CIS 351 Sample FC1 Problem Solutions

Tue 1st Feb, 2022

FC1: Functional Completeness

(a) Consider a set of gates $S = \{X, Y\}$ where X, and Y are unknown gates. If we know that the set S is functionally complete, do we *know* that the set $\{X, OR\}$ is functionally complete? Why or why not? Explain your reasoning.

We can't tell whether or not S is functionally complete. It may or may not be depending on what X is. For example, if X is NOT, then S is functionally complete. If X is AND than it is not functionally complete.

(b) Write a formal proof that the set {NOT, AND} is logically complete.

We will prove that {NOT, AND} is logically complete by building the logically complete set {NAND} using only NOT and AND gates. (We proved during lecture that {NAND} is functionally complete.)

We can construct a {NAND} gate simply by appending a NOT gate to the output of an AND gate. (*Be sure to include a picture when you answer this question on a quiz.*)

Because we can construct a logically complete set using only NOT and AND gates, we have proven that {NOT, AND} is also functionally complete.