



Estimation of the Rolling Resistance of Tires

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Abstract

Evaluation of the performance potential of an automotive conceptual design requires some initial quantitative estimate of numerous relevant parameters. Such parameters include the vehicle mass properties, frontal and plan areas, aero drag and lift coefficients, available horsepower and torque, and various tire characteristics such as the rolling resistance coefficient(s)...

A number of rolling resistance models have been advanced since Robert William Thomson first patented the pneumatic rubber tire in 1845, most of them developed in the twentieth century. Most early models only crudely approximate tire rolling resistance behavior over a limited range of operation, while the latest models overcome those limitations but often at the expense of extreme complexity requiring significant computer resources. No model extant seems well suited to the task of providing a methodology for the estimation of a tire's rolling resistance "coefficient" that is simple to use yet accurate enough for modern conceptual design evaluation.

It is the intent of this paper to suggest a methodology by which this seeming deficiency may be rectified.

Introduction

Thomson carried out crude experiments demonstrating the benefits of using pneumatic rubber tires, instead of the then conventional rigid wheels, on horse drawn conveyances in the early mid-1800's. Not only was the ride quality improved but the drawbar pull necessary to maintain motion was decreased¹. However, this qualitative indication regarding the rolling resistance of tires with respect to that of rigid wheels would represent the extent of knowledge on the matter for a long time. The prevailing level of technology of the period would prevent the general adoption of Thomson's invention.

When John Boyd Dunlop re-invented the pneumatic tire (1888, UK patent) the prevailing level of technology had risen considerably and the bicycle craze was in full swing, with the rise of the automobile just beginning. For approximately the next forty years the pneumatic rubber tire would undergo a rapid development unaided by any scientific understanding of the physical mechanisms involved. Such an understanding would await the development of machines which would allow for the testing of tire behavior in a laboratory environment.

The first tire testing machine was possibly the rotating steel drum tire tester of Becker, Fromm, and Maruhn circa 1930². These German researchers generated tire data as a prerequisite for their investigation of the great automotive problem of the time: steering "shimmy". In this they were following up on the French researcher George Broulheit who had identified the tire characteristic of "slip angle" in his investigation relating to shimmy³. A notable follower in the footsteps of these Europeans was R.D. Evans of the Goodyear Tire and Rubber Company who continued the investigation of the physical properties of tires via another drum tire tester⁴.

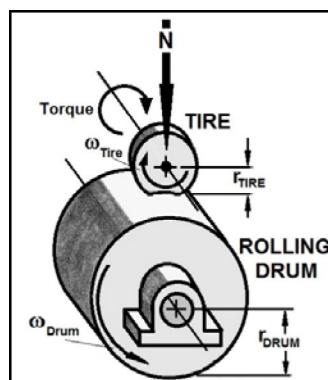


Figure 1. Rolling Drum Tire Tester.

1. Thomson published an article on the relatively low resistance to motion of "tyred" wheels with respect to rigid wheels in Mechanics Magazine (Glasgow) sometime in 1846; no copies appear to be extant today.

2. Reference [1].
3. Reference [2].
4. Reference [3].

Since tires are tested (and utilized) while within the atmosphere the measured rolling resistance has two major aspects to its composition. The primary aspect has to do with the flexing/squirming of the tread/ sidewall material as the tire rotates on the test drum, resulting in resistance due to hysteresis/friction. The secondary aspect has to do with resistance due to rotational aerodynamic drag at the external/internal tire surfaces.

Tire Testing for Rolling Resistance

The measurement of the torque “ T ” to maintain a constant angular speed “ ω_{Tire} ” of the tire tested while subject to a normal load “ N ” over a wide range of variation allowed for the collection of a vast body of data. Analysis of that data made at least two aspects of tire behavior abundantly clear: 1) There was a certain degree of “slip” between the tire and the drum dependent upon the magnitude of the torque, and 2) The torque necessary to maintain a constant speed increased with that speed.

The first observation would lead to a great many studies of how the longitudinal traction force (“ $F_x = T/Tire$ ”) is dependent upon the normal load and the longitudinal coefficient of traction (“ $F_x = \mu_x N$ ”), and how in turn the degree of slip “%S” is dependent upon “ F_x ”, *et cetera*. However, while extremely important and interesting, all of that is not relevant to the immediate subject of this paper.

What is relevant is that the second observation revealed the nature of the rolling resistance “ F_r ” to be a simple linear function of normal load⁵...

$$F_r = C_r N \quad (1)$$

...and a somewhat more complex function of velocity (“ $V = r_{TIRE} \omega_{Tire}$ ”) as illustrated by the author in the next two figures...

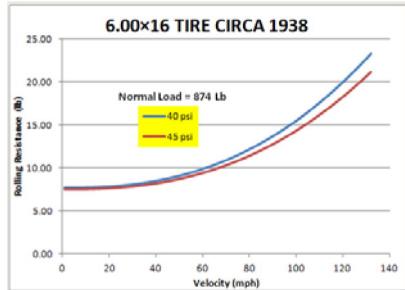


Figure 2. Rolling Resistance Force vs. Velocity.

...means the coefficient “ C_r ” must also vary with velocity, which becomes very evident over the speed range 0-140 mph (0-225 kph):

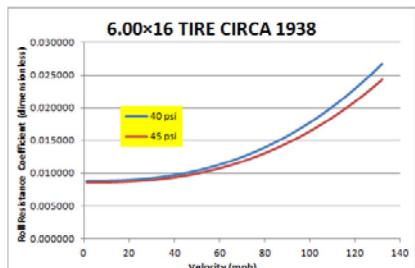


Figure 3. Rolling Resistance Coefficient vs. Velocity.

Rolling Resistance Coefficients

A slightly less simplified (includes “ V ”) formulation than Eq. 1 is⁶:

$$F_r = C_r (1 + V/100) N \quad (2)$$

This linear approximation was thought to provide reasonable accuracy up to 80 mph (129 kph). Such equations were developed for use with tables of roll resistance coefficients appropriate for passenger car bias ply tires inflated to 32-40 psi (221-276 kPa). Various such tables exist, and they all depend on the type of road surface considered and the level of technology (“LOT”) in effect at the time that the data was generated, so they all tend to be of limited application.

Such simple relationships as Eq. 2 are only useful for crude calculations valid over limited speed ranges; rolling resistance phenomena is more complex than can be captured by a simple “ C_r ” value. The phenomenon has both static (“ C_{sr} ”) and dynamic (“ C_{dr} ”) components as per an equation that was developed at the Institute of Technology in Stuttgart circa 1938, an equation which includes the vehicle velocity (“ V ”) in mph units⁷:

$$F_r = [C_{sr} + 3.24 C_{dr} (V/100)^{2.5}] N \quad (3)$$

This is perhaps the ultimate in rolling resistance modeling (or at least this author’s favorite). Taborek presents a plot of how “ C_{sr} ” and “ C_{dr} ” vary with inflation pressure “ P_i ”⁸; that plot allows for a data “pick-off” and a subsequent regression analysis of the data:

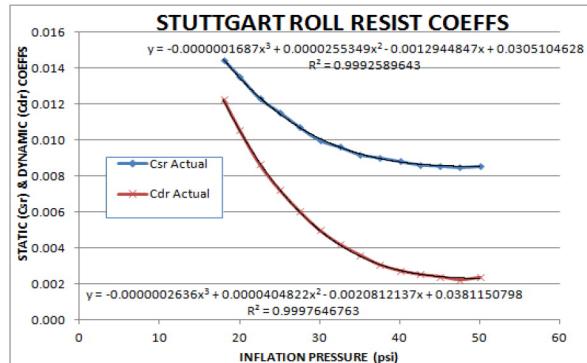


Figure 4. Rolling Resistance Coefficients vs. Inflation Pressure.

The regression analysis resulted in some very accurate expressions for “ C_{sr} ” and “ C_{dr} ” as a function of “ P_i ” for the 1938 LOT 6.00x16 reference tire⁹. These expressions were used to establish the baseline reference “ C_{sr} ” and “ C_{dr} ” values:

$$(C_{sr})_{Ref} = -0.0000001687 P_i^3 + 0.0000255349 P_i^2 - 0.0012944847 P_i + 0.0305104628 \quad (4)$$

$$(C_{dr})_{Ref} = -0.0000002636 P_i^3 + 0.0000404822 P_i^2 - 0.0020812137 P_i + 0.0381150798 \quad (5)$$

6. Reference [4], pg. 117. Reference [5], pg. 34.

7. Ibid. Ibid.

8. Ibid, pg. 118. Ibid, pg. 32.

9. The high “R²” values testify to the accuracy of the regression relations within the inflation pressure range of 15 to 50 psi (103 kPa to 345 kPa).

5. Reference [4], pg. 111.

The “ C_{sr} ” coefficient has to do mainly with the primary hysteresis/frictional aspect of rolling resistance, and “ C_{dr} ” coefficient has to do mainly with the secondary rotational aerodynamic aspect. The primary aspect is dominant at “low” velocity, while the secondary aspect becomes dominant at “high” velocity.

Establishing the Methodology

Since Equations 4 and 5 are specific to a 1938 LOT 6.00×16 tire there must be some adjustment factors added to Equations 4 & 5 to make them applicable for the estimation of the rolling resistance of tires not of that baseline LOT or tire size. Two distinctly different possible approaches to the development of such factors present themselves.

One approach would be to develop individual factors for each and every rolling resistance parameter by which a tire in question could differ with respect to the reference tire. Some of the parameters that might require evaluation could include: diameter (rim, rolling), width (rim, tire), number of plies (carcass, tread), type of material (carcass, rubber), tire volume, tire surface area, and tire surface roughness¹⁰. The total number of parameters requiring evaluation could easily exceed this list of 12 if this were the approach actually pursued.

While the first approach might result in a higher degree of accuracy, it obviously requires an intensive investment of resources which may ultimately prove not worth the effort. Hence this second approach, which seeks to develop a limited number of adjustment factors (initially just 4) which implicitly combine the various parameters into something more manageable. This approach begins with the formula for the tread contact area “ A_c ” of a tire under load¹¹:

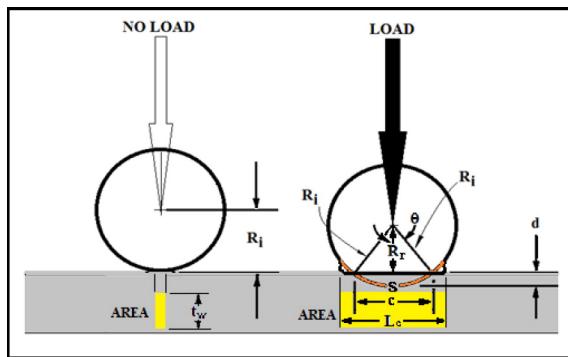


Figure 5. Tire Contact Area, Under Load.

$$A_c = L_c t_w = 1.24 R_i \cos^{-1}[(R_i - d)/R_i] t_w \quad (6)$$

Use of this equation requires knowledge of the tread width “ t_w ” and the deflection “ d ” under load “ N ”. The tread width may be determined by the “Michelin Formula”¹²:

$$t_w = 0.03937 (-0.004 AR + 1.03) S_N \quad (7)$$

The deflection may be determined by¹³:

$$d = N/K_z + d_0 \quad (8)$$

The determination of tire deflection requires knowledge of the tire vertical spring constant “ K_z ” necessitating the use of the “Rhynes Equation”¹⁴:

$$K_z = 0.00028 P_t \sqrt{(-0.004 AR + 1.03) S_N \times \left[\frac{S_N \times AR}{50} + D_R \right] + 3.45} \quad (9)$$

The parameter input values for all the equations presented so far are mainly easily determined from the tire designation, at least when the designation is a modern “P-metric” type (e.g.: P152/92R16).

However, some variation on Equation 6 is required to make it more suitable for producing factors useful for rolling resistance determination. A volume ratio factor “ f_{vol} ” to use in conjunction with “ C_{sr} ” requires not just the deflected tread volume but also some measure of the deflected sidewall volume. To this end Figure 5 and Equation 6 were utilized to develop the following formula for the tread contribution “ V_t ” to the volume ratio factor:

$$V_t = 1.24 R_i \cos^{-1}[(R_i - d)/R_i] t_w [t_t - f_{void}(t_d/32)] \quad (10)$$

And to develop the following formula for the sidewall contribution “ V_{sw} ” to the volume ratio factor:

$$V_{sw} = R_i^2 \cos^{-1}[(R_i - d)/R_i] t_{sw} \quad (11)$$

Together, Equations 10 and 11 form the basis for the volume ratio factor “ f_{vol} ”:

$$f_{vol} = \frac{(V_t + V_{sw})_{Unknown\ Tire}}{(V_t + V_{sw})_{Reference\ Tire}} \quad (12)$$

An area ratio factor “ f_{area} ” to use in conjunction with “ C_{dr} ” requires some measure of the total tire external and internal surface area. This is obtained in a similar fashion. First, the tread contribution “ A_t ” to the area ratio factor:

$$A_t = R_i (2\pi - 1.24 \cos^{-1}[(R_i - d)/R_i]) t_w \quad (13)$$

10. Reference [6], pp. 38-39. Some small liberties have been taken by this author with the order and statement of the parameters as given in the reference.

11. Reference [7], pg. 49. This equation was inspired by a concept presented by Prof. Dixon (Reference [8], pg. 85), developed by Mr. J. Todd Wasson of Performance Simulations, and refined by this author.

12. Reference [9], pg 193. Apparently Equation (7) was obtained by a regression analysis preformed using data representing a wide range of commonly available tires; the equation will not function well for tires varying greatly from the tires of that regression analysis data set (this explains why the equation did not produce reasonable results for a 152/46R8 tire!). Note that Equation (9) will also be subject to the same limitations as Equation (7).

13. Reference [7], pg. 50. The constant “ d_0 ” represents the initial non-linearity of tire load-deflection functions; this deflection axis intercept value is usually small but quite real.

14. Reference [9], pg.194.

Second, the sidewall contribution “ A_{sw} ” to the area ratio factor:

$$A_{sw} = 2R_i^2(\pi - \cos^{-1}[(R_i - d)/R_i] - 0.5 \sin(2\cos^{-1}[(R_i - d)/R_i])) \quad (14)$$

Equations 13 and 14 form the basis for the area ratio factor “ f_{area} ” in the now familiar manner:

$$f_{area} = \frac{(A_t + A_{sw})_{Unknown\ Tire}}{(A_t + A_{sw})_{Reference\ Tire}} \quad (15)$$

Equations 12 and 15 represent two of the four initial adjustment factors. For further adjustment the “ f_{LOT} ” level of technology factors, listed in Table 1, were determined per:

$$f_{LOT} = \frac{(C_r)_{Unknown\ Tire\ LOT}}{(C_r)_{Reference\ Tire\ LOT}} \quad (16)$$

For “ C_{dr} ” adjustment the “ f_{rough} ” surface roughness factors, listed in Table 2, were determined per:

$$f_{rough} = \frac{(f_{void})_{Unknown\ Tire}}{(f_{void})_{Reference\ Tire}} \quad (17)$$

These four initial factors are used to adjust the reference tire “ C_{sr} ” and “ C_{dr} ” to approximate the “ C_{sr} ” and “ C_{dr} ” of the “unknown” tire. The total equations for the estimated “ C_{sr} ” and “ C_{dr} ” values are:

$$(C_{sr})_{Unk} = f_{vol} f_{LOT} f_{R_i} (C_{sr})_{Ref} \quad (18)$$

$$(C_{dr})_{Unk} = f_{area} f_{rough} f_{LOT} (C_{dr})_{Ref} \quad (19)$$

Note that a fifth factor, “ f_{R_i} ”, has made a sudden appearance. This “wheel size” factor proved necessary upon analysis of the original results, and is calculated using a regression analysis formula obtained from those initial results:

$$f_{R_i} = 0.0701159 R_i^2 - 2.0387 R_i + 15.386 \quad (20)$$

Now, of course, the equation for the determination of estimated “ F_r ” values for the “unknown” tire (in that “ C_r ” is unknown) is:

$$[F_r]_{Unk} = [(C_{sr})_{Unk} + 3.24 (C_{dr})_{Unk} (V/100)^{2.5}] N \quad (21)$$

Given an appropriate number of estimated “ F_r ” values at various velocities “ V ” for the “unknown” tire being evaluated, a “ C_r ” value can be determined per the appropriate test standard SAE J1269 or SAE J2452 protocol¹⁵, just as if the “ F_r ” values for the “unknown” tire had been obtained from empirical rolling drum test results.

Exempli gratia: for this paper just a SAE J1269 SP (“Single Point”)

value is wanted, so Equation 21 is evaluated with “ V ” equal to 50 mph (80 kph), and the resultant “ C_r ” for the “unknown” tire is determined by:

$$(C_r)_{Unk} = \frac{[F_r]_{Unk}}{N} = (C_{sr})_{Unk} + 3.24 (C_{dr})_{Unk} \left(\frac{V}{100} \right)^{2.5} \quad (21)$$

Now that the basic methodology has been established, all that remains is the presentation of a LOT table and a void/surface roughness table that can be used for the determination of “ f_{LOT} ” and “ f_{rough} ” values to be “plugged into” Equations 18 and 19. All other values to be used in the equations presented are derived directly or indirectly from the tire designations.

The following LOT table was derived from historical “ C_r ” values presented in various sources¹⁶. For consistency the “ C_r ” values used were all for passenger car tires of about the same size as the reference tire (6.00×16, for which the “modern” equivalent designation is 152/92-16) with no “high-performance” (speed ratings of “V”, “W”, “Y”, and “Z”), no “ECO” (high fuel efficiency), and no “M/S” (Mud/Snow) or “A/T” (All Terrain) tires. To the greatest extent possible all such “extreme” tire variations were avoided for the LOT table formation. If it should be required to evaluate such an “extreme” type of tire, then some special *ad hoc* factors may be necessary for appropriate adjustment of Equations (18) and (19). The appropriate LOT factor “ f_{LOT} ” for use in Equation (18), obviating the use of Equation (16), may be obtained from the table:

Table 1. Level of Technology (LOT) Tire Factors by Year and Type

LOT FACTORS BY TIRE YEAR AND TYPE, 1938 TO 1980				RADIAL LOT FACTORS, 1980 to 2015	
YEAR	BIAS TUBE	BIAS	BIAS-BELT	RADIAL OE	RADIAL AM
1938	1.000000	COTTON			
1939	0.748906				
1940	0.764751				
1941	0.780727				
1942	0.796833				
1943	0.813070				
1944	0.829437				
1945	0.845935				
1946	0.862563				
1947	0.879322				
1948	0.896211				
1949	0.913231				
1950	0.930381	0.708405			
1951	0.947662	0.721563			
1952	0.965074	0.734821			
1953	0.982615	0.748177			
1954	1.000284	0.761633			
1955	1.018091	0.775189			
1956	1.036024	0.788843			
1957	1.054088	0.802597			
1958	1.072282	0.816451			
1959	1.090607	0.830404			
1960	1.109063	0.841046			
1961		0.851753			
1962		0.862524			
1963		0.873361			
1964		0.884262			
1965		0.895229			
1966		0.906260			
1967		0.917357			
1968		0.928518		0.736858	0.826755
1969		0.939744		0.739883	0.830148
1970		0.951035	0.878757	0.742754	0.833370
1971	STEEL	0.962391	0.889250	0.745469	0.836417
1972		0.973812	0.899802	0.748027	0.839287
1973		0.985294	0.910415	0.750426	0.841978
1974		0.996849	0.921088	0.752662	0.844487
1975		0.008465	0.931821	0.754734	0.846812
1976	ARAMID	1.020145	0.942614	0.756641	0.848951
1977		1.031891	0.953467	0.758379	0.850901
1978		1.036691	0.957902	0.759946	0.852659
1979		1.048534	0.968845	0.761341	0.854225
1980		1.060442	0.979848	0.762561	0.855594

15. The ISO equivalents are ISO 28580 and ISO 18164, respectively. On average for passenger car tires, SAE J1269 (SP) produces “ C_r ” values about 6% greater than SAE J2452 (SMERF) per information in Reference [10], pp. 55-56.

16. Reference [11] mainly, but also from Reference [10] and References [12] through [16].

The void factor “ f_{void} ” for use in [Equation 10](#), and the surface roughness factor “ f_{rough} ” (obviating the use of [Equation 17](#)) to be used in [Equation 19](#) may be obtained from the following “Land-to-Sea Ratio” table¹⁷:

Table 2. Void and Surface Roughness Factors

Typical void percents:	Land:Sea Ratio	Net Factor	Void Factor	Roughness Factor
General touring tyres - 30%	70:30	0.70	0.30	1.00
High performance tyres - 35%	65:35	0.65	0.35	1.17
Ultra high performance tyres - 38%	62:38	0.62	0.38	1.27
Winter tyres - 40%	60:40	0.60	0.40	1.33
4x4 tyres - 40%	60:40	0.60	0.40	1.33

The tread depth value “ t_d ” to be used in [Equation 10](#) may be obtained from the following table¹⁸:

Table 3. Typical Tread Depths by Tire Type

Tread Depth (1/32 in)	%Pop	Interpretation
9	5.10%	This is probably OE.
10	42.00%	This is usually OE.
10.5	4.30%	This is probably AM (Replacement).
11	29.00%	High milage, AM (Replacement).
11.5	1.40%	High milage, Aquatread AM.
12	5.80%	High milage, All Season (A/S) AM.
13	12.30%	Mud & Snow (M/S), All Terrain AM.

(The US legal minimum tread depth is generally 2/32 of an inch)

Lastly, the following table (and the reference from which it was drawn) may help in obtaining necessary tire parametric data from the tire designation system if the system is not “P-metric”¹⁹

Table 4. Historical Tire Designation Systems

ERA	SYSTEM	EXAMPLE	R	P _i (psi)	COMMENT
1900-1920	tire dia x sect wd	30 in x 3½ in	100	100	English
1920-1968	sect wd x rim dia	6.00 x 16	92	40	English (“Imperial”)
1920-1968	sect wd x rim dia	6.50 x 16	82	40	English (“Imperial”)
1920-1968	sect wd x rim dia	205 - 15	82	40	European (Metric-English)
1920-1968	sect wd x rim dia	205 R 15	82	40	European (Metric-English)
1968-1976	Alpha-Numeric	GR78-14	78	32	“Universal”
1976-now	P-metric	P235/55R16	55	35	Universal
1976-now	P-metric	235/55R16	55	35	The “P” is often dropped (US)

If the tire designation system in question should be “Alpha-Numeric”, then interpretation of the designation will be greatly facilitated by reference to the “Alpha-Numeric” Table²⁰:

Table 5. Alpha-Numeric Tire Designation System

APPROXIMATE ALPHA-NUMERIC TIRE SECTION WIDTH AS FUNCTION OF LOAD @ PSI (CODE) & ASPECT RATIO													
	A	B	C	D	E	F	G	H	J	K	L	M	N
N@24 psi =	900	980	1050	1120	1190	1280	1380	1510	1580	1620	1680	1780	1880
AR 78	13" Whl	6.60	7.05	7.45	7.70								
	14" Whl	6.45	6.65	7.05	7.35	7.65	7.90	8.35	8.70	8.80			
	15" Whl	6.35	6.95	7.15	7.35	7.70	8.05	8.55	8.70		8.85		9.80
AR 70	13" Whl	7.30	7.35	7.80	8.00								
	14" Whl			7.85	8.05	8.30	8.75	9.10	9.50		9.75		
	15" Whl	6.60		7.50	7.70	7.90	8.15	8.60	8.95	9.35	9.40	9.60	
AR 60	13" Whl	7.85	8.35	8.60	8.85								
	14" Whl		8.00	8.45	8.65	9.30	9.55	9.85	10.25	10.45		11.10	
	15" Whl	7.80	8.25			8.70	9.40	9.70	10.05	10.25		10.50	
AR 50	13" Whl		9.15	9.40	9.85								
	14" Whl					10.20	10.95	11.35				12.55	12.85
	15" Whl					9.50	10.35	11.15			11.65		12.65
N@32psi =	1060	1150	1230	1320	1400	1500	1620	1770	1860	1900	1970	2090	2210
AR 78	13" Whl	6.60	7.05	7.45	7.70								
	14" Whl	6.45	6.65	7.05	7.35	7.65	7.90	8.35	8.70	8.80			
	15" Whl	6.35		6.95	7.15	7.35	7.70	8.05	8.55	8.70		8.85	9.80
AR 70	13" Whl	7.30	7.35	7.80	8.00								
	14" Whl			7.85	8.05	8.30	8.75	9.10	9.50		9.75		
	15" Whl	6.60		7.50	7.70	7.90	8.15	8.60	8.95	9.35	9.40	9.60	
AR 60	13" Whl	7.85	8.35	8.60	8.85								
	14" Whl		8.00	8.45	8.65	9.30	9.55	9.85	10.25	10.45		11.10	
	15" Whl	7.80	8.25			8.70	9.40	9.70	10.05	10.25		10.50	
AR 50	13" Whl		9.15	9.40	9.85								
	14" Whl					10.20	10.95	11.35				12.55	12.85
	15" Whl					9.50	10.35	11.15			11.65		12.65
N@40psi =	1200	1300	1400	1490	1580	1700	1830	2010	2100	2150	2230	2370	2500
AR 78	13" Whl	6.60	7.05	7.45	7.70								
	14" Whl	6.45	6.65	7.05	7.35	7.65	7.90	8.35	8.70	8.80			
	15" Whl	6.35		6.95	7.15	7.35	7.70	8.05	8.55	8.70		8.85	9.80
AR 70	13" Whl	7.30	7.35	7.80	8.00								
	14" Whl			7.85	8.05	8.30	8.75	9.10	9.50		9.75		
	15" Whl	6.60		7.50	7.70	7.90	8.15	8.60	8.95	9.35	9.40	9.60	
AR 60	13" Whl	7.85	8.35	8.60	8.85								
	14" Whl		8.00	8.45	8.65	9.30	9.55	9.85	10.25	10.45		11.10	
	15" Whl	7.80	8.25			8.70	9.40	9.70	10.05	10.25		10.50	
AR 50	13" Whl		9.15	9.40	9.85								
	14" Whl					10.20	10.95	11.35				12.55	12.85
	15" Whl					9.50	10.35	11.15			11.65		12.65

Validation

The methodology presented herein could not be rigorously validated without an expenditure of time and effort which was beyond the resources of this author. However, a limited sort of “validation” was undertaken which, while not constituting an absolute proof of the methodology’s validity, does give some reason for confidence.

The “validation” consisted of taking 90 tires for which an empirically obtained “ C_r ” value, along with normal load and inflation pressure, is presented in the literature. This involved a span of wheel sizes 13 to 16 in (330 to 406 mm) for bias, bias-belted, and radial tires over the years 1958 to 2008. Those tires were then treated as being of “unknown” “ C_r ” value and, utilizing the rolling resistance estimating methodology put forth herein, corresponding estimated values were determined; the spreadsheet used for these determinations is presented in the [Appendix](#). The comparison of the estimated with empirical “ C_r ” values indicated that the methodology is **accurate on average within a range of about ±8%** (but with a **maximum range of variation of +32%/-21%** due to possible data point outliers); the R^2 correlation coefficient between estimated and empirical is a reasonable .86 value. The following plot of estimated versus empirical values makes clear the strong but far from perfect relationship:

17. Reference [17].

18. Reference [10], pg. 65.

19. Reference [18], pp. 92-97.

20. Ibid, pg. 93.

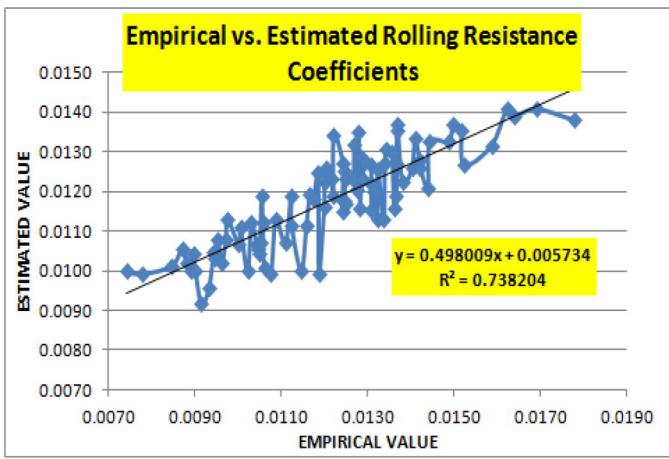


Figure 6. Calculated vs. Measured Roll Resistance Coefficients

Summary/Conclusions

The sought after methodology for estimation of a tire's rolling resistance " C_r " was obtained and seems reasonably accurate, but it is clear that there is a good deal of room for improvement. For use of the methodology as it stands, is important that the limitations of the methodology be kept in mind: the methodology is only valid for ordinary passenger car tires operating within the tire design range of load and inflation pressure (inflation pressure must also be kept within the 15 to 50 psi, a.k.a. 103 to 345 kPa, bounds of the Stuttgart rolling resistance coefficients regression analysis). Also, any tire undergoing evaluation had best be of a time period toward the more recent end of 1938-2015 bounds of the LOT table, and for a 13 to 16 inch rim size.

It should be noted that the attempted analysis was compromised due to the very wide time range considered. It is very difficult to cull from the general literature useful data for periods earlier than the 1970's, and the further back in time one looks the more intractable the problem becomes. The proliferation of useful data today is due to the modern concerns regarding fuel economy and environmental pollution, but earlier in automotive history the concerns were more basic. Early tire design was mainly driven by the need to improve wear and durability, with the emphasis slowly shifting with time toward performance considerations (hydroplaning, traction, etc.) as those earliest concerns were met. Early rolling resistance tire data is therefore scarce, and tends to be fragmentary and very general in nature, often lacking in the specificity with regard to tire type, test protocols, *et cetera*, to be useful even when found. If this paper's methodology were to be redeveloped within a narrower and more modern focus, then the accuracy of the resultant estimated figures should improve considerably.

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Definitions/Abbreviations

- AM** - "After-Market" or Replacement tire classification.
- A/S** - "All Season" tire classification.
- A/T** - "All Terrain" (On/Off-Road) tire classification.
- ECO** - Ecology "friendly" (low rolling resistance, etc.) tire classification.
- LOT** - Level of Technology.
- M/S** - "Mud and Snow" tire classification..
- OE** - "Original Equipment" tire classification.
- SMERF** - Standard Mean Equivalent Rolling Force, the "native" SAE J2452 methodology.
- SP** - Single Point, the "native" SAE J1269 methodology.
- SRC** - Standard Reference Condition, the common point SAE J1269/ SAE J2452 methodology.

Symbolism

Symbol	Definition	Units
A_c	Tire-ground gross contact area.	in ² (Eq. 6)
R_k	Tire section aspect ratio.	dimensionless
A_{sw}	Tire sidewall aero surface.	in ² (Eq. 14)
A_t	Tire tread aero surface.	in ² (Eq. 13)
C_{dr}	Stuttgart "dynamic" rolling resistance coefficient.	mph ^{2.5} (Eq. 3)
C_r	Tire rolling resistance coefficient.	dimensionless
C_{sr}	Stuttgart "static" rolling resistance coefficient.	dimensionless
$(C_a)_{ref}$	Reference tire "dynamic" rolling resistance coefficient.	mph ^{2.5} (Eq. 3)
$(C_a)_{unk}$	Unknown tire "dynamic" rolling resistance coefficient.	mph ^{2.5} (Eq. 3)
$(C_i)_{unk}$	Unknown tire rolling resistance coefficient.	dimensionless
$(C_s)_{ref}$	Reference tire "static" rolling resistance coefficient.	dimensionless
$(C_s)_{unk}$	Unknown tire "static" rolling resistance coefficient.	dimensionless
c	Tire sector chord.	Figure 5
D_r	Wheel rim nominal diameter.	mm (Eq. 9), in (spreadsheet)
d	Tire vertical deflection under load.	in (Eq. 6)
d_0	Tire deflection function "y-intercept".	in (Eq. 8)
F_r	Tire rolling resistance force.	lb (Eq. 1, 2, & 3)
F_x	Tire longitudinal traction force.	lb
$[F]_{unk}$	Unknown tire rolling resistance force.	lb (Eq. 20 & 21)
f_{area}	Tire rolling resistance area factor.	dimensionless
f_{tot}	Tire level of technology factor.	dimensionless
f_{ri}	"Wheel size" (inflated no-load radius) factor.	dimensionless
f_{rough}	Tire surface roughness factor.	dimensionless
f_{void}	Tire tread void factor.	dimensionless
f_{vol}	Tire rolling resistance volume factor.	dimensionless
K_z	Tire vertical stiffness.	kg/mm (Eq. 9), lb/in (Eq. 8)
L_c	Tire-ground contact area length.	in (Eq. 6)
N	Tire normal load.	lb (Eq. 1, 2, 3, 20, & 21)
θ	One half of tire sector included angle.	Figure 5
P_i	Tire inflation pressure.	kPa (Eq. 9), psi (Eq. 4 & 5)
R^2	Pearson correlation coefficient.	dimensionless
R_i	Tire inflated no-load radius.	in (Eq. 6)
R_r	Tire rolling radius.	Figure 5
r_{drum}	Tire test rolling drum radius.	Figure 1
r_{tire}	Tire rolling radius.	Figure 1
s	Tire tread arc segment.	Figure 5
s_h	Tire nominal section height.	in (spreadsheet)
s_n	Tire nominal section width.	mm (Eq. 9 & 7), in (spreadsheet)
$\%s$	Tire longitudinal "slip".	dimensionless
T	Torque on test tire.	Figure 1
t_d	Tire tread depth.	1/32 th in (Eq. 10)
t_t	Tire tread thickness.	in (Eq. 10)
t_w	Tire-ground contact area width.	in (Eq. 6 & 8)
μ_x	Tire longitudinal coefficient of traction.	dimensionless
V	Tire translational velocity.	mph (Eq. 2 & 3)
V_{sw}	Tire deflected sidewall volume.	in ³ (Eq. 11)
V_t	Tire deflected tread volume.	in ³ (Eq. 10)
ω_{drum}	Angular velocity of tire test rolling drum.	Figure 1
ω_{tire}	Angular velocity of test tire on rolling drum.	Figure 1

APPENDIX

The “**Rolling Resistance Coefficient (C_r) Estimation Spreadsheet**” is as follows:

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The proportion of the population that has β_1 is assumed to vary in direct proportion to β_2 , and the proportion that has β_3 is assumed to vary in direct proportion to β_4 .

¹ The inherent (and much more basic) concern is the extent to which other urban areas benefit from the presence of skilled labor.

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