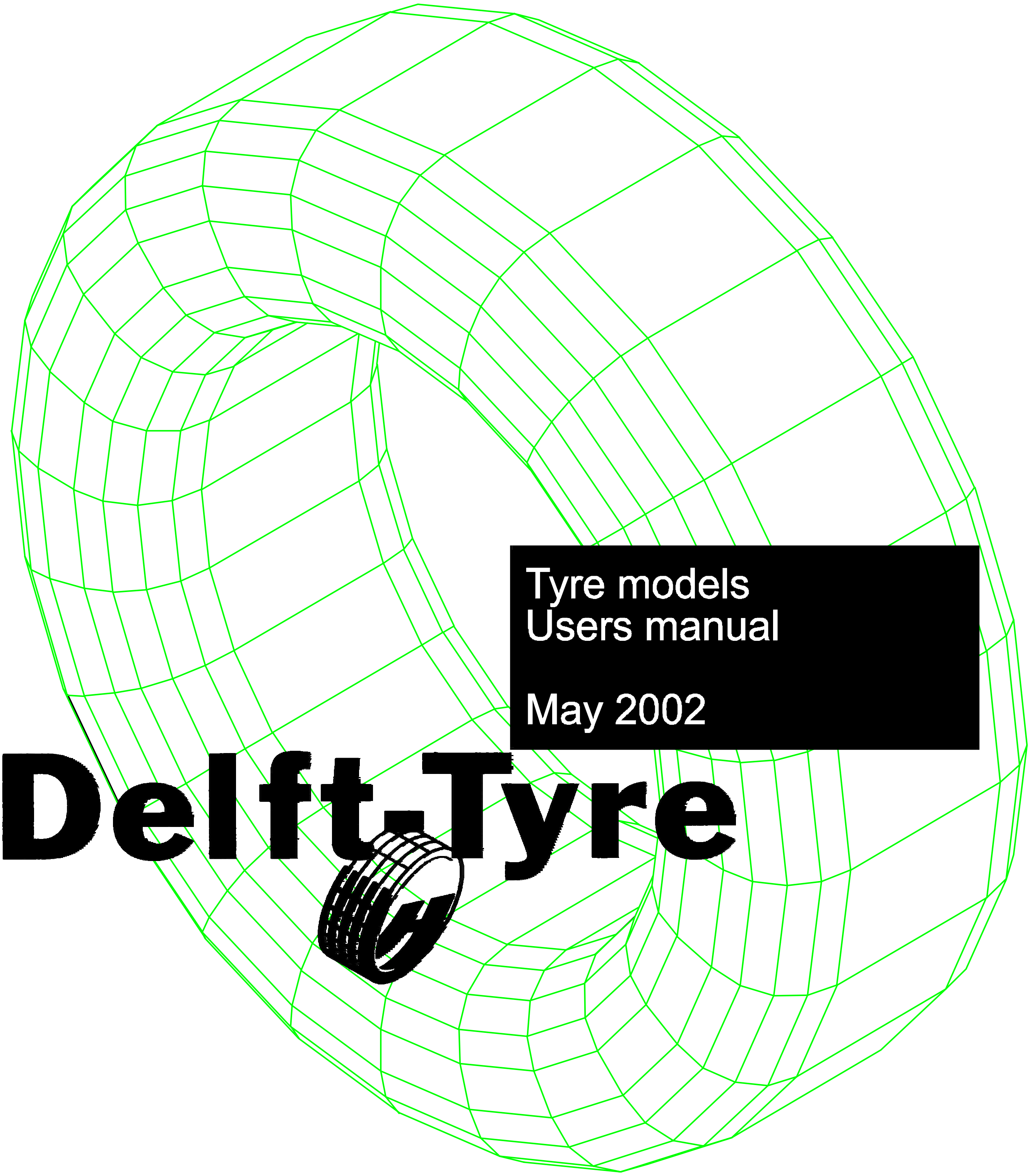

Tyre models
Users manual

May 2002

MF-Tool, MF-MCTool, MF-Tyre, MF-MCTyre & SWIFT-Tyre are part of the DELFT-TYRE product line, developed at TNO Automotive, Delft, The Netherlands.

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This manual includes the following sections:

Delft-Tyre

Using the MF-Tyre Model

Overview

The Magic-Formula (MF-Tyre) tire model is developed by TNO Automotive. MF-Tyre is the premier handling model available in ADAMS/Tire.

This chapter includes the following sections:

About MF-Tyre

The MF-Tyre model uses a method known as the Magic Formula to calculate the steady-state behavior of a tire. The Magic Formula is actually a set of mathematical formula based on the physical background of the tire, road, and the tire-to-road contact.

The Magic Formula tyre model aims at an accurate description of the steady-state behaviour of a tyre by providing a set of mathematical formulae, which are partly based on a physical background. The Magic Formula calculates the forces (F_x, F_y) and moments (M_x, M_y, M_z) acting on the tyre under pure and combined slip conditions, using longitudinal and lateral slip (κ, α), wheel camber (γ) and the vertical force (F_z) as input quantities. In addition to the Magic Formula description, a set of differential equations is defined, representing the transient behaviour of the tyre with respect to handling at frequencies up to 8 Hz.

Further information can be found on the internet site: www.delft-tyre.com.

MF-Tyre Version 5.2

Compared to MF-Tyre 5.1, following items have been changed or introduced:

- The scaling factors for the shifts have been defined such that conicity and plysteer effects can be easily switched off.
- Into the modelling of combined cornering and braking/traction E factors have been introduced, making the modelling more accurate.
- The rolling resistance torque has become a function of forward speed.
- The influence of the camber on the peak F_x has been introduced.

Figure 1 lists the additional parameters.

Table 1. New Parameters Introduced in MF-Tyre 5.2

Name:	Name used in tire property file:	Explanation:	Default Value:
$\lambda_{\gamma x}$	LGAX	Scale factor of camber for Fx	1
$\lambda_{\gamma y}$	LGAY	Scale factor of camber force stiffness	1
λ_{Vmx}	LVMX	Scale factor of Mx vertical shift	1
p_{Dx3}	PDX3	Variation of friction Mux with camber	0
r_{Ex1}	REX1	Curvature factor of combined Fx	0
r_{Ex2}	REX2	Curvature factor of combined Fx with load	0
r_{Hy2}	RHY2	Shift factor for combined Fy reduction with load	0
r_{Ey1}	REY1	Curvature factor of combined Fy	0
r_{Ey2}	REY2	Curvature factor of combined Fy with load	0
q_{sy3}	QSY3	Rolling resistance torque depending on speed	0
q_{sy4}	QSY4	Rolling resistance torque depending on speed^4	0

Furthermore, LONGVL should be defined and have a positive value. When the default values are used or omitted, the tire model is fully backward compatible with MF-Tyre version 5.1.

Tire-Road Interaction

The tire-road contact forces are mainly dependent of the tire mechanical properties (that is, stiffness and damping), the road condition (that is, the friction coefficient between tire and road, the road structure), and the motion of the tire relative to the road (that is, the amount and direction of slip).

The major control and disturbance forces on a vehicle arise from the contact of the tires with the road. The vertical loads transfer the weight of the vehicle to the road. Due to the compliance of the tires, a vehicle is cushioned against disturbances by small road irregularities. The traction and braking forces arise from the longitudinal tire forces. Lateral forces are required to control the direction of travel of the vehicle. The lateral behaviour of tires is therefore dominant in vehicle handling. Proper description of the dynamic behaviour of a vehicle requires an accurate model of the tire-road contact forces and moments generating properties under all of these different conditions.

Figure 1. Tire Factors

Tyre factors

	(Quasi) steady state		*Vibratory state*
in-plane	load carrying capacity braking/driving performance rolling resistance	radial deflection longitudinal slip and distortion	cushioning capacity dynamic coupling
out-of-plane	cornering performance lateral shift of Fz	lateral slip and distortion	phase shifts and destabilisation

primary effects
secondary effects

interactions between
in- and out-of-plane behaviour

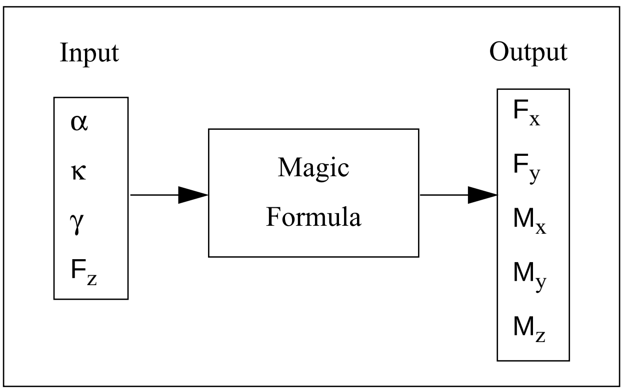
Tire behaviour results from a combination of several aspects. Factors may be distinguished which concern the primary tasks of the tire which involve (often important) secondary effects. In Figure 1 these factors have been brought in matrix form. A distinction has been made between (quasi) steady-state and vibratory behaviour and besides between in-plane and out-of-plane aspects. The primary task factors are shaded in green. The remaining secondary factors are not shaded.

The requirements to transmit forces in the three perpendicular directions (F_x, F_y and F_z) and to cushion the vehicle against road irregularities involve secondary factors like radial, lateral and longitudinal distortions and slip.

Although considered as secondary factors, some of the quantities involved have to be treated as input variables into the system which generate the forces. Figure 2 presents the input and output vectors. In this diagram the tire is assumed to be uniform and to move over a flat road surface. The input vector results from motions of the wheel relative to the road. It is advantageous to recognize the fact that, for small deviations from the straight-ahead motion, in-plane and out-of-plane motions of the assumedly symmetric wheel-tire system are uncoupled.

The forces and moments are considered as output quantities of the tire model. They are assumed to act on a rigid disc with inertial properties equal to those of the undeflected tire. The forces may differ from the corresponding forces acting on the road due to the vibrations of the tire relative to the wheel rim. Braking and traction torques are considered as acting on the rotating disc.

Figure 2. Input and Output Variables of the Magic Formula Tire Model

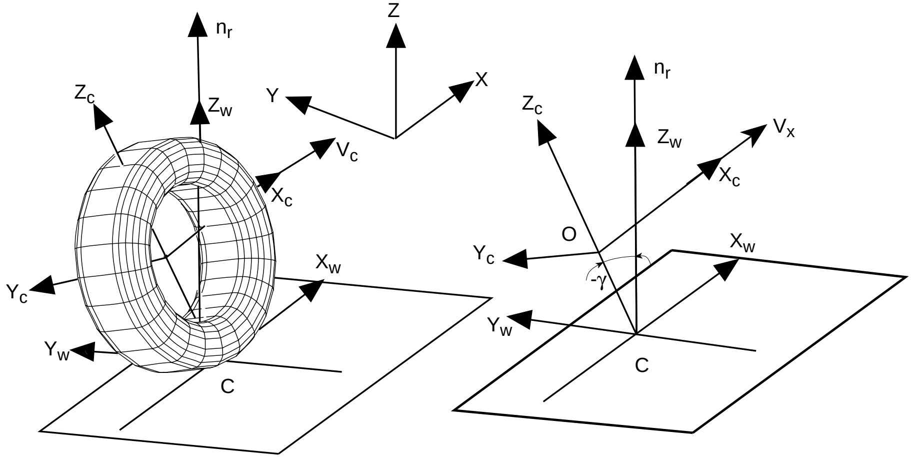


Axis Systems and Definitions

W-Axis System

MF-Tyre conforms to the TYDEX STI conventions described in the TYDEX-Format [1] and the Standard Tire Interface [2]. Two TYDEX coordinate systems with ISO orientation are particularly important, the C- and W-axis systems as detailed in Figure 3.

Figure 3. TYDEX C- and W-Axis Systems Used in MF-Tyre, According to TYDEX



The C-axis system is fixed to the wheel carrier with the longitudinal x_c-axis parallel to the road and in the wheel plane (x_c-z_c-plane). The origin O of the C-axis system is the wheel center.

The origin of the W-axis system is the road contact-point (or point of intersection) C defined by the intersection of the wheel plane, the plane through the wheel spindle and the road tangent plane. The orientation of the W-axis system agrees to ISO. The forces and torques calculated by MF-MCTyre, which depend on the vertical wheel load F_z along the z_w-axis and the slip quantities, are projected in the W-axis system. The x_w-y_w-plane is the tangent plane of the road in the contact point C.

The camber angle is defined by the inclination angle between the wheel plane and the normal n_r to the road plane (x_w-y_w-plane).

Units

Next to the convention to the TYDEX W-axis system, all units of the parameters and variables used in MF-Tyre agree to the SI units. In Table 2 provides an overview of the most important parameters and variables, see also Definitions on page 47.

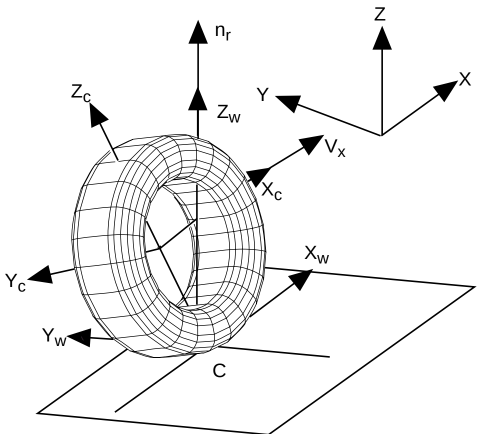
Table 2. SI Units Used in MF-Tyre

Variable Type:	Name:	Abbreviation:	Unit:
angle	slip angle camber angle	α γ	radians
force	longitudinal force lateral force vertical load	F_x F_y F_z	Newton
moment	overturning moment rolling resistance moment self aligning moment	M_x M_y M_z	Newton.meter
speed	longitudinal speed lateral speed longitudinal slip speed lateral slip speed	V_x V_y V_{sx} V_{sy}	meters per second
rotational speed	tire rolling speed	Ω	radians per second

The Contact-Point C and the Normal Load

The radius of curvature of the road profile is considered large as compared to the radius of the tire. The tire is assumed to have only a single contact point (C) with the road profile. Furthermore, for calculating the motion of the tire relative to the road, the road is approximated by its tangent plane at the point on the road below the wheel centre (see Figure 4). The tangent plane is an accurate approximation of the road, as long as the road radius of curvature is not too small (that is, not smaller than 2 meters).

Figure 4. Contact Point C (Intersection Between Normal-to-Road Tangent and Wheel Plane)



The normal load F_z of the tire is calculated with:

$$F_z = C_z\rho + K_z \cdot \dot{\rho} \tag{1}$$

with ρ the tire deflection and $\dot{\rho}$ the deflection velocity of the tire.

Table 3. Normal Load

Name:	Name Used in Tire Property File:	Explanation:
R_o	UNLOADED_RADIUS	Free tire radius
C_z	VERTICAL_STIFFNESS	Tire vertical stiffness
K_z	VERTICAL_DAMPING	Tire vertical damping

The Effective Tire Rolling Radius

The loaded tire radius R which is defined by the distance of the wheel centre to the centre of tire contact (see Figure 5).

The effective rolling radius R_e (at free rolling of the tire) is defined by:

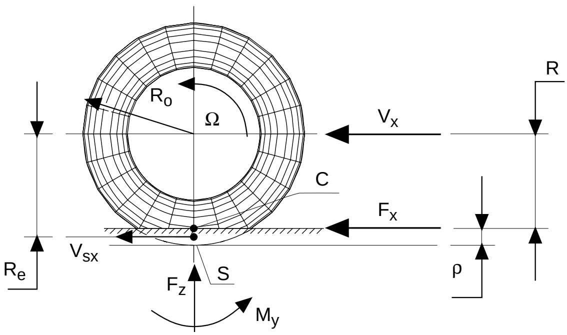
$$R_e = \frac{V_x}{\Omega} \tag{2}$$

For radial tires the effective rolling radius decreases with increasing vertical load at low loads, but around its nominal load the influence of the vertical load is small, see Figure 6.

When assuming a constant vertical tire stiffness C_z, the radial tire deflection ρ can be calculated with:

$$\rho = \frac{F_z}{C_z} \tag{3}$$

Figure 5. Effective Rolling Radius and Longitudinal Slip



For the estimation of the effective rolling radius R_e a Magic Formula approach is chosen. The equation of the effective rolling radius R_e reads:

$$R_e = R_0 - \rho_{F_{z0}}(D \arctan(B\rho^d) + F\rho^d) \tag{4}$$

in which R_0 is the unloaded radius and the nominal tire deflection $\rho_{F_{z0}}$ is defined by:

$$\rho_{F_{z0}} = \frac{F_{z0}}{C_z} \tag{5}$$

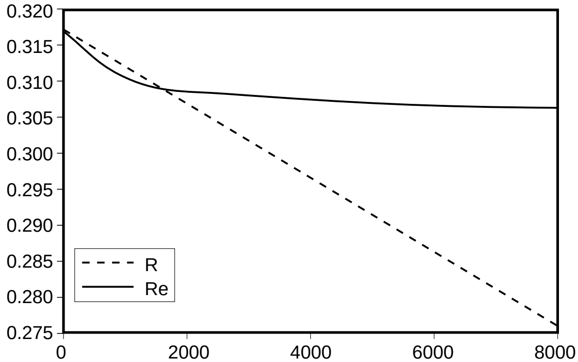
and the dimensionless radial tire deflection ρ^d can be calculated with:

$$\rho^d = \frac{\rho}{\rho_{F_{z0}}} \tag{6}$$

For a large range of tires, appropriate coefficient values are:

- <u>3,...,B,...,12</u>
  stretches the ordinate of the arctangent function, a large value of *B* means a high slope at F_z=0 ;
- <u>0.2,...,D,...0.4</u>
  defines the shift from the asymptote at high wheel loads;
- <u>0.03,...,F,...,0.25</u>
  defines the ratio between tire radial deformation r and effective tire deformation. Low values are obtained for extremely stiff tires.

Figure 6. The Tire Effective Rolling Radius as a Function of the Vertical Load (B=8.4, D=0.27 and F=0.045)



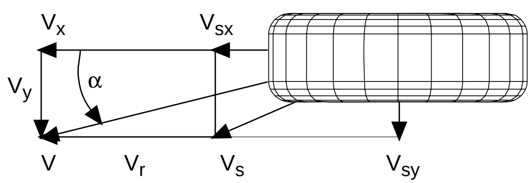
In Figure 7 an example of the effective rolling radius is shown for a passenger car tire. The approximation of R_e is made with the proposed formula with: B = 8.4, D = 0.27 and F = 0.045.

Table 4. Effective Rolling Radius Parameters

Name:	Name used in tire property file:	Explanation:
F_{z0}	FNOMIN	Nominal wheel load
B	BREFF	Low load stiffness eff. rolling radius
D	DREFF	Peak value of effective rolling radius
F	FREFF	High load stiffness effective rolling radius

Tire Slip Quantities

Figure 7. Slip Quantities at Combined Cornering and Braking/Traction



The longitudinal slip speed is defined as:

$$V_{sx} = V_x - \Omega R_e \tag{7}$$

and the lateral slip speed:

$$V_{sy} = V_y \tag{8}$$

The practical slip quantities κ and α are defined as:

$$\kappa = -\frac{V_{sx}}{V_x} \tag{9}$$

$$\tan\alpha = \frac{V_{sy}}{|V_x|} \tag{10}$$

with V_{sx} and V_{sy} the components of the slip speed which may be defined as the velocity of point *S* in the W-axis system (see Figure 7).

With Ω denoting the rotational speed of the tire, the linear rolling speed becomes:

$$V_r = R_e\Omega \tag{11}$$

The Magic Formula Tire Model (MF-Tyre)

Introduction

For a given pneumatic tire and road condition, the tire forces due to slip follow a typical characteristic. The characteristics can be accurately approximated by a special mathematical function which is known as the "Magic Formula". The parameters in the Magic Formula depend on the type of the tire and the road conditions. These parameters can be derived from experimental data obtained from tests. The tire is rolled over a road at various loads, orientations and motion conditions.

The Magic Formula tire model is mainly of an empirical nature and contains a set of mathematical formula, which are partly based on a physical background. The Magic Formula calculates the forces (F_x, F_y) and moments (M_x, M_y, M_z) acting on the tire at pure and combined slip conditions, using longitudinal and/or lateral slip (κ, α), wheel camber γ and the vertical force F_z as input quantities. The model takes into account plysteer and conicity. An extension has been provided that describes transient and oscillatory tire behaviour for limited frequencies smaller than 8 Hz and wavelengths larger than the tire circumference.

History of the Magic Formula

Through the initiative of Volvo Car Corp. a cooperate effort was started in the mid-eighties with the Delft University of Technology to develop a tire model that accurately describes the tire's ability to have horizontal forces generated between road and tire.

The first Magic Formula version was presented in 1987 [3]. The basic idea of using the sine and arcsine functions was described for mainly pure slip conditions. Further 'prototype' formula were proposed for combined slip conditions.

In the second version [4], presented in 1989 the formula for combined cornering conditions, based on physical background, were improved and tire relaxations lengths were introduced in order to have a first order approach of the transient tire behaviour. This model was improved on the description for combined slip calculations in 1993 [5].

Bayle e.o. [6] proposed to have a more empirical approach, reducing the complexity of the force calculations under combined slip conditions and yielding a considerably higher calculation speed. Their method improved the calculation speed during the calculation of the Magic Formula parameters and during simulation calculations.

The latest version [7] combines the advantage of the previous versions and has been modified for the following aspects:

- The self aligning torque has been made dependent on the side force by a new approach using the pneumatic trail in pure and combined slip conditions;
- The forces under combined slip conditions are calculated according to the proposal of Bayle [6];
- Formulae describing overturning moment have been introduced;
- The transient tire behaviour has been improved to enable zero speed;
- Loading variations to tire lift off situations;
- The parameters used in formulae are dimensionless improving manipulations with tire characteristics and parameter calculations ("fitting");
- Scaling factors are introduced for vehicle-tire optimization purposes.

Learning the Basics of the Magic Formula

The general form (sine version) of the formula reads:

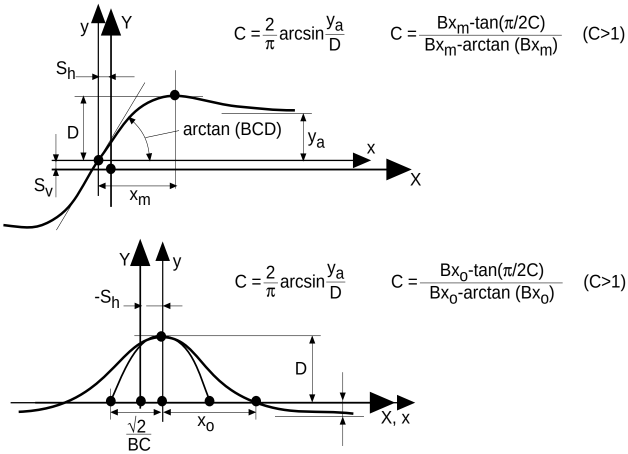
$$Y(x) = D\sin[C\arctan\{Bx - E(Bx - \arctan(Bx))\}] \tag{12}$$

where $Y(x)$ is either, F_x or F_y.

The self aligning moment M_z is calculated by using the lateral force F_y and the pneumatic trail t, which is based on a cosine type of Magic Formula:

$$Y(x) = D\cos[C\arctan\{Bx - E(Bx - \arctan(Bx))\}] \tag{13}$$

Figure 8. Curves Produced by the Sine and Cosine Versions of the Magic Formula



When the formula is used to calculate the forces generated by the tire, the following variables should serve as input for the Magic Formula:

Input Variables

Longitudinal slip	κ	[-]
Slip angle	α	[rad]
Camber angle	γ	[rad]
Normal wheel load	F_z	[N]

In case the complete model including transient properties is used, the transient tire quantities are employed instead of the wheel slip quantities κ and α.

Output Variables (in contact point C)

Longitudinal force	F_x	[N]
Lateral force	F_y	[N]
Overturning couple	M_x	[Nm]
Rolling resistance torque	M_y	[Nm]
Aligning torque	M_z	[Nm]

Basic Tire Parameters

Nominal (rated) load	F_{z0}	[N]
Unloaded tire radius	R_0	[m]
Tire belt mass	m_{belt}	[kg]

Furthermore, the normalized vertical load increment df_z is defined:

$$df_z = \frac{F_z - F'_{z0}}{F'_{z0}} \qquad [-] \tag{14}$$

with the possibly adapted nominal load (using the user scaling factor λ_{Fz0}):

$$F'_{z0} = F_{z0} \cdot \lambda_{F_{z0}} \tag{15}$$

Tire Model Parameters

In the subsequent sections, formulae are given with non-dimensional parameters a_{ijk} with the following values and connections:

Table 5. Tire Model Parameters

Parameter:		Definition:
a =	p	Force at pure slip
	q	Moment at pure slip
	r	Force at combined slip
	s	Moment at combined slip
i =	B	Stiffness factor
	C	Shape factor
	D	Peak value
	E	Curvature factor
	K	Slip stiffness = BCD
	H	Horizontal shift
	V	Vertical shift
	s	Moment at combined slip
	t	Transient tire behavior
j =	x	Along the longitudinal axis
	y	Along the lateral axis
	z	About the vertical axis

Table 5. Tire Model Parameters *(continued)*

Parameter:	Definition:
k =	1, 2, ...

User Scaling Factors

For the user convenience a set of scaling factors is available to examine the influence of changing a number of important overall parameters. The default value of these factors is one. The following factors have been defined:

Table 6. Scaling Coefficient, Pure Slip

Name:	Name used in tire property file:	Explanation:
λ_{Fzo}	LFZO	Scale factor of nominal (rated) load
λ_{Cx}	LCX	Scale factor of F_x shape factor
$\lambda_{\mu x}$	LMUX	Scale factor of F_x peak friction coefficient
λ_{Ex}	LEX	Scale factor of F_x curvature factor
λ_{Kx}	LKX	Scale factor of F_x slip stiffness
λ_{Hx}	LHX	Scale factor of F_x horizontal shift
λ_{Vx}	LVX	Scale factor of F_x vertical shift
$\lambda_{\gamma x}$	LGAX	Scale factor of camber for F_x
λ_{Cy}	LCY	Scale factor of F_y shape factor
$\lambda_{\mu y}$	LMUY	Scale factor of F_y peak friction coefficient
λ_{Ey}	LEY	Scale factor of F_y curvature factor
λ_{Ky}	LKY	Scale factor of F_y cornering stiffness
λ_{Hy}	LHY	Scale factor of F_y horizontal shift
λ_{Vy}	LVY	Scale factor of F_y vertical shift
$\lambda_{\gamma y}$	LGAY	Scale factor of camber for F_y
λ_{t}	LTR	Scale factor of Peak of pneumatic trail
λ_{Mr}	LRES	Scale factor for offset of residual torque
$\lambda_{\gamma z}$	LGAZ	Scale factor of camber for M_z
λ_{Mx}	LMX	Scale factor of overturning couple
λ_{vMx}	LVMX	Scale factor of Mx vertical shift
λ_{My}	LMY	Scale factor of rolling resistance torque

Table 7. Scaling Coefficients, Combined Slip

Name:	Name used in tire property file:	Explanation:
$\lambda_{x\alpha}$	LXAL	Scale factor of alpha influence on F_x
$\lambda_{y\kappa}$	LYKA	Scale factor of alpha influence on F_x
$\lambda_{Vy\kappa}$	LVYKA	Scale factor of kappa induced F_y
λ_s	LS	Scale factor of Moment arm of F_x

Table 8. Scaling Coefficinets, Transient Response

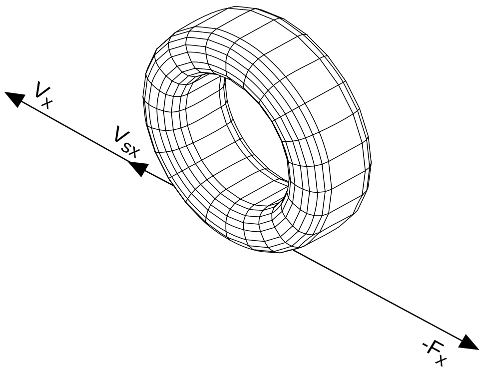
Name:	Name used in tire property file:	Explanation:
$\lambda_{\sigma\kappa}$	LSGKP	Scale factor of Relaxation length of Fx
$\lambda_{\sigma\alpha}$	LSGAL	Scale factor of Relaxation length of Fy
λ_{gyr}	LGYR	Scale factor of gyroscopic torque

Steady-State: Magic Formula

Steady-State Pure Slip

Formula: Longitudinal Slip (Pure Slip)

Figure 9. Longitudinal Slip Condition (Pure Braking/Traction)



$$F_x = F_{x0}(\kappa, F_z) \tag{16}$$

$$F_{x0} = D_x \sin[C_x \arctan\{B_x\kappa_x - E_x(B_x\kappa_x - \arctan(B_x\kappa_x))\}] + S_{Vx} \tag{17}$$

$$\kappa_x = \kappa + S_{Hx} \tag{18}$$

$$\gamma_x = \gamma \cdot \lambda_{\gamma x} \tag{19}$$

with coefficients:

$$C_x = p_{Cx1} \cdot \lambda_{Cx} \tag{20}$$

$$D_x = \mu_x \cdot F_z \tag{21}$$

$$\mu_x = (p_{Dx1} + p_{Dx2} df_z) \cdot (1 - p_{Dx3} \cdot \gamma_x{}^2) \lambda_{\mu x} \tag{22}$$

$$E_x = (p_{Ex1} + p_{Ex2} df_z + p_{Ex3} df_z^2) \cdot \{1 - p_{Ex4} \operatorname{sgn}(\kappa_x)\} \cdot \lambda_{Ex} (\ \leq 1)\ ) \tag{23}$$

$$K_x = F_z \cdot (p_{Kx1} + p_{Kx2} df_z) \cdot \exp(p_{Kx3} df_z) \cdot \lambda_{Kx} \tag{24}$$

$$\left(K_x = B_x C_x D_x = \frac{\partial F_{x0}}{\partial \kappa_x} \ \text{at} \ \kappa_x = 0\right)$$

$$B_x = K_x / (C_x D_x) \tag{25}$$

$$S_{Hx} = (p_{Hx1} + p_{Hx2} \cdot df_z) \lambda_{Hx} \tag{26}$$

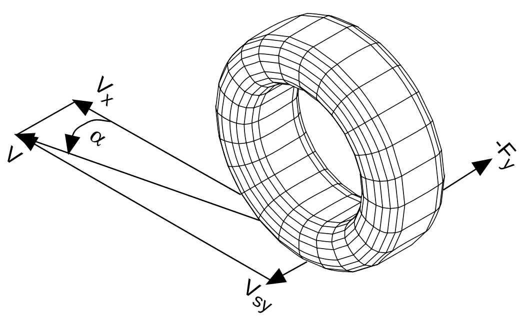
$$S_{Vx} = F_z \cdot (p_{Vx1} + p_{Vx2} df_z) \cdot \lambda_{Vx} \cdot \lambda_{\mu x} \tag{27}$$

Table 9. Longitudinal Coefficients, Pure Slip

Name:	Name used in tire property file:	Explanation:
p_{Cx1}	PCX1	Shape faxtor Cfx for longitudinal force
p_{Dx1}	PDX1	Longitudinal friction Mux at Fznom
p_{Dx2}	PDX2	Variation of friction Mux with load
p_{Dx3}	PDX3	Variation of friction Mux with camber
p_{Ex1}	PEX1	Longitudinal curvature Efx at Fznom
p_{Ex2}	PEX2	Variation of curvature Efx with load
p_{Ex3}	PEX3	Variation of curvature Efx with load squared
p_{Ex4}	PEX4	Factor in curvature Efx while driving
p_{Kx1}	PKX1	Longitudinal slip stiffness Kfx/Fz at Fznom
p_{Kx2}	PKX2	Variation of slip stiffness Kfx/Fz with load
p_{Kx3}	PKX3	Exponent in slip stiffness Kfx/Fz with load
p_{Hx1}	PHX1	Horizontal shift Shx at Fznom
p_{Hx2}	PHX2	Variation of shift Shx with load
p_{Vx1}	PVX1	Vertical shift Svx/Fz at Fznom
p_{Vx2}	PVX2	Variation of shift Svx/Fz with load

Formula: Lateral Slip (Pure Slip)

Figure 10. Lateral Slip Condition Excluding Aligning Torque (Pure Cornering)



$$F_y = F_{y0}(\alpha, \gamma, F_z) \tag{28}$$

$$F_{y0} = D_y \sin[C_y \arctan\{B_y\alpha_y - E_y(B_y\alpha_y - \arctan(B_y\alpha_y))\}] + S_{Vy} \tag{29}$$

$$\alpha_y = \alpha + S_{Hy} \tag{30}$$

the scaled camber angle:

$$\gamma_y = \gamma \cdot \lambda_{\gamma y} \tag{31}$$

with coefficients:

$$C_y = p_{Cy1} \cdot \lambda_{Cy} \tag{32}$$

$$D_y = \mu_y \cdot F_z \tag{33}$$

$$\mu_y = (p_{Dy1} + p_{Dy2}df_z) \cdot (1 - p_{Dy3}\gamma_y^2) \cdot \lambda_{\mu y} \tag{34}$$

$$E_y = (p_{Ey1} + p_{Ey2}df_z) \cdot \{1 - (p_{Ey3} + p_{Ey4}\gamma_y)\operatorname{sgn}(\alpha_y)\} \cdot \lambda_{Ey}(\quad \leq 1) \tag{35}$$

$$\begin{aligned} K_y &= p_{Ky1}F_{z0}\sin[2\arctan\{F_z/(p_{ky2}F_{z0}\lambda_{F_{z0}})\}] \cdot (1 - p_{Ky3}|\gamma_y|) \cdot \lambda_{F_{z0}} \cdot \lambda_{Ky} \\ &\quad ((= B_yC_yD_y = \frac{\partial F_{y0}}{\partial \alpha_y} \text{ at } \alpha_y = 0) \end{aligned} \tag{36}$$

$$B_y = K_y/(C_yD_y) \tag{37}$$

$$S_{Hy} = (P_{Hy1} + P_{Hy2}df_z) \cdot \lambda_{Hy} + P_{Hy3}\gamma_y \tag{38}$$

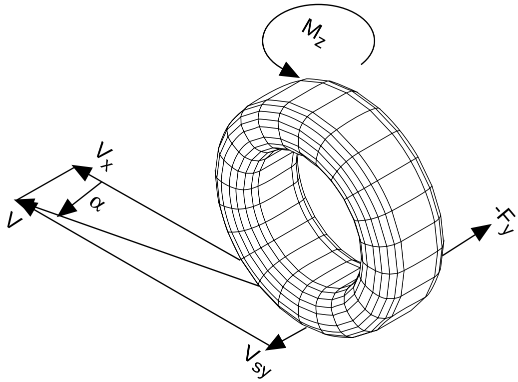
$$S_{Vy} = F_z \cdot \{(p_{Vy1} + p_{Vy2}df_z) \cdot \lambda_{Vy} + (p_{Vy3} + p_{vy4} \cdot df_z) \quad \cdot \gamma_y\} \cdot \lambda_{\mu y} \tag{39}$$

Table 10. Lateral Coefficients, Pure Slip

Name:	Name used in tire property file:	Explanation:
p_{Cy1}	PCY1	Shape factor Cfy for lateral forces
p_{Dy1}	PDY1	Lateral friction Muy
p_{Dy2}	PDY2	Variation of friction Muy with load
p_{Dy3}	PDY3	Variation of friction Muy with squared camber
p_{Ey1}	PEY1	Lateral curvature Efy at Fznom
p_{Ey2}	PEY2	Variation of curvature Efy with load
p_{Ey3}	PEY3	Zero order camber dependency of curvature Efy
p_{Ey4}	PEY4	Variation of curvature Efy with camber
p_{Ky1}	PKY1	Maximum value of stiffness Kfy/Fznom
p_{Ky2}	PKY2	Load at which Kfy reaches maximum value
p_{Ky3}	PKY3	Variation of Kfy/Fznom with camber
p_{Hy1}	PHY1	Horizontal shift Shy at Fznom
p_{Hy2}	PHY2	Variation of shift Shy with load
p_{Hy3}	PHY3	Variation of shift Shy with camber
p_{Vy1}	PVY1	Vertical shift in Svy/Fz at Fznom
p_{Vy2}	PVY2	Variation of shift Svy/Fz with load
p_{Vy3}	PVY3	Variation of shift Svy/Fz with camber
p_{Vy4}	PVY4	Variation of shift Svy/Fz with camber and load

Formula: Aligning Torque (Pure Slip)

Figure 11. Lateral Slip Condition Including Aligning Torque (Pure Cornering)



$$M_z' = M_{z0}(\alpha, \gamma, F_z) \tag{40}$$

$$M_{z0} = -t \cdot F_{y0} + M_{zr} \tag{41}$$

with the pneumatic trail:

$$t(\alpha_t) = D_t \cos[C_t \arctan\{B_t\alpha_t - E_t(B_t\alpha_t - \arctan(B_t\alpha_t))\}]\cos(\alpha) \tag{42}$$

$$\alpha_t = \alpha + S_{Ht} \tag{43}$$

the residual torque:

$$M_{zr}(\alpha_r) = D_r \cos[\arctan(B_r\alpha_r)]\cos(\alpha) \tag{44}$$

$$\alpha_r = \alpha + S_{Hr} \tag{45}$$

$$S_{Hf} = S_{Hy} + S_{Vy}/K_y \tag{46}$$

the scaled camber angle:

$$\gamma_z = \gamma \cdot \lambda_{\gamma z} \tag{47}$$

with coefficients:

$$B_t = (q_{Bz1} + q_{Bz2} df_z + q_{Bz3} df_z^2) \cdot (1 + q_{Bz4}\gamma_z + q_{Bz5}|\gamma_z|) \cdot \lambda_{Ky}/\lambda_{\mu y} \tag{48}$$

$$C_t = q_{Cz1} \tag{49}$$

$$D_t = F_z \cdot (q_{Dz1} + q_{Dz2} df_z) \cdot (1 + q_{Dz3}\gamma_z + q_{Dz4}\gamma_z^2) \cdot (R_0/F_{z0}) \cdot \lambda_t \tag{50}$$

$$E_t = (q_{Ez1} + q_{Ez2} df_z + q_{Ez3} df_z^2) \tag{51}$$

$$\left\{1 + (q_{Ez4} + q_{Ez5}\gamma_z) \cdot \left(\frac{2}{\pi}\right) \cdot \arctan(B_t \cdot C_t \cdot \alpha_t)\right\} \qquad \leq 1$$

$$S_{Ht} = q_{Hz1} + q_{Hz2} df_z + (q_{Hz3} + q_{Hz4} \cdot df_z)\gamma_z \tag{52}$$

$$B_r = q_{Bz9} \cdot \lambda_{Ky}/\lambda_{\mu y} + q_{Bz10} \cdot B_y \cdot C_y \tag{53}$$

$$D_r = F_z \cdot ((q_{Dz6} + q_{Dz7} \cdot df_z) \quad \cdot \lambda_r + (q_{Dz8} + q_{Dz9} \cdot df_z) \quad \cdot \gamma_z) \cdot R_o \cdot \lambda_{\mu\gamma} \tag{54}$$

An approximation for the aligning stiffness reads:

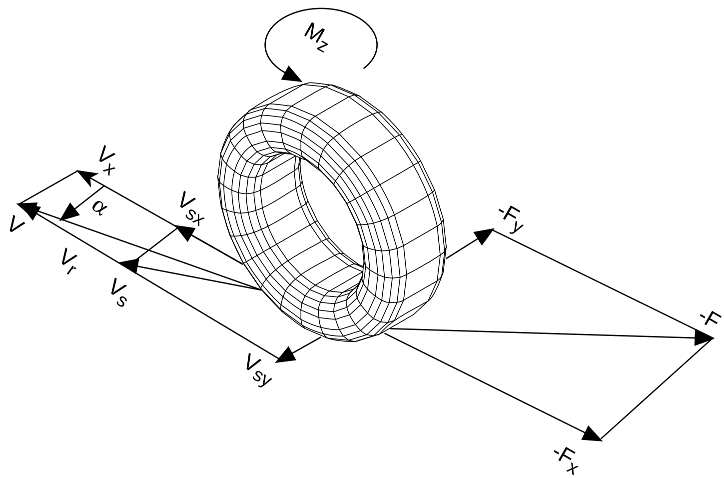
$$K_z = -t \cdot K_y \qquad \left(\quad \approx -\frac{\partial M_z}{\partial \alpha} \text{ at } \alpha \right) = 0) \tag{55}$$

Table 11. Aligning Coefficients, Pure Slip

Name:	Name used in tire property file:	Explanation:
q_{Bz1}	QBZ1	Trail slope factor for trail Bpt at Fznom
q_{Bz2}	QBZ2	Variation of slope Bpt with load
q_{Bz3}	QBZ3	Variation of slope Bpt with load squared
q_{Bz4}	QBZ4	Variation of slope Bpt with camber
q_{Bz5}	QBZ5	Variation of slope Bpt with absolute camber
q_{Bz9}	QBZ9	Slope factor Br of residual torque Mzr
q_{Bz10}	QBZ10	Slope factor Br of residual torque Mzr
q_{Cz1}	QCZ1	Shape factor Cpt for pneumatic trail
q_{Dz1}	QDZ1	Peak trail Dpt = Dpt*(Fz/Fznom*R0)
q_{Dz2}	QDZ2	Variation of peak Dpt with load
q_{Dz3}	QDZ3	Variation of peak Dpt with camber
q_{Dz4}	QDZ4	Variaion of peak Dpt with camber squared.
q_{Dz6}	QDZ6	Peak residual torque Dmr = Dmr/ (Fz*R0)
q_{Dz7}	QDZ7	Variation of peak factor Dmr with load
q_{Dz8}	QDZ8	Variation of peak factor Dmr with camber
q_{Dz9}	QDZ9	Variation of peak factor Dmr with camber and load
q_{Ez1}	QEZ1	Trail curvature Ept at Fznom
q_{Ez2}	QEZ2	Variation of curvature Ept with load
q_{Ez3}	QEZ3	Variation of curvature Ept with load squared
q_{Ez4}	QEZ4	Variation of curvature Ept with sign of Alpha-t
q_{Ez5}	QEZ5	Variation of Ept with camber and sign Alpha-t
q_{Hz1}	QHZ1	Trail horizontal shift Sht at Fznom
q_{Hz2}	QHZ2	Variation of shift Sht with load
q_{Hz3}	QHZ3	Variation of shift Sht with camber
q_{Hz4}	QHZ4	Variation of shift Sht with camber and load

Magic Formula Steady-State Combined Slip

Figure 12. Combined Slip Condition (Combined Braking/Traction and Cornering)



Formula: Longitudinal Slip (Combined Slip)

$$F_x = F_{x0} \cdot G_{x\alpha}(\alpha, \kappa, F_z) \tag{56}$$

with $G_{x\alpha}$ a weighting function.

We write:

$$F_x = D_{x\alpha} \cos[C_{x\alpha} \arctan\{B_{x\alpha}\alpha_s - E_{x\alpha}(B_{x\alpha}\alpha_s - \arctan(B_{x\alpha}\alpha_s))\}] \tag{57}$$

$$\alpha_s = \alpha + S_{Hx\alpha} \tag{58}$$

with coefficients:

$$B_{x\alpha} = r_{Bx1}\cos[\arctan\{r_{Bx2}\kappa\}] \cdot \lambda_{x\alpha} \tag{59}$$

$$C_{x\alpha} = r_{Cx1} \tag{60}$$

$$D_{x\alpha} = \frac{F_{xo}}{\cos[C_{x\alpha}\arctan\{B_{x\alpha}S_{Hx\alpha} - E_{x\alpha}(B_{x\alpha}S_{Hx\alpha} - \arctan(B_{x\alpha}S_{Hx\alpha}))\}]} \tag{61}$$

$$E_{x\alpha} = r_{Ex1} + r_{Ex2}df_z \tag{62}$$

$$S_{Hx\alpha} = r_{Hx1} \tag{63}$$

The weighting function follows as:

$$G_{x\alpha} = \frac{\cos[C_{x\alpha}\arctan\{B_{x\alpha}\alpha_s - E_{x\alpha}(B_{x\alpha}\alpha_s - \arctan(B_{x\alpha}\alpha_s))\}]}{\cos[C_{x\alpha}\arctan[B_{x\alpha}S_{Hxa} - E_{x\alpha}(B_{x\alpha}S_{Hx\alpha} - \arctan(B_{x\alpha}S_{Hx\alpha}))]]} \tag{64}$$

Table 12. Longitudinal Coefficients, Combined Slip

Name:	**Name used in tire property file:**	**Explanation:**
r_{Bx1}	RBX1	Slope factor for combined slip Fx reduction
r_{Bx2}	RBX2	Variation of slope Fx reduction with kappa
r_{Cx1}	RCX1	Shape factor for combined slip Fx reduction
r_{Ex1}	REX1	Curvature factor of combined Fx
r_{Ex2}	REX2	Curvature factor of combined Fx with load
r_{Hx1}	RHX1	Shift factor for combined slip Fx reduction

Formula: Lateral Slip (Combined Slip)

$$F_y = F_{y0} \cdot G_{y\kappa}(\alpha, \kappa, \gamma, F_z) + S_{Vy\kappa} \tag{65}$$

with G_{yk} a weighting function and S_{Vyk} the 'κ-induced' side force can be written:

$$F_y = D_{y\kappa}\cos[C_{y\kappa}\arctan\{B_{y\kappa}\kappa_s - E_{y\kappa}(B_{y\kappa}\kappa_s - \arctan(B_{y\kappa}\kappa_s))\}] + S_{Vy\kappa} \tag{66}$$

$$\kappa_s = \kappa + S_{Hy\kappa} \tag{67}$$

with coefficients:

$$B_{y\kappa} = r_{By1}\cos[\arctan\{r_{By2}(\alpha - r_{By3})\}] \cdot \lambda_{y\kappa} \tag{68}$$

$$C_{y\kappa} = r_{Cy1} \tag{69}$$

$$D_{y\kappa} = \frac{F_{yo}}{\cos[C_{y\kappa}\arctan\{B_{y\kappa}S_{Hy\kappa} - E_{yk}(B_{y\kappa}S_{Hy\kappa} - \arctan(B_{y\kappa}S_{Hy\kappa}))\}]} \tag{70}$$

$$E_{y\kappa} = r_{Ey1} + r_{Ey2}df_z \tag{71}$$

$$S_{Hy\kappa} = r_{Hy1} + r_{Hy2}df_z \tag{72}$$

$$S_{Vy\kappa} = D_{Vy\kappa}\sin[r_{Vy5}\arctan(r_{Vy6}\kappa)] \cdot \lambda_{Vy\kappa} \tag{73}$$

$$D_{Vy\kappa} = \mu_y F_z \cdot (r_{Vy1} + r_{Vy2}df_z + r_{Vy3}\gamma) \cdot \cos[\arctan(r_{Vy4}\alpha)] \tag{74}$$

The weighting function appears to read:

$$G_{y\kappa} = \frac{\cos[C_{y\kappa}\arctan\{B_{y\kappa}\kappa_s - E_{y\kappa}(B_{y\kappa}\kappa_s - \arctan(B_{y\kappa}\kappa_s))\}]}{\cos[C_{y\kappa}\arctan\{B_{y\kappa}S_{Hy\kappa} - E_{y\kappa}(B_{y\kappa}S_{Hy\kappa} - \arctan(B_{y\kappa}S_{Hy\kappa}))\}]} \tag{75}$$

Table 13. Lateral Coefficients, Combined Slip

Name:	Name used in tire property file:	Explanation:
r_{By1}	RBY1	Slope factor for combined Fy reduction
r_{By2}	RBY2	Variation of slope Fy reduction with alpha
r_{By3}	RBY3	Shift term for alpha in slope Fy reduction
r_{Cy1}	RCY1	Shape factor for combined Fy reduction
r_{Ey1}	REY1	Curvature factor of combined Fy
r_{Ey2}	REY2	Curvature factor of combined Fy with load
r_{Hy1}	RHY1	Shift factor for combined Fy reduction
r_{Hy2}	RHY2	Shift factor for combined Fy reduction with load
r_{Vy1}	RVY1	Kappa induced side force Svyk/Muy*Fz at Fznom
r_{Vy2}	RVY2	Variation of Svyk/Muy*Fz with load
r_{Vy3}	RVY3	Variation of Svyk/Muy*Fz with camber
r_{Vy4}	RVY4	Variation of Svyk/Muy*Fz with alpha
r_{Vy5}	RVY5	Variation of Svyk/Muy*Fz with kappa
r_{Vy6}	RVY6	Variation of Svyk/Muy*Fz with atan (kappa)

Formula: Aligning Torque (Combined Slip)

$$M_z' = -t \cdot F_y' + M_{zr} + s \cdot F_x \tag{76}$$

with:

$$t = t(\alpha_{t,eq}) \tag{77}$$

$$= D_t \cos[C_t \arctan\{B_t\alpha_{t,eq} - E_t(B_t\alpha_{t,eq} - \arctan(B_t\alpha_{t,eq}))\}]\cos(\alpha)$$

$$F'_{y,\gamma=0} = F_y - S_{Vy\kappa} \tag{78}$$

$$M_{zr} = M_{zr}(\alpha_{r,eq}) = D_r \cos[\arctan(B_r\alpha_{r,eq})]\cos(\alpha) \tag{79}$$

$$s = \{s_{sz1} + s_{sz2}(F_y/F_{z0}) + (s_{sz3} + s_{sz4}df_z)\gamma\} \cdot R_0 \cdot \lambda_s \qquad (80)$$

with the arguments:

$$\alpha_{t,\,eq} = \arctan\sqrt{\tan^2\alpha_t + \left(\frac{K_x}{K_y}\right)^2\kappa^2} \cdot \mathrm{sgn}(\alpha_t) \qquad (81)$$

$$\alpha_{r,\,eq} = \arctan\sqrt{\tan^2\alpha_r + \left(\frac{K_x}{K_y}\right)^2\kappa^2} \cdot \mathrm{sgn}(\alpha_r) \qquad (82)$$

Table 14. Aligning Torque, Combined Slip

Name:	Name used in tire property file:	Explanation:
s_{sz1}	SSZ1	Nominal value of s/R0 effect of Fx on Mz
s_{sz2}	SSZ2	Variation of distance s/R0 with Fy/Fznom
s_{sz3}	SSZ3	Variation of distance s/R0 with camber
s_{sz4}	SSZ4	Variation of distance s/R0 with load and camber

Formula: Overturning Moment

$$M_x = R_o \cdot F_z \cdot \{q_{Sx1} \cdot \lambda_{Vmx} + (-q_{Sx2} \cdot \gamma + q_{Sx3} \cdot F_y/F_{z0}) \cdot \lambda_{Mx}\} \qquad (83)$$

Table 15. Overturning Coefficients

Name:	Name used in tire property file:	Explanation:
q_{sx1}	QSX1	Lateral force induced overturning couple
q_{sx2}	QSX2	Camber induced overturning couple
q_{sx3}	QSX3	Fy induced overturning couple

Formula: Rolling Resistance Torque

$$M_y = R_o \cdot F_z \cdot \{q_{Sy1} + q_{Sy2}F_x/F_{z0} + q_{Sy3}|V_x/V_{ref}| + q_{Sy4}(V_x/V_{ref})^4\} \qquad (84)$$

If q_{sy1} and q_{sy2} are both zero, then the following is true (as in *MF-Tyre 5.0*):

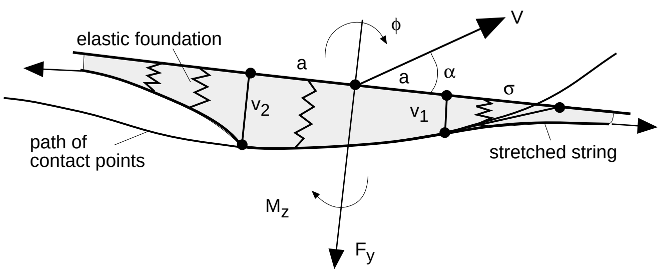
$$M_y = R_0(S_{Vx} + K_x \cdot S_{Hx}) \qquad (85)$$

Table 16. Rolling Coefficients

Name:	Name used in tire property file:	Explanation:
q_{sy1}	QSY1	Rolling resistance torque coefficient
q_{sy2}	QSY2	Rolling resistance torque depending on Fx
q_{sy3}	QSY3	Rolling resistance torque depending on speed
q_{sy4}	QSY4	Rolling resistance torque depending on speed^4
V_{ref}	LONGVL	Measurement speed

Transient Behavior

Figure 13. Stretched String Model for Transient Tire Behavior



Transient Model Equations

The present version, using slip speeds instead of α and κ, allows starting from stand-still. First-order lag of tire longitudinal and lateral deformations u and v are introduced through relaxation lengths σ_k and σ_a, see Figures 13:

$$\sigma_\kappa \frac{du}{dt} + |V_x| u = -\sigma_\kappa V_{sx} \tag{86}$$

$$\sigma_\alpha \frac{dv}{dt} + |V_x| v = \sigma_\alpha V_{sy} \tag{87}$$

These differential equations are based on the assumption that the contact points near the leading edge remain in the adhesion with the road surface (no sliding). The relaxation lengths (in this version not considered to decrease with increasing composite deformation slip) are functions of the vertical load and camber angle represented in a similar way as the slip stiffnesses K_x (Eq. 12) and K_y (Eq. 23).

$$\sigma_\kappa = F_z \cdot (p_{Tx1} + p_{Tx2} df_z) \cdot \exp(-p_{Tx3} df_z) \cdot (R_0 / F_{z0}) \cdot \lambda_{\sigma\kappa} \tag{88}$$

$$\sigma_\alpha = p_{Ty1} \sin[2 \arctan\{F_z / (p_{Ty2} F_{z0} \lambda_{F_{z0}})\}] \cdot (1 - p_{Ky3}|\gamma|) \cdot R_0 \lambda_{F_{z0}} \lambda_{\sigma\alpha} \tag{89}$$

The practical tire deformation slip quantities are defined as:

$$\kappa' = \frac{u}{\sigma_\kappa} \cdot \mathrm{sign}(V_x) \tag{90}$$

$$\tan\alpha' = \frac{v}{\sigma_\alpha} \tag{91}$$

Equations (56), (65), (76), (83), and (84) are subsequently used with arguments κ’ and α’ from Equations (90) and (91) instead of the longitudinal and lateral wheel slip quantities κ and α (Equations (9) and (10)).

$$F_x = F_x(\alpha', \kappa', F_z) \tag{92}$$

$$F_y = F_y(\alpha', \kappa', \gamma, F_z) \tag{93}$$

$$M_z' = M_z'(\alpha', \kappa', \gamma, F_z) \tag{94}$$

The Gyroscopic Couple

This moment due to tire inertia acting about the vertical axis reads:

$$M_{z,\,gyr} = c_{gyr} m_{belt} V_{rl} \frac{d\nu}{dt} \cos[\arctan(B_r \alpha_{r,\,eq})] \tag{95}$$

with parameter (in addition to the basic tire parameter m_{belt}):

$$c_{gyr} = q_{Tz1} \cdot \lambda_{gyr} \tag{96}$$

and

$$\cos[\arctan(B_r \alpha_{r,\,eq})] = 1 \tag{97}$$

for pure cornering conditions.

The total aligning torque now becomes:

$$M_z = M_z' + M_{z,\,gyr} \tag{98}$$

Table 17. Coefficients, Transient Response

Name:	Name used in tire property file:	Explanation:
p_{Tx1}	PTX1	Relaxation length SigKap0/Fz at Fznom
p_{Tx2}	PTX2	Variation of SigKap0/Fz with load
p_{Tx3}	PTX3	Variation of SigKap0/Fz with exponent of load
p_{Ty1}	PTY1	Peak value of relaxation length Sig_alpha
p_{Ty2}	PTY2	Shape factor for Sig_alpha
q_{Tz1}	QTZ1	Gyroscopic torque constant
M_{belt}	MBELT	Belt mass of the wheel

Switching from a Simple to a Complex Tire Model

MF-Tyre enables the user to switch from a simple tire model (for example only calculations for steady state pure cornering slip conditions) to tire model for transient combined slip situations. The parameter USE_MODE of the MF-Dataset determines the type of use of the tire model. In the Table 18 the possible options of USE_MODE are indicated. Note that the maximum valid USE_MODE depends on the tire test data used to determine the MF-Dataset parameters (that is, if only tire test data for pure cornering is fitted, the calculation of the contact forces under combined cornering and braking/traction slip is not possible unless the user adds the required additional parameters).

Table 18. The Different USE_MODE Values of MF-Tyre.

USE MODE:	State:	Slip conditions	MF-Tyre output (forces and torques)
0	spring	-	*0, 0, F_z, 0, 0, 0*
1	steady state	pure longitudinal	*F_x, 0, F_z, 0, M_y, 0*
2	steady state	pure lateral	*0, F_y, F_z, M_x, 0, M_z*
3	steady state	longitudinal and lateral (not combined)	*F_x, F_y, F_z, M_x, M_y, M_z*
4	steady state	combined slip forces	*F_x, F_y, F_z, M_x, M_y, M_z*
11	transient	pure longitudinal	*F_x, 0, F_z, 0, M_y, 0*
12	transient	pure lateral	*0, F_y, F_z, M_x, 0, M_z*
13	transient	longitudinal and lateral (not combined)	*F_x, F_y, F_z, M_x, M_y, M_z*
14	transient	combined slip forces	*F_x, F_y, F_z, M_x, M_y, M_z*

Some Practical Aspects

Rolling Resistance Torque

For a free rolling wheel at a constant forward velocity without camber and slip angle a drag force (rolling resistance) is generated. Passenger car tires usually have a rolling resistance coefficient between 0.7-1.2%; for truck tires the rolling resistance force is usually around 0.5% to 0.7% of the vertical load. Note that the parameter q_{sy1} in equation (80) determines the rolling resistance factor. According to the ISO sign convention this drag force as well as the rolling resistance torque M_y have negative signs ($q_{sy1} > 0$).

In order to reach equilibrium between the force and the torque on the wheel, in general a small negative value for the longitudinal slip is obtained.

Typical Tire Characteristics

For pure slip conditions (either longitudinal or lateral) three typical graphs can be made:

1. F_x as a function of the longitudinal slip κ;
2. F_y as a function of the slip angle α;
3. M_z as a function of the slip angle α.

In Figures 14 and 15, examples of these characteristics valid for the W-axis system are shown.

Figure 14. Longitudinal Force as a Function of Longitudinal Slip

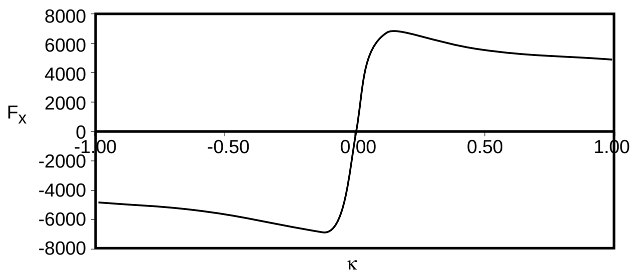
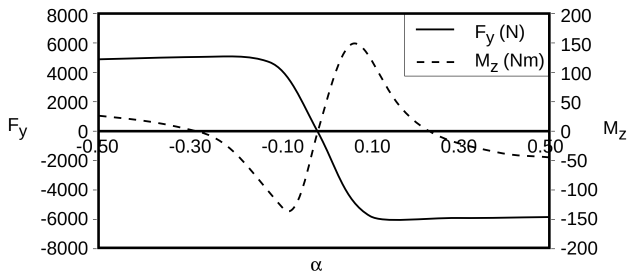


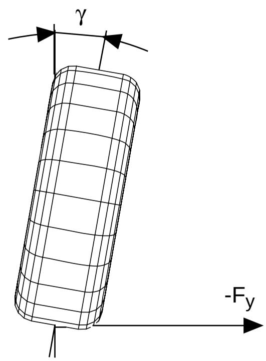
Figure 15. The Lateral Force and Self-Aligning Torque as a Function of the Slip Angle



Effect of Camber Angle

According to the W-axis system, an increase of the camber angle causes a decrease of the lateral force, as shown in Figure 16.

Figure 16. Tire Camber Angle and the Positive Direction of the Lateral Force According to the W-Axis System (Rear View)



Tire Model Output at Extreme Input Values

At extreme large input values, like a vertical load more than 3 times the nominal tire load, a real physical tire might puncture or go to pieces. In the tire model measures have been taken to avoid calculation errors or a computer simulation break down. Depending on your simulation software the tire model warns the user when the input exceeds the validity range of the MF-Dataset.

The tire property files, generated by MF-Tool, contain maxima and minima values for the tire model input, defining the validity range of the MF-Dataset:

- Fzmin and Fzmax for the vertical load F_z
- Alpmin and Alpmax for the slip angle a
- Cammin and Cammax for the camber angle g
- Kpumin and Kpumax for the longitundinal slip k.

In general the tire model fixes the B, C, D, E and shift factors when exceeding the upper mentioned limits at the corresponding limit. For vertical loads smaller than Fzmin the output of the tire model is equal to the output of the tire model for Fzmin proportionally scaled to zero output.

Standard Tire Interface (STI)

As a result of the First International Colloquium on Tire Models for Vehicle Dynamics Analysis on October 21-22, 1991, the international Tire Workshop working group was established (TYDEX).

The working group concentrated on tire measurements and tire models used for vehicle simulation purposes. For most vehicle dynamics studies people usually develop their own tire models. Since all car manufacturers and their tire suppliers have the same goal (that is development of tires to improve dynamic safety of the vehicle) standardisation in tire behaviour description should be aimed for.

In TYDEX two expert groups were defined with following goals:

- The first expert group (Tire Measurements - Tire Modelling) has as its main goal to specify an interface between tire measurements and tire models. The work shall include a description of the test conditions. The interface could be described as a definition of a method or format to describe tire measurement data in such a way that it contains all necessary items to fit tire models to the underlying data. The format shall also allow for a description of the test conditions.
- The second expert group (Tire Modelling - Vehicle Modelling) has as its main goal to specify an interface between tire models and simulation tools. Intentionally, use of this interface will ensure that a wide range of simulation software can be linked to a wide range of tire software available.

Both expert groups consist of participants of vehicle industry (passenger cars and trucks), tire manufacturers, other suppliers and research laboratories. The large number of participants indicates that there is a need for this kind of 'standardization' work. DVR is strongly involved in TYDEX.

At the Second International Colloquium on Tire Models for Vehicle Dynamics Analysis on February 19 and 20, 1997 the final documents on both interfaces have been presented [9]. The TYDEX-Format [1] describes a standard format for the exchange of tire testing and modelling data; the second document describes the standard interface between tire model and vehicle model, called the Standard Tire Interface (STI) [2].

At the moment, a concept for the description of the Tire Modeling - Vehicle Modeling interface have been developed and will be tested within the different companies. This interface is named the Standard Tire Interface (STI) [2].

The Standard Tire Interface prescribes a subroutine call with a number of subroutine arguments to pass all relevant information from tire models to multi-body programs and vice versa. The subroutine represents a shell around tire software and is fixed to the axle hub which is modelled by the multi-body programs.

MF-Datasets and MF-Tool

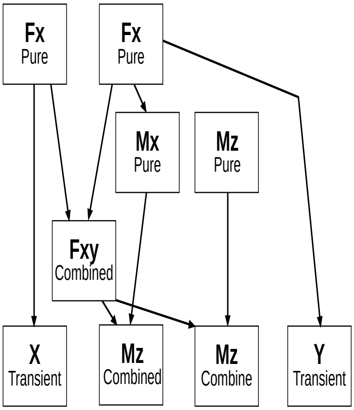
The final objective of the user is to optimize vehicle behaviour (including tire behaviour) using the potential of simulation software. Because the tire properties determine to a great extent the vehicle behaviour, a tire model without proper tire data will be useless in most cases. For full optimization purposes the engineer requires the availability of datasets under a large range of conditions.

Tire Measurements

Tire characteristics can be well described by the Magic Formula tire model. The formulae are specified by a set of Magic Formula parameters that represent the characteristics in a compact form. The parameters depend on the type of the tire and the road conditions and can be obtained from outdoor and/or laboratory tests.

Calculation of Magic Formula Parameters

Figure 17. Fit Process



Calculation of parameters from the measurement data is performed with regression techniques (also known as parameter fitting ref [8]). In such a so called fitting procedure, the results from measurements under pure slip conditions have to be used first to determine the Magic Formula parameters for side force, self aligning torque and longitudinal force and in a second step the parameters for combined slip conditions, see Figure 17. The pure cornering measurements must include the influence of camber. The parameters for transient cornering and braking are based on the steady state pure cornering and braking properties.

The MF-Tool+ software of Mf-Tyre offers the engineer a user-friendly tool to determine the MF-Tyre parameters (MF-Datasets) out of any Force and Moment tyre test data. Next to software also MF-Datasets can be selected out of existing Libraries. See www.delft-tyre.com.

Definitions

General

Table 19. General Definitions

Term:	Definition:
Inertial coordinate system	Inertial space according to ISO
Road tangent plane	Plane with the normal unit vector n_r (tangent to the road) in C.
Wheel centre O	Centre of the wheel
C-axis system	Coordinate system mounted on the wheel carrier at the Wheel center orientation accoriding ISO.
Wheel plane	The plane in the wheel centre that is formed by the wheel when considered a rigid disc with zero width.
Contact point C	Contact point between tyre and road, defined as the intersection of the wheel plane and the projection of the wheel axis onto the road plane.
W-axis system	Coordinate system at the tyre contact point C, orienation according ISO.

Tire Kinematics

Table 20. Tire Kinematics Definitions

Abbreviation:	Definition:	Units:
R_0	Unloaded tire radius	[m]
R	Loaded tire radius	[m]
R_e	Effective tire radius	[m]
r_t	Tire cross section radius (half tyre width)	[m]
ρ	Radial tire deflection	[m]
ρ^d	Dimensionless radial tire deflection	[-]
ρ_{Fz0}	Radial tire deflection at nominal load	[m]
m_{belt}	Tire belt mass	[kg]
Ω	Rotational velocity of the wheel	[$rads^{-1}$]
h_α	Distance wheel centre to road plane	[m]

Slip Quantities

Table 21. Slip Quantities Definitions

Abbreviation:	Definition:	Units:
V	Vehicle speed	[ms^{-1}]
V_{sx}	Slip speed in x-direction	[ms^{-1}]
V_{sy}	Slip speed in y-direction	[ms^{-1}]
V_s	Resulting slip speed	[ms^{-1}]

Table 21. Slip Quantities Definitions

Abbreviation:	Definition:	Units:
V_x	Rolling speed in x-direction	[ms^{-1}]
V_y	Lateral speed of tire contact center	[ms^{-1}]
V_r	Linear speed of rolling	[ms^{-1}]
κ	Longitudinal slip	[-]
α	Slip angle	[rad]
γ	Camber angle	[rad]

Forces and Moments

Table 22. Force and Moment Definitions

Abbreviation:	Definition:	Units:
F_z	vertical wheel load	[N]
F_{z0}	nominal (rated) load	[N]
df_z	dimensionless vertical load	[-]
F_x	longitudinal force	[N]
F_y	lateral force	[N]
F_z	nominal load	[N]
M_x	overturning couple	[Nm]
M_y	braking/driving moment	[Nm]
M_z	aligning moment	[Nm]

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Delft-Tyre

Using the MF-MCTyre Model

Overview

The Magic-Formula (MF-MCTyre) tire model is developed by TNO Automotive. With respect to the standard Magic Formula (MF-Tyre), this tire model is better suited for very large camber angles. Typical applications are motorcycles and vehicle roll-over.

This chapter includes the following sections:

About MF-MCTyre

The MF-MCTyre model uses a method known as the Magic Formula to calculate the steady-state behavior of a tire. The Magic Formula is actually a set of mathematical formula based on the physical background of the tire, road, and the tire-to-road contact.

The Magic Formula tyre model aims at an accurate description of the steady-state behaviour of a tyre by providing a set of mathematical formulae, which are partly based on a physical background. The Magic Formula calculates the forces (F_x, F_y) and moments (M_x, M_y, M_z) acting on the tyre under pure and combined slip conditions, using longitudinal and lateral slip (κ, α), wheel camber (γ), and the vertical force (F_z) as input quantities. In addition to the Magic Formula description, a set of differential equations is defined, representing the transient behaviour of the tyre with respect to handling at frequencies up to 8 Hz.

Further information can be found on the internet site: www.delft-tyre.com. This chapter concentrates on the Magic Formula tyre model for motorcycle tires, the MF-MCTyre 1.1 subroutine containing this Magic Formula tire model version and its transient extensions.

What's New in Version 1.1

Compared to MF-MCTyre 1.0, following items have been changed/introduced:

- The scaling factors for the shifts have been defined such that conicity and plysteer effects can be easily switched off.
- Into the modelling of combined cornering and braking/traction E factors have been introduced, making the modelling more accurate.
- The rolling resistance torque has become a function of forward speed.
- The influence of the camber in the peak F_x has been introduced.

In Table 24 the additional parameters have been listed.

Table 24. New Parameters Introduced in MF-MCTyre 1.1

Name:	Name used in tire property file:	Explanation:	Default value:
$\lambda_{\gamma x}$	LGAX	Scale factor of camber for Fx	1
$\lambda_{\gamma y}$	LGAY	Scale factor of camber force stiffness	1
λ_{Vmx}	LVMX	Scale factor of Mx vertical shift	1
p_{Dx3}	PDX3	Variation of friction Mux with camber	0
r_{Ex1}	REX1	Curvature factor of combined Fx	0
r_{Ex2}	REX2	Curvature factor of combined Fx with load	0
r_{Hy2}	RHY2	Shift factor for combined Fy reduction with load	0
r_{Ey1}	REY1	Curvature factor of combined Fy	0
r_{Ey2}	REY2	Curvature factor of combined Fy with load	0
q_{sy3}	QSY3	Rolling resistance torque depending on speed	0
q_{sy4}	QSY4	Rolling resistance torque depending on speed^4	0

Furthermore, LONGVL should be defined and have a positive value. When the default values are used, the tire model is fully backward compatible with MF-MCTyre 1.0.

Tire-Road Interaction

The tire-road contact forces are mainly dependent of the tire mechanical properties (stiffness and damping), the road condition (the friction coefficient between tire and road, the road structure), and the motion of the tire relative to the road (the amount and direction of slip).

The major control and disturbance forces on a vehicle arise from the contact of the tires with the road. The vertical loads transfer the weight of the vehicle to the road. Due to the compliance of the tires, a vehicle is cushioned against disturbances by small road irregularities. The traction and braking forces arise from the longitudinal tire forces. Lateral forces are required to control the direction of travel of the vehicle. The lateral behaviour of tires is, therefore, dominant in vehicle handling. Proper description of the dynamic behaviour of a vehicle requires an accurate model of the tire-road contact forces and moments generating properties under all of these different conditions.

Figure 18. Tire Factors

Tyre factors

	(Quasi) steady state		*Vibratory state*
in-plane	load carrying capacity braking/driving performance rolling resistance	radial deflection longitudinal slip and distortion	cushioning capacity dynamic coupling
out-of-plane	cornering performance lateral shift of Fz	lateral slip and distortion	phase shifts and destabilisation

primary effects
secondary effects

interactions between
in- and out-of-plane behaviour

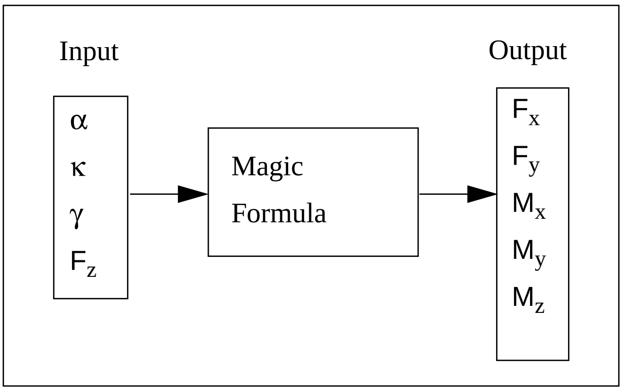
Tire behaviour results from a combination of several aspects. Factors may be distinguished which concern the primary tasks of the tire which involve (often important) secondary effects. In Figure 18 these factors have been brought in matrix form. A distinction has been made between (quasi) steady-state and vibratory behaviour and besides between in-plane and out-of-plane aspects. The primary task factors are shaded in green. The remaining secondary factors are not shaded.

The requirements to transmit forces in the three perpendicular directions (F_x, F_y, and F_z) and to cushion the vehicle against road irregularities involve secondary factors such as, radial, lateral, and longitudinal distortions and slip.

Although considered as secondary factors, some of the quantities involved have to be treated as input variables into the system which generate the forces. Figure 19 presents the input and output vectors. In this diagram the tire is assumed to be uniform and to move over a flat road surface. The input vector results from motions of the wheel relative to the road. It is advantageous to recognize the fact that, for small deviations from the straight-ahead motion, in-plane and out-of-plane motions of the assumedly symmetric wheel-tire system are uncoupled.

The forces and moments are considered as output quantities of the tire model. They are assumed to act on a rigid disc with inertial properties equal to those of the undeflected tire. The forces may differ from the corresponding forces acting on the road due to the vibrations of the tire relative to the wheel rim. Braking and traction torques are considered as acting on the rotating disc.

Figure 19. Input and Output Variables of the Magic Formula Tire Model

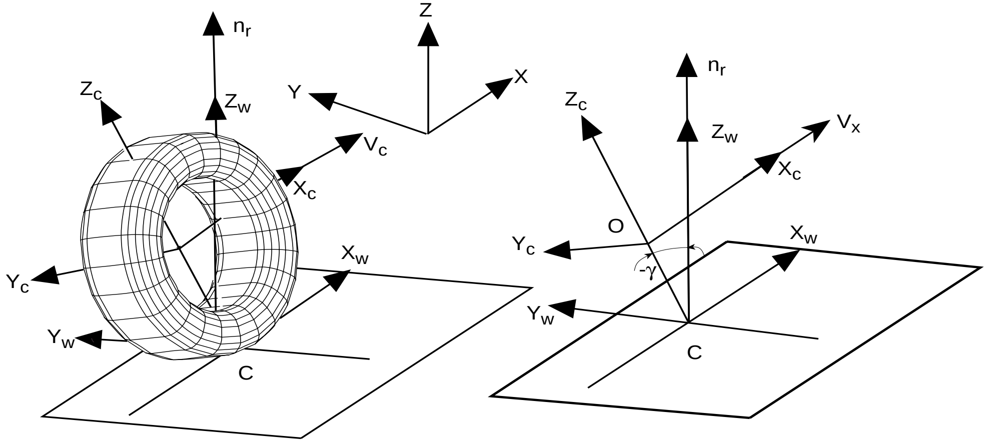


Axis Systems and Definitions

W-Axis System

MF-Tyre conforms to the TYDEX STI conventions described in the TYDEX-Format [1] and the Standard Tire Interface [2]. Two TYDEX coordinate systems with ISO orientation are particularly important, the C- and W-axis systems as detailed in Figure 20.

Figure 20. TYDEX C- and W-Axis Systems Used in MF-Tyre, According to TYDEX



The C-axis system is fixed to the wheel carrier with the longitudinal x_c-axis parallel to the road and in the wheel plane (x_c-z_c-plane). The origin O of the C-axis system is the wheel center.

The origin of the W-axis system is the road contact-point (or ‘point of intersection’) C defined by the intersection of the wheel plane, the plane through the wheel spindle and the road tangent plane. The orientation of the W-axis system agrees to ISO. The forces and torques calculated by MF-MCTyre, which depend on the vertical wheel load F_z along the z_w-axis and the slip quantities, are projected in the W-axis system. The x_w-y_w-plane is the tangent plane of the road in the contact point C.

The camber angle is defined by the inclination angle between the wheel plane and the normal n_r to the road plane (x_w-y_w-plane).

Units

Next to the convention to the TYDEX W-axis system, all units of the parameters and variables used in MF-MCTyre agree to the SI units. Table 25 provides an overview of the most important parameters and variables, see also, Definitions on page 97.

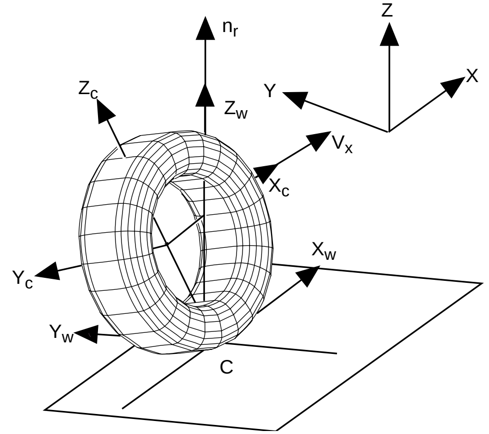
Table 25. SI Units Used in MF-Tyre

Variable type:	Name:	Abbreviation:	Unit:
Angle	Slip angle camber angle	a γ	Radians
Force	Longitudinal force Lateral force Vertical load	F_x F_y F_z	Newton
Moment	Overturning moment Rolling resistance moment Self aligning moment	M_x M_y M_z	Newton.meter
Speed	Longitudinal speed Lateral speed Longitudinal slip speed Lateral slip speed	V_x V_y V_{sx} V_{sy}	Meters per second
Rotational Speed	Tire rolling speed	Ω	Radians per second

The Contact-Point C and the Normal Load

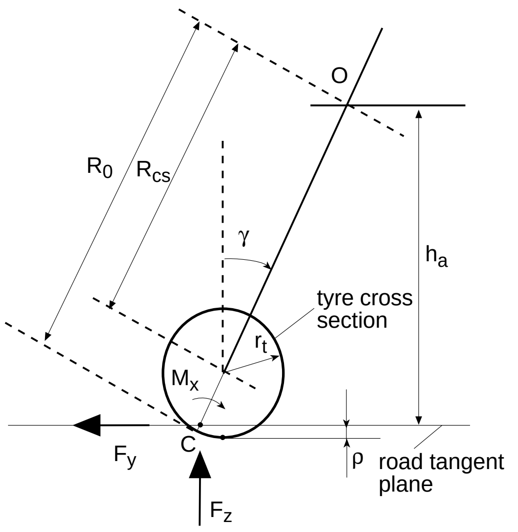
The radius of curvature of the road profile is considered large as compared to the radius of the tire. The tire is assumed to have only a single contact point (C) with the road profile. Furthermore, for calculating the motion of the tire relative to the road, the road is approximated by its tangent plane at the point on the road below the wheel centre (see Figure 21). The tangent plane is an accurate approximation of the road, as long as the road radius of curvature is not too small (that is, not smaller than 2 meters).

Figure 21. Contact Point C (Intersection Between Normal-to-Road Tangent and Wheel Plane)



Formula: Normal Load

Figure 22. Tire Normal Load Calculated with the Tire Compression



The normal compression ρ of the tire on the road can be defined by the tire free radius R_0, the cross section tire radius $r_t = 0.5W$ and the axle height h_a to the road tangent plane (see Figures 22):

$$\rho' = r_t + (R_0 - r_t)\cos\gamma - h_a \qquad \rho = \max(0, \rho') \tag{99}$$

The normal load F_z of the tire is calculated with:

$$F_z = C_z \rho + K_z \cdot \dot{\rho} \tag{100}$$

with $\dot{\rho}$ the deflection velocity of the tire.

Table 26. Normal Load

Name:	Name used in tire property file:	Explanation:
R_o	UNLOADED_RADIUS	Free tire radius
W	WIDTH	Nominal section width of the tire
C_z	VERTICAL_STIFFNESS	Tire vertical stiffness
K_z	VERTICAL_DAMPING	Tire vertical damping

The Effective Tire Rolling Radius

The loaded tire radius R which is defined by the distance of the wheel centre to the centre of tire contact (see Figure 23).

The effective rolling radius R_e (at free rolling of the tire) is defined by:

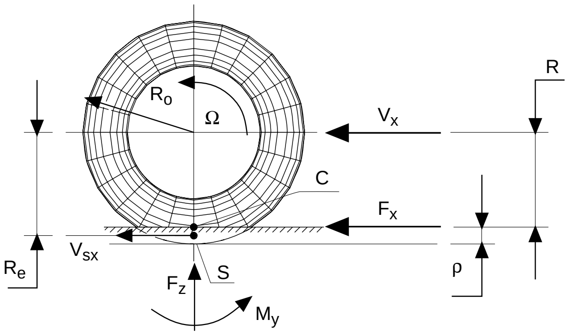
$$R_e = \frac{V_x}{\Omega} \tag{101}$$

For radial tires the effective rolling radius decreases with increasing vertical load at low loads, but around its nominal load the influence of the vertical load is small, see Figure 24.

When assuming a constant vertical tire stiffness C_z, the radial tire deflection ρ can be calculated with:

$$\rho = \frac{F_z}{C_z} \tag{102}$$

Figure 23. Effective Rolling Radius and Longitudinal Slip



For the estimation of the effective radius R_e a Magic Formula approach is chosen. The equation of the effective radius R_e reads:

$$R_e = R_0 - \rho_{F_{z0}}(D\arctan(B\rho^d) + F\rho^d) \tag{103}$$

in which R_0 is the unloaded free radius and the nominal tire deflection $\rho_{F_{z0}}$ is defined by:

$$\rho_{F_{z0}} = \frac{F_{z0}}{C_z} \tag{104}$$

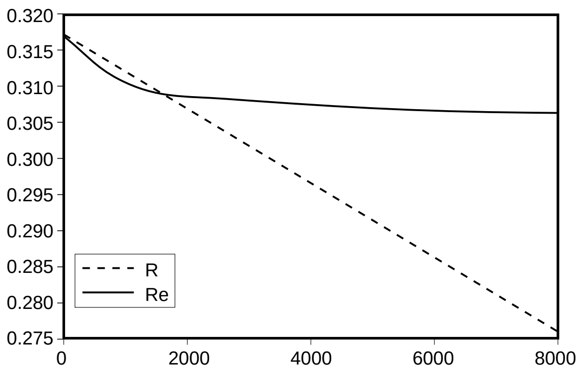
and the dimensionless radial tire deflection ρ^d can be calculated with:

$$\rho^d = \frac{\rho}{\rho_{F_{zo}}} \tag{105}$$

For a large range of tires, appropriate coefficient values are:

- <u>3,...,B,...,12</u>
  stretches the ordinate of the arctangent function, a large value of *B* means a high slope at F_z=0.
- <u>0.2,...,D,...0.4</u>
  defines the shift from the asymptote at high wheel loads.
- <u>0.03,...,F,...,0.25</u>
  defines the ratio between tire radial deformation ρ and effective tire deformation. Low values are obtained for extremely stiff tires.

Figure 24. The Tire Effective Rolling Radius as a Function of the Vertical Load (B=8.4, D=0.27 and F=0.045)



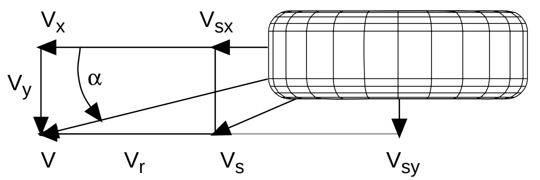
In Figure 24, an example of the effective tire rolling radius is shown for a passenger car tire. The approximation of R_e is made with the proposed formula with: B = 8.4, D = 0.27, and F = 0.045.

Table 27. Effective Rolling Radius Parameters

Name:	Name used in tire property File:	Explanation:
F_{z0}	FNOMIN	Nominal wheel load
B	BREFF	Low load stiffness effective rolling radius
D	DREFF	Peak value of effective rolling radius
F	FREFF	High load stiffness eff. rolling radius

Tire Slip Quantities

Figure 25. Slip Quantities at Combined Cornering and Braking/Traction



The longitudinal slip speed is defined as:

$$V_{sx} = V_x - \Omega R_e \tag{106}$$

and the lateral slip speed:

$$V_{sy} = V_y \tag{107}$$

The practical slip quantities κ and α are defined as:

$$\kappa = -\frac{V_{sx}}{V_x} \tag{108}$$

$$\tan\alpha = \frac{V_{sy}}{|V_x|} \tag{109}$$

with V_{sx} and V_{sy} the components of the slip speed that may be defined as the velocity of point *S* in the W-axis system (see Figure 25).

With Ω denoting the rotational speed of the tire, the linear rolling speed becomes:

$$V_r = R_e\Omega \tag{110}$$

The Magic Formula Tire Model (MF-MCTyre)

Introduction

For a given pneumatic tire and road condition, the tire forces due to slip follow a typical characteristic. The characteristics can be accurately approximated by a special mathematical function which is known as the “Magic Formula.” The parameters in the Magic Formula depend on the type of the tire and the road conditions. These parameters can be derived from experimental data obtained from tests. The tire is rolled over a road at various loads, orientations and motion conditions.

The Magic Formula tire model is mainly of an empirical nature and contains a set of mathematical formula, which are partly based on a physical background. The Magic Formula calculates the forces (F_x, F_y) and moments (M_x, M_y, M_z) acting on the tire at pure and combined slip conditions, using longitudinal and/or lateral slip (κ, α), wheel camber γ, and the vertical force F_z as input quantities. The model takes into account plysteer and conicity. An extension has been provided that describes transient and oscillatory tire behaviour for limited frequencies smaller than 8 Hz and wavelengths larger than the tire circumference.

History of the Magic Formula

Through the initiative of Volvo Car Corporation, a cooperate effort was started in the mid-eighties with the Delft University of Technology to develop a tire model that accurately describes the tire's ability to have horizontal forces generated between road and tire.

The first Magic Formula version was presented in 1987 [3]. The basic idea of using the sine and arcsine functions was described for mainly pure slip conditions. Further prototype formulas were proposed for combined slip conditions.

In the second version [4], presented in 1989 the formula for combined cornering conditions, based on physical background, were improved and tire relaxations lengths were introduced in order to have a first order approach of the transient tire behaviour. This model was improved on the description for combined slip calculations in 1993 [5].

Bayle e.o. [6] proposed to have a more empirical approach, reducing the complexity of the force calculations under combined slip conditions and yielding a considerably higher calculation speed. Their method improved the calculation speed during the calculation of the Magic Formula parameters and during simulation calculations.

The latest version [7] combines the advantage of the previous versions and has been modified for the following aspects:

- The self aligning torque has been made dependent on the side force by a new approach using the pneumatic trail in pure and combined slip conditions.
- The forces under combined slip conditions are calculated according to the proposal of Bayle [6].
- Formulae describing overturning moment have been introduced.
- The transient tire behaviour has been improved to enable zero speed.
- Loading variations to tire lift off situations.
- The parameters used in formulae are dimensionless improving manipulations with tire characteristics and parameter calculations (fitting).
- Scaling factors are introduced for vehicle-tire optimization purposes.

The Magic Formula tire models only considered passenger car and truck tires, i.e. tires for which camber angles larger than 10 degrees are exeptional. First developments of a Magic Formula tire model applicable for motor cycle tires, which were initiated for pure cornering slip conditions by de Vries [8]. Based on his experience and the knowledge of MF-Tyre 5.0, the MF-MCTyre 1.0 had been developed.

Allthough MF-Tyre was the basis for the development of MF-MCTyre, the differences between the two models do not allow interchange of tire model coefficients. In MF-MCTyre 1.1 small improvements have been made with reference to MF-MCTyre 1.0.

Learning the Basics of the Magic Formula

The general form (sine version) of the formula reads:

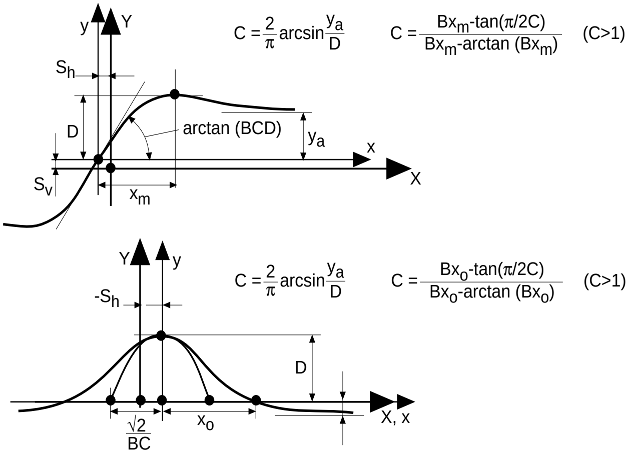
$$Y(x) = D\sin[C\arctan\{B \cdot X - E(BX - \arctan(B \cdot X))\}] \tag{111}$$

where Y(x) is either: F_x or F_y.

The self-aligning moment M_z is calculated by using the lateral force F_y and the pneumatic trail t, which is based on a cosine type of Magic Formula:

$$Y(x)) = D\cos[C\arctan\{Bx - E(Bx - \arctan(Bx))\}] \tag{112}$$

Figure 26. Curves Produced by the Sine and Cosine Versions of the Magic Formula



When the formula is used to calculate the forces generated by the tire, the following variables should serve as input for the Magic Formula:

Input Variables

Longitudinal slip	κ	[-]
Slip angle	α	[rad]
Camber angle	γ	[rad]
Normal wheel load	F_z	[N]

In case the complete model including transient properties is used, the transient tire quantities are employed instead of the wheel slip quantities κ and α (cf. Paragraph 4.4).

Output Variables (in contact point C)

Longitudinal force	F_x	[N]
Lateral force	F_y	[N]
Overturning couple	M_x	[Nm]
Rolling resistance torque	M_y	[Nm]
Aligning torque	M_z	[Nm]

Basic Tire Parameters

Nominal (rated) load	F_{z0}	[N]
Unloaded tire radius	R_0	[m]
Tire belt mass	m_{belt}	[kg]

Furthermore the normalized vertical load increment df_z is defined:

$$df_z = \frac{F_z - F'_{z0}}{F'_{z0}} \quad [-] \tag{113}$$

with the possibly adapted nominal load (using the user scaling factor λ_{Fz0}):

$$F'_{z0} = F_{z0} \cdot \lambda_{F_{z0}} \tag{114}$$

Tire Model Parameters

In the subsequent sections, formulae are given with non-dimensional parameters a_{ijk} with the following values and connections:

Parameter:		Definition:
a =	p	Force at pure slip
	q	Moment at pure slip
	r	Force at combined slip
	s	Moment at combined slip
i =	B	Stiffness factor
	C	Shape factor
	D	Peak value
	E	Curvature factor
	K	Slip stiffness = BCD
	H	Horizontal shift
	V	Vertical shift
	s	Moment at combined slip
	t	Transient tire behavior
j =	x	Along the longitudinal axis
	y	Along the lateral axis
	z	About the vertical axis
k =	1, 2, ...	

User Scaling Factors

For your convenience, a set of scaling factors is available to examine the influence of changing a number of important overall parameters. The default value of these factors is one. The following factors have been defined:

Table 28. Scaling Coefficient, Pure Slip

Name:	Name used in tire property file:	Explanation:
λ_{Fzo}	LFZO	Scale factor of nominal load
λ_{Cx}	LCX	Scale factor of F_x shape factor
$\lambda_{\mu x}$	LMUX	Scale factor of F_x peak friction coefficient
λ_{Ex}	LEX	Scale factor of F_x curvature factor
λ_{Kx}	LKX	Scale factor of F_x slip stiffness
λ_{Vx}	LVX	Scale factor of F_x vertical shift
$\lambda_{\gamma x}$	LGAX	Scale factor of camber for F_x
λ_{Cy}	LCY	Scale factor of F_y shape factor
$\lambda_{\mu y}$	LMUY	Scale factor of F_y peak friction coefficient
λ_{Ey}	LEY	Scale factor of F_y curvature factor
λ_{Ky}	LKY	Scale factor of F_y cornering stiffness
$\lambda_{C\gamma}$	LCC	Scale factor of camber shape factor
$\lambda_{K\gamma}$	LKC	Scale factor of camber stiffness (K-factor
$\lambda_{E\gamma}$	LEC	Scale factor of camber curvature factor
λ_{Hy}	LHY	Scale factor of F_y horizontal shift
$\lambda_{\gamma y}$	LGAY	Scale factor of camber force stiffness
λ_{t}	LTR	Scale factor of peak of pneumatic trail
λ_{Mr}	LRES	Scale factor of peak of residual torque
$\lambda_{\gamma z}$	LGAZ	Scale factor of camber torque stiffness
λ_{Mx}	LMX	Scale factor of overturning couple
λ_{vMx}	LVMX	Scale factor of Mx vertical shift
λ_{My}	LMY	Scale factor of rolling resistance torque

Table 29. Scaling Coefficients, Combined Slip

Name:	Name used in tire property file:	Explanation:
$\lambda_{x\alpha}$	LXAL	Scale factor of alpha influence on Fx
$\lambda_{y\kappa}$	LYKA	Scale factor of kappa influence on Fy
$\lambda_{Vy\kappa}$	LVYKA	Scale factor of kappa induced Fy
λ_{s}	LS	Scale factor of moment arm of Fx

Table 30. Scaling Coefficinets, Transient Response

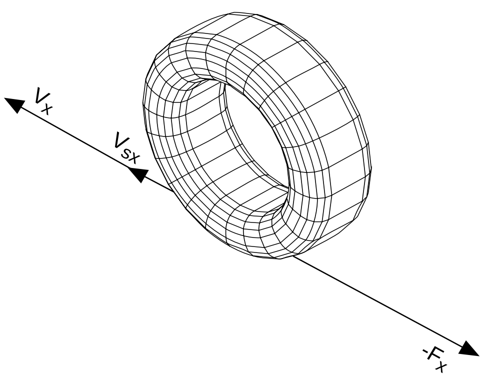
Name:	Name used in tire property file:	Explanation:
$\lambda_{\sigma\kappa}$	LSGKP	Scale factor of relaxation length of Fx
$\lambda_{\sigma\alpha}$	LSGAL	Scale factor of relaxation length of Fy
λ_{gyr}	LGYR	Scale factor of gyroscopic torque

Steady-State: Magic Formula

Steady-State Pure Slip

Formula: Longitudinal Slip (Pure Slip)

Figure 27. Longitudinal Slip Condition (Pure Braking/Traction)



$$F_x = F_{x0}(\kappa, \gamma, F_z) \tag{115}$$

$$F_{x0} = D_x \sin[C_x \arctan\{B_x\kappa_x - E_x(B_x\kappa_x - \arctan(B_x\kappa_x))\}] + S_{Vx} \tag{116}$$

$$\kappa_x = \kappa + S_{Hx} \tag{117}$$

the scaled camber angle:

$$\gamma_x = \gamma \cdot \lambda_{\gamma x} \tag{118}$$

with coefficients:

$$C_x = p_{Cx1} \cdot \lambda_{Cx} \tag{119}$$

$$D_x = \mu_x \cdot F_z \tag{120}$$

$$\mu_x = (p_{Dx1} + p_{Dx2} df_z) \cdot (1 - p_{Dx3} \cdot \gamma_x^{\ 2}) \lambda_{\mu x} \tag{121}$$

$$E_x = (p_{Ex1} + p_{Ex2} df_z + p_{Ex3} df_z^2) \cdot \{1 - p_{Ex4} \operatorname{sgn}(\kappa_x)\} \cdot \lambda_{Ex} \ (\leq 1) \tag{122}$$

$$K_x = F_z \cdot (p_{Kx1} + p_{Kx2} df_z) \cdot \exp(p_{Kx3} df_z) \cdot \lambda_{Kx} \tag{123}$$

$$\left(K_x = B_x C_x D_x = \frac{\partial F_{x0}}{\partial \kappa_x} \text{ at } \kappa_x = 0 \right)$$

$$B_x = K_x / (C_x D_x) \tag{124}$$

$$S_{Hx} = -(q_{sy1} F_z \lambda_{My} + S_{Vx}) / K_x \tag{125}$$

See also the definition of the rolling resistance torque M_y in Eqs. (186).

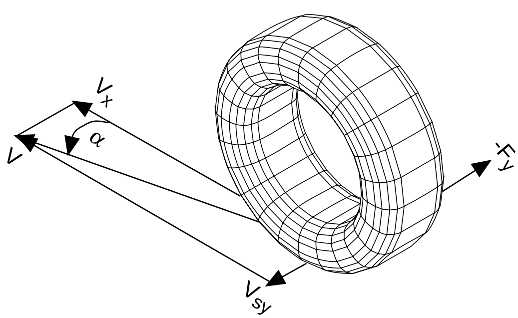
$$S_{Vx} = F_z \cdot (p_{Vx1} + p_{Vx2} df_z) \cdot \lambda_{Vx} \cdot \lambda_{\mu x} \tag{126}$$

Table 31. Longitudinal Coefficients, Pure Slip

Name:	Name used in tire property file:	Explanation:
p_{Cx1}	PCX1	Shape faxtor Cfx for longitudinal force
p_{Dx1}	PDX1	Longitudinal friction Mux at Fznom
p_{Dx2}	PDX2	Variation of friction Mux with load
p_{Dx3}	PDX3	Variation of friction Mux with camber
p_{Ex1}	PEX1	Longitudinal curvature Efx at Fznom
p_{Ex2}	PEX2	Variation of curvature Efx with load
p_{Ex3}	PEX3	Variation of curvature Efx with load squared
p_{Ex4}	PEX4	Factor in curvature Efx while driving
p_{Kx1}	PKX1	Longitudinal slip stiffness Kfx/Fz at Fznom
p_{Kx2}	PKX2	Variation of slip stiffness Kfx/Fz with load
p_{Kx3}	PKX3	Exponent in slip stiffness Kfx/Fz with load
p_{Vx1}	PVX1	Vertical shift Svx/Fz at Fznom
p_{Vx2}	PVX2	Variation of shift Svx/Fz with load

Formula: Lateral Slip (Pure Slip)

Figure 28. Lateral Slip Condition Excluding Aligning Torque (Pure Cornering)



$$F_y = F_{y0}(\alpha, \gamma, F_z) \tag{127}$$

$$F_{y0} = D_y \sin(C_y \arctan\{B_y\alpha_y - E_y(B_y\alpha_y - \arctan(B_y\alpha_y))\} \tag{128}$$

$$+ \quad C_\gamma \arctan\{B_\gamma\gamma_y - E_\gamma(B_\gamma\gamma_y - \arctan(B_\gamma\alpha_\gamma))\})$$

$$\alpha_y = \alpha + S_{Hy} \qquad (C_y + C_\gamma < 2) \tag{129}$$

the scaled camber angle:

$$\gamma_y = \gamma \cdot \lambda_{\gamma y} \tag{130}$$

with coefficients:

$$C_y = p_{Cy1} \cdot \lambda_{Cy} \tag{131}$$

$$D_y = \mu_y \cdot F_z \tag{132}$$

$$\mu_y = p_{Dy1} \cdot \exp(p_{Dy2} df_z) \cdot (1 - p_{Dy3}\gamma_y^2) \cdot \lambda_{\mu y} \tag{133}$$

$$E_y = \{p_{Ey1} + p_{Ey2}\gamma_y^2 + (p_{Ey3} + p_{Ey4}\gamma_y) \cdot \mathrm{sign}(\alpha_y)\} \cdot \lambda_{Ey} \qquad (\leq 1) \tag{134}$$

$$K_y = p_{Ky1} F_{zo} \sin\left[p_{Ky2} \arctan\left\{\frac{F_z}{(p_{Ky3} + p_{Ky4}\gamma_y^2) F_{zo} \lambda_{Fzo}}\right\}\right] \tag{135}$$

$$(1 - p_{Ky5}\gamma_y^2) \cdot \lambda_{Fzo} \cdot \lambda_{Ky}$$

$$\left(K_y = B_y C_y D_y = \frac{\partial F_{yo}}{\partial \alpha_y} \; at \qquad \alpha_y = 0\right)$$

$$B_y = K_y/(C_y D_y) \tag{136}$$

$$S_{Hy} = p_{Hy1} \cdot \lambda_{Hy} \tag{137}$$

$$C_\gamma = p_{Cy2} \cdot \lambda_{C\gamma} \tag{138}$$

$$K_\gamma = (p_{Ky6} + p_{Ky7} df_z) \cdot F_z \cdot \lambda_{K\gamma} \qquad \left(= B_\gamma C_\gamma D_\gamma = \frac{\partial F_{yo}}{\delta\gamma} \text{ at } \alpha_y = 0 \right) \tag{139}$$

$$E_\gamma = p_{Ey5} \cdot \lambda_{E\gamma} \quad (\leq 1) \tag{140}$$

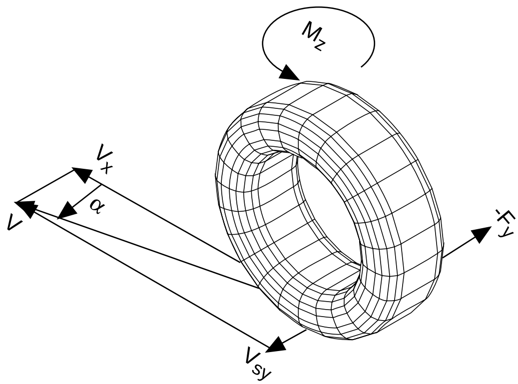
$$B_\gamma = K_\gamma/(C_\gamma D_y) \tag{141}$$

Table 32. Lateral Coefficients, Pure Slip

Name:	Name used in tire property file:	Explanation:
p_{Cy1}	PCY1	Shape factor Cfy for lateral forces
p_{Cy2}	PCY2	Shape factor Cfc for camber forces
p_{Dy1}	PDY1	Lateral friction Muy
p_{Dy2}	PDY2	Exponent lateral friction Muy
p_{Dy3}	PDY3	Variation of friction Muy with squared camber
p_{Ey1}	PEY1	Lateral curvature Efy at Fznom
p_{Ey2}	PEY2	Variation of curvature Efy with camber squared
p_{Ey3}	PEY3	Asymmetric curvature Efy at Fznom
p_{Ey4}	PEY4	Asymmetric curvature Efy with camber
p_{Ey5}	PEY5	Camber curvature Efc
p_{Ky1}	PKY1	Maximum value of stiffness Kfy/Fznom
p_{Ky2}	PKY2	Curvature of stiffness Kfy
p_{Ky3}	PKY3	Peak stiffness factor
p_{Ky4}	PKY4	Peak stiffness variation with camber squared
p_{Ky5}	PKY5	Lateral stiffness depedency with camber squared
p_{Ky6}	PKY6	Camber stiffness factor Kfc
p_{Ky7}	PKY7	Vertical load dependency of camber stiffn. Kfc
p_{Hy1}	PHY1	Horizontal shift Shy at Fznom

Formula: Aligning Torque (Pure Slip)

Figure 29. Lateral Slip Condition Including Aligning Torque (Pure Cornering)



$$M_z' = M_{zo}(\alpha, \gamma, F_z) \tag{142}$$

$$M_{zo} = -t \cdot F_{yo, \gamma = 0} + M_{zr} \tag{143}$$

with the pneumatic trail:

$$t(\alpha_t) = D_t \cos[C_t \arctan\{B_t\alpha_t - E_t(B_t\alpha_t - \arctan(B_t\alpha_t))\}]\cos(\alpha) \tag{144}$$

$$\alpha_t = \alpha + S_{Ht} \tag{145}$$

the residual torque:

$$M_{zr}(\alpha_r) = D_r \cos[\arctan(B_r\alpha_r)]\cos(\alpha) \tag{146}$$

$$\alpha_r = \alpha + S_{Hf} \tag{147}$$

the scaled camber angle:

$$\gamma_z = \gamma \cdot \lambda_{\gamma z} \tag{148}$$

with coefficients:

$$B_t = (q_{Bz1} + q_{Bz2} df_z + q_{Bz3} df_z^2) \cdot (1 + q_{Bz4}\gamma_z + q_{Bz5}|\gamma_z|) \cdot \lambda_{Ky}/\lambda_{\mu y} \tag{149}$$

$$C_t = q_{Cz1} \tag{150}$$

$$D_t = F_z \cdot (q_{Dz1} + q_{Dz2} df_z) \cdot (1 + q_{Dz3}|\gamma_z| + q_{Dz4}\gamma_z^2) \cdot (R_0/F_{z0}) \cdot \lambda_t \tag{151}$$

$$E_t = (q_{Ez1} + q_{Ez2} df_z + q_{Ez3} df_z^2) \tag{152}$$

$$\left\{1 + (q_{Ez4} + q_{Ez5}\gamma_z) \cdot \left(\frac{2}{\pi}\right) \cdot \arctan(B_t \cdot C_t \cdot \alpha_t)\right\} \quad (\leq 1)$$

$$S_{Ht} = 0 \tag{153}$$

$$B_r = q_{Bz9} \cdot \lambda_{Ky}/\lambda_{\mu y} \tag{154}$$

$$D_r = F_z \cdot (q_{Dz6} + q_{Dz7} df_z)\lambda_r + (q_{Dz8} + q_{Dz9} df_z)\gamma_z \tag{155}$$

$$+ (q_{Dz10} + q_{Dz11} df_z)\gamma_z \cdot |\gamma_z|) \cdot R_o \lambda_{\mu y}$$

$$S_{Hr} = q_{Hz1} + q_{Hz2} df_z + (q_{Hz3} + q_{Hz4} df_z)\gamma_z \tag{156}$$

An approximation for the aligning stiffness reads:

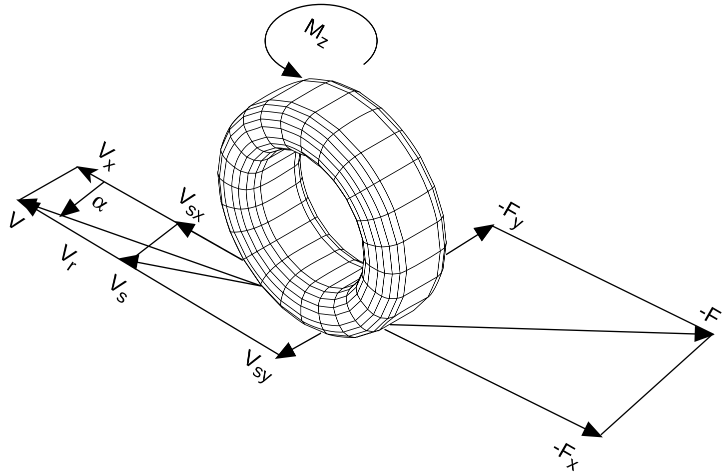
$$K_z = -t \cdot K_y \qquad \left(\approx \frac{\partial M_z}{\partial \alpha} \text{ at } \alpha = 0\right) \tag{157}$$

Table 33. Aligning Coefficients, Pure Slip

Name:	Name used in tire property file:	Explanation:
q_{Bz1}	QBZ1	Trail slope factor for trail Bpt at Fznom
q_{Bz2}	QBZ2	Variation of slope Bpt with load
q_{Bz3}	QBZ3	Variation of slope Bpt with load squared
q_{Bz4}	QBZ4	Variation of slope Bpt with camber
q_{Bz5}	QBZ5	Variation of slope Bpt with absolute camber
q_{Bz9}	QBZ9	Slope factor Br of residual torque Mzr
q_{Cz1}	QCZ1	Shape factor Cpt for pneumatic trail
q_{Dz1}	QDZ1	Peak trail Dpt + Dpt*(Fz/Fznom*R0)
q_{Dz2}	QDZ2	Variation of peak Dpt with load
q_{Dz3}	QDZ3	Variation of peak Dpt with camber
q_{Dz4}	QDZ4	Variaion of peak Dpt with camber squared.
q_{Dz6}	QDZ6	Peak residual torque Dmr + Dmr/ (Fz*R0)
q_{Dz7}	QDZ7	Variation of peak factor Dmr with load
q_{Dz8}	QDZ8	Variation of peak factor Dmr with camber
q_{Dz9}	QDZ9	Variation of peak factor Dmr with camber and load
q_{Dz10}	QDZ10	Variation of peak factor Dmr with camber squared
q_{Dz11}	QDZ11	Variation of Dmr with camber squared and load
q_{Ez1}	QEZ1	Trail curvature Ept at Fznom
q_{Ez2}	QEZ2	Variation of curvature Ept with load
q_{Ez3}	QEZ3	Variation of curvature Ept with load squared
q_{Ez4}	QEZ4	Variation of curvature Ept with sign of Alpha-t
q_{Ez5}	QEZ5	Variation of Ept with camber and sign Alpha-t
q_{Hz1}	QHZ1	Trail horizontal shift Shr at Fznom
q_{Hz2}	QHZ2	Variation of shift Shr with load
q_{Hz3}	QHZ3	Variation of shift Shr with camber
q_{Hz4}	QHZ4	Variation of shift Shr with camber and load

Magic Formula Steady-State Combined Slip

Figure 30. Combined Slip Condition (Combined Braking/Traction and Cornering)



Formula: Longitudinal Slip (Combined Slip)

$$F_x = F_{x0} \cdot G_{x\alpha}(\alpha, \kappa, F_z) \tag{158}$$

with $G_{x\alpha}$ a weighting function.

We write:

$$F_x = D_{x\alpha} \cos[C_{x\alpha} \arctan\{B_{x\alpha}\alpha_s - E_{x\alpha}(B_{x\alpha}\alpha_s - \arctan(B_{x\alpha}\alpha_s))\}] \tag{159}$$

$$\alpha_s = \alpha + S_{Hx\alpha} \tag{160}$$

with coefficients:

$$B_{x\alpha} = (r_{Bx1} + r_{Bx3}\gamma^2)\cos[\arctan\{r_{Bx2}\kappa\}] \cdot \lambda_{x\alpha} \tag{161}$$

$$C_{x\alpha} = r_{Cx1} \tag{162}$$

$$D_{x\alpha} = \frac{F_{xo}}{\cos[C_{x\alpha}\arctan\{B_{x\alpha}S_{Hx\alpha} - E_{x\alpha}(B_{x\alpha}S_{Hx\alpha} - \arctan(B_{x\alpha}S_{Hx\alpha}))\}]} \tag{163}$$

$$E_{x\alpha} = r_{Ex1} + r_{Ex2}df_z \quad (\leq 1) \tag{164}$$

$$S_{Hx\alpha} = r_{Hx1} \tag{165}$$

The weighting function follows as:

$$G_{x\alpha} = \frac{\cos[C_{x\alpha}\arctan\{B_{x\alpha}\alpha_s - E_{x\alpha}(B_{x\alpha}\alpha_s - \arctan(B_{x\alpha}\alpha_s))\}]}{\cos[C_{x\alpha}\arctan[B_{x\alpha}S_{Hxa} - E_{x\alpha}(B_{x\alpha}S_{Hx\alpha} - \arctan(B_{x\alpha}S_{Hx\alpha}))]]} \tag{166}$$

Table 34. Longitudinal Coefficients, Combined Slip

Name:	Name used in tire property file:	Explanation:
r_{Bx1}	RBX1	Slope factor for combined slip Fx reduction
r_{Bx2}	RBX2	Variation of slope Fx reduction with kappa
r_{Bx3}	RBX3	Influence of camber on stiffness for Fx combined
r_{Cx1}	RCX1	Shape factor for combined slip Fx reduction
r_{Ex1}	REX1	Curvature factor of combined Fx
r_{Ex2}	REX2	Curvature factor of combined Fx with load
r_{Hx1}	RHX1	Shift factor for combined slip Fx reduction

Formula: Lateral Slip (Combined Slip)

$$F_y = F_{y0} \cdot G_{y\kappa}(\alpha, \kappa, \gamma, F_z) + S_{Vy\kappa} \tag{167}$$

with G_{yk} a weighting function and S_{Vyk} the “κ-induced” side force can be written:

$$F_y = D_{y\kappa}\cos[C_{y\kappa}\arctan\{B_{y\kappa}\kappa_s - E_{y\kappa}(B_{y\kappa}\kappa_s - \arctan(B_{y\kappa}\kappa_s))\}] + S_{Vy\kappa} \tag{168}$$

$$\kappa_s = \kappa + S_{Hy}\kappa \tag{169}$$

with coefficients:

$$B_{y\kappa} = (r_{By1} + r_{By4}\gamma^2)\cos[\arctan\{r_{By2}(\alpha - r_{By3})\}] \cdot \lambda_{y\kappa} \tag{170}$$

$$C_{y\kappa} = r_{Cy1} \tag{171}$$

$$D_{y\kappa} = \frac{F_{yo}}{\cos[C_{y\kappa}\arctan\{B_{y\kappa}S_{Hy\kappa} - E_{yk}(B_{y\kappa}S_{Hy\kappa} - \arctan(B_{y\kappa}S_{Hy\kappa}))\}]} \tag{172}$$

$$E_{y\kappa} = r_{Ey1} + r_{Ey2}df_z \tag{173}$$

$$S_{Hy\kappa} = r_{Hy1} + r_{Hy2}df_z \qquad (\quad < 0.1 \ \ (\text{drive} - \text{slip})) \tag{174}$$

$$S_{Vy\kappa} = D_{Vy\kappa}\sin[r_{Vy5}\arctan(r_{Vy6}\kappa)] \cdot \lambda_{Vy\kappa} \tag{175}$$

$$D_{Vy\kappa} = \mu_y F_z \cdot (r_{Vy1} + r_{Vy2}df_z + r_{Vy3}\gamma) \cdot \cos[\arctan(r_{Vy4}\alpha)] \tag{176}$$

The weighting function appears to read:

$$G_{y\kappa} = \frac{\cos[C_{y\kappa}\arctan\{B_{y\kappa}\kappa_s - E_{y\kappa}(B_{y\kappa}\kappa_s - \arctan(B_{y\kappa}\kappa_s))\}]}{\cos[C_{y\kappa}\arctan\{B_{y\kappa}S_{Hy\kappa} - E_{y\kappa}(B_{y\kappa}S_{Hy\kappa} - \arctan(B_{y\kappa}S_{Hy\kappa}))\}]} \tag{177}$$

Table 35. Lateral Coefficients, Combined Slip

Name:	Name used in tire property file:	Explanation:
r_{By1}	RBY1	Slope factor for combined Fy reduction
r_{By2}	RBY2	Variation of slope Fy reduction with alpha
r_{By3}	RBY3	Shift term for alpha in slope Fy reduction
r_{By4}	RBY4	Influence of camber on stiffness of Fy combined
r_{Cy1}	RCY1	Shape factor for combined Fy reduction
r_{Ey1}	REY1	Curvature factor of combined Fy
r_{Ey2}	REY2	Curvature factor of combined Fy with load
r_{Hy1}	RHY1	Shift factor for combined Fy reduction
r_{Hy2}	RHY2	Shift factor for combined Fy reduction with load
r_{Vy1}	RVY1	Kappa induced side force Svyk/Muy*Fz at Fznom
r_{Vy2}	RVY2	Variation of Svyk/Muy*Fz with load
r_{Vy3}	RVY3	Variation of Svyk/Muy*Fz with camber
r_{Vy4}	RVY4	Variation of Svyk/Muy*Fz with alpha
r_{Vy5}	RVY5	Variation of Svyk/Muy*Fz with kappa
r_{Vy6}	RVY6	Variation of Svyk/Muy*Fz with atan (kappa)

Formula: Aligning Torque (Combined Slip)

$$M_z' = -t \cdot F'_{y,\gamma=0} + M_{zr} + s \cdot F_x \tag{178}$$

with:

$$t = t(\alpha_{t,eq}) \tag{179}$$

$$= D_t \cos[C_t \arctan\{B_t\alpha_{t,eq} - E_t(B_t\alpha_{t,eq} - \arctan(B_t\alpha_{t,eq}))\}]\cos(\alpha)$$

$$F'_{y,\gamma=0} = F_y - S_{Vy\kappa} \text{ (at } \gamma{=}0) \tag{180}$$

$$M_{zr} = M_{zr}(\alpha_{r,eq}) = D_r\cos[\arctan(B_r\alpha_{r,eq})]\cos(\alpha) \tag{181}$$

$$s = \{s_{sz1} + s_{sz2}(F_y/F_{z0}) + (s_{sz3} + s_{sz4}df_z)\gamma\} \cdot R_0 \cdot \lambda_s \tag{182}$$

with the arguments:

$$\alpha_{t,\,eq} = \arctan\sqrt{\tan^2\alpha_t + \left(\frac{K_x}{K_y}\right)^2\kappa^2} \cdot \mathrm{sgn}(\alpha_t) \tag{183}$$

$$\alpha_{r,\,eq} = \arctan\sqrt{\tan^2\alpha_r + \left(\frac{K_x}{K_y}\right)^2\kappa^2} \cdot \mathrm{sgn}(\alpha_r) \tag{184}$$

Table 36. Aligning Torque/Combined Slip

Name:	Name used in tire property file:	Explanation:
s_{sz1}	SSZ1	Nominal value of s/R0 effect of Fx on Mz
s_{sz2}	SSZ2	Variation of distance s/R0 with Fy/Fznom
s_{sz3}	SSZ3	Variation of distance s/R0 with camber
s_{sz4}	SSZ4	Variation of distance s/R0 with load and camber

Formula: Overturning Moment

$$M_x = R_o \cdot F_z \cdot \{q_{Sx1} \cdot \lambda_{Vmx} + (-q_{Sx2} \cdot \gamma + q_{Sx3} \cdot F_y/F_{z0}) \cdot \lambda_{Mx}\} \tag{185}$$

Table 37. Overturning Coefficients

Name:	Name Used in Tire Property File:	Explanation:
q_{sx1}	QSX1	Lateral force induced overturning couple
q_{sx2}	QSX2	Camber induced overturning couple
q_{sx3}	QSX3	Fy induced overturning couple

Formula: Rolling Resistance Torque

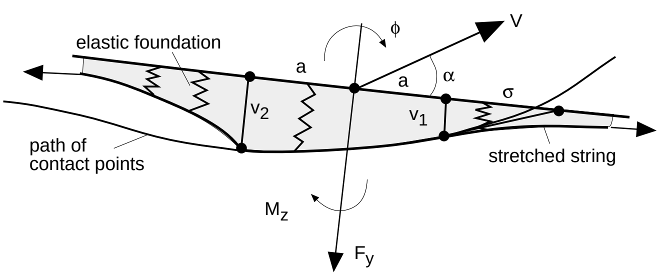
$$M_y = R_o \cdot F_z \cdot \{q_{Sy1} + q_{Sy2}F_x/F_{z0} + q_{Sy3}|V_x/V_{ref}| + q_{Sy4}(V_x/V_{ref})^4\} \quad (186)$$

Table 38. Rolling Coefficients

Name:	Name used in tire property file:	Explanation:
q_{sy1}	QSY1	Rolling resistance torque coefficient
q_{sy2}	QSY2	Rolling resistance torque depending on Fx
q_{sy3}	QSY3	Rolling resistance torque depending on speed
q_{sy4}	QSY4	Rolling resistance torque depending on speed^4
V_{ref}	LONGVL	Measurement speed

Transient Behavior

Figure 31. Stretched String Model for Transient Tire Behavior



Transient Model Equations

The present version, using slip speeds instead of a and κ, allows starting from stand-still. First-order lag of tire longitudinal and lateral deformations u and v are introduced through relaxation lengths σ_κ and σ_α, see Figures 31:

$$\sigma_\kappa \frac{du}{dt} + |V_x| u = -\sigma_\kappa V_{sx} \quad (187)$$

$$\sigma_\alpha \frac{d\nu}{dt} + |V_x|\nu = \sigma_\alpha V_{sy} \tag{188}$$

These differential equations are based on the assumption that the contact points near the leading edge remain in the adhesion with the road surface (no sliding). The relaxation lengths (in this version not considered to decrease with increasing composite deformation slip) are functions of the vertical load and camber angle represented in a similar way as the slip stiffnesses K_x (Eq. 123) and K_y (Eq. 135).

$$\sigma_\kappa = F_z \cdot (p_{Tx1} + p_{Tx2} df_z) \cdot \exp(-p_{Tx3} df_z) \cdot (R_0 / F_{z0}) \cdot \lambda_{\sigma\kappa} \tag{189}$$

$$\sigma_\alpha = p_{Ty1} \sin\left[p_{Ty2} \arctan\left\{\frac{F_z}{(p_{Ty3} + p_{Ky4}\gamma^2) F_{z0} \lambda_{F_{z0}}}\right\}\right] \tag{190}$$

$$(1 - p_{Ky5}\gamma^2) \cdot R_0 \lambda_{F_{z0}} \lambda_{\sigma\alpha}$$

The practical tire deformation slip quantities are defined as:

$$\kappa' = \frac{u}{\sigma_\kappa} \cdot \operatorname{sign}(V_x) \tag{191}$$

$$\tan\alpha' = \frac{\nu}{\sigma_\alpha} \tag{192}$$

Equations (158), (167), (178), (185), and (186) are subsequently used with arguments κ’ and α’ from Eqs. (191), (192) instead of the longitudinal and lateral wheel slip quantities κ and α (Eqs.(108), (109)).

$$F_x = F_x(\alpha', \gamma, \kappa', F_z) \tag{193}$$

$$F_y = F_y(\alpha', \kappa', \gamma, F_z) \tag{194}$$

$$M_z' = M_z'(\alpha', \kappa', \gamma, F_z) \tag{195}$$

The Gyroscopic Couple

This moment due to tire inertia acting about the vertical axis reads:

$$M_{z,\,gyr} = c_{gyr} M_{belt} V_r \cdot \frac{d\nu}{dt} \cos[\arctan(B_r \alpha_{r,\,eq})] \tag{196}$$

with parameter (in addition to the basic tire parameter m_{belt}):

$$c_{gyr} = q_{Tz1} \cdot \lambda_{gyr} \tag{197}$$

The total aligning torque now becomes:

$$M_z = M_z' + M_{z,\,gyr} \tag{198}$$

Table 39. Coefficients, Transient Response

Name:	Name used in tire property file:	Explanation:
p_{Tx1}	PTX1	Relaxation length SigKap0/Fz at Fznom
p_{Tx2}	PTX2	Variation of SigKap0/Fz with load
p_{Tx3}	PTX3	Variation of SigKap0/Fz with exponent of load
p_{Ty1}	PTY1	Peak value of relaxation length Sig_alpha
p_{Ty2}	PTY2	Shape factor for Sig_alpha
p_{Ty3}	PTY3	Value of Fznom where Sig_alpha is max. rolling resistance
q_{Tz1}	QTZ1	Gyroscopic torque constant
M_{belt}	MBELT	Belt mass of the wheel

Switching from a Simple to a Complex Tire Model

MF-MCTyre enables the user to switch from a simple tire model (for example only calculations for steady state pure cornering slip conditions) to tire model for transient combined slip situations. The parameter USE_MODE of the MF-MCDataset determines the type of use of the tire model. In the Table 40 the possible options of USE_MODE are indicated. Note that the maximum valid USE_MODE depends on the tire test data used to determine the MF-MCDataset parameters (i.e. if only tire test data for pure cornering is fit-ted, the calculation of the contact forces under combined cornering and braking/traction slip is not possible unless the user adds the required additional parameters).

Table 40. The Different USE_MODE Values of MF-MCTyre.

USE MODE:	State:	Slip conditions:	MF-Tyre output (forces and torques):
0	spring	-	$0, 0, F_z, 0, 0, 0$
1	steady state	pure longitudinal	$F_x, 0, F_z, 0, M_y, 0$
2	steady state	pure lateral	$0, F_y, F_z, M_x, 0, M_z$
3	steady state	longitudinal and lateral (not combined)	$F_x, F_y, F_z, M_x, M_y, M_z$
4	steady state	combined slip forces	$F_x, F_y, F_z, M_x, M_y, M_z$
11	transient	pure longitudinal	$F_x, 0, F_z, 0, M_y, 0$
12	transient	pure lateral	$0, F_y, F_z, M_x, 0, M_z$
13	transient	longitudinal and lateral (not combined)	$F_x, F_y, F_z, M_x, M_y, M_z$
14	transient	combined slip forces	$F_x, F_y, F_z, M_x, M_y, M_z$

Some Practical Aspects

Rolling Resistance Torque

For a free-rolling wheel at a constant forward velocity without camber and slip angle a drag force (rolling resistance) is generated. Passenger car tires usually have a rolling resistance coefficient between 0.7-1.2%; for truck tires the rolling resistance force is usually around 0.5% to 0.7% of the vertical load. Note that the parameter q_{sy1} in equation (182) determines the rolling resistance factor. According to the ISO sign convention, this drag force as well as the rolling resistance torque M_y have negative signs ($q_{sy1} > 0$).

In order to reach equilibrium between the force and the torque on the wheel, in general, a small negative value for the longitudinal slip is obtained.

Typical Tire Characteristics

For pure slip conditions (either longitudinal or lateral) three typical graphs can be made:

1. F_x as a function of the longitudinal slip κ;

2. F_y as a function of the slip angle α;

3. M_z as a function of the slip angle α.

In the Figure 32 and Figure 33 and examples of these characteristics valid for the W-axis system are shown.

Figure 32. The Longitudinal Force as a Function of the Longitudinal Slip

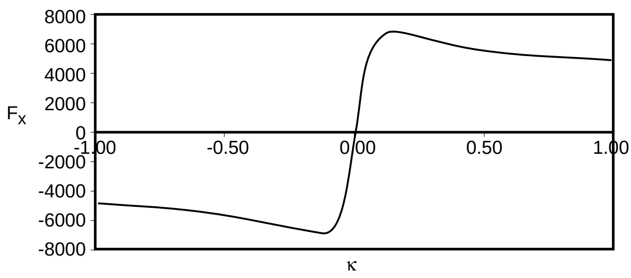
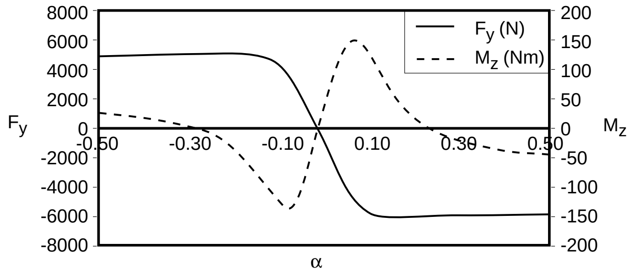


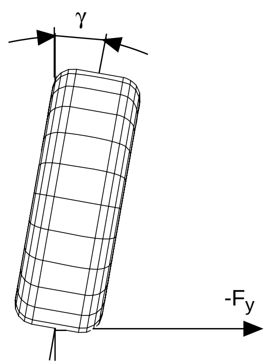
Figure 33. The Lateral Force and Self Aligning Torque as a Function of the Slip Angle



The Effect of Camber Angle

According to the W-axis system, an increase of the camber angle will cause a decrease of the lateral force, as shown in Figure 34.

Figure 34. Tire Camber Angle and the Positive Direction of the Lateral Force According to the W-Axis System (Rear View)



Tire Model Output at Extreme Input Values

At extreme large input values, such as a vertical load more than three times the nominal tire load, a real physical tire might puncture or go to pieces. In the tire model measures have been taken to avoid calculation errors or a computer simulation break down. Depending on your simulation software the tire model warns the user when the input exceeds the validity range of the MF-MCDataset.

The tire property files, generated by MF-MCTool, contain maxima and minima values for the tire model input, defining the validity range of the MF-MCDataset:

- Fzmin and Fzmax for the vertical load F_z
- Alpmin and Alpmax for the slip angle α
- Cammin and Cammax for the camber angle γ
- Kpumin and Kpumax for the longitundinal slip κ

In general, the tire model fixes the B, C, D, E and shift factors when exceeding the upper mentioned limits at the corresponding limit. For vertical loads smaller than Fzmin the output of the tire model is equal to the output of the tire model for Fzmin proportionally scaled to zero output.

Standard Tire Interface (STI)

As a result of the First International Colloquium on Tire Models for Vehicle Dynamics Analysis, October 21-22, 1991, the international Tire Workshop working group was established (TYDEX).

The working group concentrated on tire measurements and tire models used for vehicle simulation purposes. For most vehicle dynamics studies people usually develop their own tire models. Since all car manufacturers and their tire suppliers have the same goal (that is development of tires to improve dynamic safety of the vehicle), standardisation in tire behaviour description should be aimed for.

In TYDEX two expert groups were defined with following goals:

- The first expert group (Tire Measurements - Tire Modeling) has as its main goal to specify an interface between tire measurements and tire models. The work shall include a description of the test conditions. The interface could be described as a definition of a method or format to describe tire measurement data in such a way that it contains all necessary items to fit tire models to the underlying data. The format shall also allow for a description of the test conditions.
- The second expert group (Tire Modeling - Vehicle Modeling) has as its main goal to specify an interface between tire models and simulation tools. Intentionally, use of this interface will ensure that a wide range of simulation software can be linked to a wide range of tire software available.

Both expert groups consist of participants of vehicle industry (passenger cars and trucks), tire manufacturers, other suppliers and research laboratories. The large number of participants indicates that there is a need for this kind of 'standardization' work. DVR is strongly involved in TYDEX.

At the Second International Colloquium on Tire Models for Vehicle Dynamics Analysis, February 19 and 20, 1997, the final documents on both interfaces have been presented [9]. The TYDEX-Format [1] describes a standard format for the exchange of tire testing and modelling data; the second document describes the standard interface between tire model and vehicle model, called the Standard Tire Interface (STI) [2].

The Standard Tire Interface prescribes a subroutine call with a number of subroutine arguments to pass all relevant information from tire models to multi-body programs and vice versa. The subroutine represents a shell around tire software and is fixed to the axle hub which is modelled by the multi-body programs.

MF-Datasets and MF-MCTool

The final objective of the user is to optimize vehicle behaviour (including tire behaviour) using the potential of simulation software. Because the tire properties determine to a great extent the vehicle behaviour, a tire model without proper tire data will be useless in most cases. For full optimization purposes the engineer requires the availability of datasets under a large range of conditions.

Tire Measurements

Tire characteristics can be well described by the Magic Formula tire model. The formula are specified by a set of "Magic Formula parameters" which represent the characteristics in a compact form. The parameters depend on the type of the tire and the road conditions and can be obtained from outdoor and/or laboratory tests.

Calculation of Magic Formula Parameters

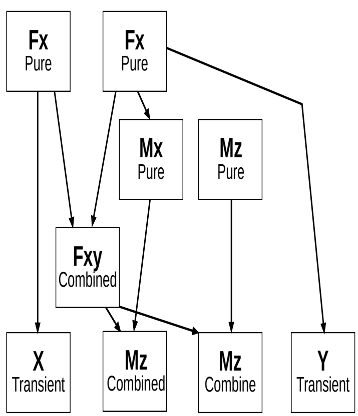


Figure 35. Fit Process

Calculation of parameters from the measurement data is performed with regression techniques (also known as parameter fitting ref [10]). In such a so called fitting procedure, the results from measurements under pure slip conditions have to be used first to determine the Magic Formula parameters for side force, self aligning torque and longitudinal force and in a second step the parameters for combined slip conditions, see Figure 35. The pure cornering measurements must include the influence of camber. The parameters for transient cornering and braking are based on the steady state pure cornering and braking properties.

The MF-Tool+ software of MF-Tyre offers the engineer a user-friendly tool to determine the MF-Tyre parameters (MF-Datasets) out of any Force and Moment tyre test data. Next to software also MF-Datasets can be selected out of existing libraries.
See www.delft-tyre.com.

Definitions

General

Table 41. General Definitions

Term:	Definition:
Inertial coordinate system	Inertial space according to ISO
Road tangent plane	Plane with the normal unit vector n_r (tangent to the road) in C.
Wheel centre O	Centre of the wheel
C-axis system	Coordinate system mounted on the wheel carrier at the Wheel center orientation accoriding ISO.
Wheel plane	The plane in the wheel centre that is formed by the wheel when considered a rigid disc with zero width.
Contact point C	Contact point between tyre and road, defined as the intersection of the wheel plane and the projection of the wheel axis onto the road plane.
W-axis system	Coordinate system at the tyre contact point C, orienation according ISO.

Tire Kinematics

Table 42. Tire Kinematic Definitions

Abbreviation:	Definition:	Units:
R_0	Unloaded tire radius	[m]
R	Loaded tire radius	[m]
R_e	Effective tire radius	[m]
r_t	Tire cross section radius (half tyre width)	[m]
ρ	Radial tire deflection	[m]
ρ^d	Dimensionless radial tire deflection	[-]
ρ_{Fz0}	Radial tire deflection at nominal load	[m]
m_{belt}	Tire belt mass	[kg]
Ω	Rotational velocity of the wheel	[rads^{-1}]
h_α	Distance wheel centre to road plane	[m]

Slip Quantities

Table 43. Slip Quantity Definitions

Abbreviation:	Definition:	Units:
V	Vehicle speed	[ms^{-1}]
V_{sx}	Slip speed in x-direction	[ms^{-1}]
V_{sy}	Slip speed in y-direction	[ms^{-1}]
V_s	Resulting slip speed	[ms^{-1}]
V_x	Rolling speed in x-direction	[ms^{-1}]
V_y	Lateral speed of tire contact center	[ms^{-1}]
V_r	Linear speed of rolling	[ms^{-1}]
κ	Longitudinal slip	[-]
α	Slip angle	[rad]
γ	Camber angle	[rad]

Forces and Moments

Table 44. Forces and Moment Definitions

Abbreviation:	Definition:	Units:
F_z	vertical wheel load	[N]
F_{z0}	nominal (rated) load	[N]
df_z	dimensionless vertical load	[-]
F_x	longitudinal force	[N]
F_y	lateral force	[N]
F_z	nominal load	[N]
M_x	overturning couple	[Nm]
M_y	braking/driving moment	[Nm]
M_z	aligning moment	[Nm]

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Using the SWIFT-Tyre Model

Overview

The SWIFT-Tyre model combines a Magic Formula slip force description with a rigid ring model and has been validated by experiments up to frequencies of 60-100 Hz. Typical applications of the SWIFT-Tyre model are: durability studies, shimmy analysis, chassis control system evaluation (that is, ABS, ESP) and cornering on uneven roads.

This chapter includes the following sections:

Introduction

The Magic Formula is a widely used and accepted method for modelling tire forces and moments under steady-state rolling conditions. At higher excitation frequencies (>1-2 Hz) relaxation effects and belt dynamics become important for the forces transmitted by the tire to the wheel centre. SWIFT combines a Magic Formula slip force calculation with a rigid ring model, thus greatly extending the frequency range where the tire model is valid. The SWIFT-Tyre model was developed in a joint cooperation between the Delft University of Technology and TNO Automotive under the guidance of Dr. Pacejka. Reference documentation can be found in References on page 107.

Dynamics

The SWIFT-Tyre model is a rigid ring model, in which the tire belt is assumed to behave like a rigid body. This means that the model is accurate in the frequency range where the bending modes of the tire belt can be neglected, which, depending on the tire properties is up to 50 – 60 Hz for lateral behaviour and up to 100 Hz for vertical and longitudinal behaviour. SWIFT has been validated using measurements of a rolling tire (7 to 40 m/s) containing frequencies up to 120 Hz. The model includes essential gyroscopic effects.

Slip Force Calculation

SWIFT uses the Magic Formula for calculation of slip forces, providing an accurate representation of measurement results which usually are available up to 15 degrees side slip, 100% brake slip and 5 degrees of camber angle for different vertical load. For efficiency reasons SWIFT uses a single point contact for slip calculation and therewith is fully compatible with MF-Tyre. Due to the introduction of a so-called phase leading network for the pneumatic trail, SWIFT is suitable for path curvature with a wavelength in the order of two times the contact length. For braking/traction applications, wavelengths as small as half the contact length are well described. The transient slip behaviour is well described up to full sliding, due to modelling of decrease in relaxation length for increased slip levels.

Road Input

The dynamic model has been validated for load variations up to 100 Hz, and the slip model for wavelengths as small as two times the contact length. SWIFT uses a single point contact model, which generally can be applied as long as the road curvature is about half of the tire curvature. To be able to cope with shorter obstacles a method of describing enveloping behaviour is applied. It is assumed that a measured road profile can be evaluated as a series of step obstacles, for which the enveloping effect of the tire is described with so-called basic functions. This method has been validated for isolated obstacles up to 10% of the tire radius, and provides an accurate prediction of vertical load, longitudinal force and wheel rotation fluctuations. Also with measured road profiles good correlation has been found with vehicle measurement data.

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Notation

The equations in the model are expressed in dimensionless quantities as much as possible. This is achieved by the introduction of various reference values that are described in this section.

The reference speed V_0 is the speed at which the contact slip characteristics are measured. This reference speed is used with the nominal tire radius (R_0) for calculation of the reference wheel rotational velocity (Ω_0) in accordance with Equation [200]:

$$\Omega_0 = \frac{V_0}{R_0} \tag{200}$$

The reference stiffness for translation C_{t0}, is derived from the nominal tire load (F_{z0}) and nominal tire radius (R_0) in accordance with Equation [201], and the reference stiffness for rotation C_{ro} is given in Equation [202]:

$$C_{t0} = \frac{F_{z0}}{R_0} \tag{201}$$

$$C_{r0} = F_{z0} R_0 \tag{202}$$

Dimensionless damping is defined in Equation [203], in which m represents the inertia, c represents the stiffness and k the damping as is common for simple mass-spring systems. Additionally reference values for damping of translations (k_{to}) and rotations (k_{ro}) are defined in Equation [204] and [205] respectively:

$$\kappa = \frac{k}{2\sqrt{mc}} \tag{203}$$

$$k_{t0} = \sqrt{\frac{m_0 F_{z0}}{R_0}} \tag{204}$$

$$k_{r0} = \sqrt{m_0 F_{z0} R_0^3} \tag{205}$$

The reference moment of inertia (I_0) is calculated using Equation [206], in which m_0 is the mass of the tire:

$$I_0 = m_0 \cdot R_0^2 \tag{206}$$

The normalised variables that occur in the equations are denoted by an overbar.

Table 46. Forces and Moments

Symbol:	Description:	Units:	Normalized with:
F_{bx}	Longitudinal belt force	[N]	-
F_{by}	Lateral belt force	[N]	-
F_{bz}	Vertical belt force	[N]	-
F_{grv}	Gravity force	[N]	-
F_{rx}	Longitudinal residual force	[N]	-
F_{ry}	Lateral residual force	[N]	-
$\overline{F}_x$	Normalised longitudinal force	-	F_{z0}
$\overline{F}_y$	Normalised lateral force	-	F_{z0}
$\overline{F}_z$	Normalised vertical force	-	F_{z0}
F_z	Vertical axle load	[N]	-
F_{z0}	Nominal tire load	[N]	-
M_{bx}	Camber belt torque	[Nm]	-
M_{by}	Wind-up belt torque	[Nm]	-
M_{bz}	Yaw belt torque	[Nm]	-
M_{rz}	Yaw residual torque	[Nm]	-

Table 47. Radii

Symbol:	Description:	Units:	Normalized with:
Dr_0	Speed radius increase	-	-
R_l	Loaded tire radius	[m]	-
R_0	Nominal tire radius	[m]	-
R_e	Effective rolling radius	[m]	-
R_W	Free tire radius	[m]	-

Table 48. Inertia

Symbol:	Description:	Units:	Normalized with:
m_0	Tire mass	[kg]	-
$\overline{m_b}$	Normalized belt mass	-	m_0
I_0	Reference moment of inertia	[kgm^2]	-

Table 49. General Coefficients

Symbol:	Description:	Units:	Normalized with:
c_{grv}	Gravity constant	-	-
q_{bVx}	Correction coefficient radial belt stiffness	-	-
$q_{bV\theta}$	Correction coefficient tangential belt stiffness	-	-
q_{Fcx}	Brake force stiffness scaling coefficient	-	-
q_{Fcy}	Side force stiffness scaling coefficient	-	-
$q_{Fz1,2}$	Vertical force coefficients	-	-
$q_{kc1,2}$	Coefficients for tread element damping characteristics	-	-
q_{re0}	Tire radius scaling coefficient	-	-
Q_v	Speed and load correction coefficient	-	-
q_{V1}	Tire growth coefficient	-	-
q_{V2}	Vertical force speed coefficient	-	-

Table 50. Displacements and Deflections

Symbol:	Description:	Units:	Normalized with:
ρ_{bx}	Longitudinal belt displacement	[m]	-
$\bar{\rho}_{bx}$	Normalized longitudinal belt displacement	-	R_0
$\rho_{b\gamma}$	Camber belt displacement	[rad]	-
ρ_{bz}	Vertical belt displacement	[m]	-
$\bar{\rho}_{bz}$	Normalized vertical belt displacement	-	R_0
$\bar{\rho}_{by}$	Normalized lateral belt displacemen	-	R_0
$\rho_{b\theta}$	Wind-up belt displacement	[rad]	-
$\rho_{b\Psi}$	Yaw belt displacement	[rad]	-
ρ^d	Dimensionless radial deflection	-	ρ_{Fz0}
ρ_{Fz0}	Nominal tire deflection	[m]	-
$\bar{\rho}_z$	Normalized vertical tire deflection	-	R_0
$\bar{\rho}_{rx}$	Normalized longitudinal residual deflection	-	R_0
$\bar{\rho}_{ry}$	Normalized lateral residual deflection	-	R_0
$\rho_{r\psi}$	Yaw residual deflection	[rad]	-
Ω	Wheel rotational speed	[rad/s]	
$\bar{\Omega}$	Normalised wheel rotation speed	-	Ω
Ω_0	Nominal wheel rotation speed	[rad/s]	-
$V_{c,sx}$	Longitudinal contact point velocity	[m/s]	-
V_0	Nominal wheel speed	[m/s]	-
V_x	Wheel speed	[m/s]	-

Table 51. Stiffness and Damping

Symbol:	Description:	Units:	Normalized with:
c_{bx}	Translation belt stiffness	[N/m]	-
$\bar{c}_{bx}$	Normalized in-plane translation belt stiffness	-	C_{t0}
$\bar{c}_{bx0}$	Normalized nominal in-plane translation belt stiffness	-	C_{t0}
c_{by}	Lateral belt stiffness	[N/m]	-
$\bar{c}_{by}$	Normalized out-of-plane translation belt stiffness	-	C_{t0}
$c_{b\gamma}$	Out-of-plane rotation belt stiffness	[N/m]	-
$\bar{c}_{b\gamma}$	Normalized out-of-plane rotation belt stiffness	-	C_{r0}
$c_{b\theta}$	Wind-up belt stiffness	[N/m]	-
$\bar{c}_{b\theta}$	Normalized in-plane rotation belt stiffness	-	C_{r0}

Force Evaluation

Rigid Ring Model

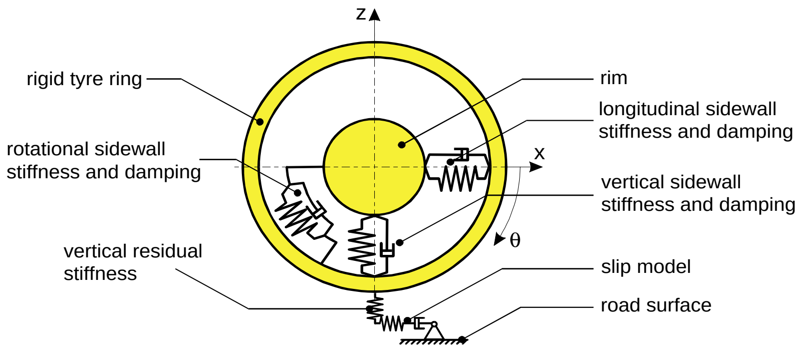
The tire belt is modelled as a rigid body with mass and moments of inertia, that is suspended with spring-damper systems to the rim. The stiffness of the springs is calculated from the frequencies of the so-called rigid body modes. The gravitational force is along the global Z-axis, and is defined in accordance with Equation [207]:

$$F_{grv} = c_{grv}\bar{m}_b m_0 \tag{207}$$

In-Plane Characteristics

Figure 36 shows a side view of the rigid-ring representation of the tire.

Figure 36. Side View of the Rigid-Ring Representation of the Tire



For the in-plane behaviour, the stiffness of the springs is dependent on the in-plane belt displacements (ρ_{bx}, ρ_{bz}) and rotating speed (Ω). The influence of speed and load is implemented by using a correction coefficient Q_v. The correction coefficient is defined by Equation [208]:

$$Q_v = |\bar{\Omega}| \cdot \sqrt{\bar{\rho}_{bx}^{\,2} + \bar{\rho}_{bz}^{\,2}} \tag{208}$$

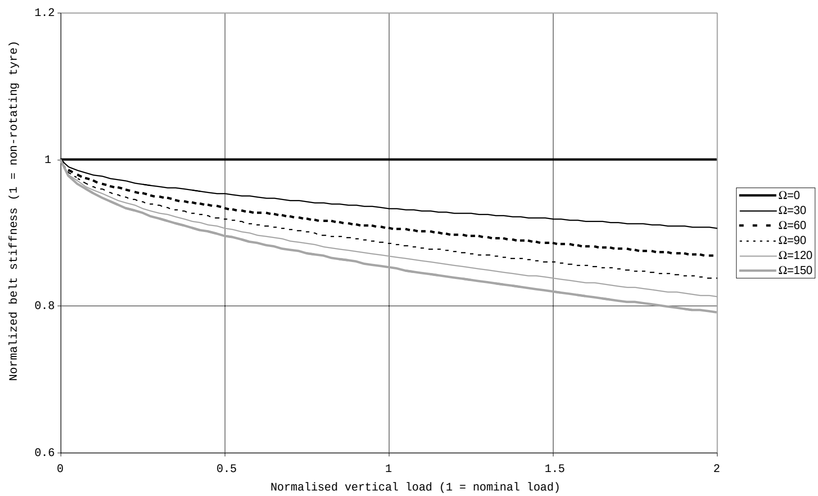
The nominal tire belt stiffness for the in-plane motions is corrected for deflection and speed in accordance with Equations [209] and [210]:

$$\bar{c}_{bx} = \bar{c}_{bx0}(1 - q_{bVx}\sqrt{Q_v}) \tag{209}$$

$$\bar{c}_{b\theta} = \bar{c}_{b\theta 0}(1 - q_{bV\theta}\sqrt{Q_v}) \tag{210}$$

This results in a dependency of the belt stiffness on speed and load typically as displayed in Figure 37.

Figure 37. Belt Stiffness for Various Speeds



The in-plane forces and torque that are transmitted by the tire belt to the rim are given in Equations [211], [212] and [213].

Longitudinal belt force:

$$F_{bx} = \bar{c}_{bx}\bar{\rho}_{bx}F_{z0} + 2\kappa_{bx}\dot{\bar{\rho}}_{bx}k_{t0} \tag{211}$$

Vertical belt force:

$$F_{bz} = \bar{c}_{bx}\bar{\rho}_{bz}F_{z0} + 2\kappa_{bx}\dot{\bar{\rho}}_{bz}k_0 \tag{212}$$

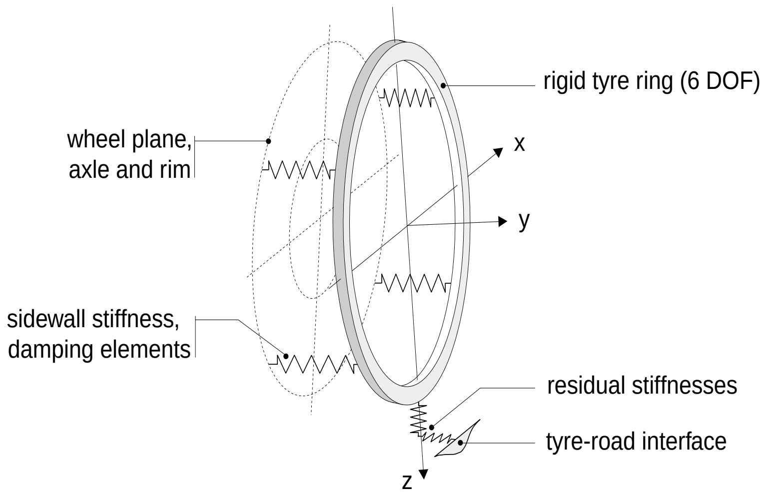
Belt wind-up torque:

$$M_{by} = \bar{c}_{b\theta}\rho_{b\theta}C_{r0} + 2\kappa_{b\theta}\dot{\rho}_{b\theta}k_{r0} \tag{213}$$

Out-of-Plane Characteristics

Figure 38 shows the out-of-plane deflection of the tire, and related specifications.

Figure 38. Out-of-Plane Deflection of the Tire



The out-of-plane stiffness is not dependent on speed and/or load. The lateral force acting on the wheel carrier is given in Equation [214]:

$$F_{by} = \bar{c}_{by}\bar{\rho}_{by}F_{z0} + 2\kappa_{by}\dot{\bar{\rho}}_{by}k_{to} \tag{214}$$

Belt camber torque is calculated using Equation [215], and the yaw torque is calculated using Equation [216]:

$$M_{bx} = \bar{c}_{b\gamma}\rho_{b\gamma}C_{r0} + 2\kappa_{b\gamma}\dot{\rho}_{b\gamma}k_{r0} \tag{215}$$

$$M_{bz} = \bar{c}_{b\gamma}\rho_{b\psi}C_{r0} + 2\kappa_{b\gamma}\dot{\rho}_{b\psi}k_{r0} \tag{216}$$

Vertical Force Characteristics

The overall vertical tire force is the tire force that results from a steady state vertical deflection of the tire. As the vertical tire belt stiffness is modelled with the Rigid Ring model, a residual spring is introduced in order to achieve the overall vertical force characteristic of the tire (see Figure 40). Changes in vertical force due to slip forces are incorporated in the vertical force calculation.

The vertical tire force is a function of deflection and speed. The tire deflection is used for the overall vertical force and is the difference between the free tire radius R_Ω and the loaded tire radius R_l . The free tire radius is a function of speed as the tire grows with speed as given in Equation [217]:

$$\Delta r_0 = q_{V1}\bar{\Omega}^2 \tag{217}$$

The free tire radius is calculated using Equation [218], and is displayed as function of the wheel speed in Figure 39:

$$R_\Omega = (q_{re0} + \Delta r_0)R_0 \tag{218}$$

The normalized vertical tire deflection is defined by Equation [219]:

$$\bar{\rho}_z = (R_\Omega - R_l)/R_0 \tag{219}$$

The overall vertical tire force is a function of the tire deflection, wheel rotation velocity and slip forces as given in Equation [220]:

$$F_z = (1 + q_{V2}|\bar{\Omega}| - (q_{Fcx}\bar{F}_x)^2 - (q_{Fcy}\bar{F}_y)^2 (q_{Fz2}\bar{\rho}_z^2 + q_{Fz1}\bar{\rho}_z) F_{z0} \qquad (220)$$

Figure 39. Free Tire Radius as Function of Wheel Speed

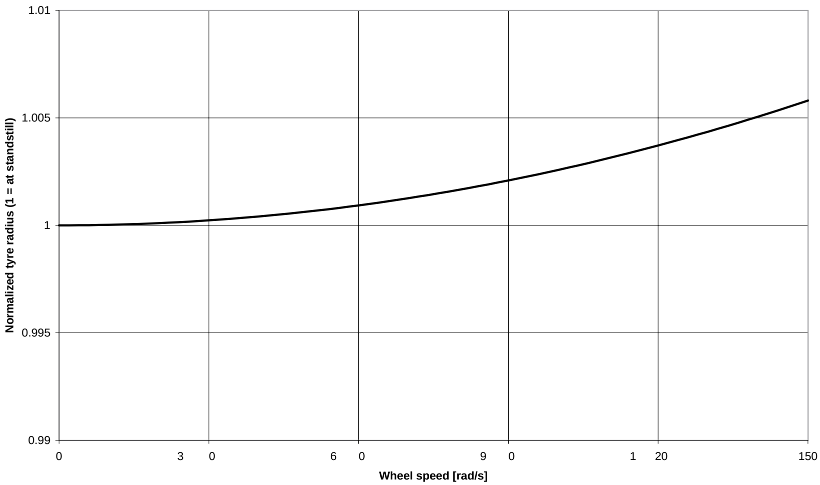
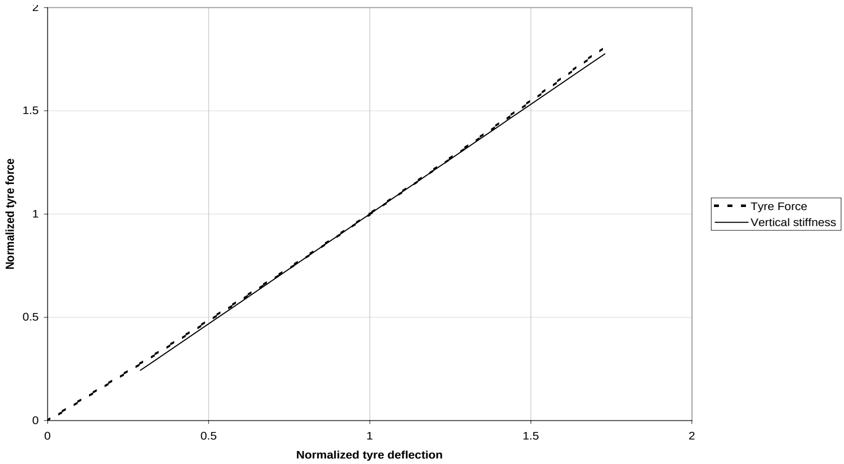


Figure 40. Vertical Tire Force and Stiffness



As shown in Figure 40, the vertical tire force as function of the deflection is a parabola. The vertical stiffness value in the tire property file C_z is the stiffness at the nominal tire load. The relation between C_z, q_{Fz1} and q_{Fz2} is given by equation [221]:

$$C_z = \frac{F_{z0}}{R_0}\sqrt{q_{Fz1}^2 + 4q_{Fz2}} \tag{221}$$

The vertical force characteristic for different wheel speeds is shown in Figure 41 on page 120.

Figure 41. Wheel Load as Function of Tire Deflection at Different Wheel Speeds

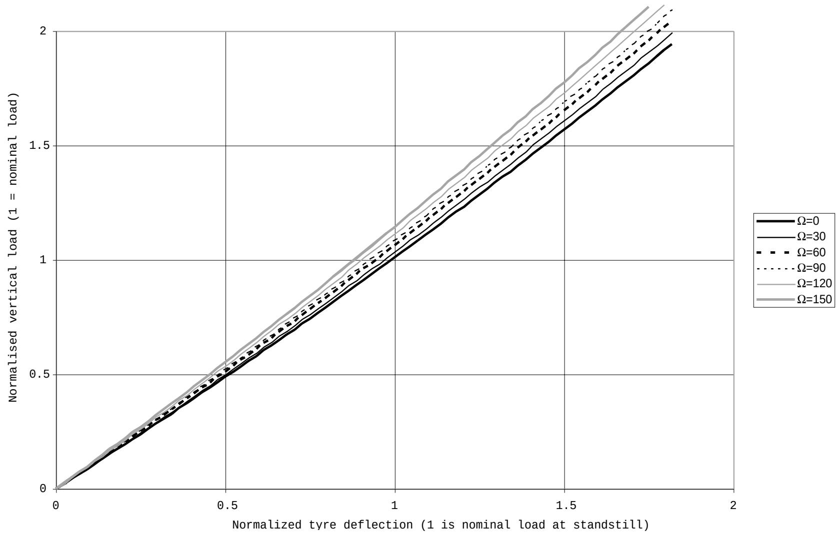
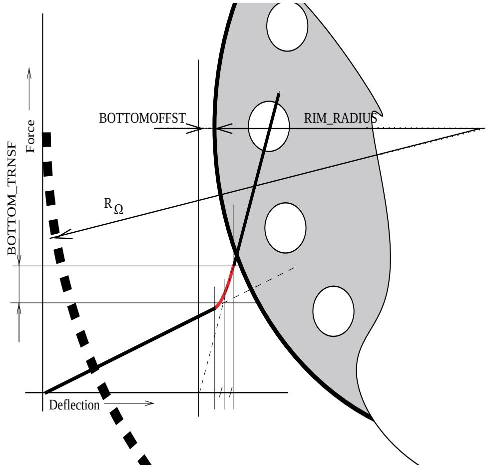


Figure 42. SWIFT with Bottoming Characteristics



SWIFT incorporates bottoming effects for load-case studies as shown in Figure 42. Bottoming occurs when the deflection of the tire results in contact of the tire tread band with the wheel rim, a radius that generally will be somewhat larger than the rim radius (RIM_RADIUS). The assumption is made that the bottoming characteristics are independent from the normal vertical spring curve. Three parameters are required to define the bottoming characteristics:

BOTTOM_STIFF: Defines the linear vertical stiffness of the tyre-wheel assembly when the tyre is bottoming. As a first estimate a value of ten times the vertical stiffness may be appropriate.

BOTTOM_OFFST: Defines (in combination with RIM_RADIUS) the maximum radius where bottoming can start to occur, see Figure 42. The actual point where the vertical force starts to increase is the point of intersection between the normal vertical spring and bottoming spring curve.

BOTTOM_TRNSF: Defines the size of the transition range where the normal spring curve is smoothly changed into the bottoming spring curve, see Figure 42. The unit of this parameter is force.

The Effective Tire Rolling Radius

The effective tire-rolling radius R_e is estimated using a Magic Formula approach. Equation [222] holds the formula for the effective tire-rolling radius:

$$R_e = R_\Omega - \rho_{Fz0}(D\arctan((B\rho^d) + F\rho^d)) \tag{222}$$

The nominal tire deflection ρ_{Fz0} is defined by Equation [223] (C_z = radial tire stiffness), and the dimensionless radial deflection is calculated using Equation [224]:

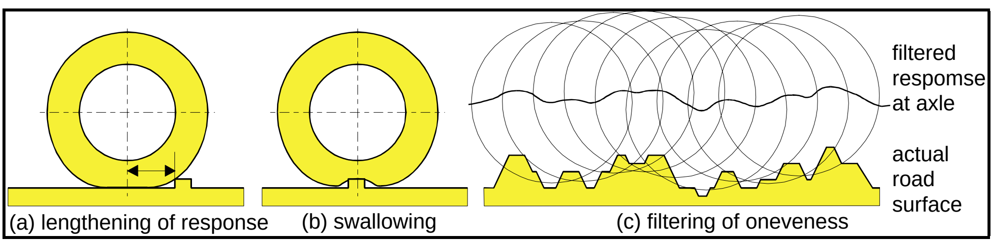
$$\rho_{Fz0} = \frac{F_{z0}}{C_z} \tag{223}$$

$$\rho^d = \frac{\rho_z}{\rho_{Fz0}} \tag{224}$$

Effective Road Input

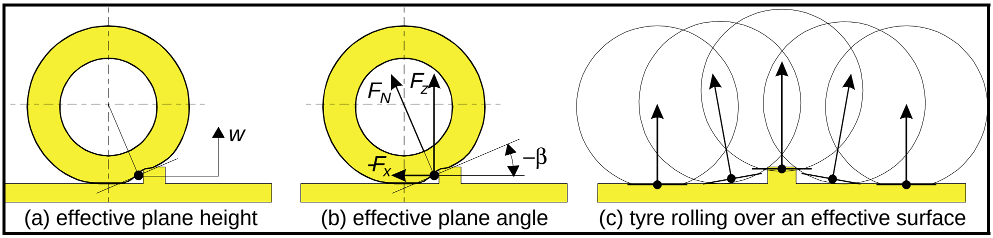
The standard single point contact model is valid for vertical road input for wavelengths larger then the contact length (>0.2m). For short wavelength obstacles, the enveloping behaviour of the tire needs to be described more accurately. The enveloping properties of the tire are described in SWIFT by so-called basic functions. This method is incorporated for steps in road height, and stochastic road input is treated as a sequence of steps. The phenomena that occur when a tire is rolling on an uneven road are illustrated in Figure 43.

Figure 43. Tire Enveloping Behaviour



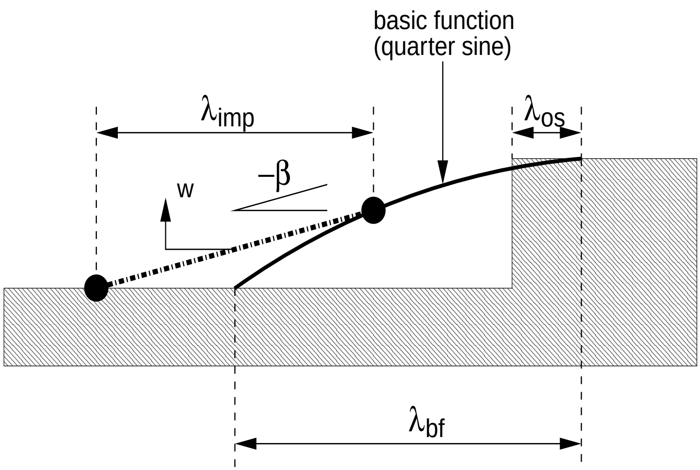
The basic function concept uses a quarter sine function to describe the response of the tire to a step obstacle. The length of the basic function corresponds to the lengthening effect, and the swallowing effect is taken care of by using a two-point contact. Additionally, the rolling radius variations that result from rolling over an obstacle is modelled. A cleat obstacle is converted into an effective height and plane angle as shown in Figure 44.

Figure 44. Effective Inputs



The effective inputs (plane height w and plane angle β) are calculated using the basic function (or curve) is demonstrated in Figure 45. The basic function relates to the response of a rigid wheel, and the parameters depend on the tire radius R_0 and obstacle height h only.

Figure 45. Example for Effective Inputs with Basic Curve



The height of the sine wave is equal to the step height and the width is approximately equal to the width of the rigid wheel response. The effective road surface is obtained by 'travelling' over the basic curve with a two-point tire-road interface. The distance between the two points (shift) is slightly smaller than the contact length of the tire. The effective plane height is obtained from the average height at the edges of the contact patch. The effective plane angle is the slope of the line through the two-point tire-road interface with respect to the horizontal.

The length (or width) of the basic function can be approximated by a Rigid Wheel response (λ_{RIGID}). The length of the basic function (λ_{bf}) is calculated in accordance with Equation [225]:

$$\lambda_{bf} = q_{\lambda bf} \lambda_{RIGID} \tag{225}$$

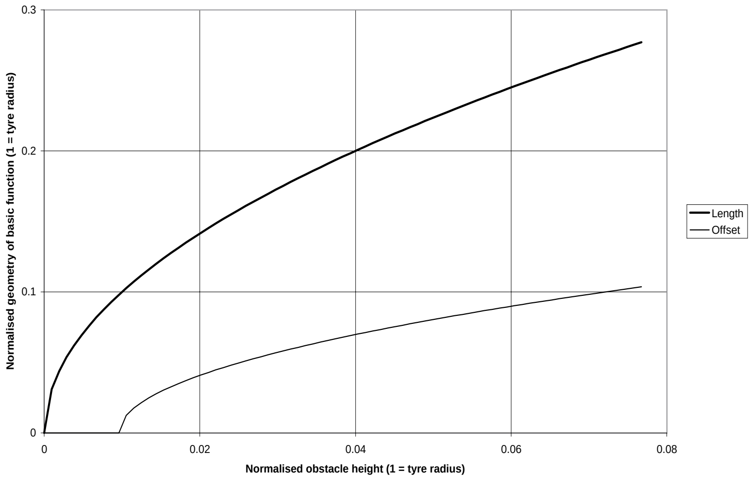
The offset λ_{os} of the basic functions occurs when a threshold value for the height is exceeded. In general, the dependency on the height resembles the Rigid Wheel response. This is incorporated in the functions of Equation [226]:

$$\lambda_{os} = 0 \qquad |h| \le q_{\lambda os1} R_0 \tag{226}$$

$$\lambda_{os} = q_{\lambda os2} \lambda_{RIGID} (|h| - q_{\lambda os1} R_0) \qquad |h| > q_{\lambda os1} R_0$$

Both the length of the basic function and the offset of the basic function are displayed as function of the obstacle height in Figure 46 on page 125 .

Figure 46. Basic Function Length and Offset as Function of Obstacle Heightt

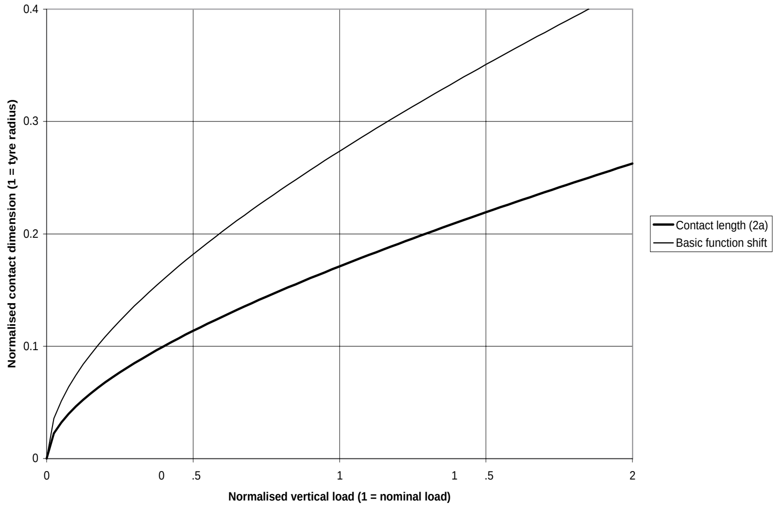


The next step in determining the effective input is made by taking two points and run over the basic function (see Figure 47 on page 126). The distance between the two points is indicated as the shift of the basic function. The distance between the points (λ_{imp}) depends on the contact length as is displayed in Figure 4.28 of [1], and the shift is calculated using Equation [227]:

$$\lambda_{imp} = 2(q_{\lambda imp1} \cdot a + q_{\lambda imp2} \cdot a^2) \tag{227}$$

Both the contact length and the shift of the basic function are displayed in Figure 47 on page 126.

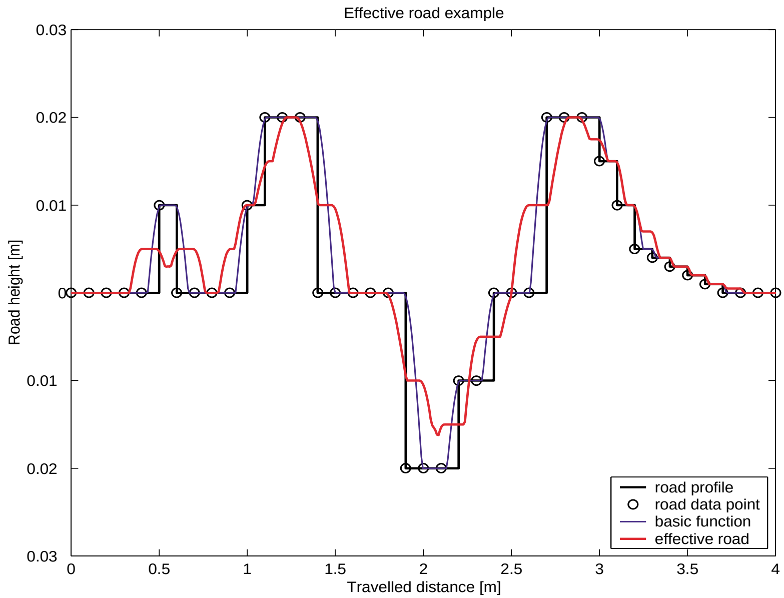
Figure 47. Contact Length and Shift of the Basic Function as Function of Load



This method of effective inputs can not only be used for discrete obstacles, but also for measured road data having a random character, Figure 48 on page 127 gives an illustration. In this example the road height is specified every 0.1 meter. Using the stepwise changes in road height the basic functions can be calculated and using the two-point-tire-road interface model finally the effective road height is obtained.

The SWIFT-Tyre model samples the road using a fixed interval. This value is specified by ROAD_INCREMENT in the [MODEL] section of the tire property file, as seen on page 139. Typically this value is in the range of 0.1–0.2 meter or larger; values below 0.01 meter are ignored. If the road data has a fixed sample interval, then the most accurate results will be obtained when ROAD_INCREMENT is set equal to the sample interval of the road data. In the example of Figure 48 the value of ROAD_INCREMENT is set to 0.1 meter, the actual road data used in this example can be found in Road Property File Example on page 146.

Figure 48. Filtering of Road Data Using Basic Functions

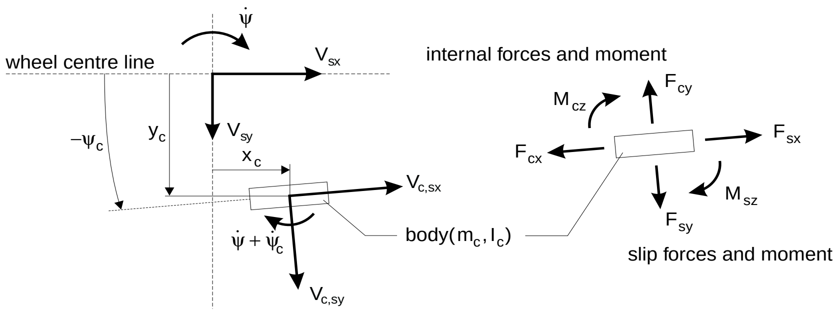


Contact Model

Residual Stiffness

The contact patch is modelled as a body with mass and inertia, and has three degrees of freedom: longitudinal, lateral and yaw motion, as depicted in Figure 49. The contact patch is connected to the rigid ring body of the tire belt with residual spring-damper systems. The slip forces are applied to the contact patch, and the transient of slip forces is modelled following the relaxation length concept, with an elaborate model for the aligning moment calculation.

Figure 49. Contact Patch Model



The forces and torque that are transmitted through residual springs from the contact patch to the Rigid Ring are given by Equations [228] to [230]:

$$F_{rx}= \bar{c}_{rx}\bar{\rho}_{rx}F_{z0} + 2\kappa_{rx}\dot{\bar{\rho}}_{rx}k_{t0} \tag{228}$$

$$F_{ry}= \bar{c}_{ry}\bar{\rho}_{ry}F_{z0} + 2\kappa_{ry}\dot{\bar{\rho}}_{ry}k_{t0} \tag{229}$$

$$M_{rz}= \bar{c}_{r\psi}\rho_{r\psi}C_{r0} + 2\kappa_{r\psi}\dot{\rho}_{r\psi}k_{r0} \tag{230}$$

Transient Slip Behaviour

At change of slip, it takes a certain distance to build up the forces in the contact area. This transient behaviour is incorporated in the model, and is referred to as relaxation length. The contact length is the main determining factor for the transient properties in the contact patch and it is a function of vertical load in accordance with Equation [231]:

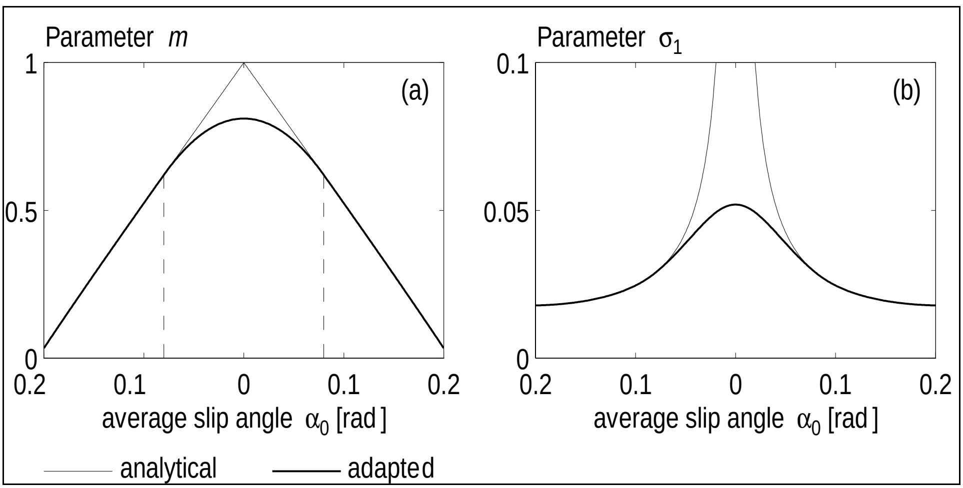
$$a= \left(q_{a2}\sqrt{\bar{F}_z^2} + q_{a1}\sqrt{\bar{F}_z}\right)R_0 \tag{231}$$

The resulting function for the contact length is displayed in Figure 47 on page 126. The relaxation length in the contact area (σ_c) is a function of the adhesion level (m) in accordance with Equation [232], which is similar to Equation 3.28 in reference [2]:

$$\sigma_c = m \cdot a \tag{232}$$

The value of m is also used in the so-called Phase leading network that is applied in the aligning moment calculation (see reference [2]). In order to prevent numeric instability around zero slip (when m approaches 1), the value is modified within a small band of slip as displayed in Figure 50.

Figure 50. Modification of Adhesion Coefficient in Calculation



The band of modification is determined by Equation [233], where α_{sl} is the slip value where full sliding is assumed:

$$\alpha_{min} = q_{\alpha min} \alpha_{sl} \tag{233}$$

In addition to the common relaxation length system, the longitudinal relaxation length system is extended to Equation [234] to increase damping at lower speeds. The damping parameters are applied in the model through Equation [235]. In both equations, the forward speed (V_x) is used instead of the rolling speed (V_{cr}) to increase the robustness of the model:

$$\left(\sigma_c + \frac{k_{cx}}{c_{cx}}|V_x|\right)\zeta_{cx} + |V_x|\zeta_{cx} = -V_{c,sx} - \frac{k_{cx}}{c_{cx}}\dot{V}_{c,sx} \tag{234}$$

$$\frac{k_{cx}}{c_{cx}} = \frac{\dot{q}_{kc1}}{1 + q_{kc2}|V_x|} \tag{235}$$

Switching from Simple to Complex Tire Model

The SWIFT software incorporates a switch for tire complexity selection: the parameter USE_MODE in the [MODEL] section of the tire property file. Instead of full Rigid Ring tire dynamics, the switch can be set for transient behaviour only, or steady state behaviour. Under these conditions the SWIFT model behaves just as MF-Tyre. The optional settings are given in Table 52.

Table 52. Various Options for the Value of USE_MODE

	Steady state:	Transient:	SWIFT:
Vertical only	0	0	20
Longitudinal only	1	11	21
Lateral only	2	12	22
Long. and lat. uncombined	3	13	23
Combined slip	4	14	24

In case of transient behaviour, the belt stiffness is taken into account as well as the contact length for the calculation of the tire relaxation length. The longitudinal relaxation length is calculated from the SWIFT stiffness using Equation [236], which incorporates a scaling factor:

$$\sigma_{\kappa} = \left(\alpha + K_x\left(\frac{1}{c_{bx}} + \frac{1}{c_{rx}} + \frac{R_e^{2}}{C_{b\theta}}\right)\right)\lambda_{\sigma\kappa} \tag{236}$$

Similarly, the lateral relaxation length is calculated from the SWIFT stiffness using Equation [237] (note that K_y is a negative quantity), including a scaling factor:

$$\sigma_{\alpha} = \left(\alpha - K_y\left(\frac{1}{c_{by}} + \frac{1}{c_{ry}} + \frac{R_e^{2}}{C_{b\gamma}}\right)\right)\lambda_{\sigma\alpha} \tag{237}$$

Tire Model Parameters

The tire parameters that are defined in the tire property file are related to the model structure in Figure 51. For each of the parameters, a reference to the equations in this manual is given. The full context of the parameters can be understood by looking up the appropriate equations in this guide. The SWIFT-specific parameters are listed below. MF-Tyre indicates that the parameter or group of parameters is also used for MF-Tyre. For more information on those parameters see Using the MF-Tyre Model on page 1.

Figure 51. Tire Parameters in Model Structure

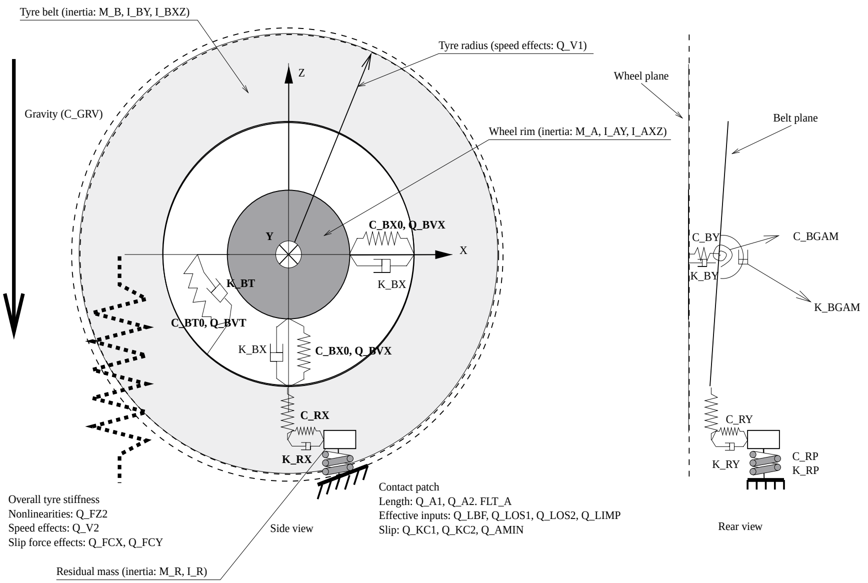


Table 53. Definition of Parameters in Tire Property File

Tire parameter:	User manual notation:	Notes Tire characteristic (if applicable):
[MODEL]		
LONGVL	V_0	See Equation [200], derived from Test Trailer measurement conditions
ROAD_INCREMENT	n.a	Sample interval road data
ROAD_DIRECTION	n.a	>0 driving in positive x-direction <0 driving in negative x-direction
[DIMENSION]		
UNLOADED_RADIUS	R_0	
[SHAPE]		MF-Tyre
[INERTIA]		
MASS	m_0	Tire mass
I_AY	n.a	Value to be added to wheel rim (multiply with $I_0=m_0R_0^2$)
I_AXZ	n.a	Value to be added to wheel rim (multiply with $I_0=m_0R_0^2$)
I_BY	n.a	Tire belt moment of inertia about Y-axis / Belt wind-up frequency — gyroscopic effects
I_BXZ	n.a	Tire belt moment of inertia about X and Z-axis / Belt camber and yaw frequency — gyroscopic effects
I_R	n.a.	Residual mass moment of inertia about Z-axis, tuned value for optimal performance
M_A	n.a.	Value to be added to wheel rim (multiply with m0)
M_B	m_b	See Table 48 on page 110 /Translation belt frequencies

Table 53. Definition of Parameters in Tire Property File *(continued)*

Tire parameter:	User manual notation:	Notes Tire characteristic (if applicable):
M_R	n.a.	Residual mass, tuned value for optimal performance
C_GRV	C_{grv}	See Equation [207]
[CONTACT_PATCH]		
Q_A2	q_{a2}	See Equation [231] / Relaxation length & enveloping behaviour
Q_A1	q_{a1}	See Equation [231] / Relaxation length & enveloping behaviour
Q_LBF	q_{lbf}	See Equation [225] / Enveloping behaviour
Q_LOS1	q_{los1}	See Equation [226] /Enveloping behaviour
Q_LOS2	q_{los2}	See Equation [226] Enveloping behaviour
Q_LIMP1	q_{limp1}	See Equation[227] Enveloping behaviour
Q_LIMP2	q_{limp2}	See Equation[228] Enveloping behaviour
Q_KC1	q_{kc1}	See Equation [235], tuned value for optimal performance / Tire damping for low speed
Q_KC2	q_{kc2}	See Equation [235], tuned value for optimal performance /Tire damping for low speed
Q_AMIN	q_{amin}	See Equation [233], tuned value for optimal performance / Aligning moment for short wavelength around slip=0
FLT_A	n.a.	Contact length filter, tuned value for optimal performance / Contact length for load variations
[VERTICAL]		
VERTICAL_STIFFNESS	C_z	Flat Planksee Equation [221] / Vertical stiffness + vertical belt frequency
VERTICAL_DAMPING		MF-Tyre
BREFF		MF-Tyre

Table 53. Definition of Parameters in Tire Property File *(continued)*

Tire parameter:	User manual notation:	Notes Tire characteristic (if applicable):
DREFF		MF-Tyre
FREFF		MF-Tyre
FNOMIN	F_{z0}	see Equation [201], defined value for measurement programme /Tire load rating
Q_RE0	q_{re0}	see Equation [218] / Free tire and effective tire radius
Q_V1	q_{V1}	see Equation [217] /Tire growth due to speed
Q_V2	q_{V2}	see Equation [220] / Speed effect on vertical stiffness
Q_FZ2	q_{Fz2}	See Equation [220] / Progessiveness of vertical load for deflection
Q_FCX	q_{Fcx}	See Equation [220] / Decrease in vertical stiffness due to brake slip force
Q_FCY	q_{Fcy}	See Equation [220] / Decrease in vertical stiffness due to side slip force
[LONG_SLIP_RANGE]		MF-Tyre
[SLIP_ANGLE_RANGE]		MF-Tyre
[INCLINATION_ANGLE_RANGE]		MF-Tyre
[VERTICAL_FORCE_RANGE]		MF-Tyre
[SCALING_COEFFICIENTS]		MF-Tyre
[LONGITUDINAL_COEFFICIENTS]		MF-Tyre
[OVERTURNING_COEFFICIENTS]		MF-Tyre
[LATERAL_COEFFICIENTS]		MF-Tyre
[ROLLING_COEFFICIENTS]		MF-Tyre
[ALIGNING_COEFFICIENTS]		MF-Tyre

Table 53. Definition of Parameters in Tire Property File *(continued)*

Tire parameter:	User manual notation:	Notes Tire characteristic (if applicable):
[STRUCTURAL]		
C_BX0	c_{bxo}	See Equation [209] / In-plane translation belt frequency — longitudinal relaxation length
C_RX	c_{rx}	See Equation [228], tuned value for optimal performance /Longitudinal relaxation length
C_BT0	c_{bq0}	see Equation [210] / Belt wind-up rotation frequency — longitudinal relaxation length
C_BY	c_{by}	See Equation [214] /Out-of-plane translation belt frequency — lateral relaxation length
C_RY	c_{ry}	See Equation [229] /Out-of-plane translation belt frequency — lateral relaxation length
C_BGAM	c_{bg}	See Equation [215] /Belt Camber / yaw rotation frequency — lateral relaxation length
C_RP	c_{ry}	See Equation [230], tuned value for optimal performance / Aligning moment at short wavelength
K_BX	k_{bx}	See Equation [211] /damping for in-plane translation belt frequency
K_RX	k_{rx}	See Equation [228], tuned value for optimal performance
K_BT	k_{bq}	See Equation [213] /damping for wind-up belt frequency
K_BY	k_{by}	See Equation [214] /damping for lateral belt frequency
K_RY	k_{ry}	See Equation [229]
K_BGAM	k_{bg}	See Equation [215] /Belt Camber / yaw rotation frequency — lateral relaxation length

Table 53. Definition of Parameters in Tire Property File *(continued)*

Tire parameter:	User manual notation:	Notes Tire characteristic (if applicable):
K_RP	k_{rp}	See Equation [230], tuned value for optimal performance
Q_BVX	q_{bvx}	See Equation [209] /Change of in-plane belt translation frequency with speed
Q_BVT	q_{bVq}	See Equation [210] /Change of wind-up belt rotation frequency with speed

Tire Property File Example

```
[[MDI_HEADER]
FILE_TYPE                ='tir'
FILE_VERSION             =3.0
FILE_FORMAT              ='ASCII'
! : TIRE_VERSION :      SWIFT-Tyre 1.0
! : COMMENT :           New File Format v3.0
! : COMMENT :           Tire                    205/60 R15
! : COMMENT :           Manufacturer            DELFT-TYRE
! : COMMENT :           Nom. section with  (m)   0.205
! : COMMENT :           Nom. aspect ratio  (-)    60
! : COMMENT :           Infl. pressure    (Pa)   220000
! : COMMENT :           Rim radius         (m)    0.19
! : COMMENT :           Measurement ID           DELFT-TYRE
! : COMMENT :           Test speed        (m/s)  16.667
! : COMMENT :           Road surface            Asphalt
! : COMMENT :           Road condition          Dry
! : FILE_FORMAT :     ASCII
! : USER :                      MF-Tool
! : Generated by :           TNO
! : Copyright TNO, Tue Aug 07 16:33:34 2001
!
! USE_MODE specifies the type of calculation performed:
!       0:    Fz only, no Magic Formula evaluation
!       1:    Fx,My only
!       2:    Fy,Mx,Mz only
!       3:    Fx,Fy,Mx,My,Mz uncombined force/moment calculation
!       4:    Fx,Fy,Mx,My,Mz combined force/moment calculation
!     +10:   including relaxation behaviour
!     +20:   including rigid ring dynamics
!     *-1:    mirroring of tyre characteristics
!
!    example: USE_MODE = -12 implies:
!       -calculation of Fy,Mx,Mz only
!       -including relaxation effects
!       -mirrored tyre characteristics
!
$----------------------------------------------------------------units
[UNITS]
LENGTH                  ='meter'
```

```
FORCE                         ='newton'
ANGLE                         ='radians'
MASS                          ='kg'
TIME                          ='second'
$---------------------------------------------------------------model
[MODEL]
PROPERTY_FILE_FORMAT          ='SWIFT-TYRE'
TYPE                             ='CAR'
FITTYP                           = 21
USE_MODE                         = 24
MFSAFE1                          = -5280
MFSAFE2                          = 0
MFSAFE3                          = 150
LONGVL                           = 16.667
VXLOW                            = 1
ROAD_INCREMENT                   = 0.1
ROAD_DIRECTION                   = 1
$----------------------------------------------------------dimensions
[DIMENSION]
UNLOADED_RADIUS                  = 0.3135
WIDTH                                  = 0.205
ASPECT_RATIO                     = 0.6
RIM_RADIUS                       = 0.19
RIM_WIDTH                        = 0
$---------------------------------------------------------------shape
[SHAPE]
{radial width}
 1.0    0.0
 1.0    0.4
 1.0    0.9
 0.9    1.0
$-------------------------------------------------------------inertia
[INERTIA]
MASS                             = 9.3
I_AY                             = 0.109406207
I_AXZ                            = 0.0711140344
I_BY                             = 0.695823475
I_BXZ                            = 0.356664234
I_R                              = 0.0547031034
M_A                              = 0.23655914
```

```
M_B                          = 0.76344086
M_R                          = 0.107526882
C_GRV                        = -9.81
$-------------------------------------------------------contact_patch
[CONTACT_PATCH]
Q_A2                         = 0.0353429027
Q_A1                         = 0.135228475
Q_LBF                        = 1
Q_LOS1                       = 0.01
Q_LOS2                       = 0.4
Q_LIMP1                      = 0.8
Q_LIMP2                      = 0.0
Q_KC1                        = 0.106328549
Q_KC2                        = 6.6668
Q_AMIN                       = 0.3
FLT_A                        = 2000
$-------------------------------------------------------------vertical
[VERTICAL]
VERTICAL_STIFFNESS           = 196261
VERTICAL_DAMPING             = 50
BREFF                        = 9
DREFF                        = 0.23
FREFF                        = 0.01
FNOMIN                       = 4000
Q_RE0                        = 0.997448166
Q_V1                         = 7.15073791e-005
Q_V2                         = 2.4892
Q_FZ2                        = 14.3468
Q_FCX                        = 0
Q_FCY                        = 0
BOTTOM_OFFST                 = 0.01
BOTTOM_TRNSF                 = 1000
BOTTOM_STIFF                 = 2E+6
$----------------------------------------------------long_slip_range
[LONG_SLIP_RANGE]
KPUMIN                       = -1.5
KPUMAX                       = 1.5
$---------------------------------------------------slip_angle_range
[SLIP_ANGLE_RANGE]
ALPMIN                       = -1.5708
```

```
ALPMAX                      = 1.5708
$---------------------------------------------inclination_slip_range
[INCLINATION_ANGLE_RANGE]
CAMMIN                      = -0.2619
CAMMAX                      = 0.2619
$-----------------------------------------------vertical_force_range
[VERTICAL_FORCE_RANGE]
FZMIN                       =   0
FZMAX                       = 9000
$------------------------------------------------------------scaling
[SCALING_COEFFICIENTS]
LFZO                        = 1
LCX                         = 1
LMUX                        = 1
LEX                         = 1
LKX                         = 1
LHX                         = 0
LVX                         = 0
LGAX                        = 1
LCY                         = 1
LMUY                        = 1
LEY                         = 1
LKY                         = 1
LHY                         = 0
LVY                         = 0
LGAY                        = 1
LTR                         = 1
LRES                        = 0
LGAZ                        = 1
LXAL                        = 1
LYKA                        = 1
LVYKA                       = 1
LS                          = 1
LSGKP                       = 1
LSGAL                       = 1
LGYR                        = 1
LMX                         = 1
LVMX                        = 1
LMY                         = 1
$-------------------------------------------------------longitudinal
```

```
[LONGITUDINAL_COEFFICIENTS]
PCX1                        = 1.6846
PDX1                        = 1.2096
PDX2                        = -0.03705
PDX3                        = 0
PEX1                        = 0.34446
PEX2                        = 0.095439
PEX3                        = -0.020488
PEX4                        = 0
PKX1                        = 21.512
PKX2                        = -0.16314
PKX3                        = 0.24502
PHX1                        = -0.0016331
PHX2                        = 0.001517
PVX1                        = 0
PVX2                        = 0
RBX1                        = 12.35
RBX2                        = -10.767
RCX1                        = 1.0918
REX1                        = 0
REX2                        = 0
RHX1                        = 0.0066313
PTX1                        = 1
PTX2                        = 0
PTX3                        = 0
$----------------------------------------------------------overturning
[OVERTURNING_COEFFICIENTS]
QSX1                        = 0
QSX2                        = 0
QSX3                        = 0
$--------------------------------------------------------------lateral
[LATERAL_COEFFICIENTS]
PCY1                        = 1.1931
PDY1                        = -0.99006
PDY2                        = 0.14522
PDY3                        = -11.231
PEY1                        = -1.0026
PEY2                        = -0.53683
PEY3                        = -0.083107
PEY4                        = -4.7866
```

```
PKY1                    = -14.946
PKY2                    = 2.1297
PKY3                    = -0.028283
PHY1                    = 0.0033518
PHY2                    = -0.00053863
PHY3                    = 0.07452
PVY1                    = 0.044552
PVY2                    = -0.023557
PVY3                    = -0.53156
PVY4                    = 0.03923
RBY1                    = 6.461
RBY2                    = 4.1957
RBY3                    = -0.015164
RCY1                    = 1.0812
REY1                    = 0
REY2                    = 0
RHY1                    = 0.0086257
RHY2                    = 0
RVY1                    = 0.053266
RVY2                    = -0.073458
RVY3                    = 0.51728
RVY4                    = 35.444
RVY5                    = 1.9
RVY6                    = -10.715
PTY1                    = 1
PTY2                    = 1
$---------------------------------------------------rolling resistance
[ROLLING_COEFFICIENTS]
QSY1                    = 0.01
QSY2                    = 0
QSY3                    = 0
QSY4                    = 0
$-------------------------------------------------------------aligning
[ALIGNING_COEFFICIENTS]
QBZ1                    = 8.9644
QBZ2                    = -1.1064
QBZ3                    = -0.8422
QBZ4                    = 0
QBZ5                    = -0.22733
QBZ9                    = 18.465
```

```
QBZ10                       = 0
QCZ1                        = 1.1805
QDZ1                        = 0.099556
QDZ2                        = -0.00074773
QDZ3                        = 0.0065197
QDZ4                        = 13.053
QDZ6                        = -0.0079448
QDZ7                        = 0.00019609
QDZ8                        = -0.29569
QDZ9                        = -0.0089855
QEZ1                        = -1.6085
QEZ2                        = -0.3592
QEZ3                        = 0
QEZ4                        = 0.17433
QEZ5                        = -0.8957
QHZ1                        = 0.0067668
QHZ2                        = -0.0018847
QHZ3                        = 0.14697
QHZ4                        = 0.0042775
SSZ1                        = 0.043285
SSZ2                        = 0.0013747
SSZ3                        = 0.73146
SSZ4                        = -0.23758
QTZ1                        = 0.05
MBELT                       = 9.3
$------------------------------------------------------------structural
[STRUCTURAL]
C_BX0                       = 121.3872
C_RX                        = 391.875
C_BT0                       = 61.9617225
C_BY                        = 40.049625
C_RY                        = 62.7
C_BGAM                      = 20.3349282
C_RP                        = 55.8213716
K_BX                        = 0.113761382
K_RX                        = 0.45504553
K_BT                        = 0.0398641872
K_BY                        = 0.141974205
K_RY                        = 0.45504553
K_BGAM                      = 0.0185199476
```

```
K_RP                     = 0.416698821
Q_BVX                    = 3.9567458
Q_BVT                    = 3.9567458
```

Road Property File Example

In the road property file the road height is specified as a function of traveled distance. In a road data file the left and right track data may be specified; the appropriate track data is selected depending on the role of the tire in the model.

SWIFT uses a zero-order sample and hold when evaluating the road profile, as shown in Figure 48 on page 127. Changes in the height of the road profile are interpreted as steps. For maximum accuracy it is important that the sample points coincide with the data provided by the user, otherwise interpolated data will be used. So you should use road data with a fixed sample interval and specify this value for ROAD_INCREMENT in the [MODEL] section of the tire property file. Typically, the road sample interval should be in the range of 0.1-0.2 meters or larger. For the road data given below, the value of ROAD_INCREMENT should be set to 0.1 meter.

```
$---------------------------------------------------------MDI_HEADER
[MDI_HEADER]
FILE_TYPE            =  'rdf'
FILE_VERSION         =  5.00
FILE_FORMAT          =  'ASCII'
(COMMENTS)
{comment_string}
'polyline style road description'
$--------------------------------------------------------------UNITS
[UNITS]
 MASS                = 'kg'
 LENGTH              = 'meter'
 TIME                = 'sec'
 ANGLE               = 'degree'
 FORCE               = 'newton'
$--------------------------------------------------------------MODEL
[MODEL]
 METHOD              = '2D'
 ROAD_TYPE           = 'poly_line'
$---------------------------------------------------------PARAMETERS
[PARAMETERS]
 OFFSET                            =   0
 ROTATION_ANGLE_XY_PLANE           =   0
 MU                                =   1
```

```
$
(XZ_DATA)
-10000    0          0
     0         0       0
     0.1       0       0
     0.2       0       0
     0.3       0       0
     0.4       0       0
     0.5    0.01     0.01
     0.6       0       0
     0.7       0       0
     0.8       0       0
     0.9       0       0
       1    0.01     0.01
     1.1    0.02     0.02
     1.2    0.02     0.02
     1.3    0.02     0.02
     1.4       0       0
     1.5       0       0
     1.6       0       0
     1.7       0       0
     1.8       0       0
     1.9     -0.02        -0.02
       2    -0.02        -0.02
     2.1    -0.02        -0.02
     2.2    -0.01        -0.01
     2.3    -0.01        -0.01
     2.4       0       0
     2.5       0       0
     2.6       0       0
     2.7    0.02     0.02
     2.8    0.02     0.02
     2.9    0.02     0.02
       3   0.015     0.015
     3.1    0.01     0.01
     3.2   0.005     0.005
     3.3   0.004     0.004
     3.4   0.003     0.003
     3.5   0.002     0.002
     3.6   0.001     0.001
```

```
    3.7       0       0
    3.8       0       0
    3.9       0       0
    4.0       0       0
10000      0        0
```