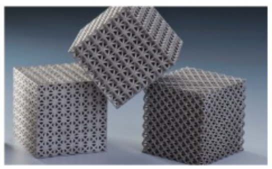
Material structures give rise to rich wave dynamics.

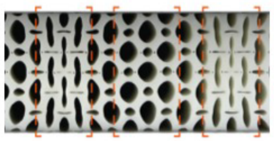
$$c_0^2 u_{xx} = u_{tt}$$

Linear Waves



$$c_0^2 u_{xx} = u_{tt} + \beta u_{xxxx}$$

Dispersive Waves



$$c_0^2 u_{xx} = u_{tt} + \beta u_{xxxx} - \sigma u_x u_{xx}$$

Weak Nonlinearity (Solitons)

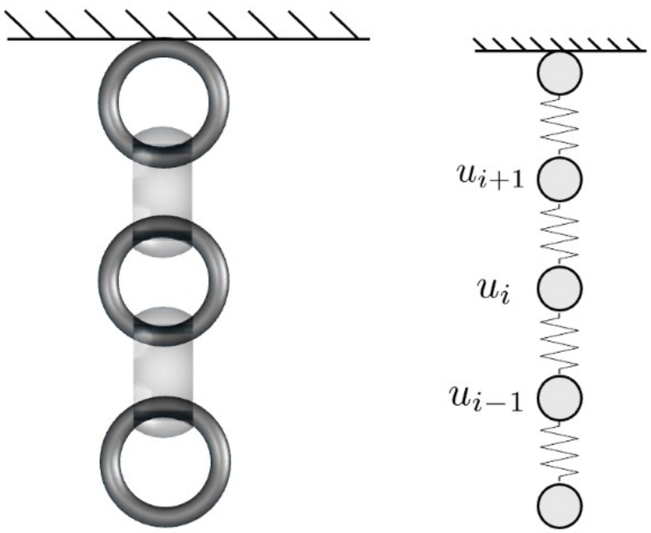
$$u_{tt} = u_x^{n-1} u_{xx} + u_x^{n-3} u_{xx}^3 + \cdots$$

Strong Nonlinearity (Compactons)

$$u_{tt} = \frac{1}{m} \sum_i F_i \cdots$$

Discrete Interacting Particles

Wave Dynamics and Soliton Behaviour



Convert a discrete system of equations into well-studied PDEs by using long-wavelength approximation.

Equation of motion:

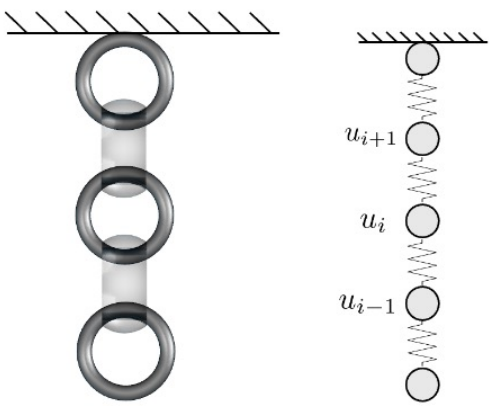
$$\ddot{u}_i = K(u_{i+1} - u_i)^n - K(u_i - u_{i-1})^n$$

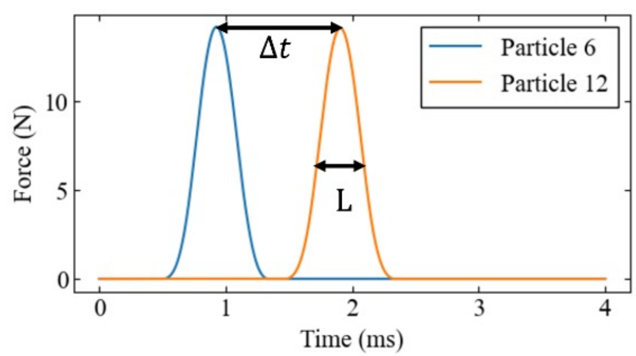
Transform into PDE:

$$u_{tt} = Ka^{n+1}\left\{u_x^n + \frac{na^2}{24}\left[(n-1)u_x^{n-2}u_{xx}^2 - 2u_x^{n-1}u_{xxx}\right]\right\}$$

Nesterenko, V. (Springer Science & Business Media, 2013)

Wave Dynamics and Soliton Behaviour





Solitary wave speed:

$$V_s = \left(\frac{2c_n^2}{n+1}\right)^{\frac{1}{n+1}} (-v_{max})^{\frac{n-1}{n+1}}$$

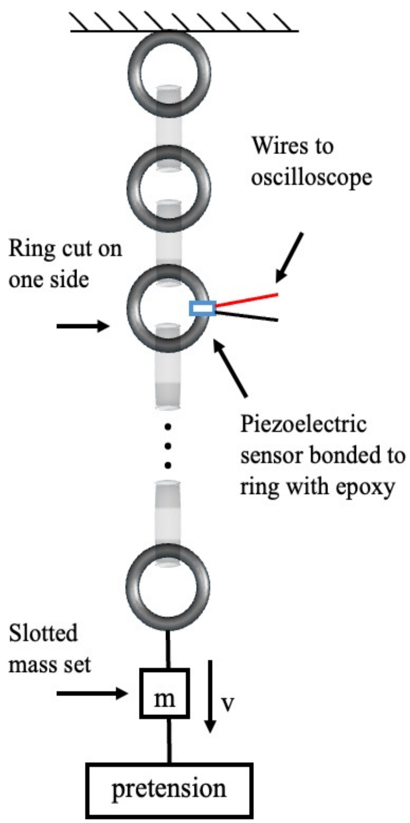
Characteristic length of soliton:

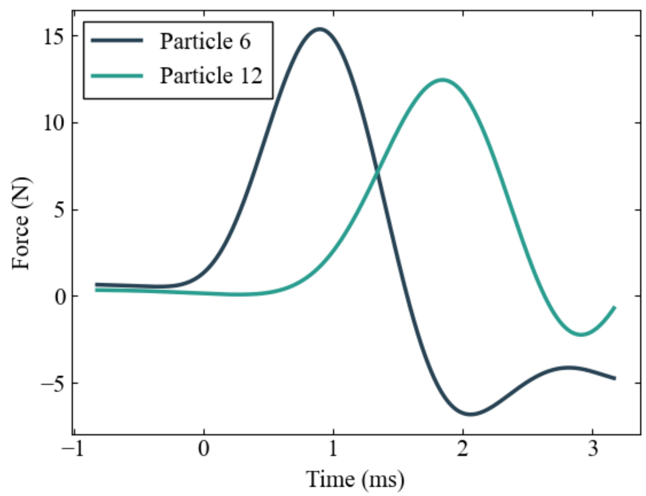
$$L_n = \frac{\pi a}{n-1}\sqrt{\frac{n(n+1)}{6}}$$

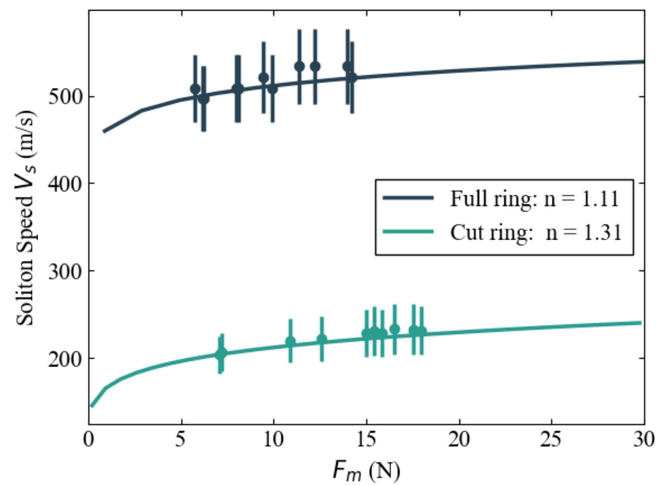
Soliton behvaviour is a function of the nonlinear exponent n and the initial conditions of the impulse.

Nesterenko, V. (Springer Science & Business Media, 2013)

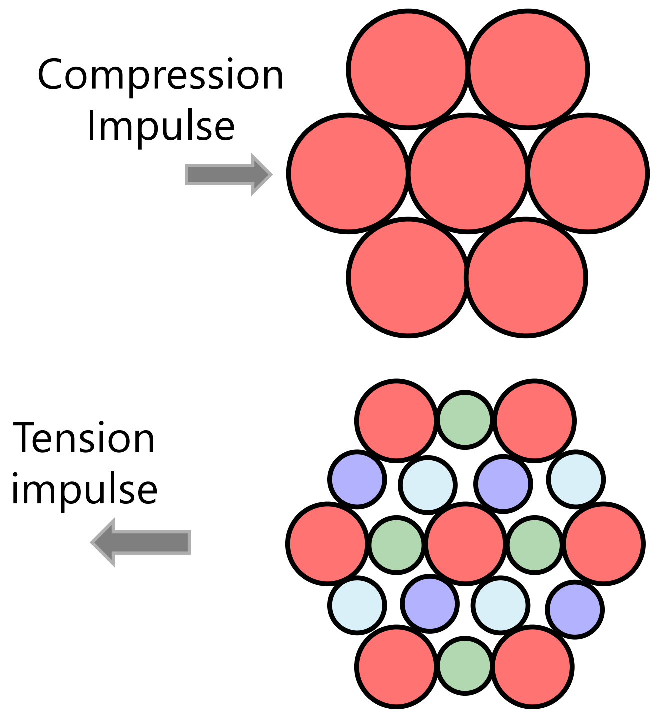
Dynamics in 1D Polycatenated Materials



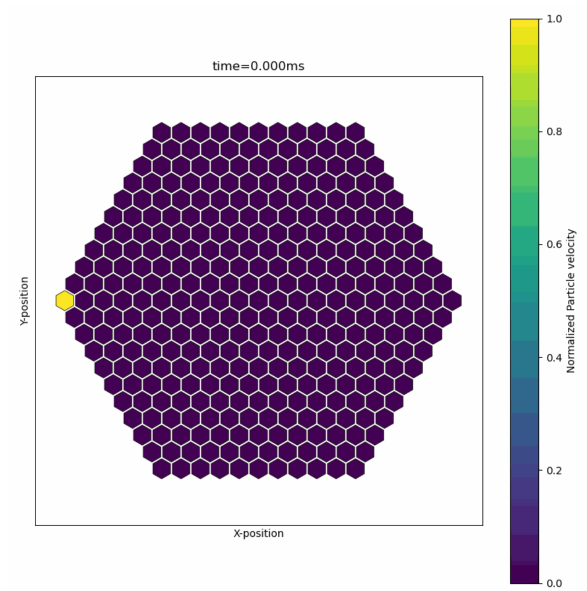




Dynamics in 2D Polycatenated Materials

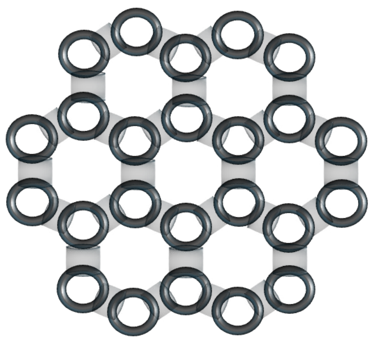


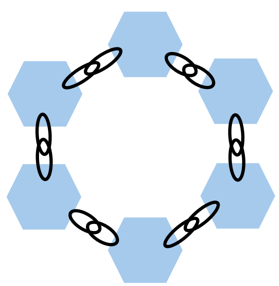
Different interaction due to the topology



- Simulation of wavefront propagation in hexagonal lattice (Reo)
- Simulation of soliton propagation/collision in 2D networks/ junctions (Henry)

Dynamics in 2D Polycatenated Materials

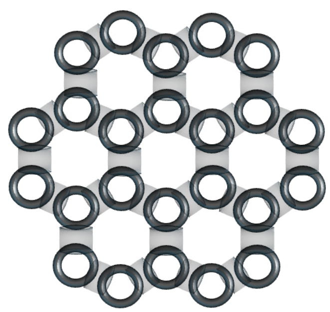


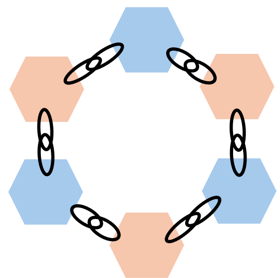


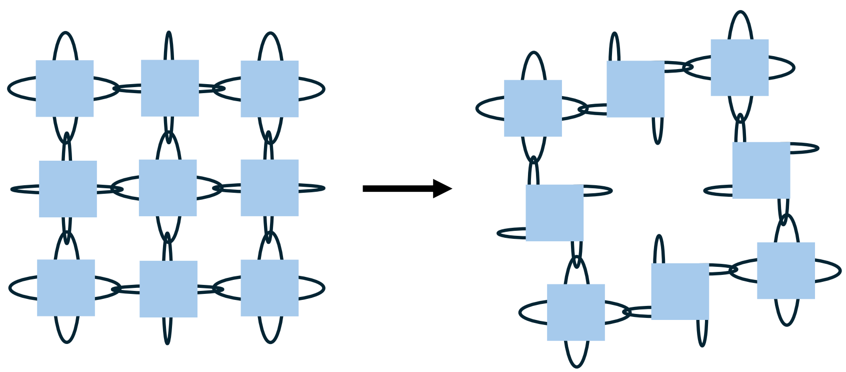
- Previous challenge: lattice too flexible/ hard to constrain in a lattice in 2D experiment

- New setup:
  - Lattice consists rigid cylinders with flat surfaces
  - Connected with ring chains
  - Can control on-site potential by embedding magnet etc.
  - Fabricate with metal 3D printing

(Naïve) Plan for the Project







1. 2D network/square lattice and check print quality/surface roughness
2. Mechanical characterization
3. Set up piezosensor to measure force as a function of time
4. Set up synchronized way to excite multiple points on the boundary
5. Compare soliton propagation speed, wavefront profile etc. with simulation
6. Other fun things:
   - Introduce defects, vacancies, fracture
   - Broken inversion symmetry
   - Introduce magnets in the lattice pieces and add external magnetic fields