Recently, I have read the 3.2 Analysis of General Picking Sequences

And I feels a little confused about the equtation below:

$$\frac{c_i(X_i) - x \cdot c_i(e_1)}{w_i} \leq \frac{c_i(X_j) + y \cdot c_i(e_1)}{w_j}.$$

Let $\rho : (0, k/w_i] \to \mathbb{R}^+$ be a continuous function such that $\rho(\alpha)$ represents the cost of the item agent i is consuming, when $s_i(t)$ reaches α. In particular, we have

$$\rho(\alpha) = c_i(e_z), \quad \text{for } \alpha \in \left(\frac{z-1}{w_i}, \frac{z}{w_i}\right], \text{ where } z \in \{1, 2, \ldots, k\}.$$

Similarly, we define $\rho' : (0, k'/w_j] \to \mathbb{R}^+$ be a continuous function such that $\rho'(\alpha)$ represents the cost of the item agent j is consuming, under the cost function of agent i, when $s_j(t)$ reaches α. Hence we have

$$\rho'(\alpha) = c_i(e'_z), \quad \text{for } \alpha \in \left(\frac{z-1}{w_j}, \frac{z}{w_j}\right], \text{ where } z \in \{1, 2, \ldots, k'\}.$$

By definition of ρ and ρ', we have

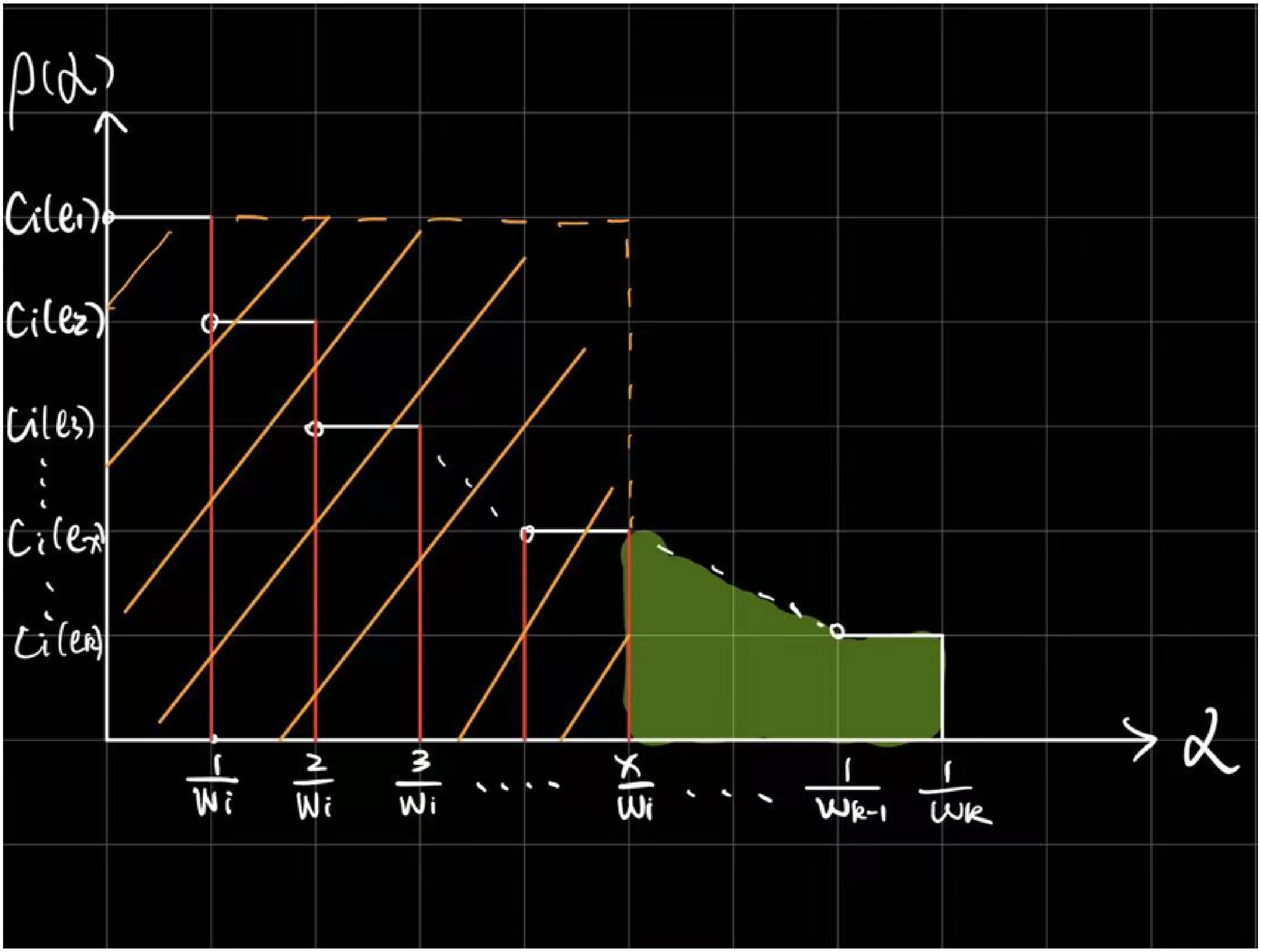
$$\frac{c_i(X_i) - x \cdot c_i(e_1)}{w_i} = \int_{\frac{x}{w_i}}^{\frac{k}{w_i}} \rho(\alpha) d\alpha, \quad \text{and} \quad \frac{c_i(X_j) + y \cdot c_i(e_1)}{w_j} = \int_0^{\frac{k'}{w_j}} \rho'(\alpha) d\alpha + \frac{y}{w_j} \cdot c_i(e_1).$$

Using the condition given in the theorem at $t = m$, we obtain the following immediately.

In my understanding, given the condition

$$c_i(e_1) \geq c_i(e_2) \cdots c_i(e_x) \cdots \geq c_i(e_k)$$

the expression $\rho(\alpha)$ could be shown as follows:



In this figure, I interpret the green area as the integral $\int_{\frac{x}{w_i}}^{\frac{k}{w_i}} \rho(\alpha)d\alpha$, and the yellow square area as $\frac{x \cdot c_i(e_1)}{w_i}$.Assuming $c_i(X_i) = \sum_{j=1}^{k} \frac{c_i(e_j)}{w_i}$, which represents the cumulative sum of all such rectangles, I thus hypothesize that $\frac{c_i(X_i) - x \cdot c_i(e_1)}{w_i} \leq \int_{\frac{x}{w_i}}^{\frac{k}{w_i}} \rho(\alpha)d\alpha$, and I suspect that the equality in this equation is achieved solely when all $c_i(e)$ are identical.