```
f[x_, y_] = 9*x^4 + 9*x^6 + 20*x^3*y^2 - 36*x^2*y^3 + 5*y^4 +
  10*y^6
g1[x_] = 1 - 4*x^4 - 9*x^6 + 16*x^2*y^2 - 8*x^3*y^3 - 15*y^4 - 2*y^6
g2[x_] = 2 - 5*x^2 - 4*x^6 - 20*x*y^2 - 16*y^3*x^3 - 8*y^4 - 8*y^6
g3[x_] = 1 - 6*x^2 - 4*x^6 - 24*x*y - 15*y^2 + 4*x^3*y^2 - y^4

dfdx[x_, y_] = D[f[x, y], {x, 1}]
dfdy[x_, y_] = D[f[x, y], {y, 1}]
d2fdx2[x_, y_] = D[f[x, y], {x, 2}]
d2fdy2[x_, y_] = D[f[x, y], {y, 2}]
d2fdxdy[x_, y_] = D[dfdx[x, y], {y, 1}]
```

$$9 x^4 + 9 x^6 + 20 x^3 y^2 - 36 x^2 y^3 + 5 y^4 + 10 y^6$$

$$1 - 4 x^4 - 9 x^6 + 16 x^2 y^2 - 8 x^3 y^3 - 15 y^4 - 2 y^6$$

$$2 - 5 x^2 - 4 x^6 - 20 x y^2 - 16 x^3 y^3 - 8 y^4 - 8 y^6$$

$$1 - 6 x^2 - 4 x^6 - 24 x y - 15 y^2 + 4 x^3 y^2 - y^4$$

$$36 x^3 + 54 x^5 + 60 x^2 y^2 - 72 x y^3$$

$$40 x^3 y - 108 x^2 y^2 + 20 y^3 + 60 y^5$$

$$108 x^2 + 270 x^4 + 120 x y^2 - 72 y^3$$

$$40 x^3 - 216 x^2 y + 60 y^2 + 300 y^4$$

$$120 x^2 y - 216 x y^2$$

```
(* Поиск "обычного" экстремума *)

NSolve[dfdx[x, y] == 0 && dfdy[x, y] == 0 && g1[x] >= 0 && g2[x] >= 0 &&
  g3[x] >= 0 {x, y}]
```

$$\left\{\{x \to -0.190185, y \to 0.223433\}, \left\{x \to 1.9913 \times 10^{-155}, y \to -2.81955 \times 10^{-155}\right\}, \{x \to 0., y \to 0.\}, \{x \to 0., y \to 0.\}, \{x \to 0., y \to 0.\}, \{x \to 0., y \to 0.\}, \{x \to 0., y \to 0.\}, \{x \to 0., y \to 0.\}, \{x \to 0., y \to 0.\}\right\}$$

```
Eigenvalues[
 {{d2fdx2[-0.19018520115156`, 0.2234333131892448`],
   d2fdxdy[-0.19018520115156`, 0.2234333131892448`]},
  {d2fdxdy[-0.19018520115156`, 0.2234333131892448`],
   d2fdy2[-0.19018520115156`, 0.2234333131892448`]}}]
```

$$\{5.05493, -1.01552\}$$

```
(*Собственные числа разных знаков,
значит в данной точке отсутствует экстремум*)
```

```
Eigenvalues[
 {{d2fdx2[1.9913048617620597`*^-155, -2.819550834045552`*^-155],
   d2fdxdy[1.9913048617620597`*^-155, -2.819550834045552`*^-155]},
  {d2fdxdy[1.9913048617620597`*^-155, -2.819550834045552`*^-155],
   d2fdy2[1.9913048617620597`*^-155, -2.819550834045552`*^-155]}}]
```

$$\left\{4.76992014346018 \times 10^{-308}, 4.282518656675393 \times 10^{-308}\right\}$$

```
(*Собственные числа оба положительны,
значит в данной точке подтверждается минимум =>
  добавим в рабочий список*)
(* Найдем условные экстремумы с помощью функции Лагранжа и
 добавим в рабочий список *)
```

```
L1[x_, y_, λ] = f[x, y] - λ * g1[x]
dL1dx[x_, y_, λ] = D[L1[x, y, λ], {x, 1}]
dL1dy[x_, y_, λ] = D[L1[x, y, λ], {y, 1}]
solution1 =
 N[Solve[dL1dx[x, y, λ] == 0 && dL1dy[x, y, λ] == 0 && g1[x] == 0 &&
    g2[x] ≥ 0 && g3[x] ≥ 0, {x, y, λ}]]
```

$$9\,x^4 + 9\,x^6 + 20\,x^3\,y^2 - 36\,x^2\,y^3 + 5\,y^4 + 10\,y^6 - \left(1 - 4\,x^4 - 9\,x^6 + 16\,x^2\,y^2 - 8\,x^3\,y^3 - 15\,y^4 - 2\,y^6\right)\lambda$$

$$36\,x^3 + 54\,x^5 + 60\,x^2\,y^2 - 72\,x\,y^3 - \left(-16\,x^3 - 54\,x^5 + 32\,x\,y^2 - 24\,x^2\,y^3\right)\lambda$$

$$40\,x^3\,y - 108\,x^2\,y^2 + 20\,y^3 + 60\,y^5 - \left(32\,x^2\,y - 24\,x^3\,y^2 - 60\,y^3 - 12\,y^5\right)\lambda$$

$$\{\{x \to -0.723941, y \to 0.686265, \lambda \to 3.96145\},\ \{x \to -0.635636, y \to 0.198157, \lambda \to -1.53207\}\}$$

```
L2[x_, y_, λ] = f[x, y] - λ * g2[x]
dL2dx[x_, y_, λ] = D[L2[x, y, λ], {x, 1}]
dL2dy[x_, y_, λ] = D[L2[x, y, λ], {y, 1}]
solution2 =
 N[Solve[dL2dx[x, y, λ] == 0 && dL2dy[x, y, λ] == 0 && g1[x] ≥ 0 &&
    g2[x] == 0 && g3[x] ≥ 0, {x, y, λ}]]
```

$$9\,x^4 + 9\,x^6 + 20\,x^3\,y^2 - 36\,x^2\,y^3 + 5\,y^4 + 10\,y^6 - \left(2 - 5\,x^2 - 4\,x^6 - 20\,x\,y^2 - 16\,x^3\,y^3 - 8\,y^4 - 8\,y^6\right)\lambda$$

$$36\,x^3 + 54\,x^5 + 60\,x^2\,y^2 - 72\,x\,y^3 - \left(-10\,x - 24\,x^5 - 20\,y^2 - 48\,x^2\,y^3\right)\lambda$$

$$40\,x^3\,y - 108\,x^2\,y^2 + 20\,y^3 + 60\,y^5 - \left(-40\,x\,y - 48\,x^3\,y^2 - 32\,y^3 - 48\,y^5\right)\lambda$$

```
{{x → 0.29115, y → -0.45406, λ → -0.598501}}
```

```
L3[x_, y_, λ] = f[x, y] - λ * g3[x]
dL3dx[x_, y_, λ] = D[L3[x, y, λ], {x, 1}]
dL3dy[x_, y_, λ] = D[L3[x, y, λ], {y, 1}]
solution3 =
 N[Solve[dL3dx[x, y, λ] == 0 && dL3dy[x, y, λ] == 0 && g1[x] ≥ 0 &&
    g2[x] ≥ 0 && g3[x] == 0, {x, y, λ}]]
```

$$9\,x^4 + 9\,x^6 + 20\,x^3\,y^2 - 36\,x^2\,y^3 + 5\,y^4 + 10\,y^6 - \left(1 - 6\,x^2 - 4\,x^6 - 24\,x\,y - 15\,y^2 + 4\,x^3\,y^2 - y^4\right)\lambda$$

$$36\,x^3 + 54\,x^5 + 60\,x^2\,y^2 - 72\,x\,y^3 - \left(-12\,x - 24\,x^5 - 24\,y + 12\,x^2\,y^2\right)\lambda$$

$$40\,x^3\,y - 108\,x^2\,y^2 + 20\,y^3 + 60\,y^5 - \left(-24\,x - 30\,y + 8\,x^3\,y - 4\,y^3\right)\lambda$$

```
{{x → -0.214904, y → 0.445027, λ → -0.191767},
 {x → -0.126186, y → -0.164426, λ → -0.0162118},
 {x → 0.120895, y → 0.168112, λ → -0.0088363}}
```

```
(* Найдем крайние точки множества Ω и добавим в рабочий
 список *)
```

```
NSolve[g1[x] == 0 && g2[x] == 0 && g3[x] ≥ 0, {x, y}]
```

```
{}
```

```
NSolve[g1[x] == 0 && g2[x] ≥ 0 && g3[x] == 0, {x, y}]
```

```
{{x → -0.342702, y → 0.564399}, {x → -0.617825, y → 0.116666}}
```

```
NSolve[g1[x] ≥ 0 && g2[x] == 0 && g3[x] == 0, {x, y}]
```

```
{{x → 0.25183, y → -0.482867}, {x → 0.587631, y → -0.0971887}}
```

```
(* Итоговый рабочий список *)
```

```
{{x → 1.9913048617620597`*^-155, y → -2.819550834045552`*^-155},
 {x → -0.7239406851447917`, y → 0.6862652473100752`},
 {x → -0.6356364678976296`, y → 0.1981567891209579`},
 {x → 0.2911503119201686`, y → -0.4540596991744629`},
 {x → -0.21490447201056134`, y → 0.44502698388391937`},
 {x → -0.12618638388693054`, y → -0.16442590253394845`},
 {x → 0.12089481794262269`, y → 0.16811170567680522`},
 {x → -0.34270205094294615`, y → 0.5643989451384949`},
 {x → -0.6178248221860946`, y → 0.1166655070907164`},
 {x → 0.25183022228358937`, y → -0.48286746202239184`},
 {x → 0.5876309957441543`, y → -0.09718870545923704`}}
```

$$\left\{\left\{x \to 1.9913 \times 10^{-155}, y \to -2.81955 \times 10^{-155}\right\},\right.$$
$$\{x \to -0.723941, y \to 0.686265\}, \{x \to -0.635636, y \to 0.198157\},$$
$$\{x \to 0.29115, y \to -0.45406\}, \{x \to -0.214904, y \to 0.445027\},$$
$$\{x \to -0.126186, y \to -0.164426\}, \{x \to 0.120895, y \to 0.168112\},$$
$$\{x \to -0.342702, y \to 0.564399\}, \{x \to -0.617825, y \to 0.116666\},$$
$$\left.\{x \to 0.25183, y \to -0.482867\}, \{x \to 0.587631, y \to -0.0971887\}\right\}$$

```
(* Рассчитаем значения функции для элементов *)
answer = {f[1.9913048617620597`*^-155, -2.819550834045552`*^-155],
  f[-0.7239406851447917`, 0.6862652473100752`],
  f[-0.6356364678976296`, 0.1981567891209579`],
  f[0.2911503119201686`, -0.4540596991744629`],
  f[-0.21490447201056134`, 0.44502698388391937`],
  f[-0.12618638388693054`, -0.16442590253394845`],
  f[0.12089481794262269`, 0.16811170567680522`],
  f[-0.34270205094294615`, 0.5643989451384949`],
  f[-0.6178248221860946`, 0.1166655070907164`],
  f[0.25183022228358937`, -0.48286746202239184`],
  f[0.5876309957441543`, -0.09718870545923704`]}
```

$$\{4.57514002775847 \times 10^{-618}, -3.75047, 1.75625, 0.757764, 0.10803,$$
$$0.0076323, 0.00466883, -0.0472507, 1.72678, 0.768584, 1.49392\}$$

```
(*Получили результат *)
min = {x → -0.7239406851447917`, y → 0.6862652473100752`}
max = {x → -0.6356364678976296`, y → 0.1981567891209579`}
```

$\{x \to -0.723941, y \to 0.686265\}$

$\{x \to -0.635636, y \to 0.198157\}$