



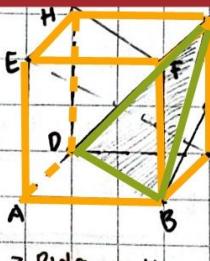
catatan utbk matematika

by :  @_membasuh

this book belongs to _____

dimensi tiga

1. diagonal



- > Bidang diagonal
- = ABGH, DBFH, GCAE, EFDC, ...
- = ada 6

> Diagonal sisi
= AF, BE, BG, CF, ...
= ada 12

> Diagonal ruang
= AG, EC, BH, DF
= ada 4

2. titik garis bidang

- > Titik (•)
- = A, B, C, D, ...
- > Garis (—)
- = AB, BC, ...
- > Bidang (□ ■)
- = ABCD

3. kedudukan titik, garis, dan bidang

1 Kedudukan titik dg garis / bidang

= terletak pada garis / bidang

contoh: A pada AB

C pada ABCB

2 Kedudukan garis/garis

= Berimpit

contoh: P O Q

PQ dg PQ

= Sejajar

contoh: AB dg EF

AB dg HC

3 Berpotongan

| (tgk lurus) X (tdk tgk lurus)

contoh: AB dg BC (\perp)

4 Berilangan

syarat: 1) tidak sebridang

2) tidak sejajar

contoh: AB dg CG (\perp)

5 Kedudukan garis dengan bidang

> terletak pada

: AB dgn ABCD

> sejajar

: AB dgn EFGH

: AC dgn EFGH

> berpotongan (menembus)

: AB dg BFGP (\perp)

: DB dg BCFG (tdk \perp)

: CE dg BDG (\perp)

6 Kedudukan 2 bidang

> Berimpit (nempel)

ABD dgn ABCD

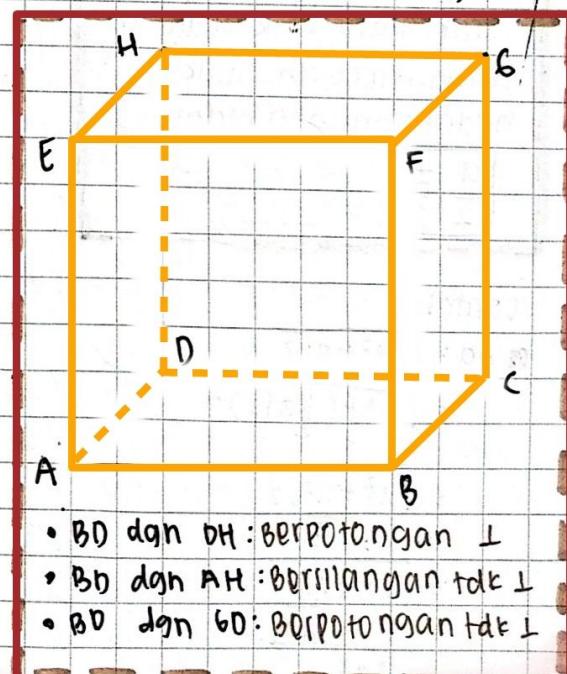
> Sejajar

ABCD dgn EFGH

> Berpotongan

ABCD dgn BCFG (\perp)

ABCD dgn ABHG (tdk \perp)



statistika

data berkelompok

1. rata-rata / mean

$$\textcircled{a} \quad \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$x_i - x_s$

$$\textcircled{b} \quad \bar{x} = \bar{x}_s + \frac{\sum f_i d_i}{\sum f_i}$$

$$\textcircled{c} \quad \bar{x} = \bar{x}_s + \left(\frac{\sum f_i c_i}{\sum f_i} \right) C$$

banyak

contoh soal

BERAT BADAN								
41-50	5	5	15	0	3	1	1	
51-60	6	6	36	0	3	1	1	
61-70	10	10	100	0	3	1	1	
71-80	13	13	130	0	3	1	1	
81-90	6	6	60	0	3	1	1	
91-100	4	4	40	0	3	1	1	
Tb								
Ta - Tb								

jumlah c = $\textcircled{50}$

teripatas-topi bawah

$\textcircled{69,15}$

note \bar{x}_s : bilah f paling banyak / nilai tengah.

$\textcircled{70,5}$

$$\textcircled{c} \quad \bar{x} = \bar{x}_s + \left(\frac{\sum f_i c_i}{\sum f_i} \right) C$$

$$69,15 + 14/50 \cdot 5 = 69,4$$

2. modus

$$M_o = (Tb) + \left(\frac{d_1}{d_1 + d_2} \right) C$$

sejauh dg frekuensi atunya

Tb: peralihan $\frac{x+y}{2}$

data urut: $(x-0,5)$

sejauh dg frekuensi bawahnya

contoh soal

nilai	F
41-50	2
51-60	5
61-70	10
71-80	13
81-90	6
91-100	4

$$\textcircled{d} \quad Tb: \frac{71+70}{2} = 70,5 /$$

$$Tb = 71 - 0,5 = 70,5$$

$$\textcircled{e} \quad \text{modus: } Tb + \left(\frac{d_1}{d_1 + d_2} \right) C$$

$$70,5 + \frac{3}{10} \cdot 10$$

$$70,5 + 3$$

$$= 73,5 //$$

contoh soal

INTERVAL	F
40-49	1
45-49	2
50-59	3
55-59	6
60-64	7
65-69	5
70-74	K
75-79	7

modus 72,83; k?

$$72,83 = 69,5 + \left(\frac{k-5}{k-5+k-2} \right) 5$$

$$72,83 = 69,5 + \frac{5k-25}{2k-12}$$

$$3,33 = \frac{5k-25}{2k-12}$$

$$\frac{10}{3} = \frac{5k-25}{2k-12}$$

$$20k - 120 = 15k - 75$$

$$5k = 45 //$$

$$k = 9 //$$

rataan

CARA A:

$$\frac{\sum f_i x_i}{\sum f_i} = \frac{2935}{50} : 59,4$$

CARA B:

$$\frac{\sum f_i + \sum f_i d_i}{\sum f_i} = \frac{50 + 70}{50} = 59,4$$

3. Median

$$M_d = Tb + \left(\frac{Y_2 N - f_{\text{ks}}}{f_{\text{me}}} \right) c$$

contoh soal

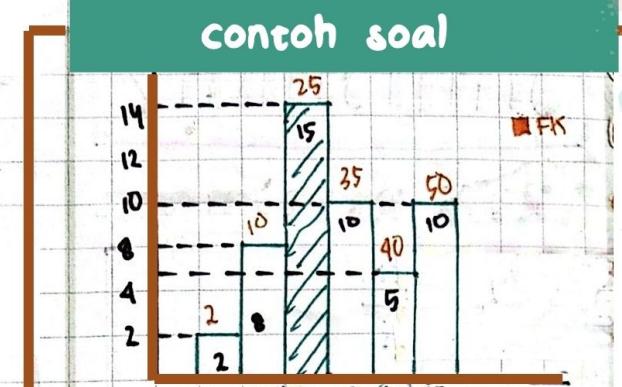
BERAT	F	FK
52-54	4	4
55-57	5	9
58-60	3	12
61-63	2	14
64-66	6	20

* med = $\frac{1}{2} N$
 $\therefore \frac{1}{2} 20 = 10$ //

letak med.
di 10.

$$M_d = 57,5 + \left(\frac{10-9}{3} \right) 3$$

$$= 57,5 + 1$$

$$= 58,5$$


$$\textcircled{1} \text{ MODUS} = 49,5 + \left(\frac{7}{12} \right) 5 \quad C = Ta - Tb \\ 54,5 - 49,5$$

$$= 49,5 + \frac{35}{12} //$$

$$\textcircled{2} \text{ MEAN} = \bar{x}_s + \left(\frac{\sum f_i c_i}{\sum f_i} \right) c \\ = 52 + \left(\frac{3}{50} \right) 5$$

$$= 52 + \frac{15}{50}$$

$$= 52 + 0,3 = 52,3$$

$$\textcircled{3} \text{ MEDIAN} = Q_2 \\ = Tb + \left(\frac{Y_2 N - f_{\text{ks}}}{f_{\text{me}}} \right) c \\ = 49,5 + \left(\frac{25 - 10}{15} \right) 5 \\ = 49,5 + \frac{15}{3} \\ = 49,5 + 5 = 54,5$$

4. kuartil (dibagi 4)

$$\textcircled{1} Q_1 = Tb + \left(\frac{Y_1 N - f_{\text{ks}}}{f_{Q1}} \right) c$$

$$\rightarrow \\ \text{DESIIL} = 1/10 \\ \text{PERSENTIL} : \\ /100$$

$$\textcircled{2} Q_2 = M_d$$

$$\textcircled{3} Q_3 = Tb + \left(\frac{3/4 N - f_{\text{ks}}}{f_{Q3}} \right) c$$

contoh soal

INTERVAL	F	fk
40-44	12	12
45-49	20	32
50-54	15	47
55-59	30	77
60-64	12	89
65-69	11	100

$$\frac{1}{4} N = 100 = 25 \text{ (Q bawah)}$$

$$\frac{3}{4} N \cdot 100 = 75 \text{ (Q atas)}$$

$$\textcircled{1} 49,5 + \left(\frac{25 - 12}{20} \right) 5$$

$$49,5 + \frac{13}{4} =$$

$$\textcircled{2} 54,5 + \left(\frac{75 - 47}{30} \right) 5$$

$$54,5 + \frac{28}{6} =$$

$$54,5 + 4,67 = 59,1$$

$$\textcircled{3} \text{ DESIL KUARTIL}$$

$$\frac{3/50}{10} = 15/11$$

$$= Tb + \left(\frac{3/10 N - f_{\text{ks}}}{f_{\text{me}}} \right) c$$

$$= 40,5 + \left(\frac{15 - 12}{15} \right) 5$$

$$= 40,5 + \frac{5}{3}$$

$$= 40,5 + 1,67$$

$$= 51,16$$

• SIMPANGAN RATA-RATA

$$SR: \frac{\sum f_i (x_i - \bar{x})}{N}$$

• SIMPANGAN BESAR

$$SB: \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{N}}$$

• RAGAM

$$\text{Var: } \frac{\sum f_i (x_i - \bar{x})^2}{N}$$

permutasi kombinasi

1. kaidah pencacahan

- a. penjumlahan → atau
b. perkalian — dan

contoh soal

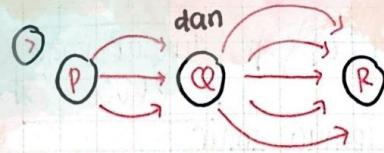
⑦ 2 mobil, 3 motor, 2 sepeda, dan 1 becak. Berapa cara pergi ke sekolah?
 $2+3+2+1 = 8$ cara

⑦ Agus punya 4 kemeja & 3 celana. Berapa banyak cara Agus berpakaian?
 $4 \times 3 = 12$ cara

⑦ 4, 5, 6, 7, 8.
 disusun bilangan genap dr tiga angka. Tidak boleh sama
 $\boxed{4 \ 3 \ 3} \quad 4 \times 3 \times 3 = 36$

⑦ 4, 5, 6, 7, 8, 0
 Bil. genap 3 angka berbeda.
 $\boxed{5 \ 4 \ 1} \quad 5 \times 4 \times 1 = 20$
> Belakang nol
 $\boxed{9 \ 4 \ 3} \quad 4 \times 9 \times 3 = 98$
> Belakang tdk nol

⑦ 2, 3, 5, 6, 8, 9
 Bil. 3 angka berlainan.
 - Bil. lebih besar dr 500
 $\boxed{4 \ 5 \ 9} \quad 4 \times 5 \times 9 = 80$
 - Bil. genap lebih dr 500
 I) $\boxed{4 \ 4 \ 1} = 16$ $\boxed{4 \ 3 \ 2} = 16$
 II) $\boxed{2 \ 4 \ 2} = 16$ $\boxed{6 \ 8} = 16$



Banyak cara pergi & kembali tdk boleh sama.

§ (Berangkat : $3 \times 5 = 15$ pulang : $2 \times 4 = 8$) $\times 120$, //

⑦ 3 orang duduk berjajar.
 Banyak kemungkinan posisi duduk
 $\boxed{3 \ 2 \ 1} = 6$, //

2. kombinasi (C)

$$nCr = \frac{n!}{(n-r)!r!}$$

↪ **DIBALIK-BALIK**
sama

n = unsur yg tersedia
 r = unsur yg diambil

⑦ 10 orang berjabat tangan, tdk dobel. Banyak x jabat tangan
 $10C_2 = \frac{10!}{8!2!} : \frac{4 \cdot 9 \cdot 8!}{8!2} = 45$, //

⑦ dari 5 orang akan dipilih 3 sbg pengurus kelas. Banyak cara pemilihan
 $5C_3 = \frac{5 \cdot 4 \cdot 3 \cdot 2!}{2! \cdot 3!} = 10$, //

⑦ dari 7 pria dan 4 wanita, hendak dibuat sebuah panitia 9 orang. Banyak cara membentuk panitia jika pria min 2.

$$7C_2 \cdot 4C_2 = \frac{7!}{5!2!} \cdot \frac{4!}{2!2!} = 126$$

$$7C_3 \cdot 4C_1 = \frac{7!}{4!3!} \cdot \frac{4!}{3!1!} = 280$$

$$7C_4 \cdot 4C_0 = \frac{7!}{3!4!} \cdot \frac{4!}{4!0!} = 35$$

(491)

⑦ seorang murid diminta mengerjakan 9 dr soal ulangan, no 1-5 harus dikerjakan. Banyak pilihan yg dapat diambil?

$$(10-5)(9-5)$$

$$= 5C_5 \cdot \frac{5!}{1!4!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4!} = 5$$

3. permutasi

dibolak-balik
BEDA

① permutasi semua unsur

$$P = n!$$

② permutasi sebagian unsur

$$nPr = \frac{n!}{(n-r)!}$$

ex: 10 orang duduk
rekre, ketua, bend.

③ permutasi siklus / melingkar

$$P = (n-1)!$$

ex. sebuah keluarga
dudur melingkar

④ permutasi van unsur sama

$$P = \frac{n!}{k! l! m!}$$

ex. Banyak cara
menyusun kata dr
huruf.

contoh soal

$$2, 3, 5, 8, 9$$

nomor disusun dr yg terkecil.

no 53829 BERADA pada urutan

① angka pertama

2	4	3	2	1
---	---	---	---	---

$$= 8 \times 3 \times 2 = 48 //$$

② angka 2

1	1	3	2	1
---	---	---	---	---

$$= 6 \times 1 = 6 //$$

③ angka 3

1	1	1	2	1
---	---	---	---	---

$$= 2 \times 1 = 2 //$$

④ angka 4

1	1	1	1	1
---	---	---	---	---

$$= 1 //$$

$$48 + 6 = 54 //$$

rak buku 5 macam. 2 harus sejajar
4! 2!

contoh soal

① arkan akan membuat password email
yg terdiri dr 5 huruf & 3 angka
berbeda. Huruf dr kata nama &
angka {0,1,2,3,4,5}

$$\frac{5!}{2!} \cdot \boxed{6 \ 5 \ 4} = 5 \cdot 4 \cdot 3 \cdot 1 \cdot 120
= 60.120 = 7200 //$$

② Ayah ibu dan 4 orang anak
akan dudur melingkar. Tent temung-
kinan dudur bila ayah ibu harus
berselisihan

① Ayah + ibu dihitung 1.

$$(5-1)! = 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24 //$$

② Ayah ibu bisa tuker posisi (kanan &
kin)

$$2! \times 24 = 48 //$$

③ Hal 99 ; 3 //

ada 5 baris @ 6 kursi'. Banyak
cara agar ibu & anak duduk
dlm 1 baris.

① BARIS SAMA

$$SC_1 = \frac{5!}{4!1!} = 5 \cdot \underbrace{?}_{5 \times 30} = 150 //$$

② DUDUK

$$GP_2 = \frac{6!}{1!2!} = 30$$



limit trigonometri

$$\lim_{x \rightarrow a} f(x)$$

cara penyelesaian

- 1) Langsung substitusikan
- 2) Jika hasilnya $\frac{0}{0}$ / ∞/∞ pakai :
- PEMFAKTORAN / turunan / $\sin^3 x$ tan dicoret

contoh soal

$$1. \lim_{x \rightarrow \frac{\pi}{4}} \sin 2x = \sin 90^\circ = 1 //$$

$$2. \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{\sin^2 x - \cos^2 x} \left(\frac{0}{0} \right)$$

Jawab : PEMFAKTORAN

$$\begin{aligned} & \frac{\sin x - \cos x}{(\sin x - \cos x)(\sin x + \cos x)} \\ & \frac{1}{(\sin x + \cos x)} = \frac{1}{\sin 45^\circ + \cos 45^\circ} \\ & = \frac{1}{\frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ & \approx \frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{2} \end{aligned}$$

$$3. \lim_{x \rightarrow 0} \frac{\sin 4x}{1 - \sqrt{1-x}} \left(\frac{0}{0} \right)$$

JAWAB : turunan

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{4 \cos 4x}{0 - (-1)} \left(\frac{0}{0} \right) \\ & = \frac{4 \cos 0^\circ}{2\sqrt{1-0}} = \frac{4 \cdot 1}{2} = 8 // \end{aligned}$$

the king turunan akar

$$y = \sqrt{f(x)}$$

$$y' = \frac{f'(x)}{2\sqrt{f(x)}}$$

Contoh:

$$y = \sqrt{1-2x}$$

$$y = \frac{-2}{2\sqrt{1-2x}}$$

$\tan x$

$\sec x$

$\cot x$

$-\csc x \cot x$

$\sec x \tan x$

$-\csc^2 x$

$$\cos 2x = 1 - 2 \sin^2 x$$

$$\cos 4x = 1 - 2 \sin^2 2x$$

$$\cos 6x = 1 - 2 \sin^2 3x$$

$$y = \frac{2}{\sqrt{3}} x$$

$$y' = \frac{2}{\sqrt{3}}$$

contoh soal

$$1. \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x}$$

► PEMFAKTORAN

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} \quad (\text{RUMUS ada di belakang})$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x - \sin x}$$

$$: \cos 45^\circ + \sin 45^\circ$$

$$= \frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{2} = \sqrt{2}$$

► TURUNAN

$$\begin{aligned} & \lim_{x \rightarrow \frac{\pi}{4}} \frac{-2 \sin 2x}{-\sin x - \cos x} \\ & = \frac{-2 \sin 90^\circ}{-\sin 45^\circ - \cos 45^\circ} = \frac{-2}{-\sqrt{2}} = \frac{\sqrt{2}}{1} // \end{aligned}$$

pemfaktoran

\sin
 \cos
 \tan

- $\sin 2x = 2 \sin x \cos x$
- $\cos 2x = \cos^2 x - \sin^2 x$
 $= 2 \cos^2 x - 1$
 $= 1 - 2 \sin^2 x$
- $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$
- sida wasico
- coda codrat min sindrat
- tada wata per 1 min tadrat



$$\lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{\sin^2 x - 1}$$

$$\lim_{x \rightarrow 0} \frac{1 - (x - \sin^2 x)}{\sin^2 x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin^2 x}{1 - \cos^2 2x} = \frac{\sin^2 x}{\cos^2 2x}$$

$$\lim_{x \rightarrow 0} \frac{\sin^2 x \cdot \cos^2 2x}{1 - (1 - \sin^2 2x)} = \frac{\sin^2 x \cdot \cos^2 2x}{\sin^2 2x \cdot \cos^2 2x}$$

$$= \frac{1}{2} \cdot \frac{1}{2} \cdot 1 = \frac{1}{4},$$

contoh soal

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin x} = 2$$

$$\lim_{x \rightarrow 2} \frac{\tan(ux-8)}{2(x-2)}$$

$$\frac{4(x-2)}{2(x-2)} = 2$$

$$\lim_{x \rightarrow 1} \frac{\tan^2(\sqrt{x}-1)}{2}$$

$$\lim_{x \rightarrow 1} \frac{\tan(\sqrt{x}-1) \tan(\sqrt{x}-1)}{(x-1)(x-1)}$$

$$\lim_{x \rightarrow 1} \frac{(\sqrt{x}-1)(\sqrt{x}-1)}{(\sqrt{x}-1)(\sqrt{x}+1)(\sqrt{x}-1)(\sqrt{x}+1)}$$

$$= \frac{1}{2 \cdot 2} = \frac{1}{4},$$

$$\lim_{x \rightarrow 0} \frac{\cos 4x - 1}{\tan^2 2x}$$

$$\lim_{x \rightarrow 0} \frac{1 - 2 \sin^2 2x - 1}{\tan^2 2x}$$

$$\lim_{x \rightarrow 0} \frac{-2 \sin 2x \cdot (\sin 2x)}{\tan 2x \cdot \tan 2x}$$

$$= \frac{-2 \cdot 2x \cdot 2x}{2x \cdot 2x} = -2$$

PAKAI THE KING

$$\lim_{x \rightarrow 0} \frac{-1/2 (4x)^2}{2x \cdot 2x} = \frac{-1/2 \cdot 16x^2}{4x^2} = -2,$$

the king

$$\lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{1 - \cos^2 2x} = \frac{\cos^2 x}{\cos^2 2x}$$

jawab dg

the king:

$$\lim_{x \rightarrow 0} \frac{x^2}{(2x)^2} \cdot \cos^2 2x$$

$$= \frac{1}{4} \cdot 1 = \frac{1}{4},$$

IDENTITAS TRIGONOMETRI

$$\sin^2 x + \cos^2 x = 1$$

$$\Leftrightarrow \cos^2 x = 1 - \sin^2 x$$

the king

$$1 - \cos A \rightarrow \frac{1}{2} A^2$$

$$1 - \cos^2 A \rightarrow \frac{2}{2} A^2 = A^2$$

$$1 - \cos 3A \rightarrow \frac{3}{2} A^2$$

turunan

definisi

turunan dari $f(x)$ ditulis $f'(x)$
atau $\frac{df(x)}{dx}$, $\frac{dy}{dx}$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

CONTOH

$$f(x) = 2x, f'(x) = \dots$$

$$\lim_{h \rightarrow 0} \frac{2(x+h) - 2x}{h}$$

$$\lim_{h \rightarrow 0} \frac{2x+2h-2x}{h} = 2 //$$

rumus turunan

$$a) y = ax \longrightarrow y' = a$$

$$b) y = ax^n \longrightarrow y' = nax^{n-1}$$

$$c) y = u \cdot v \longrightarrow y' = u'v + uv'$$

$$d) y = \frac{u}{v} \longrightarrow y' = \frac{u'v - uv'}{v^2}$$

$$e) y = (f(x))^n \longrightarrow y' = n(f(x))^{n-1} \cdot f'(x)$$

$$f) y = u \cdot v \cdot w \longrightarrow y' = u'vw + uvw' + uvw'$$

contoh soal

$$f(x) = (x+2)^3(x-1)(x+1)$$

$$f'(x) = \dots$$

- pakai rumus $y' = uvw + uv'w + uwv'$

$$f'(x) = 3(x+2)^2(x-1)(x+1) + (x+2)^3 1(x+1) + (x+2)^3(x-1) \cdot 1$$

$$= 3(x+2)^2(x^2-1) + (x+2)^3(x+1) + (x+2)^3(x-1)$$

$$= [x+2]^2 [3(x^2-1) + (x+2)(x+1) + (x+2)(x-1)]$$

$$= [x+2]^2 [3x^2-3 + x^2+8x+2 + x^2-2]$$

$$= [x+2]^2 [5x^2+9x-3]$$

- menggunakan $y' = u'v + uv'$

contoh soal

$$f(x) = \frac{2x-1}{4x+2}, f'(x) \dots$$

- pakai rumus $f'(x) = \frac{u'v - uv'}{\sqrt{2}}$

$$f'(x) = \frac{2 \cdot (4x+2) - (2x-1)4}{(4x+2)^2}$$

$$= \frac{8x+4 - 8x+4}{(4x+2)^2} = \frac{8}{(4x+2)^2}$$

the king

$$f(x) = \frac{ax+b}{cx+d}$$

$$f'(x) = \frac{ad-bc}{(cx+d)^2}$$

contoh soal

$$y = \frac{2x^2-3}{x^2+3}, \frac{dy}{dx} \text{ di } x=2 \text{ adalah}$$

$$y' = \frac{4x(x^2+3) - (2x^2-3)(2x)}{(x^2+3)^2}$$

$$y' = \frac{4x^3 + 12x - 4x^5 + 6x}{(x^2+3)^2} = \frac{18}{(x^2+3)^2} \text{ maka, } x=2$$

$$y' = \frac{18 \cdot 2}{(4+3)^2} = \frac{36}{49} //$$

contoh soal

$$f(x) = \frac{2x^4-1}{3x^3+4}$$

COBA PAKAI THE-KING

$$f'(x) = \frac{8x^3 \cdot 4 - 9x^2 \cdot 1}{(3x^3+4)^2}$$

$$= \frac{32x^3 - 9x^2}{(3x^3+4)^2}$$

konsep the king

$$\blacksquare y = (3x+2)^4$$

$$y' = 4(3x+2)^3 \cdot 3 \\ \cdot 12(3x+2)^3$$

$$\blacksquare y = (x+5)^5$$

$$y' = 5(x+5)^4 \cdot 1$$

contoh soal

$$f(x) = \sqrt{2x-1}$$

$$f(x) = (2x-1)^{1/2}$$

$$f'(x) = \frac{1}{2} (2x-1)^{-1/2} \cdot 2$$

$$f'(x) = (2x-1)^{-1/2} = \frac{1}{(2x-1)^{1/2}} = \frac{1}{\sqrt{2x-1}} //$$

the king

$$y = \sqrt{f(x)}$$

$$y' = \frac{f'(x)}{2\sqrt{f(x)}}$$

contoh soal

$$- f(x) = \sqrt{2x-1}$$

$$f'(x) = \frac{2}{2\sqrt{2x-1}} = \frac{1}{\sqrt{2x-1}} //$$

$$- f(x) = \sqrt{4x^2-2x}$$

$$f'(x) = \frac{4x-2}{2\sqrt{4x^2-2x}} = \frac{4x-1}{\sqrt{4x^2-2x}} //$$

$$- f(x) = 2\sqrt{x} + x$$

$$= 2x^{1/2} + x$$

$$f'(x) = 2 \cdot \frac{1}{2} x^{-1/2} + 1$$

$$= \frac{1}{x^{1/2}} + 1 = \frac{1}{\sqrt{x}} + 1$$

>pakai the king:

$$f(x) = 2\sqrt{x} + x$$

$$f'(x) = 2 \cdot \frac{1}{2\sqrt{x}} + 1 = \frac{1}{\sqrt{x}} + 1$$

aturan rantai

contoh soal

$$\bullet y = f(x) = (x^2+1)$$

$$x = g(t) = \sqrt{2t+1}$$

$$\frac{dy}{dt} ? \text{ (dibaca: turunan dy trhdp dt)}$$

jawab: $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} \rightarrow f'$

$$= 2x \cdot \frac{2}{\sqrt{2t+1}}$$

> caralain. x diganti t.

$$y = ((\sqrt{2t+1})^2 + 1)$$

$$= 2t+1+1$$

$$= 2t+2$$

$$y' = 2 //$$

turunan trigonometri

$$a) y = \sin x \rightarrow y' = \cos x. \text{ } \times$$

$$b) y = \cos x \rightarrow y' = -\sin x. \text{ } \times$$

$$c) y = \tan x \rightarrow y' = \sec^2 x. \text{ } \times$$

$$d) y = \cot x \rightarrow y' = -\operatorname{cosec}^2 x. \text{ } \times$$

$$e) y = \operatorname{sec} x \rightarrow y' = \tan x \cdot \operatorname{sec} x. \text{ } \times$$

$$f) y = \operatorname{cosec} x \rightarrow y' = -\cot x \cdot \operatorname{cosec} x. \text{ } \times$$



aplikasi turunan

persamaan garis singgung

- persamaan garis singgung
 $y = f(x)$ di titik (x_1, y_1) pd kurva

$$y - y_1 = m(x - x_1)$$

$$m = f'(x_1)$$

- persamaan garis normal:

$$m = -\frac{1}{f'(x_1)}$$

ordinat = y
absis = x

contoh soal

- ① persamaan garis singgung x, y ,
 $f(x) = 2 \sin x + 1$ di titik $(\frac{\pi}{2}, 3)$

- ① DCDK $(\frac{\pi}{2}, 3)$ pd kurva atau tidak.

$$3 = 2 \sin 90^\circ + 1$$

$$3 = 2 \cdot 1 + 1$$

$$3 = 3 \quad \text{✓ pada kurva}$$

$$\text{② } m = f'(x) = 2 \cos x$$

$$f'(\frac{\pi}{2}) = 2 \cos 90^\circ \\ = 2 \cdot 0 = 0 //$$

$$\text{③ } y - y_1 = m(x - x_1)$$

$$y - 3 = 0$$

$$y = 3 //$$

- ④ halaman 168 : 12
 PGS $f(x) = \sqrt{x^2+2x}$ di titik $\frac{x}{2}$ dg absis 2.

$$\text{① } y = \sqrt{1+2}$$

$$= \sqrt{3}$$

$$\text{② } m = f'(x) = \frac{2x+2}{2\sqrt{x^2+2x}}$$

$$= \frac{2+2}{2\sqrt{3}} = \frac{2}{\sqrt{3}} > \frac{2\sqrt{3}}{3} //$$

$$\text{③ } y - y_1 = m(x - x_1)$$

$$y - \sqrt{3} = \frac{2\sqrt{3}}{3}(x - 1)$$

$$y = \frac{2\sqrt{3}}{3}x - \frac{2\sqrt{3}}{3} + \sqrt{3} = \frac{2\sqrt{3}}{3}x - \frac{1}{3}\sqrt{3}$$

untuk diingat

- $y = ax + b \Rightarrow m = a$
- $ax + by + c = 0 \Rightarrow m = -\frac{a}{b}$
- garis sejajar $\Rightarrow m_1 = m_2$
- garis tegak lurus
 $m_1 \cdot m_2 = -1$

$m > 0$ monoton
 $m < 0$ monoton

motong y, $K \neq 0$
 motong x, $y = 0$

contoh soal

- 6) persamaan garis singgung

$y = x^2$ dan tegak lurus

$$x - 4y - 1 = 0$$

$$\text{① cari } m: \begin{aligned} x - 4y &= -1 \\ y &= \frac{1}{4}x + \frac{1}{4} \end{aligned} \quad m_1 = \frac{1}{4} //$$

$$\text{tegak lurus: } \frac{1}{4} \cdot (-4) = -1 \quad m_2 = -4 //$$

$$\text{maka } y = -4x$$

$$x = -1$$

$$\text{② } m = f'(x) = \begin{cases} -2x & x = 2 \\ -4 & x = 2 \end{cases} \quad \begin{cases} y = -x^2 \\ y = -(2^2) = -4 \end{cases} //$$

$$\text{③ } y + 4 = -4(x - 2)$$

$$y = -4x + 8 - 4$$

$$y = -4x + 4$$

$$4x + y - 4 = 0 //$$

- 7) garis singgung $y = (x^2 + 2)^2$
 melalui $(1, 9)$ memotong sumbu y di

$$\text{① } m = y' = 2(x^2 + 2) \cdot 2x$$

$$= 2(3) \cdot 2$$

$$= 6 \cdot 2 = 12$$

$$\text{② } y - y_1 = m(x - x_1)$$

$$y - 9 = 12(x - 1)$$

$$y = 12x - 12 + 9$$

$$y = 12x - 3$$

- ③ memotong di y . $x = 0$

$$y = 0 - 3$$

$$y = -3 // \quad (0, -3)$$

6. garis g menyinggung

$y = \sin x + \cos x$ di titik dg absis $\frac{\pi}{2}\sqrt{2}$
garis g memotong sumbu y dr..

① $y = \sin 90^\circ + \cos 0^\circ$

$$= 1/1$$

② $m = y' = \cos x - \sin x$

$$= \cos 90^\circ - \sin 90^\circ$$

$$= 0 - 1 = -1/1$$

③ $y - y_1 = m(x - x_1)$

$$y - 1 = -1(x - \frac{\pi}{2})$$

$$y = -x + \frac{\pi}{2} + 1$$

④ jika $x = 0$ maka:

$$y = 0 + \frac{\pi}{2} + 1$$

$$y = \frac{\pi}{2} + 1 \quad \text{maka: } (0, \frac{\pi}{2} + 1)$$

7) $y = x^3 + 2x^2 - x + 5,$

menyinggung kurva di (a, b)
sejajar dgn $y = 3x - 4 = 0$.

$$b =$$

① sejajar m

$$y = 3x + 4$$

$$m = 3 \quad m_2 = 3$$

② $m = y' = 3x^2 + 4x - 1 = 0$

$$= :$$

Fungsi naik, turun, minimal, maksimal

$$\text{naik} = f'(x) > 0$$

$$\text{turun} = f'(x) < 0$$

$$\text{stationer} = f'(x) = 0$$

$$\text{naik} > 0 \quad \text{④}$$

$$\text{turun} < 0 \quad \text{⑤}$$

max

min

CONTOH:

① $f(x) = \sin 2x, 0 \leq x \leq 2\pi$

① naik = $f'(x) > 0$

$$2\cos 2x > 0 \rightarrow \text{maka } \text{④}$$

② cari pembuat nol:

$$2\cos 2x = 0$$

$$\cos 2x = 0$$

③ $2x = 90^\circ$

$$x = 45^\circ$$

④ $2x = 270^\circ$

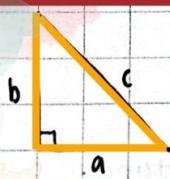
$$x = 135^\circ$$

⑤ $2x = (360 + 90)$

$$= 450^\circ$$

$$x = 225^\circ$$

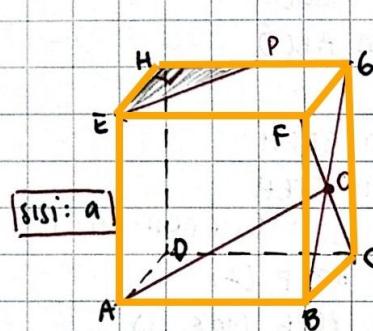
4. jarak titik ke titik



Pythagoras:
 $c^2 = a^2 + b^2$

ingat!

- diagonal sisi: $a\sqrt{2}$
- diagonal ruang: $a\sqrt{3}$



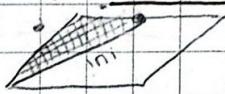
contoh:

■ EP: $\sqrt{EH^2 + HP^2}$
 $= \sqrt{a^2 + (\frac{1}{2}a)^2}$
 $= \sqrt{a^2 + \frac{1}{4}a^2}$
 $= \sqrt{\frac{5}{4}a^2} = \sqrt{\frac{5a^2}{4}} = \frac{a\sqrt{5}}{2}$

ingat!

Jarak suatu titik sudut kp tengah-tengah rusuk di hadapannya (1 bidang)

$$\frac{a\sqrt{5}}{2}$$



contoh:

■ AO: $\sqrt{AB^2 + BO^2}$
 $= \sqrt{a^2 + (\frac{1}{2}a)^2}$
 $= \sqrt{a^2 + \frac{1}{4}a^2}$
 $= \sqrt{\frac{5}{4}a^2} = \sqrt{\frac{5a^2}{4}}$
 $= \frac{a\sqrt{5}}{2}$

5. jarak titik ke garis

JARAK A ke BC = AA'

syarat: dari jarak yg paling dekat (pasti siku-siku)



1. segitiga sama kaki



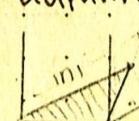
contoh soal

■ Jarak = ke AC?

JAWAB: $\sqrt{AF^2 - AF'^2}$
 $= \sqrt{(4\sqrt{2})^2 - (4\sqrt{2}/2)^2}$
 $= \sqrt{(16 \cdot 2) - (16 \cdot 2)} / 4$
 $= \sqrt{32 - 8} = \sqrt{24} = 2\sqrt{6}$

ingat!

Jarak pusat sisi ke masing-masing titik sudut sisi dihadapannya: $\frac{a}{2}\sqrt{6}$



KAMIS,
19/09
/2019

asimtot

MTK W.

1. asimtot

Asimtot tegak = penyebut 0

$$x=p$$

Asimtot datar = $y=q$

(Pakai limit) $q = \lim_{x \rightarrow \infty} f(x)$
 $0 = b^2 - 4ac$ $x \rightarrow \infty \frac{q(x)}{g(x)}$

Asimtot miring =

e.g. $f(x) = \frac{2x^2 + x - 3}{x + 1}$

+ kalau punya A. miring
tidak punya A. datar

cirinya: angka berselisih,
sudut bagi biasa

e.g. $\frac{2x-1}{(x+1)\sqrt{2x^2+x-3}}$

$$\begin{array}{r} 2x-1 \\ (x+1)\sqrt{2x^2+x-3} \\ \hline 2x^2+2x \\ -x-3 \\ -x-1 \\ \hline -2 \end{array}$$

asimtot miring = $2x-1 = y$

contoh soal

1 $f(x) = \frac{x+1}{x-2}$

titik potong sbx x

$$y=0 = x+1$$

$$x=-1 (-1, 0)$$

titik smb y

$$x=0 = -\frac{1}{2} (y)$$

$$(0, -\frac{1}{2})$$

Asimtot datar

$$\lim_{x \rightarrow \infty} \frac{x+1}{x-2}$$

Asimtot tegak

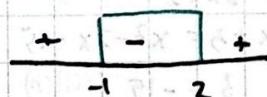
$$x-2=0$$

$$x=2$$

$$y=1$$

Daerah fungsi

$$\frac{x+1}{x-2} \begin{cases} 0 & (x=-1) \\ & (x=2) \end{cases}$$



$$f(x) < 0 = -1 < x < 2$$

$$f(x) > 0 = x < -1 \text{ atau } x > 2$$

Gambar

titik bantu

$$x=3, y = \frac{(3+1)}{(3-2)} = 4 \quad (3, 4)$$

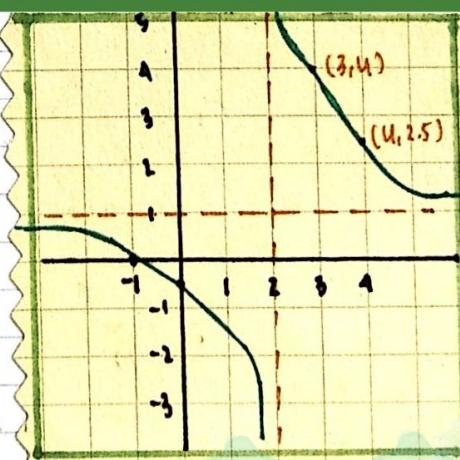
$$x=4, y = \frac{(4+1)}{(4-2)} = 2.5 \quad (4, 2.5)$$

2. menggambar grafik fungsi rasional

$$F(x) = \frac{px+q}{ax+b}$$

Langkah-langkah: $y=0$ $x=0$

- titik potong dengan sbx dan sb y
- Asimtot datar, Asimtot tegak
- daerah fungsi untuk $f(x) > 0$
 $f(x) < 0$
- cari titik bantu

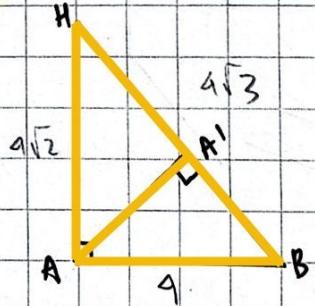


2. segitiga siku-siku

= pakai persamaan
Luas Segitiga

Contoh Jadi:

- Jarak A ke HB



Jawab:

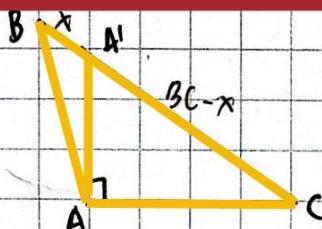
$$\frac{AB \cdot AA'}{2} = \frac{BH \cdot AA'}{2}$$

$$9 \cdot 4\sqrt{2} = 4\sqrt{3} \cdot AA'$$

$$\frac{9 \cdot 4\sqrt{2}}{4\sqrt{3}} = AA'$$

$$AA' = \frac{9\sqrt{2} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{9\sqrt{6}}{3} = 3\sqrt{6}$$

3. segitiga sembarang



Langkah awal
adalah cari x dulu

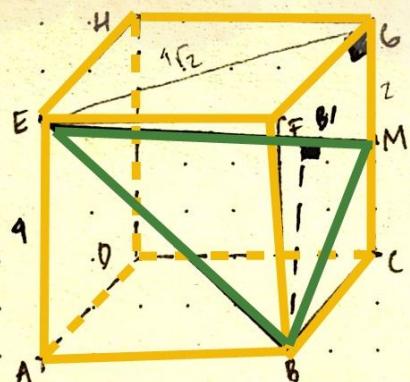
$$AA' = \sqrt{AB^2 - x^2} = \sqrt{(BC-x)^2 - AC^2}$$

3. segitiga sama sisi

$$\sin 60^\circ = \frac{\text{dppn}}{\text{mimn}}$$

$$\frac{1}{2}\sqrt{3} = \frac{\text{sisi dppn}}{\sin \text{mimn}}$$

contoh soal segitiga sembarang



Jarak B ke Em adalah:

Jawab:

$$BM = \frac{1}{2}\sqrt{5} \quad EM = \sqrt{(1\sqrt{2})^2 + 2^2} \\ = \sqrt{32+4}$$

$$BE = 4\sqrt{2} \quad = \sqrt{36} = 6$$

$$BB' = BM^2 - x^2 = BE^2 - (EM - x)^2 \\ = \left(\frac{1}{2}\sqrt{5}\right)^2 - x^2 = (4\sqrt{2})^2 - (6-x)^2$$

$$\frac{16}{4} - x^2 = (16 \cdot 2) - (36 - 12x + x^2)$$

$$\frac{80}{4} = 32 - 36 + 12x$$

$$12x = 20 + 4$$

$$x = \frac{24}{12} = 2$$

$$\text{Jadi, } BB' = \sqrt{\left(\frac{1}{2}\sqrt{5}\right)^2 - 2^2}$$

$$= \sqrt{\frac{16}{4} - 4}$$

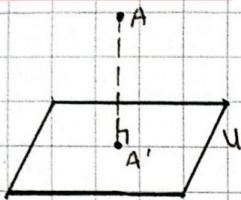
$$= \sqrt{20-4}$$

$$= \sqrt{16} = 4$$

Penjelasan: cari x , dg perbandingan pythagoras (cari BB'), kalo dulu ketemu tinggal masukan

6. jarak titik ke bidang

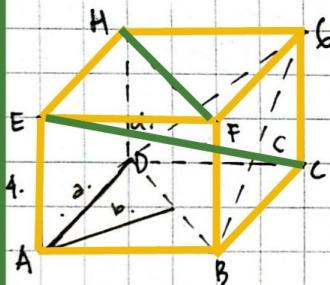
(JARAK A KE
bidang U
adlh A')



Langkah:

1. Buat bidang melalui A dan tegak lurus bidang U
2. Cari garis potong antara 2 bidang tsb \Rightarrow milai garis g
3. Jarak A ke garis g

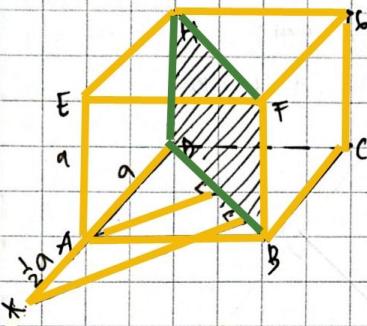
contoh soal



- a. Jarak A ke DCBHG
 $\Rightarrow \sqrt{2}a$
- b. Jarak A ke BDHF
 $\Rightarrow \frac{1}{2}\sqrt{2}a = 2\sqrt{2}$
- c. Jarak C ke BDG
 $\Rightarrow \frac{1}{3}\sqrt{3}a = \frac{4\sqrt{3}}{3}$
- d. Jarak E ke BDG
 $\Rightarrow \frac{2}{3}\sqrt{3}a = \frac{8\sqrt{3}}{3}$
- e. Jika a = 6. Jarak C ke EBG
 $\Rightarrow \frac{1}{3}\cdot 6\sqrt{3} = 2\sqrt{3}$

contoh soal

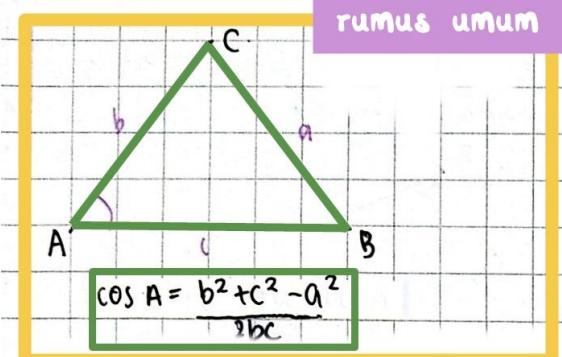
- KUBUS ABCD.EFGH - r=a cm.
K pada perpanjangan DA shg
KA = $\frac{1}{3}$ KD. Jarak K ke BDHF
adalah :



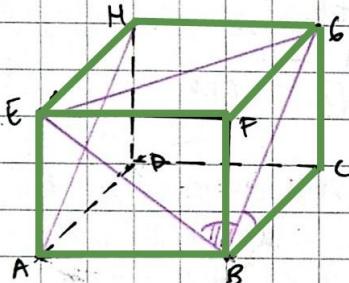
Jawab:

$$\begin{aligned} \frac{a}{\frac{1}{2}a} &= \frac{\frac{1}{2}a\sqrt{2}}{x} \\ x &= \frac{3}{2}a\sqrt{2} \times \frac{1}{2}a \\ &= \frac{3}{4}a\sqrt{2} \quad // \end{aligned}$$

dimensi tiga



1. mencari sudut diantara 2 sudut bersilangan

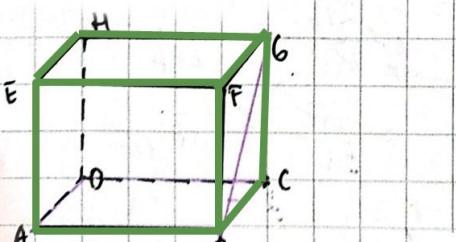


① tentukan sudut antara $AH \setminus BC$?

$$\angle(AH, BC) = \angle(BG, BC) \\ = 45^\circ$$

② sudut antara $AH \setminus BE$?
sama sisi : 60°

2. mencari sudut diantara garis dan bidang



sudut antara BG dan ABC
 $45^\circ //$

rumus umum

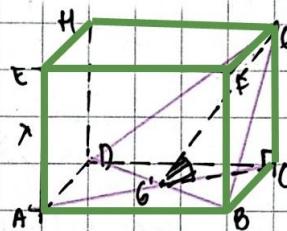
- $\sin = \frac{\text{depan}}{\text{miring}}$

- $\tan = \frac{\text{depan}}{\text{samping}}$

- $\cos = \frac{\text{samping}}{\text{miring}}$

contoh soal

1.

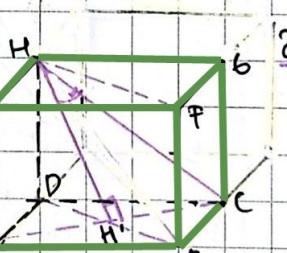


sudut antara
 $AC \setminus BDG$

Jawab:

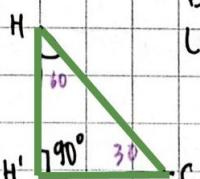
$$\begin{aligned} \tan x &= \frac{x}{\frac{1}{2}x\sqrt{2}} \quad (x>2) \\ &= \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{2\sqrt{2}}{2} \end{aligned}$$

2.



$2 = \text{arc tan } \sqrt{2}$

sudut antara
 $CH \setminus BFHD$

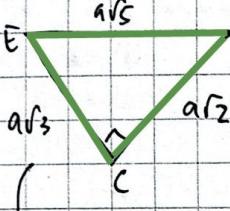
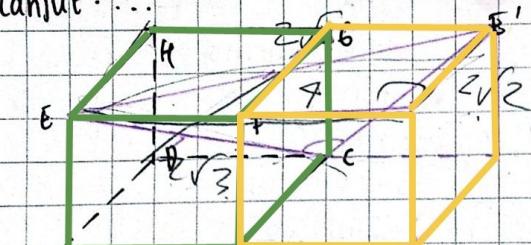


ya pokoknya hasilnya
 60° , coba beroke
tanya ya..

3. diketahui kubus ABCD-EFGH dengan
bidang alas ABCD dan bidang atas
EFHG. sudut antara di diagonal
BG dan diagonal ruang EC adalah

contoh soal

Lanjut: ...



rumus sisi miring

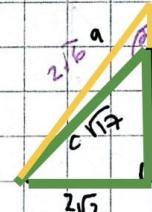
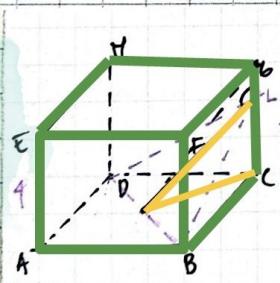
$$\begin{array}{c} a\sqrt{x+y} \\ \sqrt{x^2 + y^2} \\ a\sqrt{x^2 + y^2} \\ a\sqrt{y^2} \end{array}$$

di cari, adalah
sisi \equiv segitiga
siku \equiv .

maka $c = 90^\circ$

$$\cos \theta = \frac{a^2 + b^2 - c^2}{2ab}$$

4. Diberikan kubus ABCD.EF6H. sebuah titik L terletak pada rusuk CG, sehingga $CL : LG = 3 : 1$. Jika α sudut terbesar yg terbentuk antara rusuk CG dgn bidang BDL. maka $\cos \alpha$:



Jawab:

$$a^2 = b^2 + c^2 - 2bc \cdot \cos \alpha$$

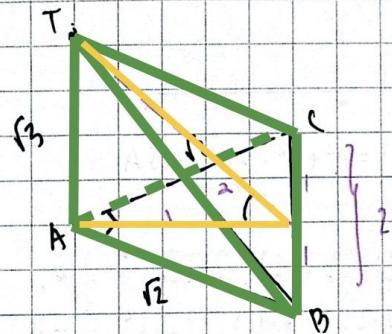
$$24 = 1 + 17 - 2\sqrt{17} \cdot \cos \alpha$$

$$-2\sqrt{17} \cos \alpha = 24 - 18$$

$$-2\sqrt{17} \cos \alpha = 6$$

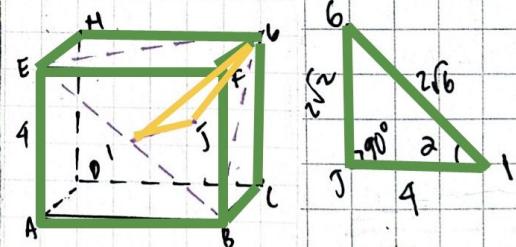
$$\cos \alpha = \frac{-6}{2\sqrt{17}} = \frac{-3}{\sqrt{17}}$$

5. Bidang empat silindris ABC dan TA \perp AC, TA \perp AB, AB \perp AC, AB : AC = $\sqrt{2}$ cm. dan TA = $\sqrt{3}$ cm. berapa sudut antara TBC β ABC adalah.



$$\tan \beta = \frac{\sqrt{3}}{1} = \sqrt{3} = 60^\circ$$

6. Pada kubus ABCD.EF6H dengan jantangan rusuk 4 cm maka nilai kosinus sudut antara BET β BCP adalah



$\cos = \frac{\text{samping}}{\text{miring}}$

$$\begin{aligned} \cos \alpha &= \frac{4}{2\sqrt{6}} = \frac{2\sqrt{4}}{2\sqrt{6}} = \frac{2}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} \\ &= \frac{2\sqrt{6}}{6} = \frac{1}{3}\sqrt{6} \end{aligned}$$

matriks

- MATRIKS = kumpulan bilangan yang disusun dalam baris dan kolom.

- BENTUK UMUM:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & \textcircled{a_{23}} & \dots & a_{2n} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{pmatrix} \xrightarrow{\substack{\text{Bentuk dr} \\ 2x3 \neq 3x1}}$$

- ORDO: banyaknya baris dan kolom

e.g. matriks $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ ordo 2×3

transpose matriks

$$A = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \xrightarrow{\rightarrow} A^T = \begin{pmatrix} a & d \\ b & e \\ c & f \end{pmatrix}$$

$$B = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \xrightarrow{\rightarrow} B^T = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$$

operasi matriks

1. PENJUMLAHAN

syarat: ordonya harus sama

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} a+p & b+q \\ c+r & d+s \end{pmatrix}$$

2. PENURANGAN

syarat: ordonya harus sama

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} a-p & b-q \\ c-r & d-s \end{pmatrix}$$

3. PERKALIAN

syarat:

$$\begin{array}{|l} (\text{baris} \times \text{kolom}) \text{ dg } (\text{baris} \times \text{kolom}) \\ (m \times n) \text{ dg } (n \times p) = m \times p \end{array}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} ap+br & bp+ds \\ cp+dr & dq+ts \end{pmatrix}$$

operasi matriks

$$\boxed{\text{MATRIK } A \cdot B \neq \text{MATRIK } B \cdot A}$$

$$\boxed{\text{DET}(A \cdot B) = \text{DET}(B \cdot A)}$$

$$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} ap+bp+cp \\ dp+eq+fr \end{pmatrix}$$

Determinan Matriks

Untuk matriks persegi

$$\text{1. ORDO } 2 \times 2 \Rightarrow A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \text{ Det } A = ad - bc$$

$$\text{2. ORDO } 3 \times 3 \Rightarrow \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \xrightarrow{\cancel{\text{det}}} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \xrightarrow{\cancel{\text{det}}}$$

$$\text{Determinan } A = (aei + bfg + cdh) - (gef + bdi + ace)$$

sifat-sifat determinan

$$\bullet \text{ Det } A^T = \text{Det } A$$

$$\bullet \text{ Det } (kA) = k^3 \cdot \text{Det } A \quad \boxed{(2 \times 2)} \\ \text{angka} = k^3 \cdot \text{Det } A \quad \boxed{(3 \times 3)}$$

$$\bullet \text{ Det } (A \cdot B) = \text{Det } A \cdot \text{Det } B$$

pembuktian sifat 1

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \rightarrow \text{Det } A = (1 \cdot 4 - 2 \cdot 3) = -2$$

$$A^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \rightarrow \text{Det } A^T = (1 \cdot 4 - 2 \cdot 3) = -2$$

pembuktian sifat 2

$$2A = \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix} \rightarrow \text{Det } 2A = 16 - 24 = \boxed{-8}$$

$$\text{Det } 2A = 2^2 \cdot (-2) = \boxed{-8}$$

pembuktian sifat 3

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad B = \begin{pmatrix} -2 & 3 \\ 1 & -1 \end{pmatrix}$$

$$\ast A \cdot B = \begin{pmatrix} 0 & 1 \\ -2 & 5 \end{pmatrix} \quad \text{det}(AB) = 0 + 2 = 2$$

$$\ast \text{ Det } A \cdot \text{Det } B = -2 \cdot 1 = 2$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

statistika

UKURAN PEMUSATAN DATA

1. mean

$$\bar{x} = \frac{\sum x_i \cdot f_i}{\sum f_i} \rightarrow \text{CARA UMUM}$$

CARA RATA-RATA SEMENTARA

$$\bar{x} = \bar{x}_s + \frac{\sum s_i f_i}{\sum f_i}$$

$$s_i = x_i - \bar{x}_s$$

contoh soal

BB (x_i)	f_i	$x_i f_i$
41	12	492
42	85	210
43	15	645
45	16	720
50	2	100
[CARA I]		2.167

$$\bar{x} = \frac{2167}{50} = 43,37 //$$

CARA II

Pilih Rataan Sementara (\bar{x}_s)

1. Nilai yang ditengah /
2. Nilai yg frekuensi banyak

aku milish $\bar{x}_s = 43$

f_i	d_i	δf_i
12	-2	-24
5	-1	-5
15	0	150
16	2	32
2	7	14
[CARA II]		17

$$\bar{x} = \bar{x}_s + \frac{\sum \delta f_i}{\sum f_i} = 43 + \frac{17}{50}$$

mean gabungan (\bar{x}_{gab})

$$\bar{x}_{gab} = \frac{\bar{x}_A \cdot n_A + \bar{x}_B \cdot n_B + \dots}{n_A + n_B + \dots}$$

contoh soal

- 9) \bar{x} rkelompok siswa = 7 [\bar{x}_A]
 \bar{x} siswa lain = 6,5 [\bar{x}_B] $n_B = 1$
 $\bar{x}_{gab} = 6,8$ \bar{x}_{gab} banyak siswa sblm digabung $\neq n_A$?
Jawab:

$$\bar{x}_{gab} = \frac{7 \cdot n_A + 6,5 \cdot 1}{n_A + 1}$$

$$6,8 = \frac{7n_A + 26}{(n_A + 1)}$$

$$6,8n_A + 26 \cdot 2 = 7n_A + 26$$

$$6,8n_A - 7n_A = 26 - 26$$

$$0,8n_A = 0$$

$$n_A = 0 //$$

THN KIRI jika jumlahnya hanya 2

$$\frac{n_A}{n_B} = \frac{\bar{x}_{gab} - \bar{x}_B}{\bar{x}_A - \bar{x}_{gab}}$$

$$\frac{n_A}{1} = \frac{6,8 - 6,5}{7 - 6,8} = \frac{0,3}{0,2} = 1,5 //$$

2. median

1. nilai tengah data

*data diurutkan terlebih dahulu

> n ganjil

$$\text{med} = \text{data ke-} \frac{n+1}{2} = x_{(\frac{n+1}{2})}$$

> n genap

$$\text{med} = \frac{x_{(\frac{n}{2})} + x_{(\frac{n}{2}+1)}}{2}$$

n = jumlah
frekuensi

CONTOH SOAL

nilai | frekuensi



3. modus

- nilai yang paling banyak muncul
- frekuensi paling besar

contoh:

nilai	F
6	12
7	10
8	3
9	20
10	6

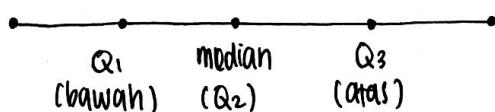
$$\text{med: } x \left(\frac{5+1}{2} \right) \\ = x_{26} \\ = 9,1$$

UKURAN PERSEBARAN DATA

• diurutkan terlebih dahulu

1. KUARTIL

data dibagi menjadi 4 bagian



contoh soal:

- (21) 12, 15, 10, 13, 11, 12, 14, 10 (genap)

→ 10, 10 | 11, 12 | 12, 13 | 14, 15

$$Q_1 \quad Q_2 \quad Q_3$$

$$Q_2 = \frac{12+12}{2} = 12$$

$$Q_1 = \frac{10+11}{2} = 10,5$$

$$Q_3 = \frac{13+14}{2} = 13,5$$

- (*) 5, 5, 6, 7, 8, 8, 9, 10, 11

→ 5, 5 | 6, 7 | 8, 8 | 9 | 10, 11

$$Q_1 \quad Q_2 \quad Q_3$$

$$Q_1 = 5,5$$

$$Q_2 = 8$$

$$Q_3 = 9,5$$

$$n=100$$

$$\text{med: } Q_2 = \frac{x_{50} + x_{51}}{2}$$

$$Q_1 = \frac{x_{25} + x_{26}}{2}$$

$$Q_3 = \frac{x_{75} + x_{76}}{2}$$

2. DESIL

= data dibagi menjadi 10 bagian

$$D_i = i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$$

$$\text{Letak } D_i = \frac{i}{10}(n+1) = \text{k.i.d}$$

$$D_i = x_k + O_{rd} (x_{k+1} - x_k)$$

contoh

- (*) 1, 2, 2, 3, 3, 4, 4, 4, 5, 5, 6, 7, 8
g. (ada 13)

ditanya: desil ke-8?

$$\text{Letak } D_8 = \frac{8}{10} (13+1) = 11,2 \quad \text{k.i.d}$$

$$= \frac{4}{5} \times 14 = 11,2$$

$$(b) D_8 = x_{11} + O_{rd} (x_{12} - x_{11}) \\ = 7 + 0,2 (1) \\ D_8 = 7,2$$

UKURAN SIMPANGAN DATA

$$1. \text{ jangkauan} = X_{\max} - X_{\min}$$

$$2. \text{ simpangan kuartil} = \frac{Q_3 - Q_1}{2}$$

$$3. \text{ simpangan rata} = SR = \frac{\sum |x_i - \bar{x}|}{n}$$

$$4. \text{ ragam/variasi} \text{ Var} = \frac{\sum (x_i - \bar{x})^2}{n}$$

$$5. \text{ simpangan baku/standar deviasi}$$

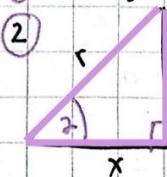
$$SB = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$



Trigonometri

pada segitiga siku-siku

1) Phytagoras



$$\begin{aligned}\sin \alpha &= \frac{y}{r} \\ \cos \alpha &= \frac{x}{r} \\ \tan \alpha &= \frac{y}{x}\end{aligned}$$

$$\csc \alpha = \frac{1}{\sin \alpha}$$

$$\sec \alpha = \frac{1}{\cos \alpha}$$

$$\cot \alpha = \frac{1}{\tan \alpha}$$

2) SUDUT

I. \sin

(90)

semua. I

180

III, \tan

- $\sin(90-\alpha) = \frac{x}{r} = \cos \alpha$
- $\cos(90-\alpha) = \frac{y}{r} = \sin \alpha$
- $\tan(90-\alpha) = \frac{x}{y} = \cot \alpha$

cara membaca sudut

dari $0-90$ semua positif

dari $90-180$ hanya \sin yg positif

dari $180-270$ hanya \tan yg positif

dari 270 hanya \csc yg positif

asalnya $+/-$, didapat dari

pencerminan yg nya negatif,

maka yg tdk mengandung yg tdk positif
 $\cos = x/r$

cermin x negatif, yg tdk ada x nya
 tdk positif, $\sin = y/r$

Teorema Sudut Berelasi

TERHADAP SUMBU

* datar (x) = 180° dan 360°

= TETAP

* tegak (y) = 90° dan 270°

= BERUBAH

sin \rightarrow cos

cos \rightarrow sin

tan \rightarrow cot

$$\text{eg: } \cdot \sin(180^\circ + \alpha) = -\sin \alpha$$

$$\cdot \cos(270^\circ + \alpha) = +\sin \alpha$$

$$\cdot \sin 210 = -\sin(180+30)$$

$$= -\sin 30$$

$$= -1/2$$

$$\cdot \cos 735 = \cos(2 \cdot 360 + 15)$$

$$= \cos 15$$

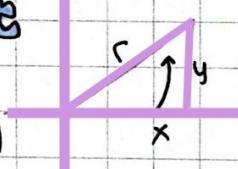
-berhimpit

0°

$$\sin 0 = \frac{0}{r} = 0$$

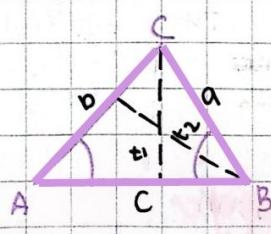
$$\cos 90^\circ = \frac{0}{r} = 0$$

$$\tan 0 = \frac{0}{0} = 0$$



-Sudut \angle Sisi

△ sembarang



$$\sin A = \frac{t}{b} \quad \sin B = \frac{t}{a}$$

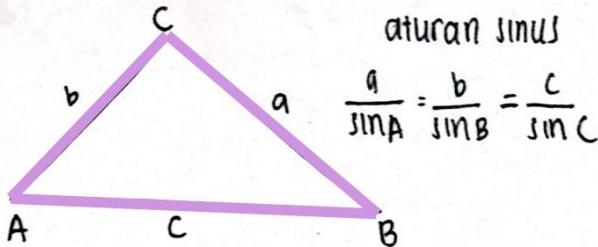
$$t = b \cdot \sin A \quad t = a \cdot \sin B$$

$$b \sin A = a \sin B$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

trigonometri

RUMUS TRIGONOMETRI \triangle



REMEMBER !

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\text{luas } \triangle = \frac{1}{2}ab \sin C$$

RUMUS IDENTITAS

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x = \tan^2 x + 1 = \sec$$

$$\sin^2 x = 1 + \cot^2 x = \operatorname{cosec}^2 x$$

RUMUS JUMLAH DAN SELISIH SUDUT

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \pm \tan \alpha \tan \beta}$$

RUMUS SUDUT RANGKAP

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

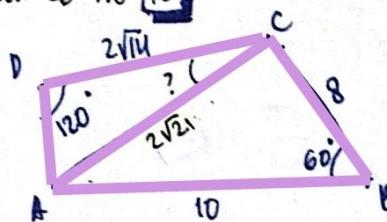
$$\sin 2x = 2\sin x \cos x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$1 - \cos 2x = 2\sin^2(\frac{1}{2}x)$$

LATIHAN SOAL

hal 26 no 10



$\angle ACD$?

CARI PANJANG AC.

$$\cos 60 = \frac{8^2 + 10^2 - x^2}{160}$$

$$\frac{1}{2} = \frac{(64 + 100 - x^2)}{160}$$

$$160 = 128 + 200 - 2x^2$$

$$2x^2 = 128 + 200 - 160$$

$$2x^2 = 168$$

$$x^2 = 84 \quad AC = \sqrt{84}$$

$$= \frac{2\sqrt{21}}{2\sqrt{21}}$$

$$\frac{\sqrt{21}}{\sin 120} = \frac{\sqrt{14}}{\sin x}$$

$$\frac{\sqrt{21}}{\frac{1}{2}\sqrt{3}} = \frac{\sqrt{14}}{\sin x}$$

$$\sin x = \sqrt{14} \cdot \frac{1}{2}\sqrt{3}$$

$$= \frac{\sqrt{21}\sqrt{14}}{\sqrt{21}\sqrt{3}}$$

$$\sin x = \frac{1}{2}\sqrt{2}$$

$$x = 45^\circ //$$

no 12

$$\tan x = -\frac{3}{4} \quad \frac{de}{ca}, m_i = 5$$

$$3\pi < x < 2\pi$$

$$\sin(\frac{\pi}{3} - x) = \dots ?$$

JAWAB =

$$\sin(60 - x) = \sin 60 \cdot \cos x - \sin x \cdot \cos 60$$

$$= \frac{1}{2}\sqrt{3} \cdot \frac{4}{5} - \left(-\frac{3}{5} \cdot \frac{1}{2}\right)$$

$$= \frac{4\sqrt{3}}{10} + \frac{3}{10} = \frac{4\sqrt{3} + 3}{10}$$

INTEGRAL

- konstan: $\int k \, dx = kx + C$
- variabel: $\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C$
- khusus x^{-1} : $\int \frac{1}{x} \, dx = \ln|x| + C$
- dipisah tambah atau kurang: blh di integral.
- dipisah kali atau bagi: ga boleh
Alternatifnya:
 - ubah sng. dipisah tambah / kurang dulu.
 - Pake metode substitusi (paket 2)
 - Pake metode persial

Integral Tentu

contoh: $\int f(x) \, dx = g(x) + C$

$$\int_a^b f(x) \, dx = g(x) \Big|_a^b \\ = g(b) - g(a)$$

$$\int_a^a f(x) \, dx = 0$$

$$\int_a^b f(x) \, dx = - \int_b^a f(x) \, dx$$

$$\int_a^c f(x) \, dx = \int_a^b f(x) \, dx + \int_b^c f(x) \, dx$$

FUNGSI GANJIL ($f(-x) = -f(x)$)

$$\text{berlaku: } \int_{-a}^a f(x) \, dx = 0 //$$

FUNGSI GENAP ($f(-x) = f(x)$)

$$\text{berlaku: } \int_{-a}^a f(x) \, dx = 2 \cdot \int_0^a f(x) \, dx$$

contoh soal

$$* f(x) = x^3$$

$$* f(-x) = (-x)^3 \\ = -x^3$$

$$f(-x) = -f(x) \text{ FUNGSI GANJIL}$$

$$* f(x) = x^2$$

$$* f(-x) = x^2$$

$$f(-x) = f(x) \text{ FUNGSI GENAP}$$

$$* f(x) = (x-3)^2$$

$$f(-x) = (-x-3)^2 \\ = -(x+3)^2 \\ = (x+3)^2$$

$$f \text{ bukan fungsi ganjil/fungsi genap}$$

contoh soal

$$\Theta \ln 3 = e^{\ln 3} \\ e \approx 2,...$$

Integral tertentu

$$\int_{-2}^2 (4x^3 - 6x^2 + 2x) \, dx \Big|_{-2}^2 \\ = [2^4 - (-2)^4] - 2[2^3 - (-2)^3] + [2^2 - (-2)^2] \\ = 0 - 32 + 0 = -32 //$$

cara ganjil genap

$$\int_{-2}^2 (4x^3 - 6x^2 + 2x) \, dx \\ \text{ganjil} \quad \text{genap} \\ 0 \quad 0$$

$$2. \int_0^2 -6x^2 \, dx$$

$$2. -2x^3 \Big|_0^2$$

$$2. -2 \cdot 8 = -32 //$$

$$\bullet \int_2^5 3f(x) \, dx = 15 \rightarrow \int_2^5 f(x) \, dx = 5$$

$$\int_2^7 2f(x) \, dx = 6 \rightarrow \int_2^7 f(x) \, dx = 3$$

$$\int_5^7 f(x) \, dx = ? \quad -2$$

$$\text{jawab: } \int_2^5 f(x) \, dx + \int_5^7 f(x) \, dx = 3$$

$$\bullet \text{jika } \int_7^5 f(x) \, dx = 2 // \quad + (-2) = 3$$

Tipe SBMPTN

$$F(x) = x^3 + 3x^2 - 5x + \underbrace{\int_{-1}^1 f(x) \, dx}_K$$

$$\int_{-1}^1 (x^3 + 3x^2 - 5x + K) \, dx = K$$

$$2x \int_0^1 3x^2 + K \, dx = K$$

$$2x \cdot x^3 + Kx \Big|_0^1 = K$$

$$2 + 2K = K$$

$$K = -2 //$$

maka

$$f(x) = x^3 + 3x^2 - 5x - 2$$

$$f(1) = 1 + 3 - 5 - 2$$

$$= -3 //$$

integral

a. SUBSTITUSI

$$\int u^n du = \frac{1}{n+1} u^{n+1} + C$$

eg: $\int (2x-3)^8 dx$

① $u \rightarrow 2x-3$ punya pangkat

• $u = 2x-3$

• $\frac{du}{dx} = 2 \cdot \frac{d}{dx} u$

• $\int \frac{1}{2} (2x-3)^8 du$

= $\frac{1}{2} \cdot \frac{1}{9} (2x-3)^9 + C$

= $\frac{1}{18} (2x-3)^9 + C$

② $\int (2x-4)(x^2-4x-5)^6 dx$

• $\frac{du}{dx} = 2x-4 \quad | \quad dx = \frac{du}{2x-4}$

• $\int (2x-4) u^6 \cdot \frac{du}{2x-4}$

• $\frac{1}{7} (x^2-4x-5)^7 + C //$

③ $\int \frac{4x}{(2x^2-5)^3} dx$

$u = 2x^2-5$

• $\frac{du}{dx} = 4x \quad | \quad dx = \frac{du}{4x}$

• $\int \frac{4x}{u^3} \cdot \frac{du}{4x}$

• $\int 4u^{-3} du$

• $-\frac{1}{2} (2x^2-5)^{-2} + C //$

$$\begin{aligned} & 4x(2x^2-5)^{-3} \\ & 4x \cdot \frac{1}{-2 \cdot 4x} (2x^2-5)^{-2} \\ & -\frac{1}{2} \end{aligned}$$

$$4. \int \frac{6x^2}{\sqrt{x^3-1}} dx$$

• $\frac{du}{dx} = 3x^2 \quad | \quad dx = \frac{du}{3x^2}$

• $\int 6x^2 \cdot u^{-1/2} \cdot \frac{du}{3x^2}$

• $2 \cdot 2 (x^3-1)^{1/2} + C$

= $4(x^3-1)^{1/2} + C$

= $4\sqrt{x^3-1} + C //$

b. PARSIAL

$$\int u \cdot dv = uv - \int u dv$$

eg: $\int x (x+1)^2 dx$

• $u = x \quad | \quad dv = (x+1)^2$
 $du = 1 \quad | \quad v = \frac{1}{3}(x+1)^3$

↑ turunan
↓ integral
↑ substitusikan ke bentuk

$$\int x(x+1)^2 dx = x \cdot \frac{1}{3}(x+1)^3 - \int \frac{1}{3}(x+1)^3 dx$$

= $\frac{1}{3}x(x+1)^3 - \frac{1}{3} \cdot \frac{1}{4}(x+1)^4 + C$

= $\frac{1}{3}x(x+1)^3 - \frac{1}{12}(x+1)^4 + C$

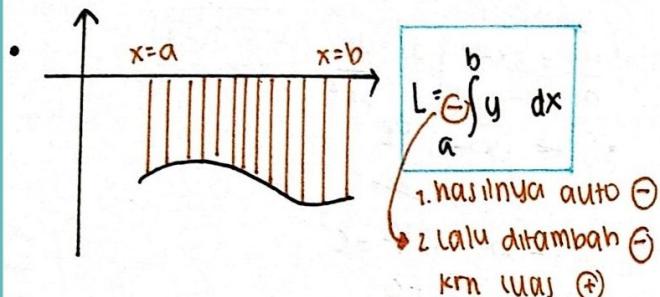
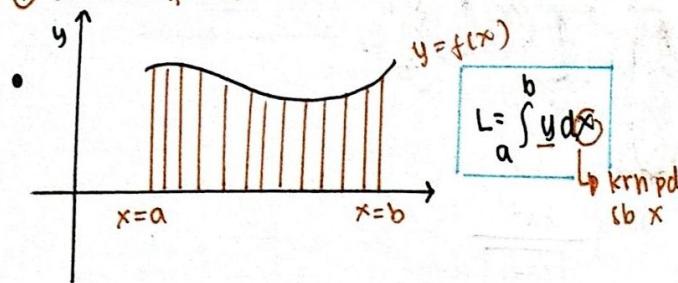
metode tanza line

x	(x+1) ²
1	$\frac{1}{3}(x+1)^3$
0	$\frac{1}{3} \cdot \frac{1}{4}(x+1)^4 = \frac{1}{12}(x+1)^4$

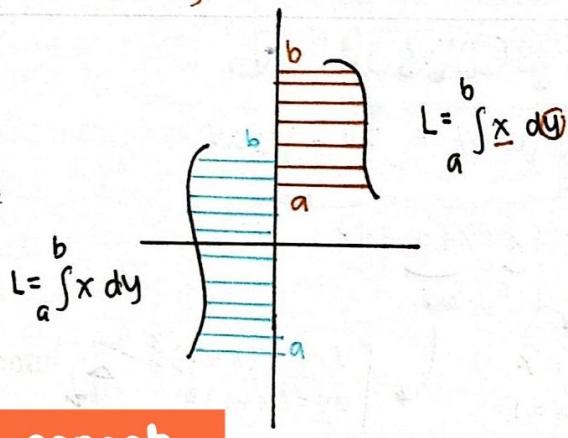
↳ $\frac{1}{3}x(x+1)^3 - \frac{1}{12}(x+1)^4 + C //$

c. LUAS KURVA

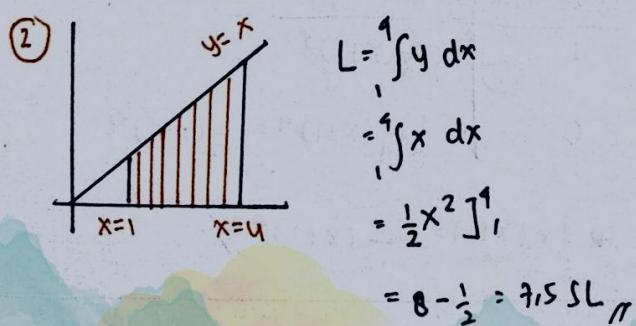
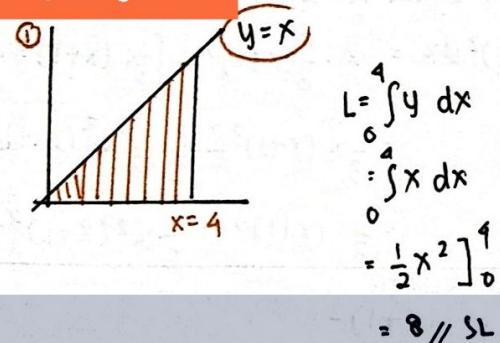
① terhadap sbx



② terhadap sb y

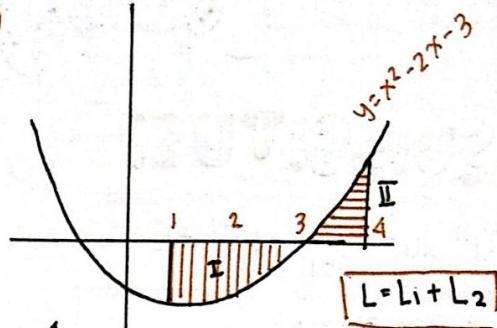


contoh



ingat! Luas semua POSITIF +++

③



$$L_1 = \int_1^3 (x^2 - 2x - 3) \, dx$$

$$= \left[\frac{1}{3}x^3 - x^2 - 3x \right]_1^3$$

$$= \left(\frac{64}{3} - 16 - 12 \right) - \left(\frac{2}{3} - 1 - 3 \right)$$

$$= \frac{37}{3} - 10 = \frac{7}{3} \text{ //}$$

$$L_2 = \int_3^4 (x^2 - 2x - 3) \, dx$$

$$= \left[\frac{1}{3}x^3 - x^2 - 3x \right]_3^4$$

$$= -\left(\frac{2}{3} - 9 - 9 \right) - \left(\frac{1}{3} - 1 - 3 \right)$$

$$= -\left(\frac{26}{3} - 14 \right) = -\left(\frac{26}{3} - \frac{42}{3} \right) = \frac{16}{3} \text{ //}$$

(b) gunanya biar luasnya jd + krn dibawah sbx hasil auto 0)

$$L = L_1 + L_2$$

$$= \frac{7}{3} + \frac{16}{3}$$

$$= \frac{23}{3}$$

daerah tertutup antara 2 kurva

