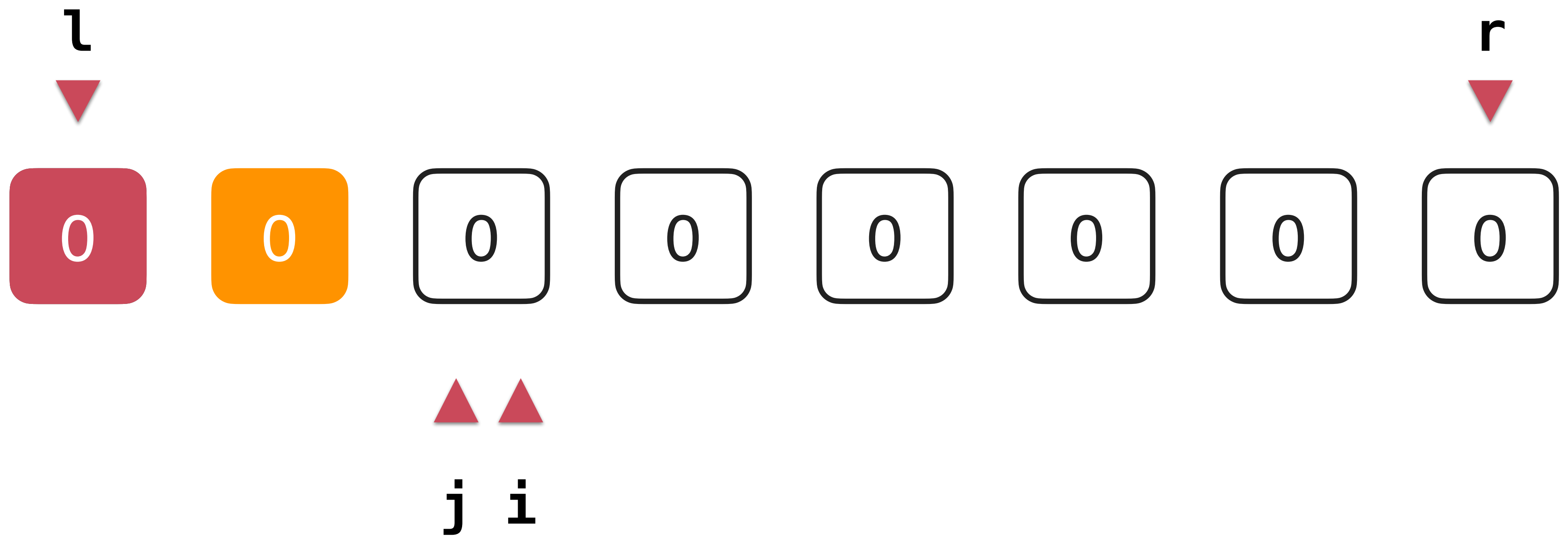
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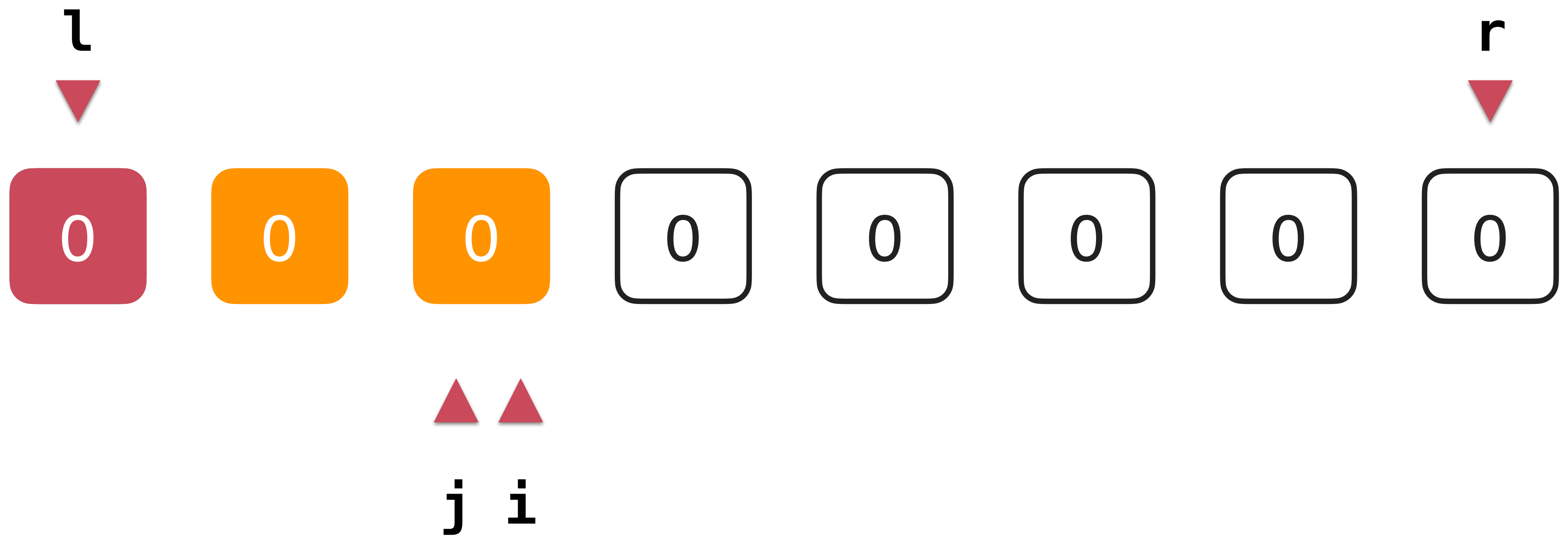
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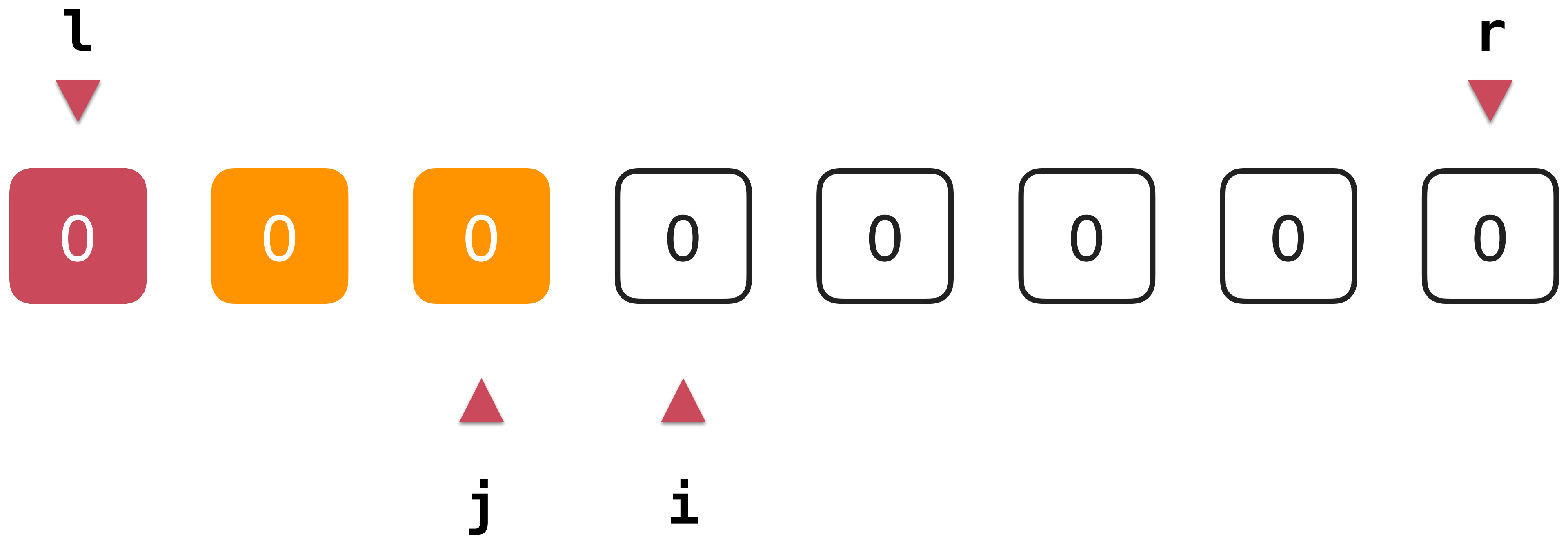


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# 解决方案

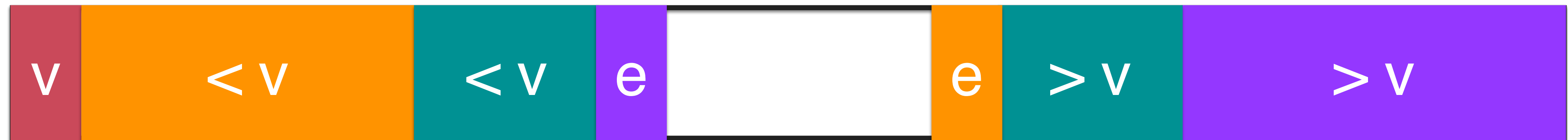
双路快速排序法

# 双路快速排序法

# Partition2

$\text{arr}[l+1 \dots i-1] < v$

$\text{arr}[j+1 \dots r] > v$



$l$



$i$



$j$



$r$

$e \geq v$

$e \leq v$

# Partition2

$\text{arr}[l+1 \dots i-1] < v$

$\text{arr}[j+1 \dots r] > v$



$l$



$i$



$j$



$r$

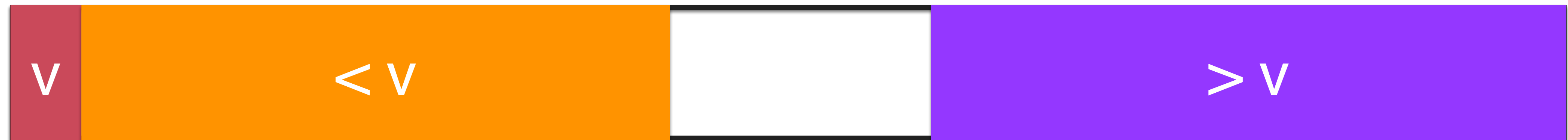
$e \geq v$

$e \leq v$

# Partition2

$\text{arr}[l+1 \dots i-1] < v$

$\text{arr}[j+1 \dots r] > v$



**$l$**



**$i$**



**$j$**



**$r$**

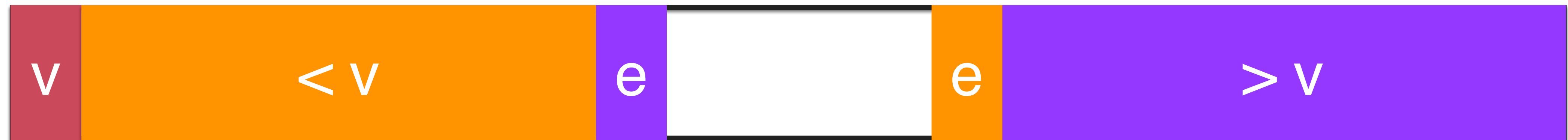
**$e \geq v$**

**$e \leq v$**

# Partition2

$\text{arr}[l+1 \dots i-1] < v$

$\text{arr}[j+1 \dots r] > v$



***l***



***i***



***j***



***r***

***e*  $\geq v$**

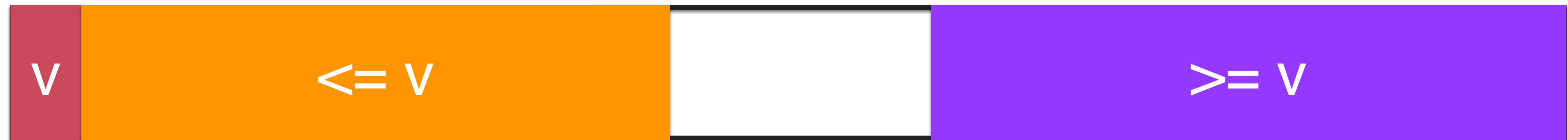
***e*  $\leq v$**



# Partition2

$\text{arr}[l+1 \dots i-1] \leq v$

$\text{arr}[j+1 \dots r] \geq v$



$l$



$i$



$j$

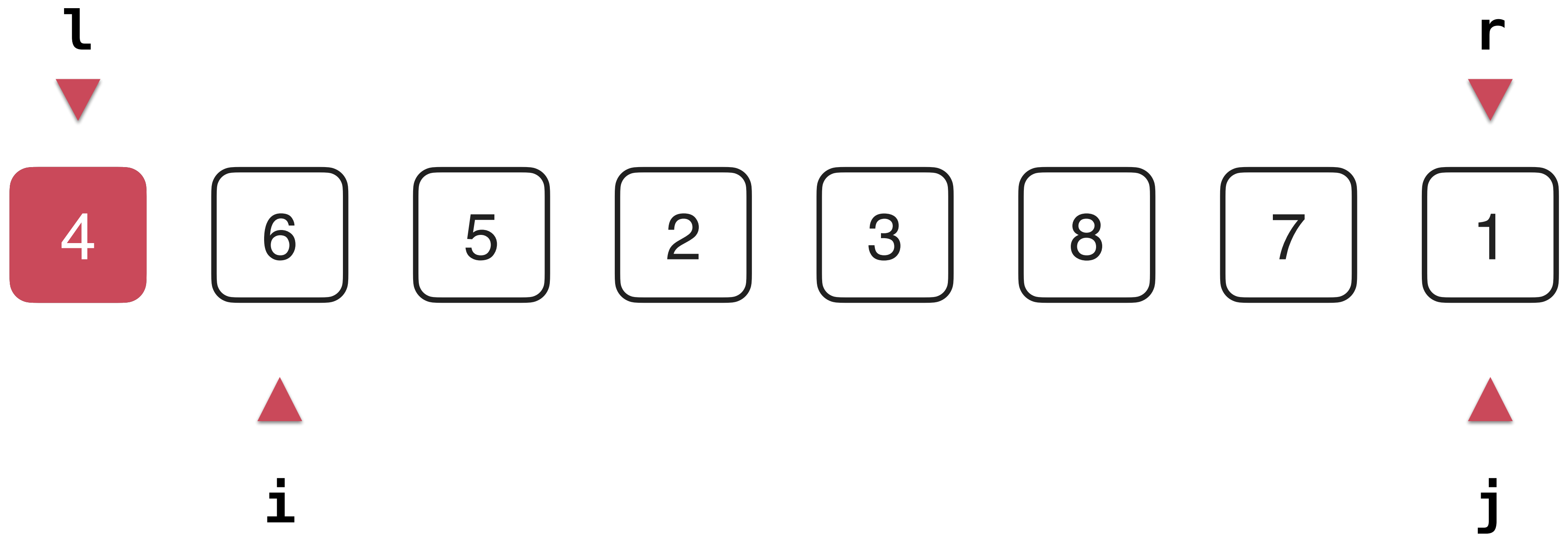


$r$

$e \geq v$

$e \leq v$

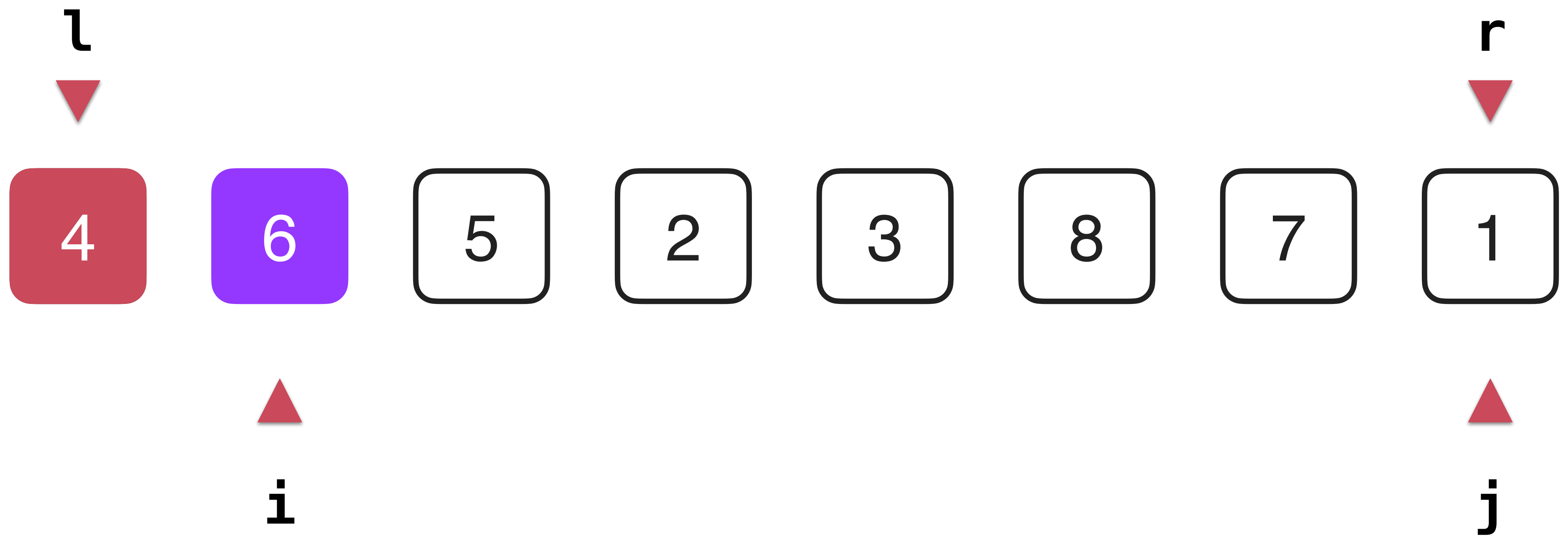
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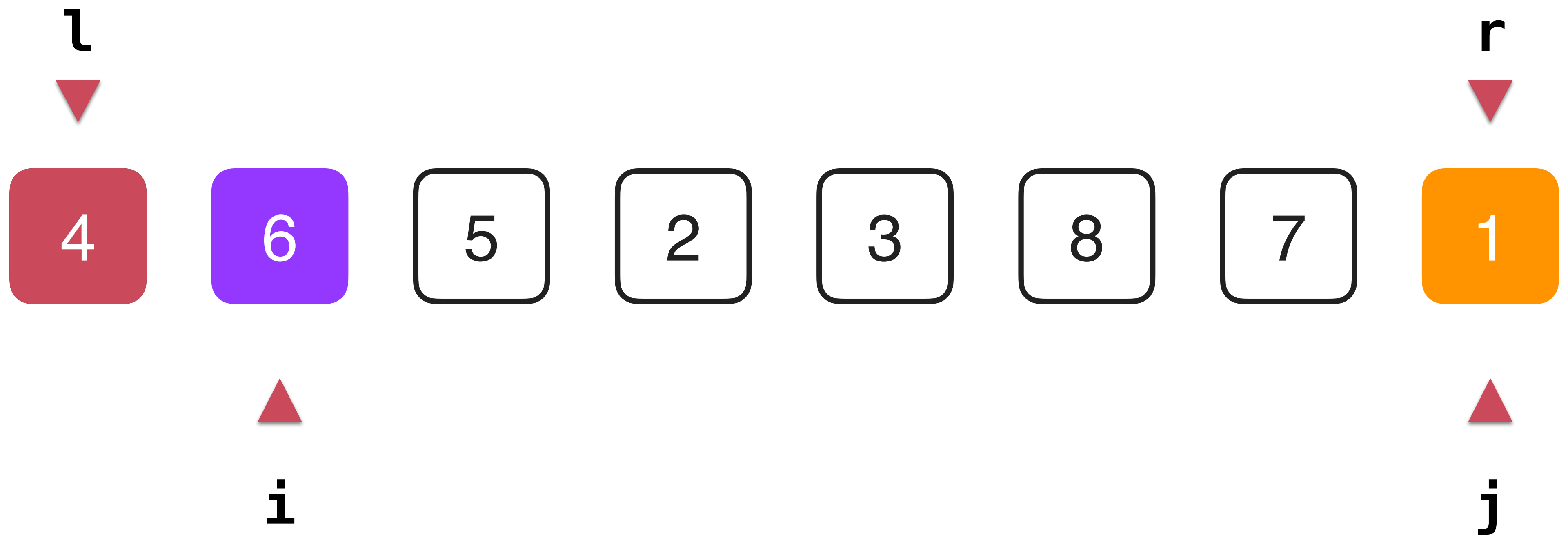
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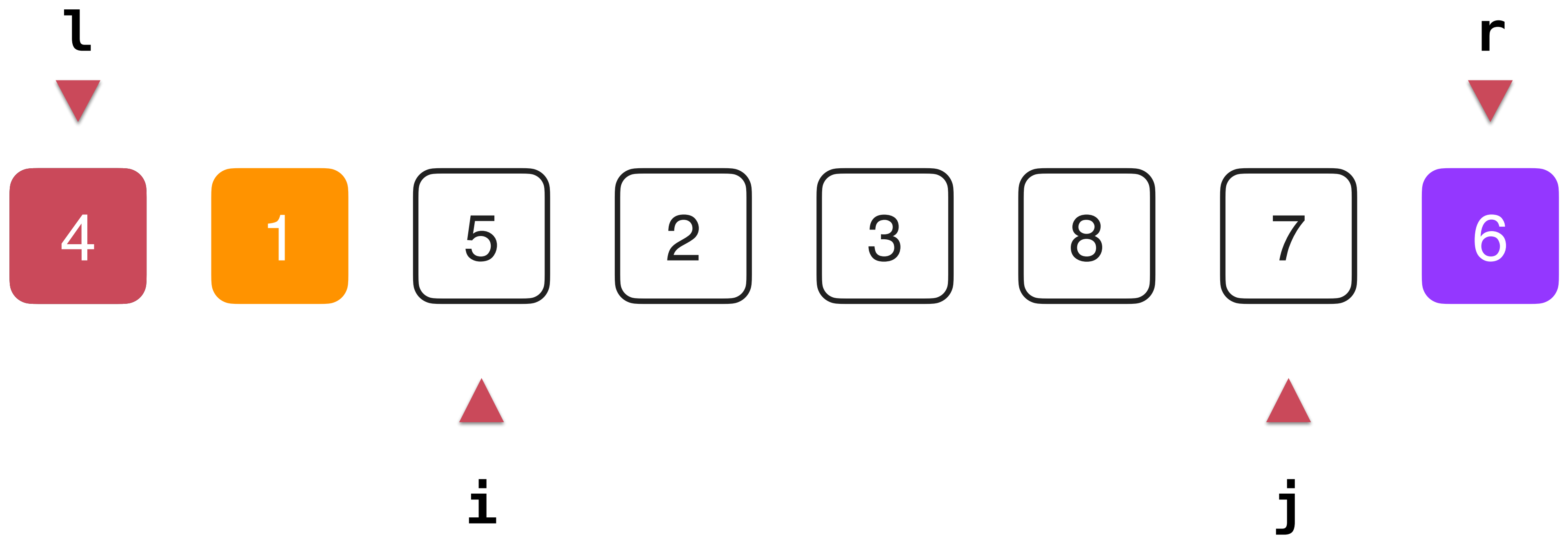
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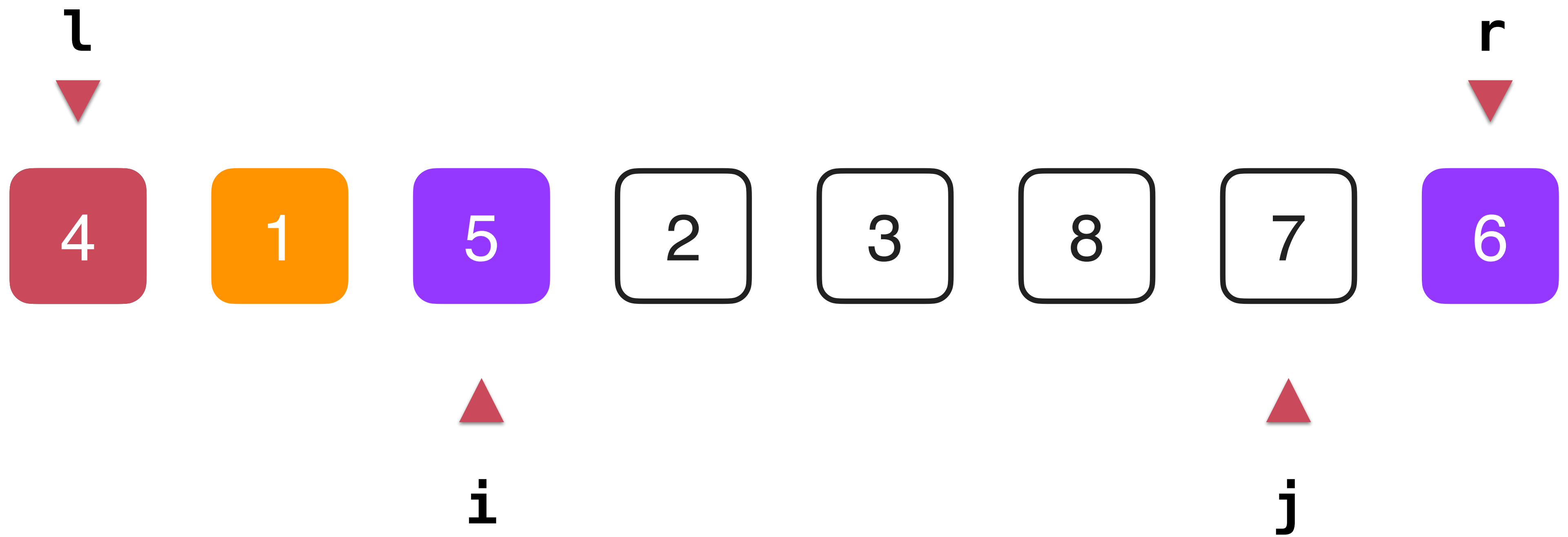
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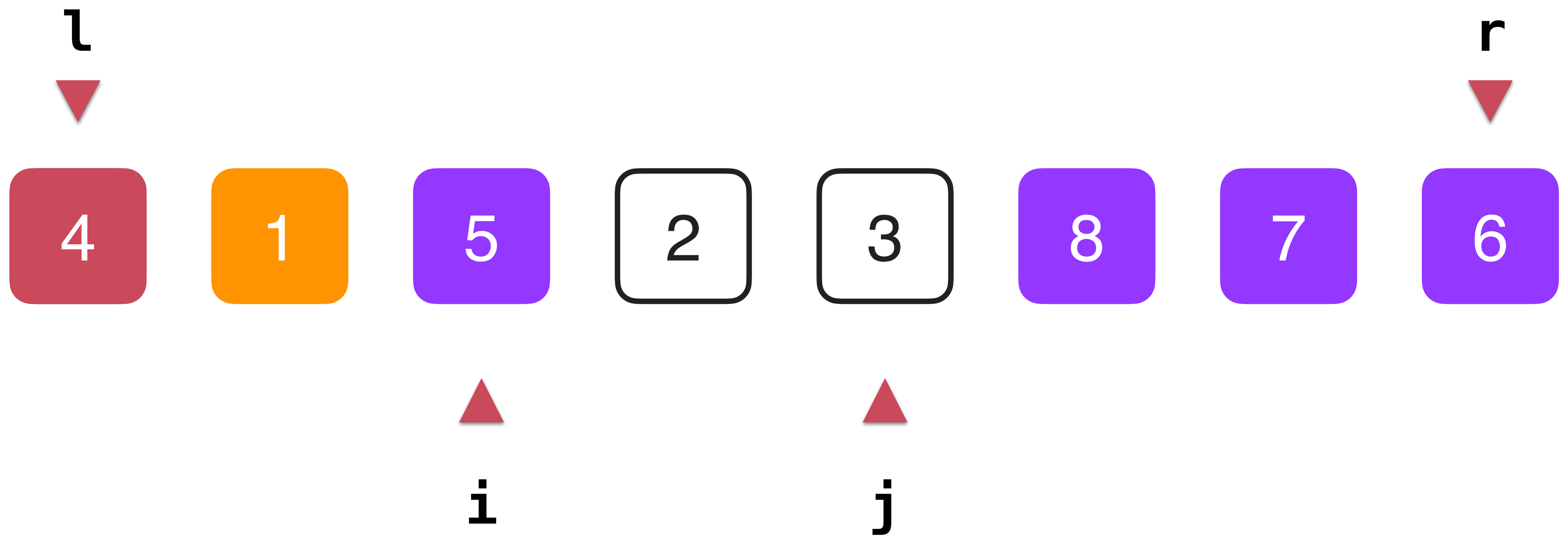
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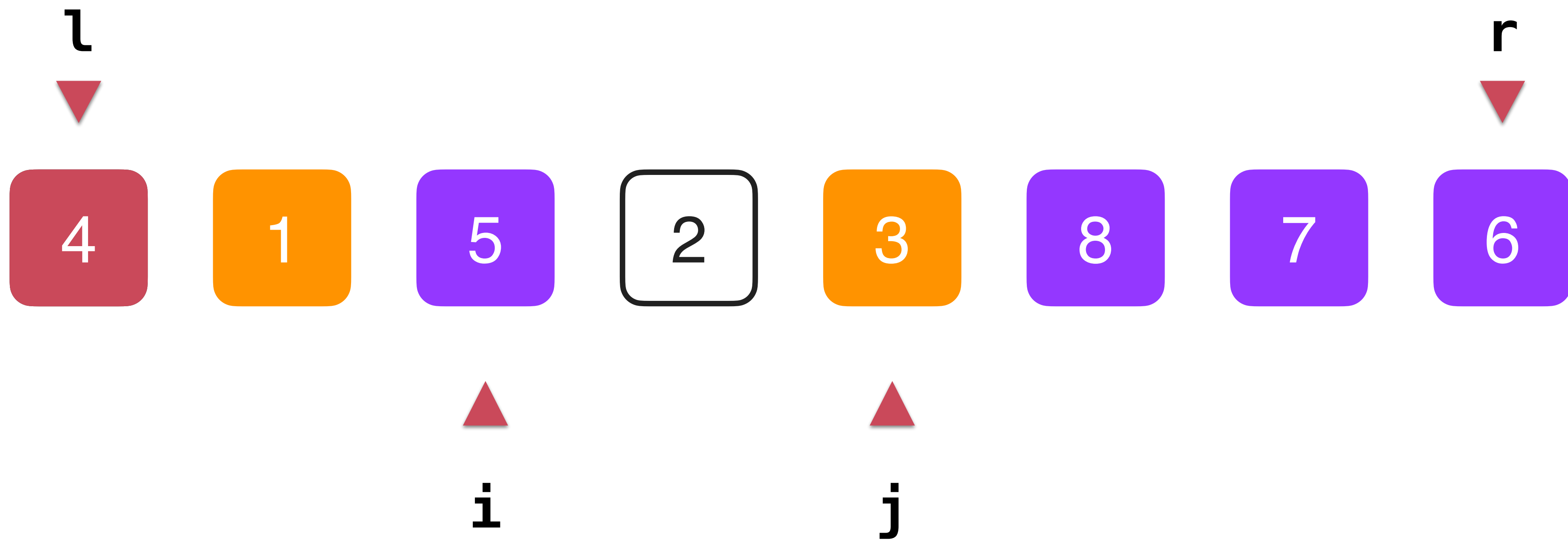
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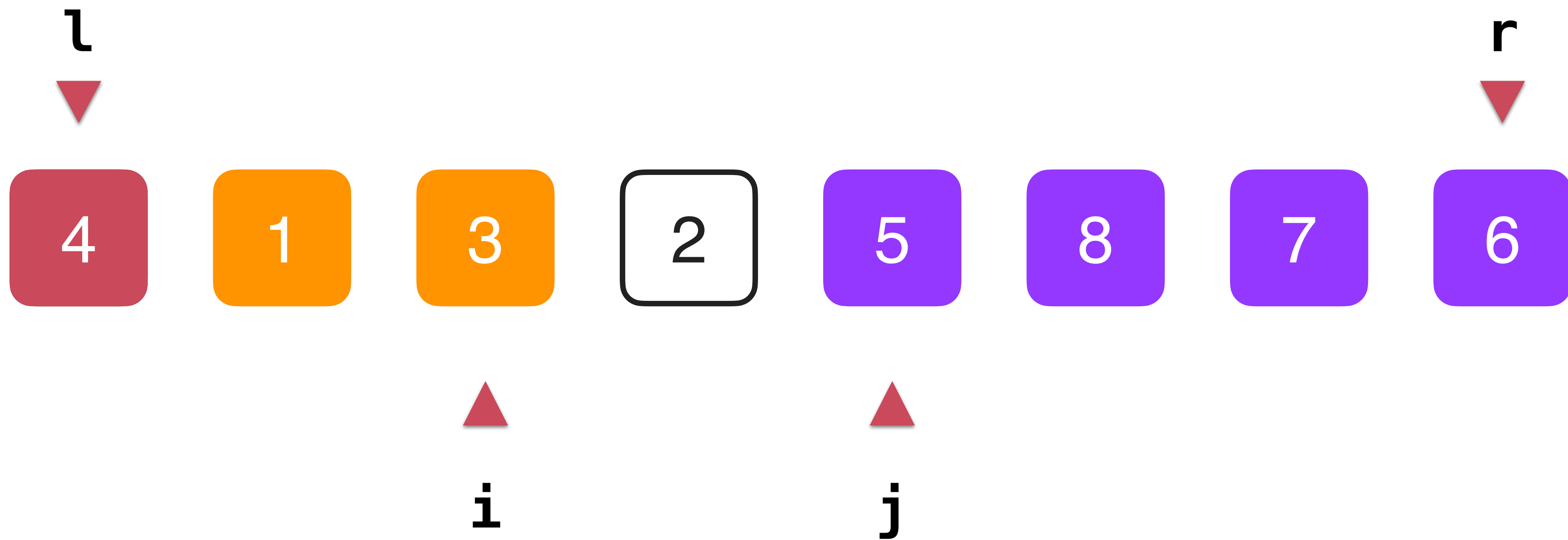


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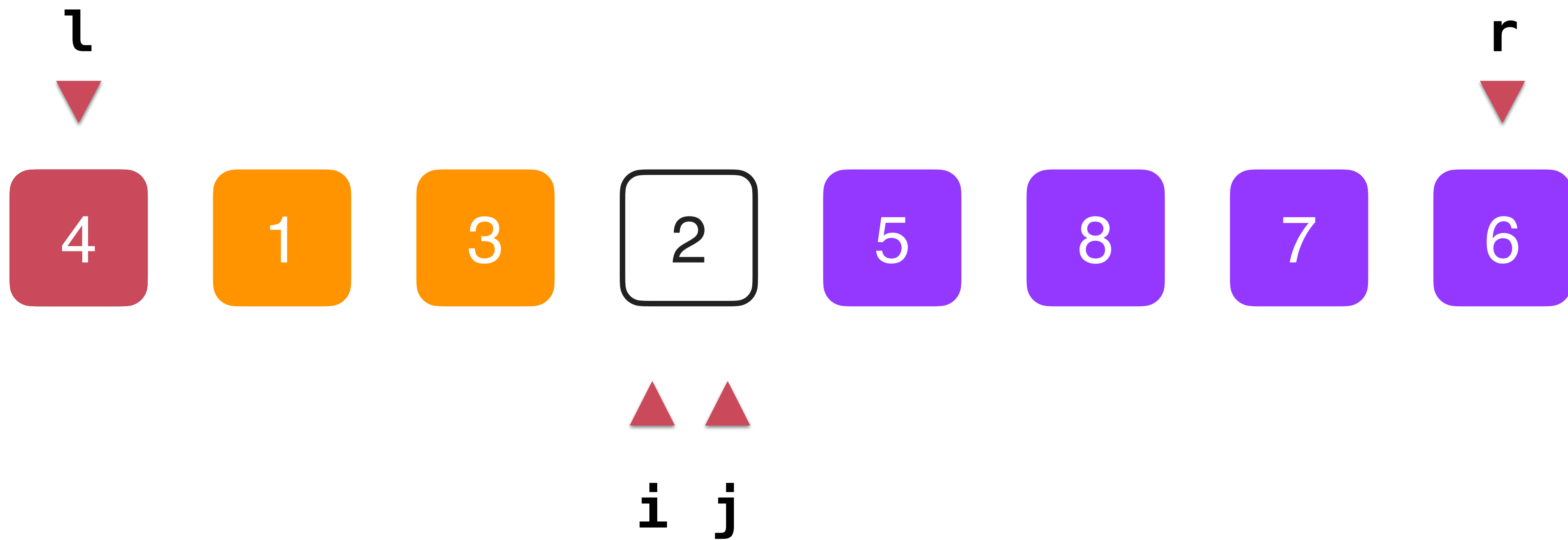
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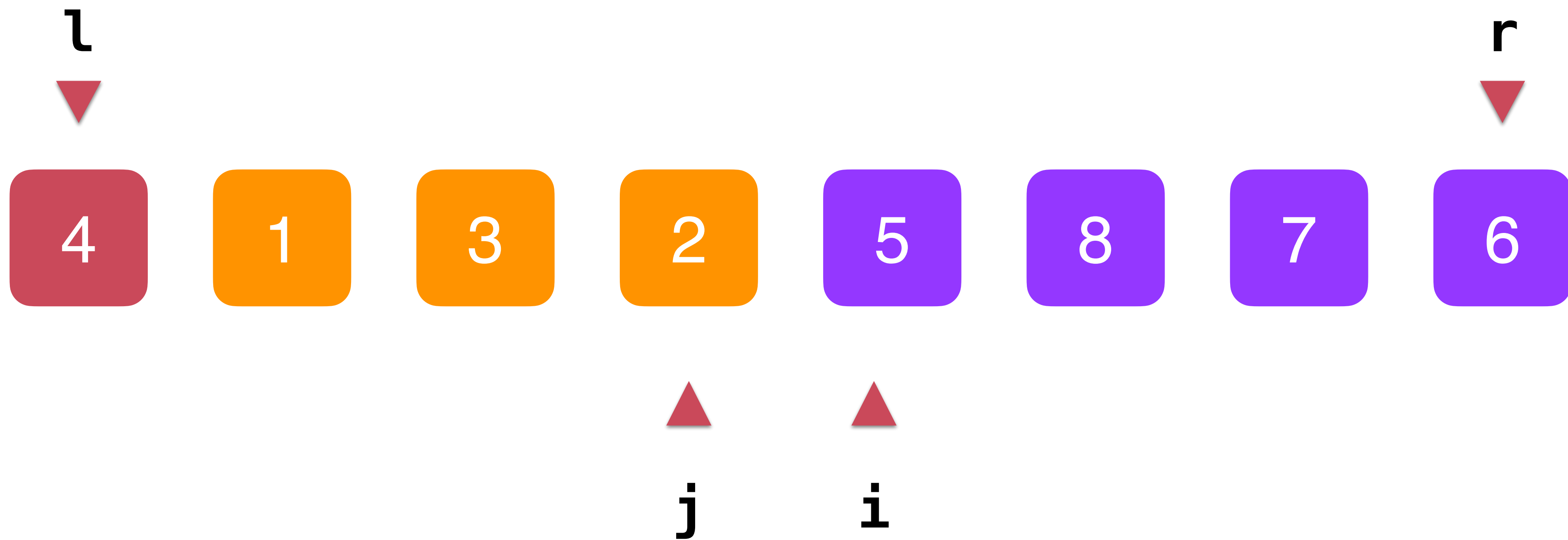
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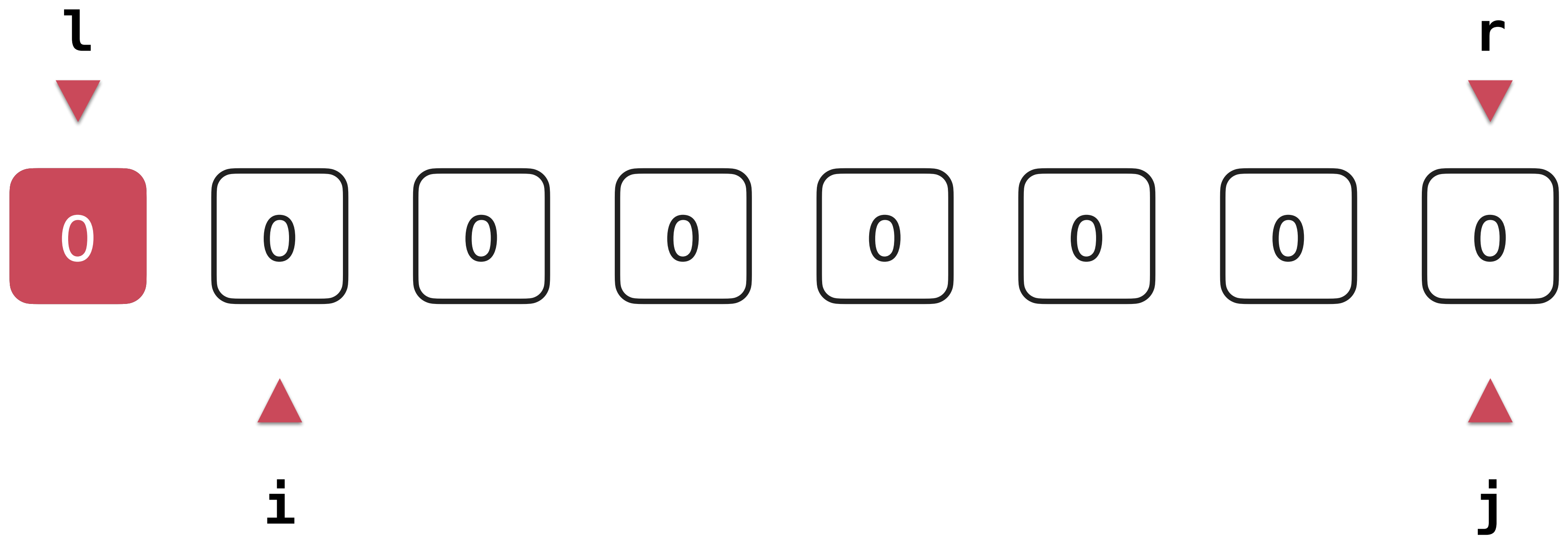
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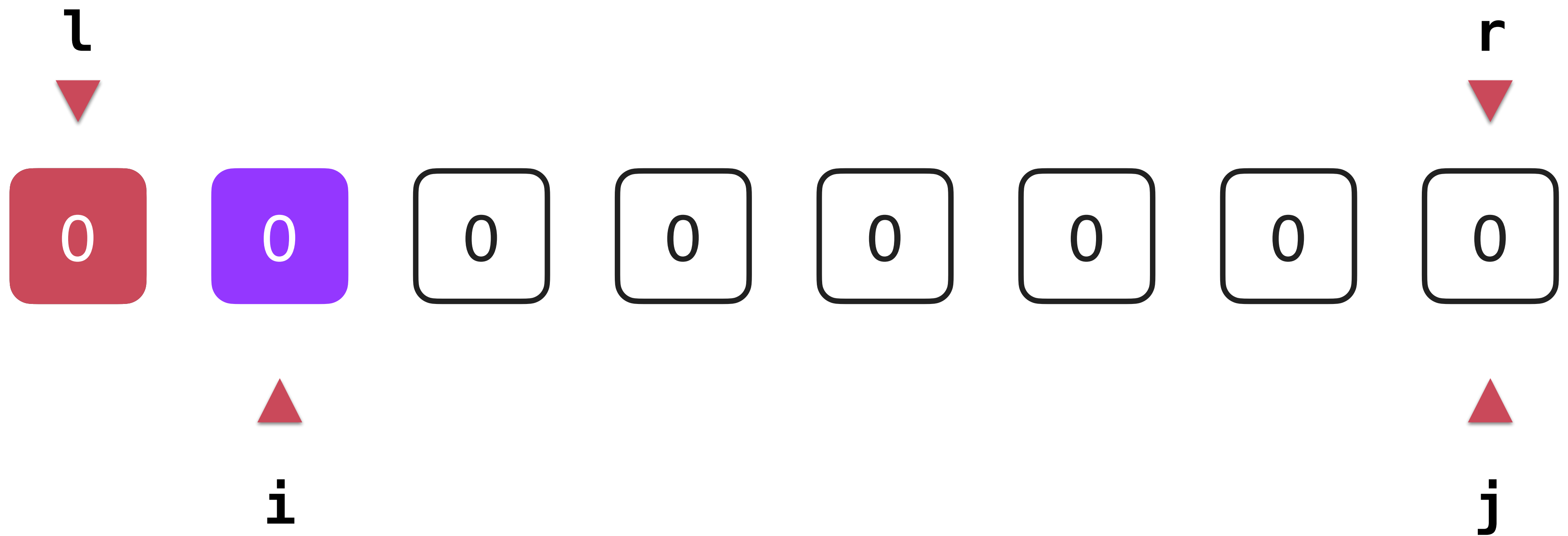
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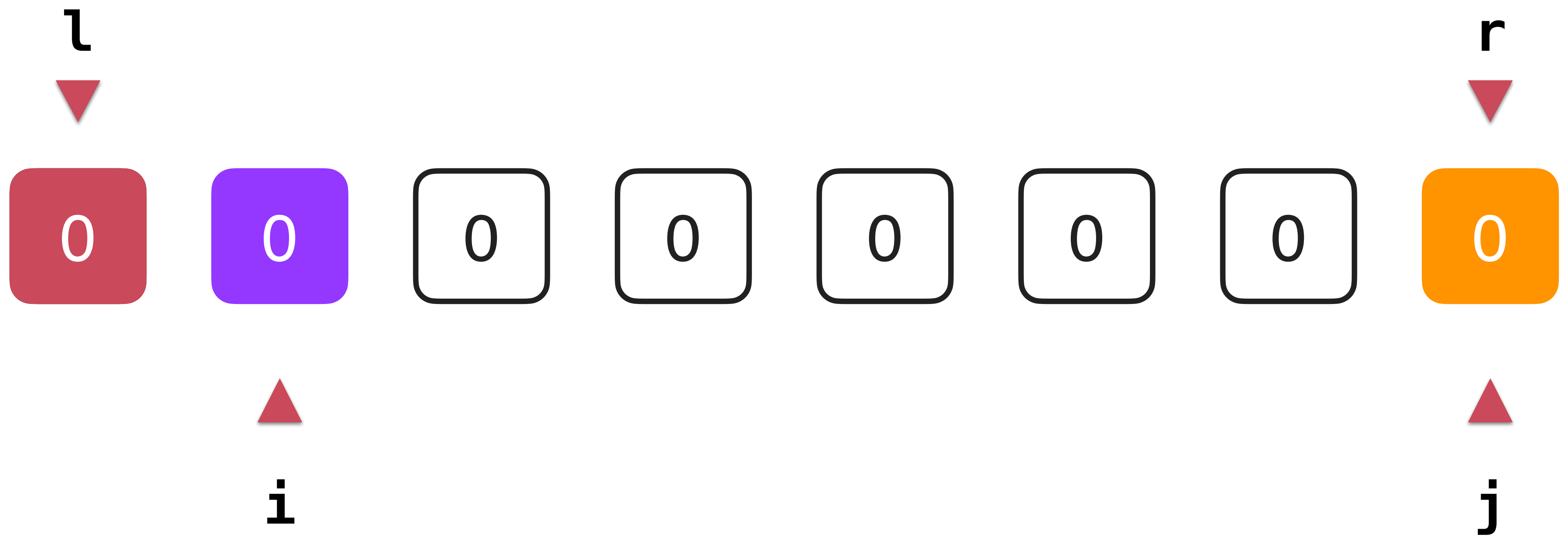
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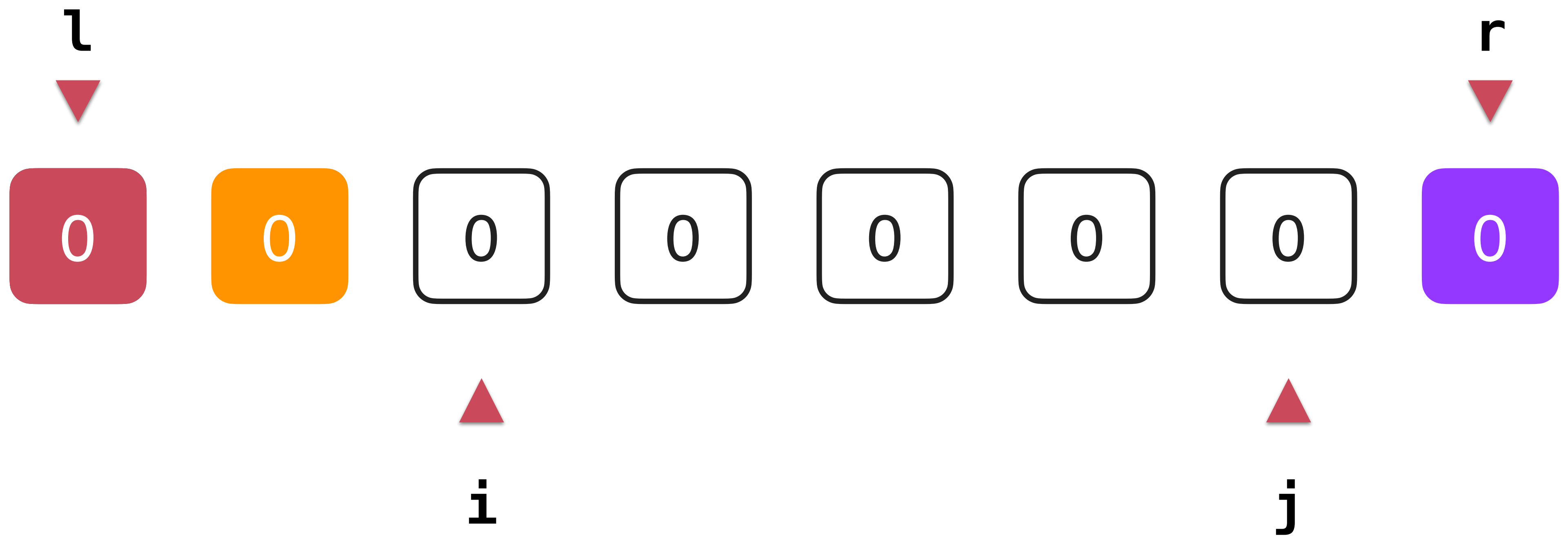
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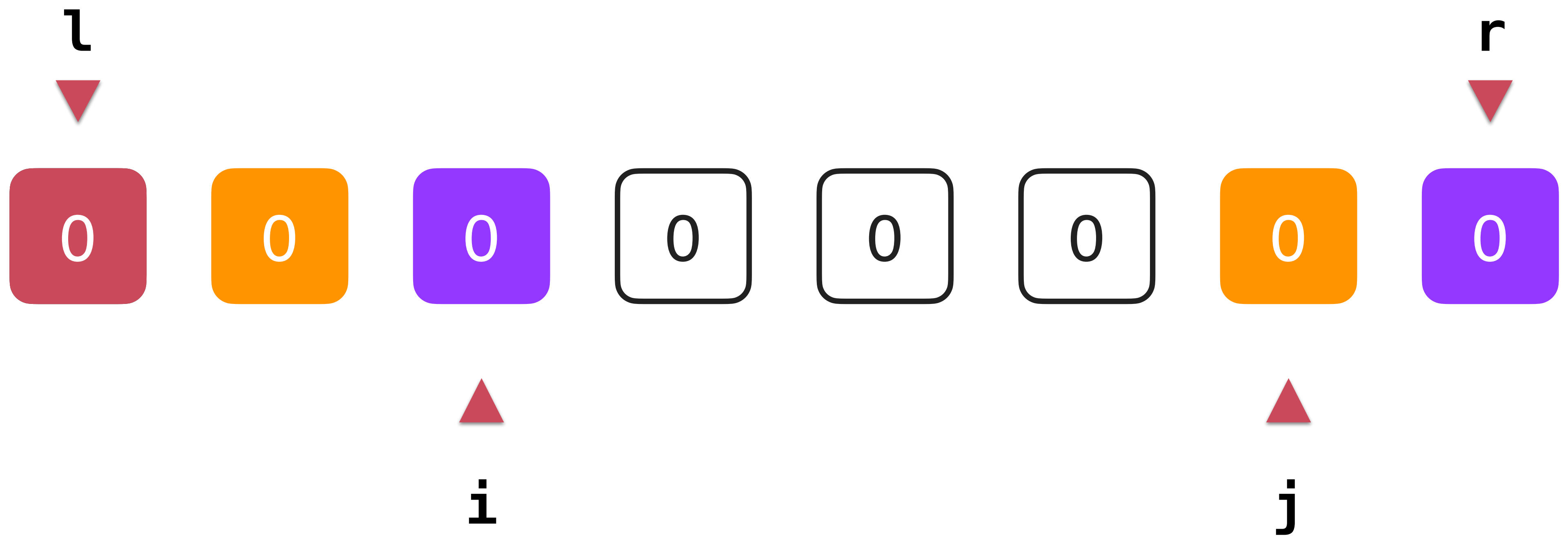
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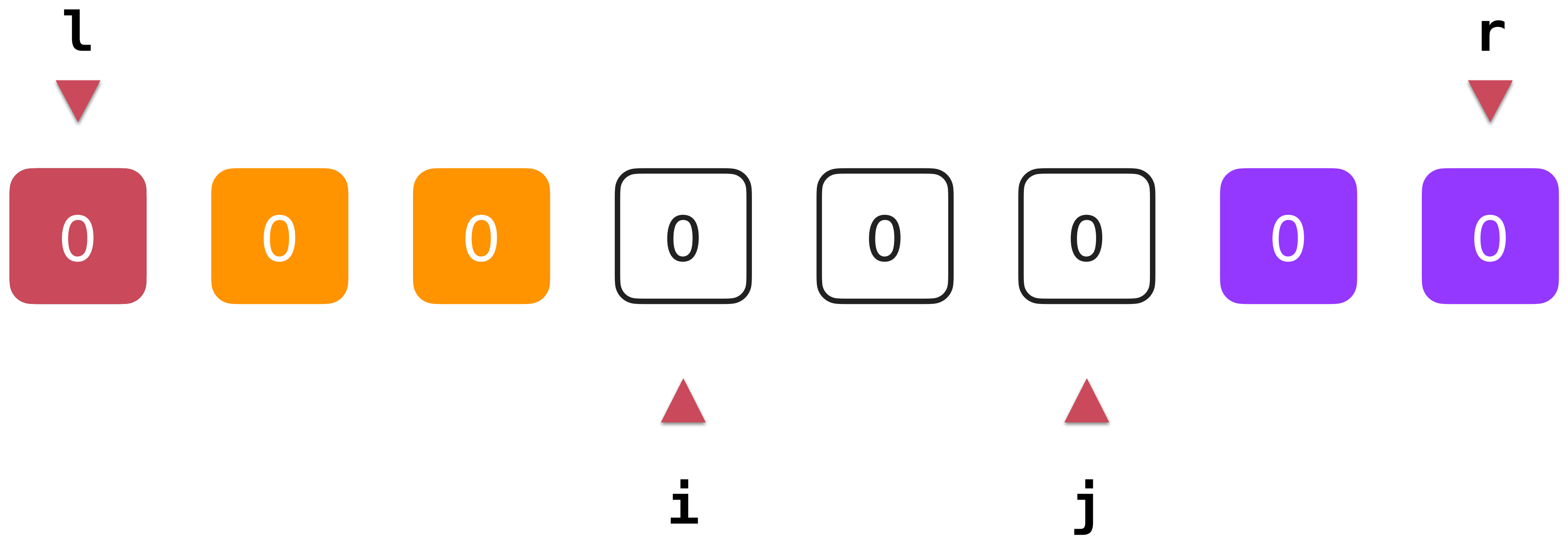


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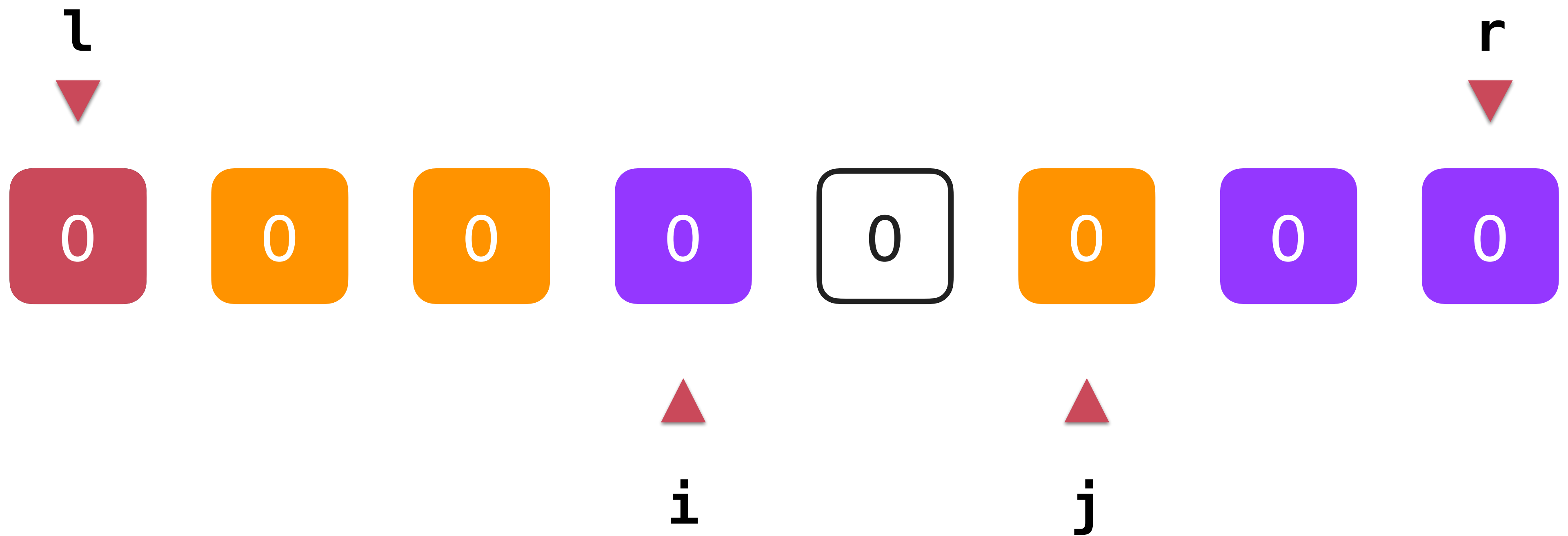
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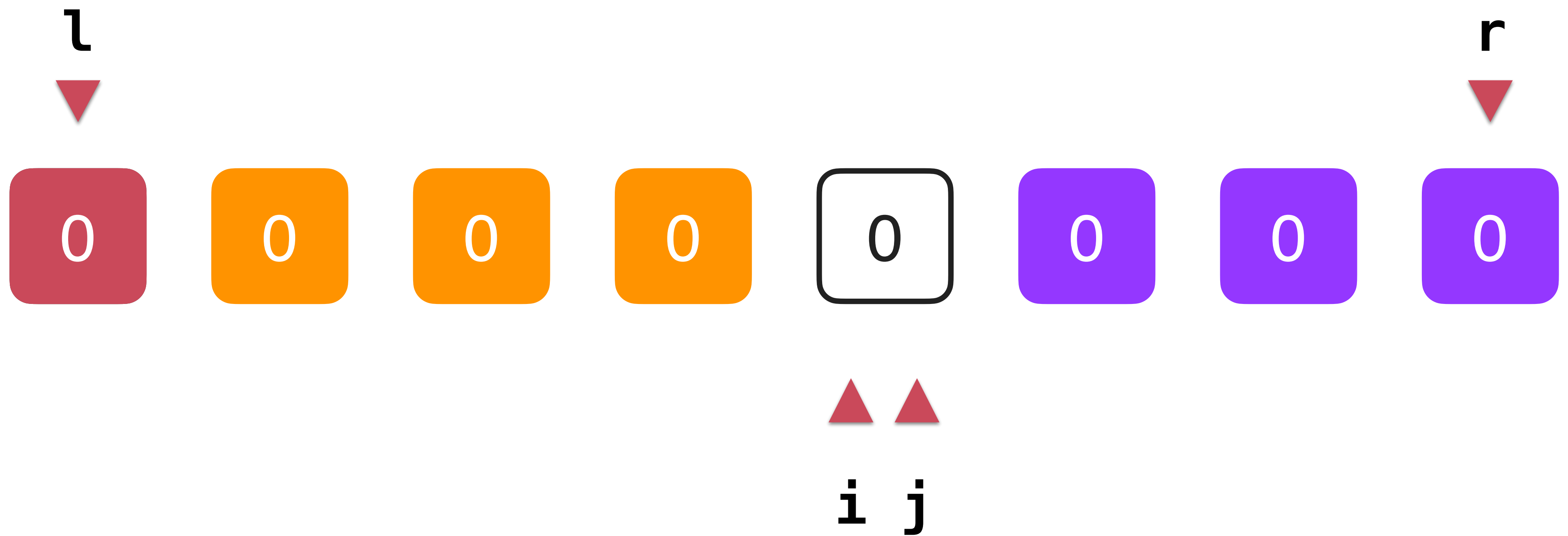
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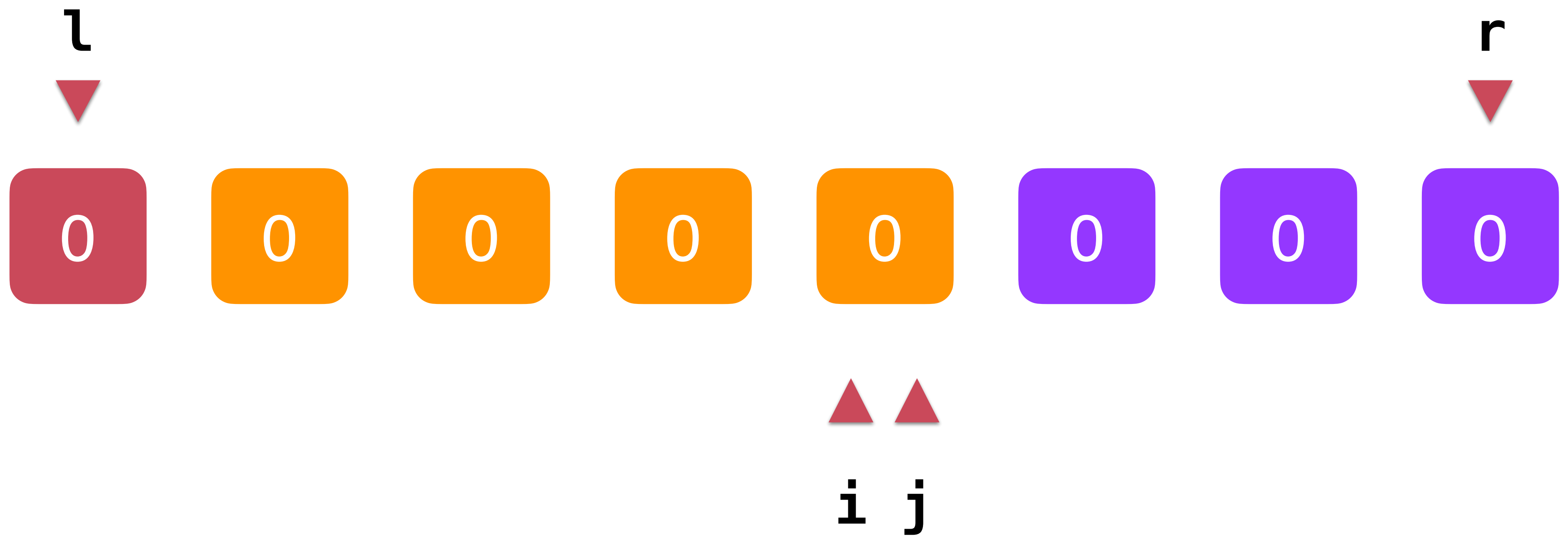
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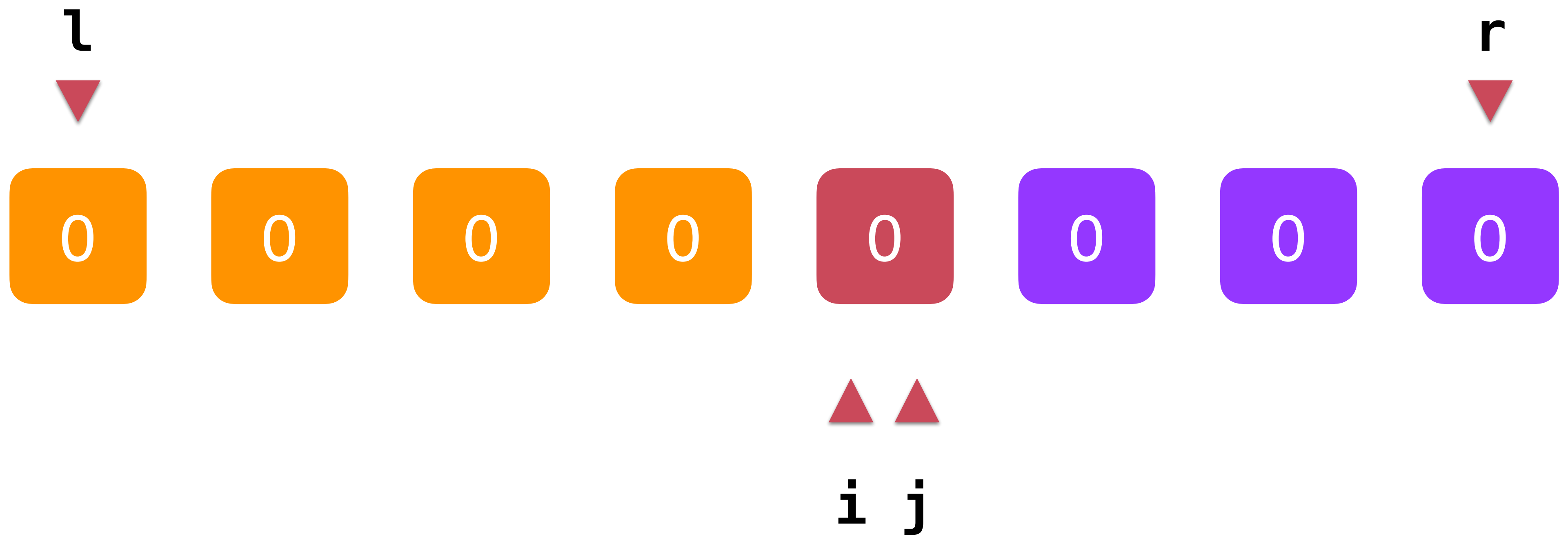
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# 实现双路快速排序法

# 实践：双路快速排序法

# 快速排序的复杂度分析



# 快速排序的复杂度分析

一共  $\log n$  层

每层总共是  $O(n)$

$O(n \log n)$

arr[0...7]

递归归并排序法

递归树

arr[0...3]

arr[4...7]

arr[0...1]

arr[2...3]

arr[4...5]

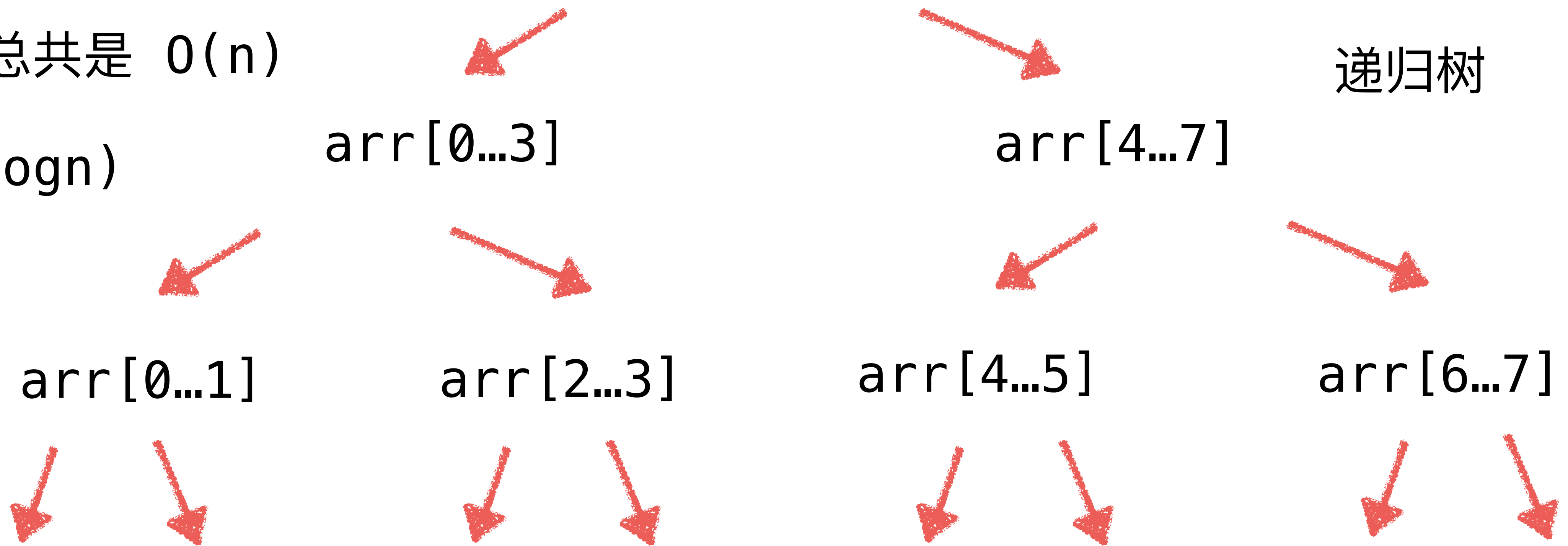
arr[6...7]

arr[0] arr[1]

arr[2] arr[3]

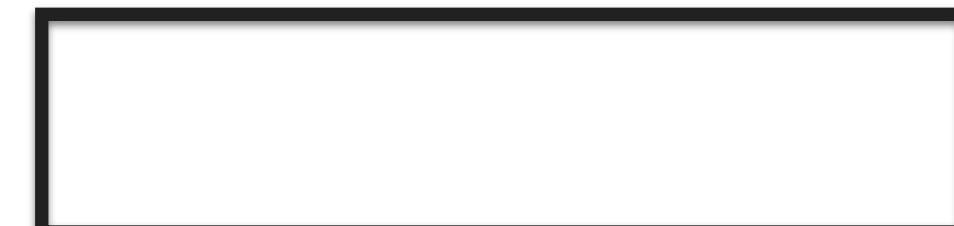
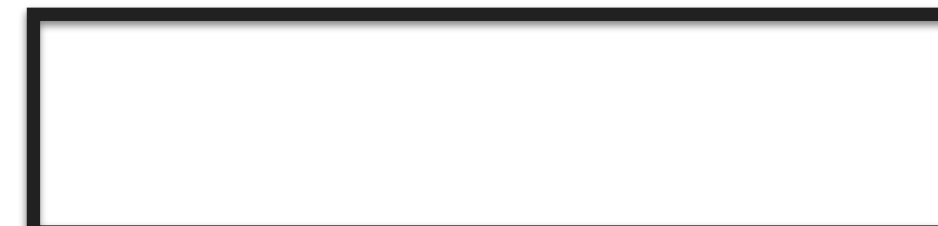
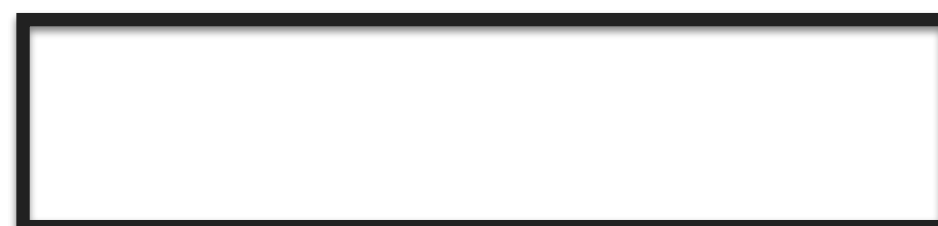
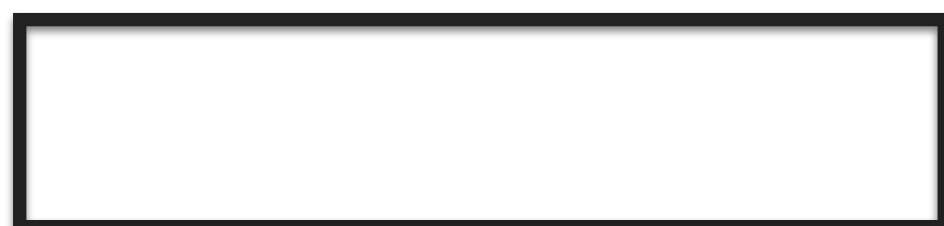
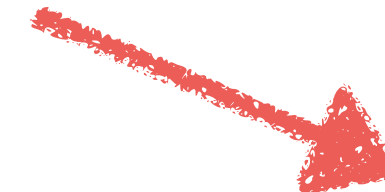
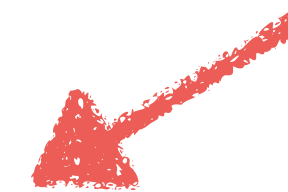
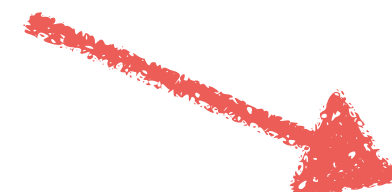
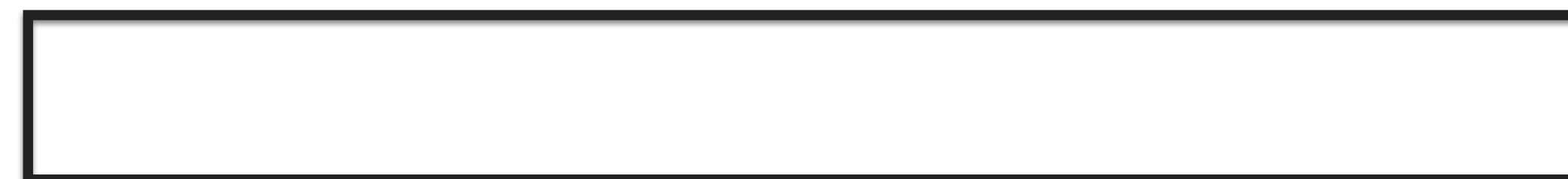
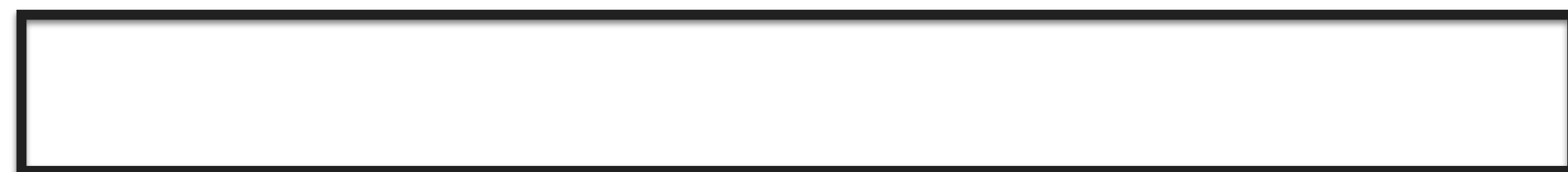
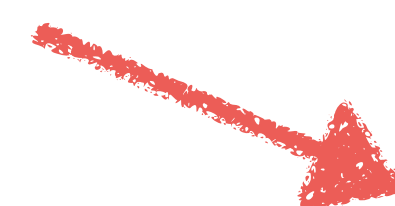
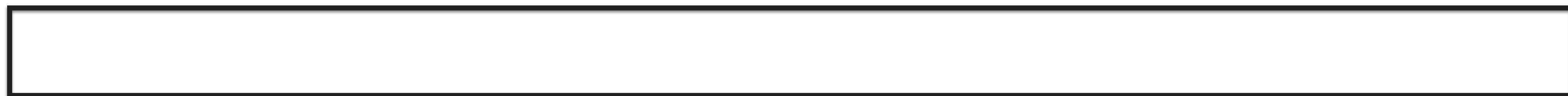
arr[4] arr[5]

arr[6] arr[7]



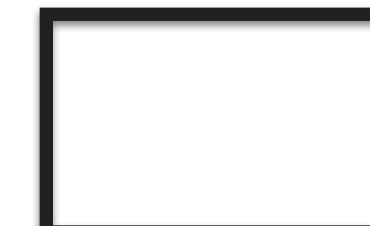
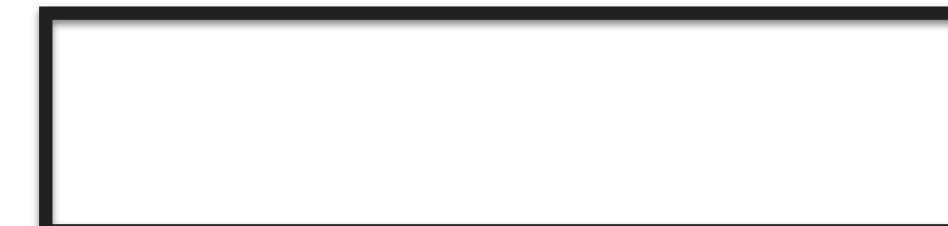
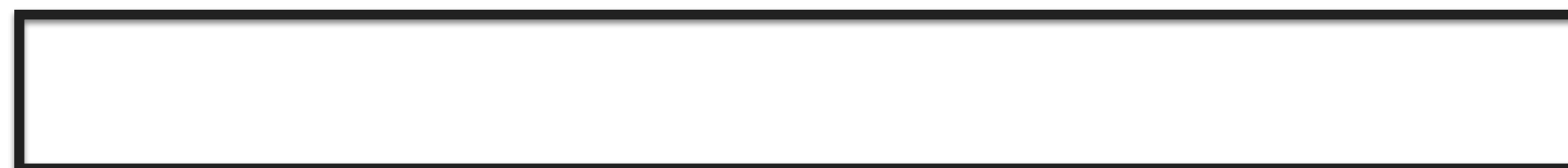
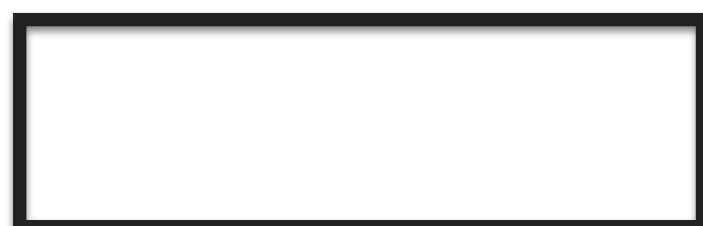
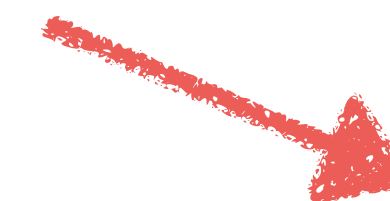
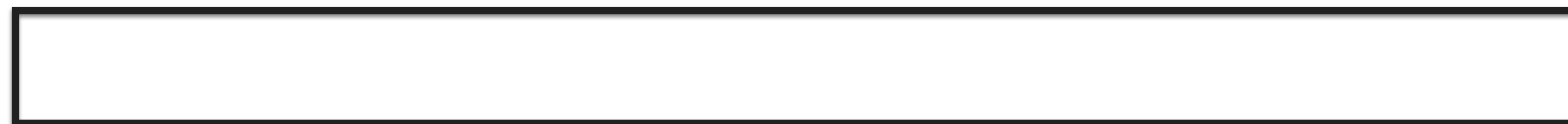
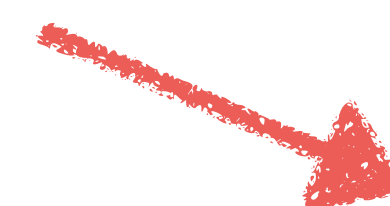
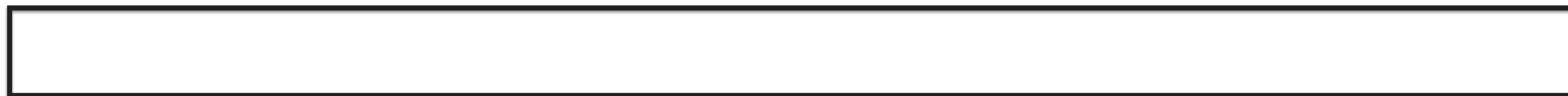
# 快速排序的复杂度分析

归并排序法



# 快速排序的复杂度分析

快速排序法



# 快速排序的复杂度分析

快速排序法

最坏复杂度

$O(n^2)$

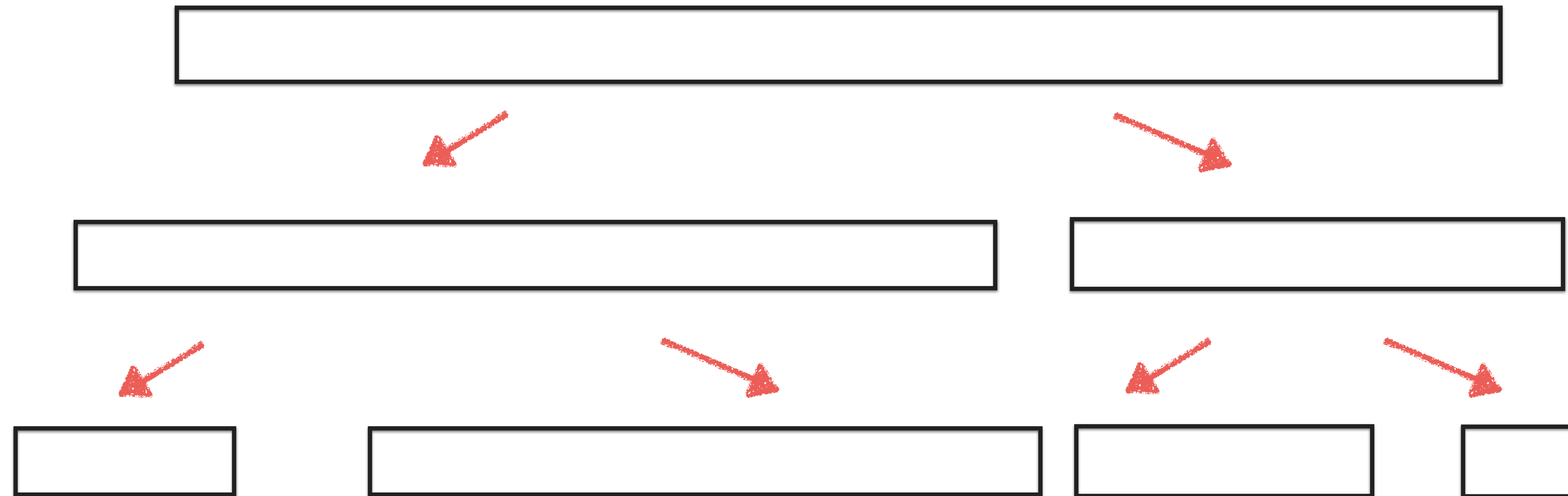
概率非常低

是一个随机算法

使用期望

# 快速排序的复杂度分析

数学期望的角度看：平分    层数的期望值： $O(\log n)$     复杂度期望值： $O(n \log n)$



# 快速排序的复杂度分析

更严谨的数学推导：参考《算法导论》

# 快速排序的复杂度分析

普通算法：看最差                      能找到一组数据 100% 恶化

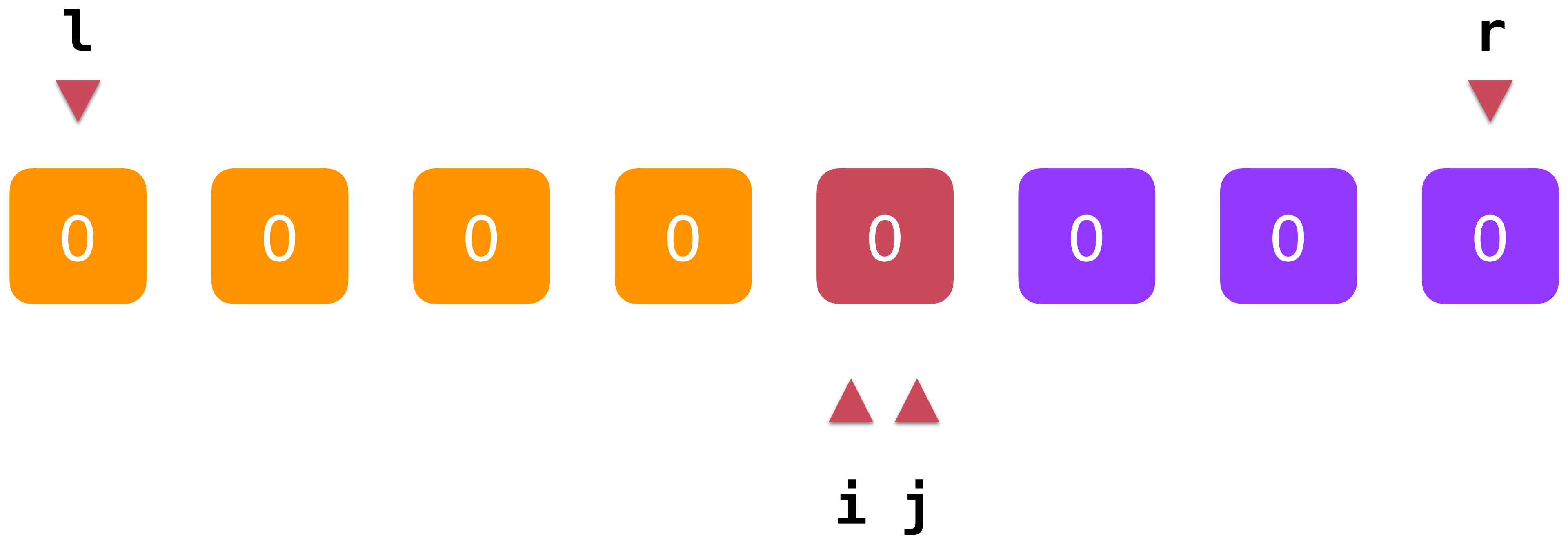
随机算法：看期望                      没有一组数据能 100% 恶化

多次调用？ 尝试均摊分析

# 三路快速排序算法



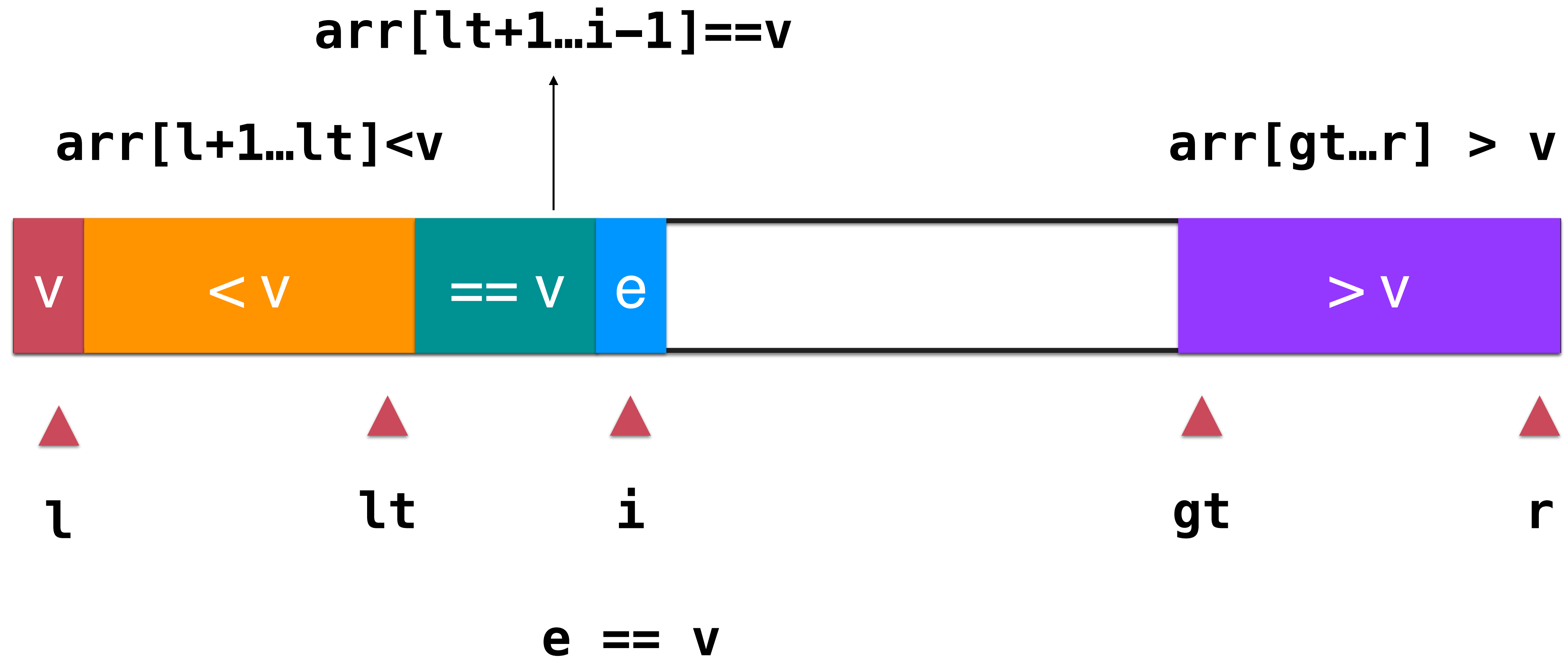
# Partition2



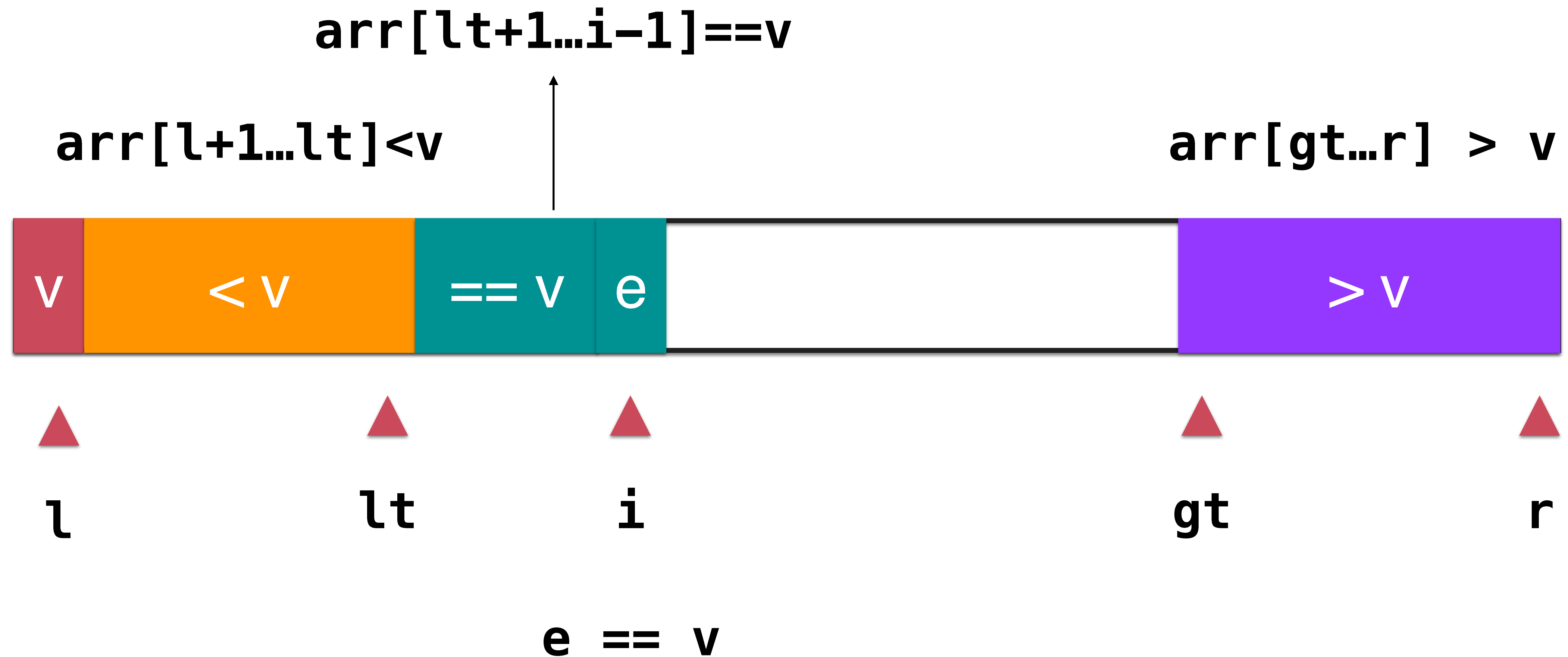
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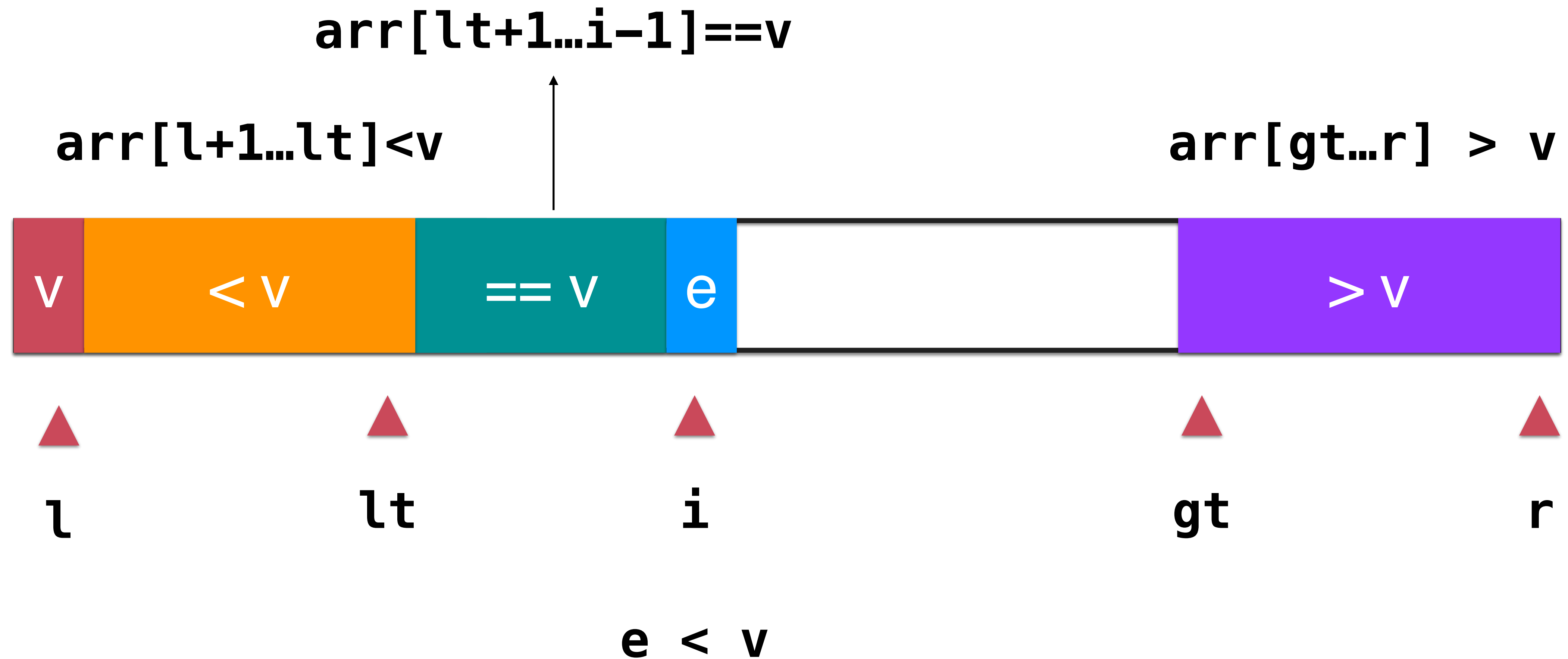
# Quick Sort 3 Ways



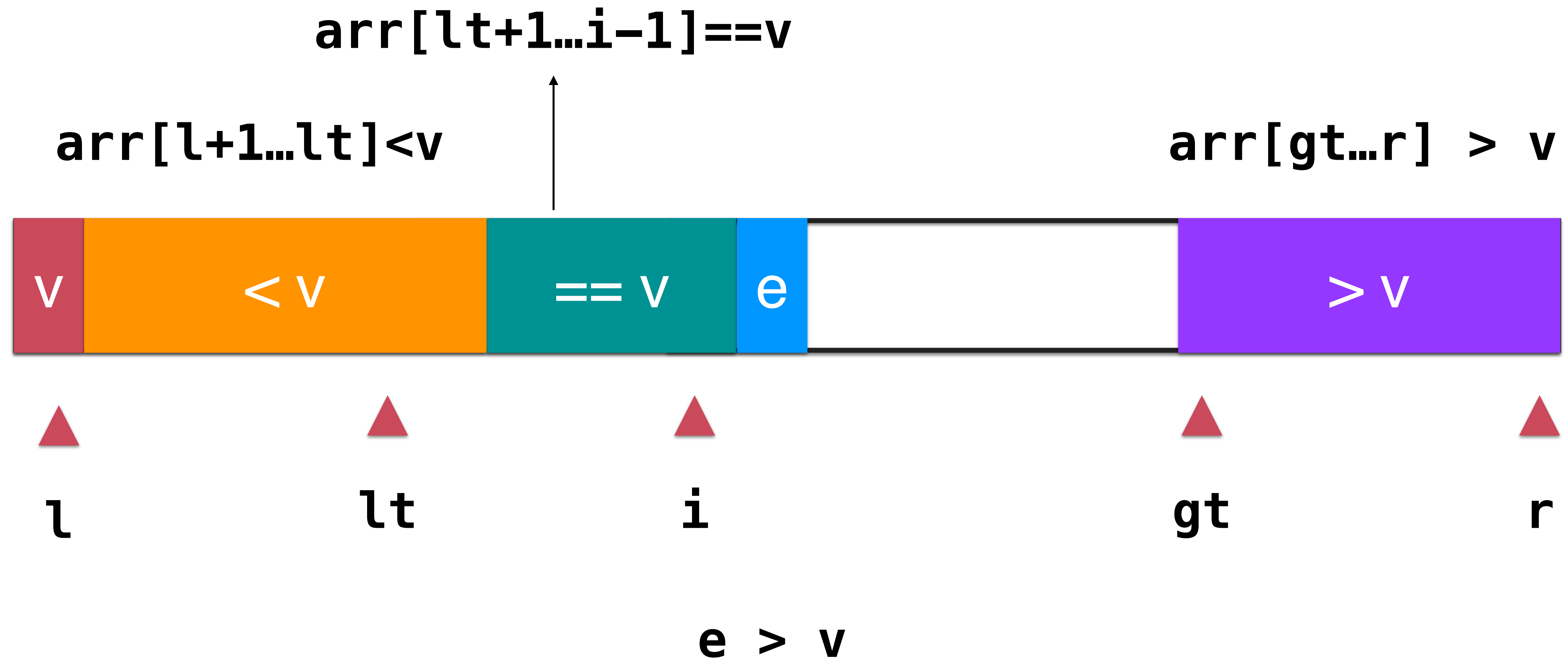
# Quick Sort 3 Ways



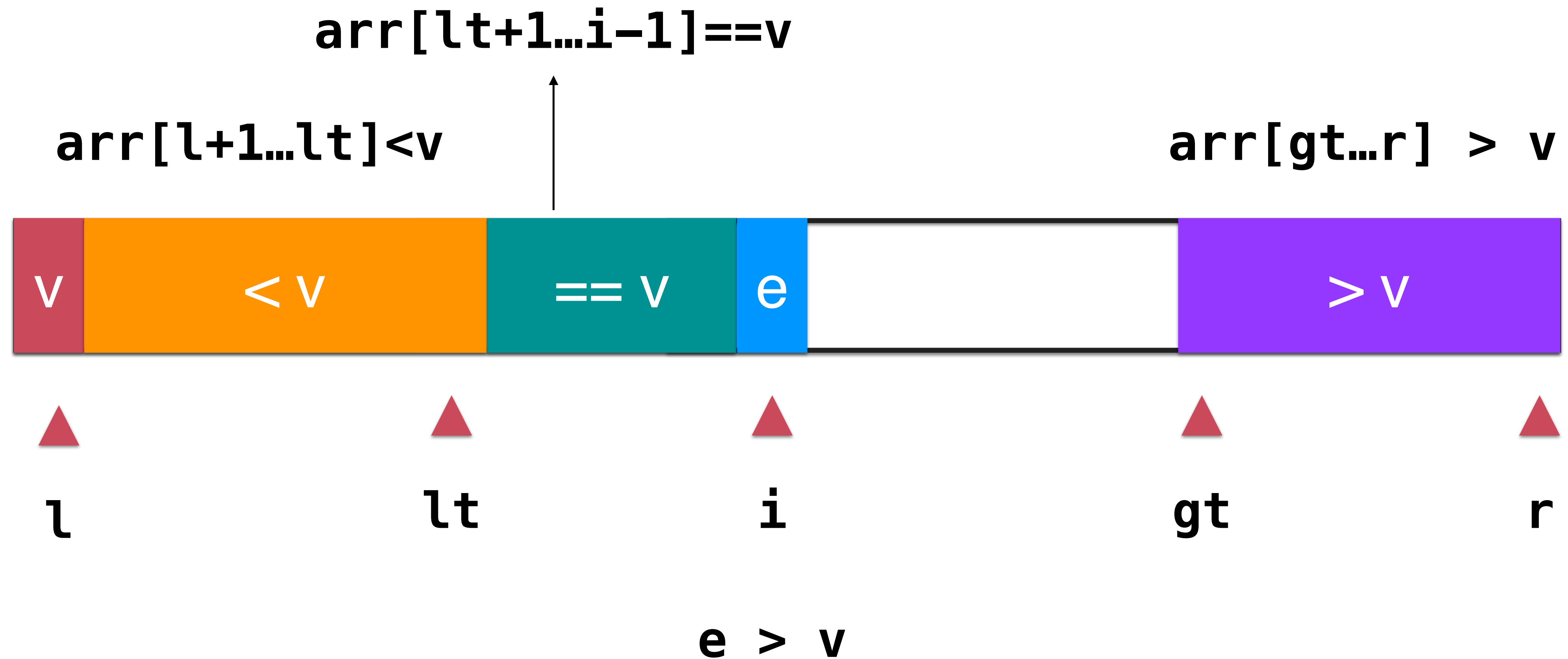
# Quick Sort 3 Ways



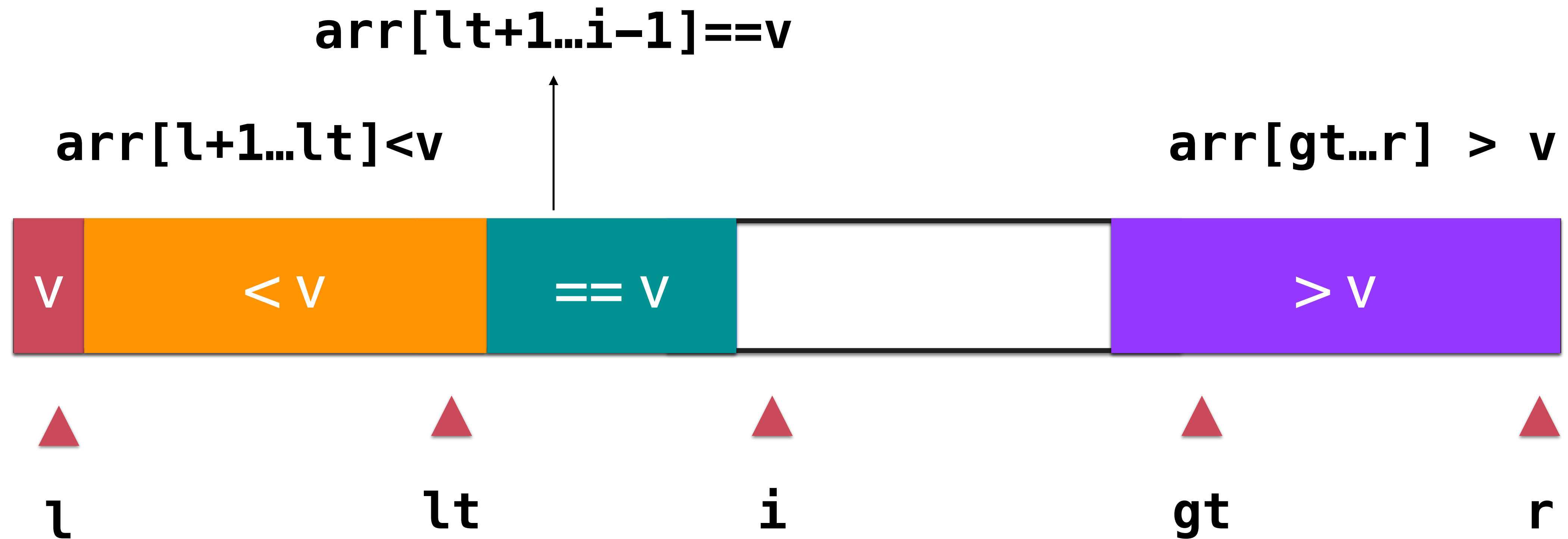
# Quick Sort 3 Ways



# Quick Sort 3 Ways

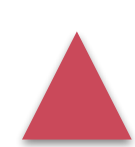


# Quick Sort 3 Ways



# Quick Sort 3 Ways

$\text{arr}[l+1 \dots lt] < v$     $\text{arr}[lt+1 \dots i-1] == v$     $\text{arr}[gt \dots r] > v$



$l$



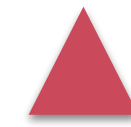
$lt$



$gt$



$r$



$i$

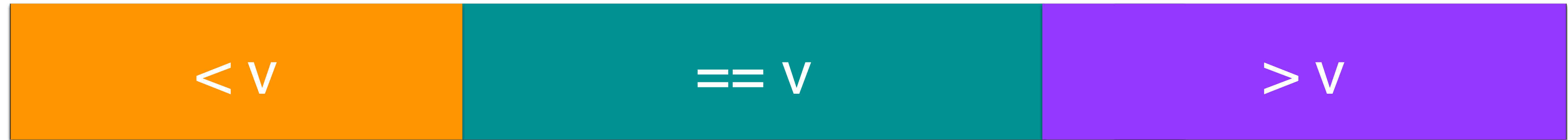


# Quick Sort 3 Ways

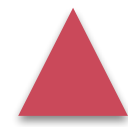
$\text{arr}[l \dots lt-1] < v$

$\text{arr}[lt \dots gt-1] == v$

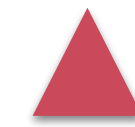
$\text{arr}[gt \dots r] > v$



$l$



$lt$



$gt$



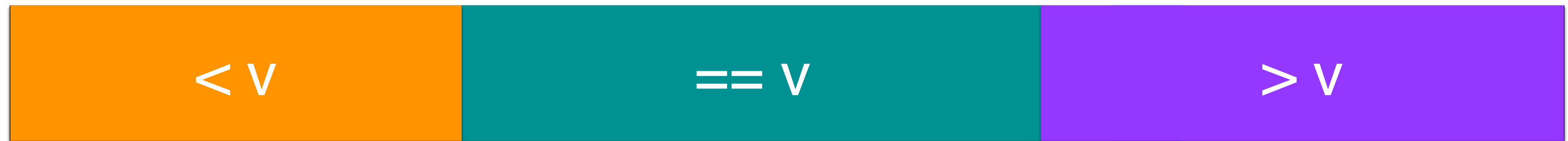
$r$

$\text{sort}(l, lt - 1)$

$\text{sort}(gt, r)$

# Quick Sort 3 Ways

$\text{arr}[l \dots lt-1] < v$        $\text{arr}[lt \dots gt-1] == v$        $\text{arr}[gt \dots r] > v$



$l$



$lt$



$gt$



$r$

所有元素都相同的数组:  $O(n)$

# 实现三路快速排序算法

实践： 实现三路快速排序算法

作业： Sort Colors

# 作业解析： Sort Colors

Select K

# Select K

给出一个无序数组，找出数组的第 K 小的元素

排序；`arr[k - 1]`       $O(n \log n)$

使用快速排序算法的思路，可以在  $O(n)$  时间完成



# Select K

4 6 2 3 1 5 7 8

2 1 3 4 5 8 7 6

< 4

4

> 4

# Select K



**p**

**k == p?**

**找到了!**

**k < p?**

**到左边找**

**k > p?**

**到右边找**

# Select K

< 4

4

> 4

$k == p?$

找到了!

$k < p?$

到左边找

$k > p?$

到右边找

复杂度分析

期望

$$n + n/2 + n/4 + \dots + 1$$

$$= 2n = O(n)$$

# Select K

<https://leetcode-cn.com/problems/zui-xiao-de-kge-shu-lcof/>

<https://leetcode-cn.com/problems/kth-largest-element-in-an-array/>

作业解析： Select K

# 快速排序算法总结

# 快速排序法总结

快速排序整体思想：

$< v$

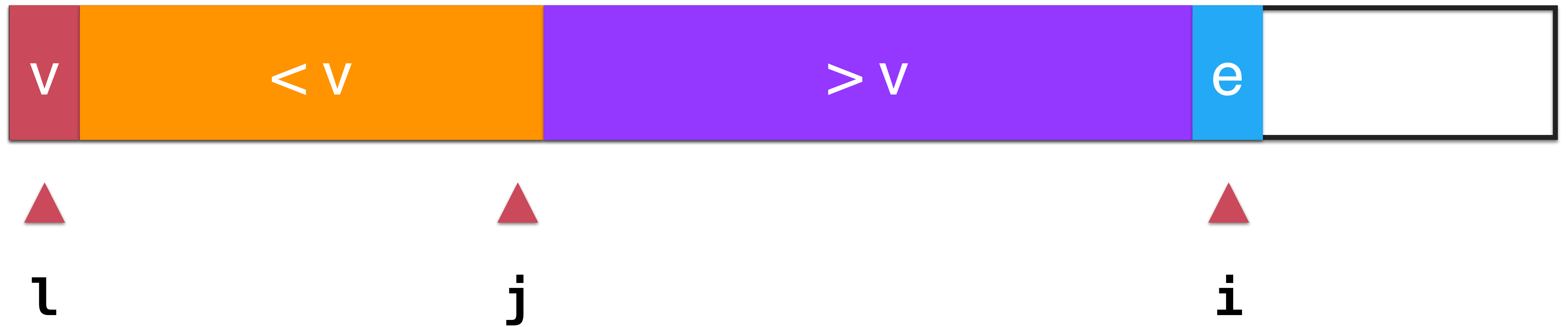
$v$

$> v$

# 快速排序法总结

单路快速排序

$\text{arr}[l+1 \dots j] < v$        $\text{arr}[j+1 \dots i-1] > v$





# 快速排序法总结

单路快速排序

完全有序的数据退化

引入随机化

# 快速排序法总结

引入随机化

所有元素一样的数据退化

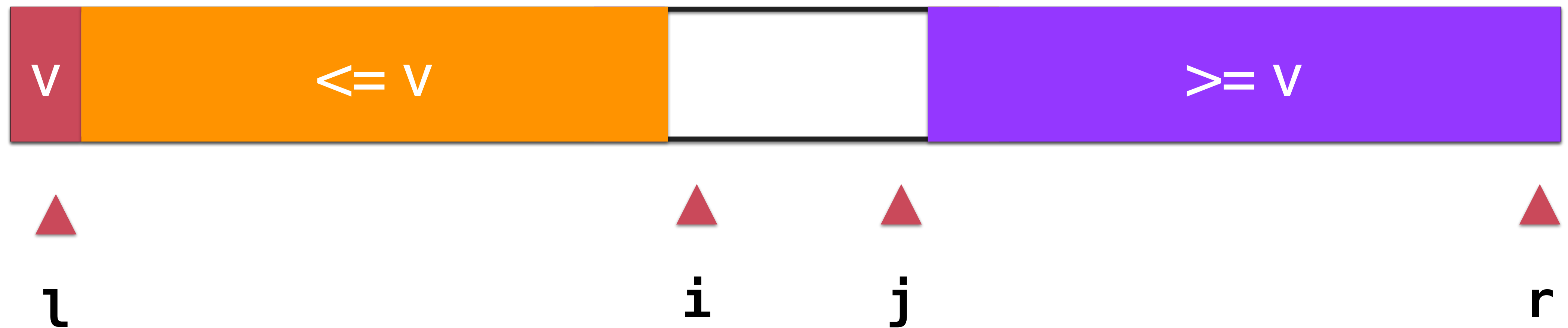
双路快速排序

# 快速排序法总结

## 双路快速排序

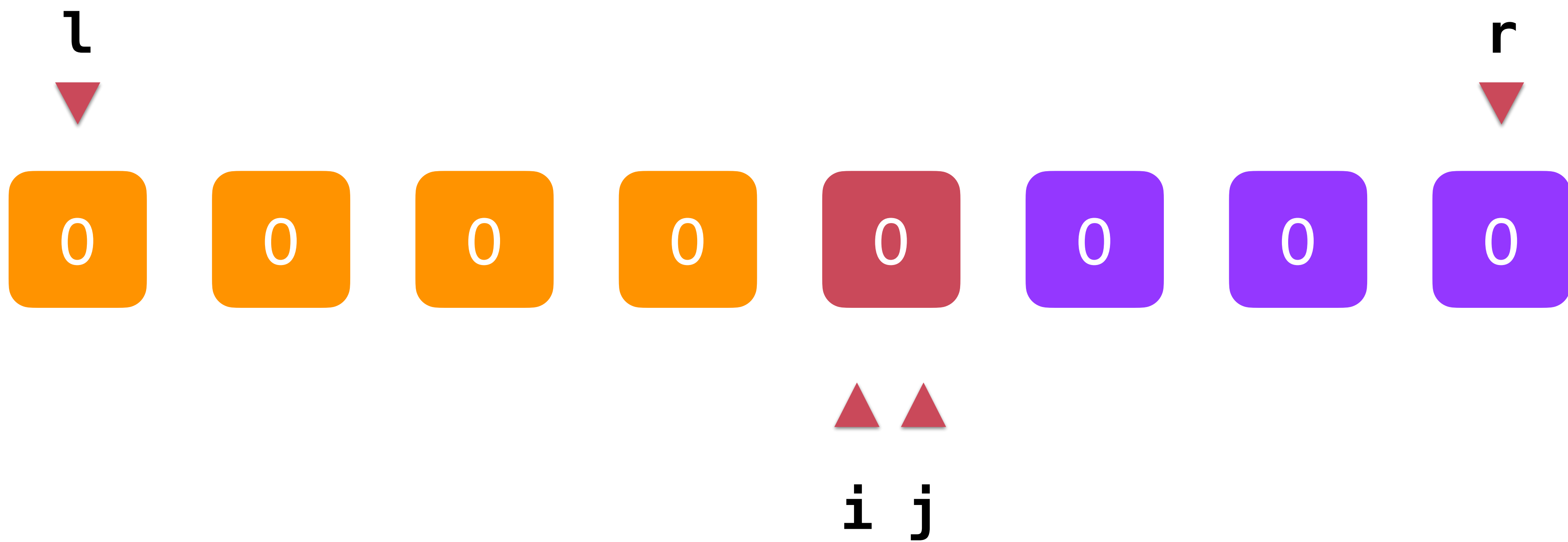
$\text{arr}[l+1 \dots i-1] \leq v$

$\text{arr}[j+1 \dots r] \geq v$



# 快速排序法总结

## 双路快速排序



# 快速排序法总结

三路快速排序

所有元素都一样的数据:  $O(n)$

$arr[l+1...i-1] == v$

$arr[l+1...lt] < v$

$arr[gt...r] > v$



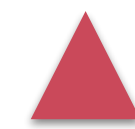
$l$



$lt$



$i$



$gt$



$r$

# 快速排序法总结

单路快速排序

随机化

双路快速排序

三路快速排序

# 快速排序法总结

普通算法：看最差

能找到一组数据 100% 恶化

随机算法：看期望

没有一组数据能 100% 恶化

多次调用？ 尝试均摊分析

# 快速排序法总结

快速排序算法的思想：

排序          SelectK



# 其他

欢迎大家关注我的个人公众号：是不是很酷



# 算法与数据结构体系课程

liuyubobobo