CMPT 440 – Spring 2019: Quantum Finite Automata

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Theoretical Background

Quantum finite automata has been categorized into distinct types of quantum state machines. The quantum state machine can be defined broadly by the 3-tuple that is $M = (S, s_0, \delta)$. Where $\delta(s, t)$ is the amplitude of the transition function from s to t and probability of the transition is $|\delta(s, t)|^2$. The probability of 1 denotes that the transition is probabilistic and 0 denoting the opposite. The Hilber space is used as a computational basis for M based on the unit circle for calculating probability. The elements of S *states* and $\hat{H}$ as *superposition states*. The unitary operator describes the operation for the superposition of states $\psi = \Sigma \alpha_i s_i$ and the amplitude also defines the probability of the superposition states as $|\langle \psi, s_i \rangle|^2$. The q-*state machine* is then written as $M = (H, s_0, U)$ A quantum state machine of with accepting states is finally written as the following:

$$M_f = (S, s_0, \delta, S_f) \tag{1}$$

Where $M = (S, s_0, \delta)$ is a QSM. This formula introduced by Gudder (2000). From this 1-way and 2-way finite automata can be defined. These are further categorized as measure-once quantum finite automata (MO-QFAs) defined as $M = (Q, \Sigma, \delta, q_o, F)$ and measure-many quantum finite automata (MM-QFAs) defined as $M = (Q, \Sigma, \delta, q_o, Q_{acc}, Q_{rej})$. The difference being, measuring after calculation and measuring after each transition receptively. Brodsky and Pippenger (2002)

An Example

The result of these quantum finite automata allow the acceptance of non-regular languages such as $L = a^n b^n$ The two way finite automata can check each condition in each direction not limited to only checking in a single direction. This allows for the start to be in some arbitrary position s_i in the string input and work from that point. This allows a check of both side at a time for which a regular DFA will not be able model without expanding the DFA by the length of the string.

References

A. Brodsky and N. Pippenger. Characterizations of 1-way quantum finite automata. *SIAM Journal on Computing*, 31(5):1456, 2002. ISSN 00975397. URL `http://online.library.marist.edu/login?url=https://search.ebscohost.com/login.aspx?dir`

S. Gudder. Basic properties of quantum automata. *Foundations of Physics*, 30 (2):301–319, Feb 2000. ISSN 1572-9516. doi: 10.1023/A:1003649201735. URL `https://doi.org/10.1023/A:1003649201735`.