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Finals :

1) Problem 1 :

Preprocessing :

① create an array1 [1, 2 ... , k], where total length is k, each element in array is set to 0.

② go through the n integers, for each integer x, if it's in the array from step 1, incress array1 [x] by 1

③ then update array1, for j from the second element to the last ($2 \sim k$), array1 [j] = array1 [j] + array1 [j-1], so that array1 [j] is the sum of all previous elements.

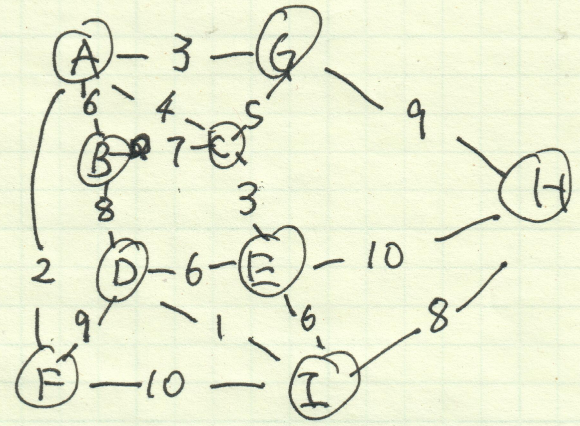
Execution :

for range [a, b], return array1 [b] − array1 [a−1]

O(1).

2) Start with A for Prim's

a) KRUSCALS



9 edges

Preprocessing/sorting:

{DI:1, AF:2, AG:3, CE:3, AC:4, CG:5, AB:6, DE:6, EI:6,

BC:7, BD:8, IH:8, DF:9, GH:9, EH:10, FI:10}

- Step 1: DI
  {DI}, no loop

- Step 2: AF
  {DI, AF}, no loop

- Step 3: AG
  {DI, AF, AG}
  no loop

- step 4: CE
  {DI, AF, AG, CE}
  no loop

- step 5: AC
  {DI, AF, AG, CE, AC}
  no loop

- step 6: CG
  LOOP, skip

- step 7: AB
  {DI, AF, AG, CE, AC, AB}
  ~~NO loop~~ No loop

- step 8: DE
  {DI, AF, AG, CE, AC, AB, DE}
  no loop

- step 9: EI
  loop, skip

- Step 10: BC
  loop, skip

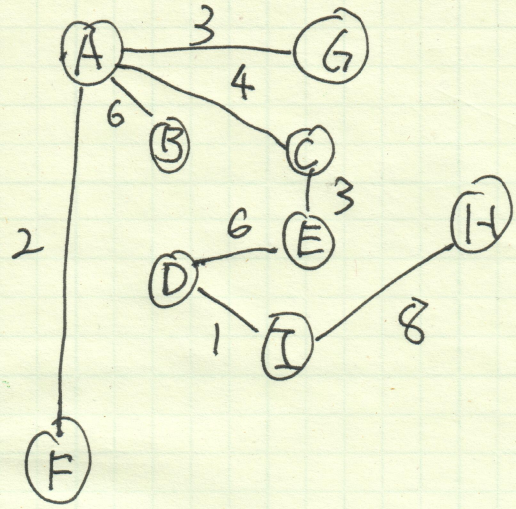
- step 11: BD
  loop, skip

- step 12: IH
  {DI, AF, AG, CE, AC, AB, DE, IH}
  no loop.
  added ~~edge~~ count = 8 = 9-1
  <<terminate>>

* result see next page.

Total weight = 1+2+3+3+4+6+6+8
= 33.

tree:



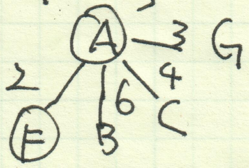
total weight = 33

KRUSCAL'S

2) b) Prim's.

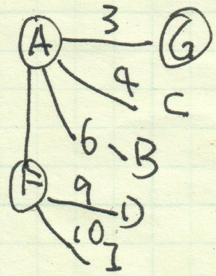
- Step 1: add F

{AF}



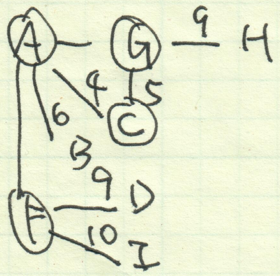
- step 2: add G

{AF, AG}



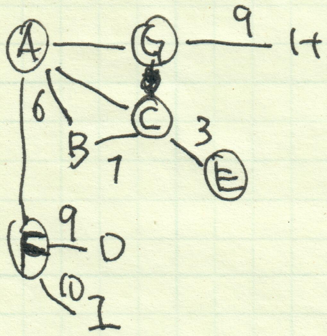
- step 3: add C

{AF, AG, AC}



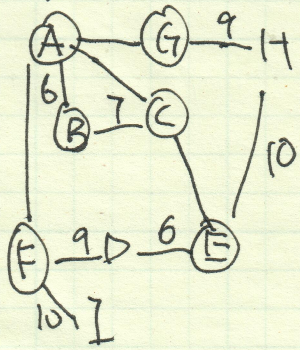
- Step 4: add E

{AF, AG, AC, CE}

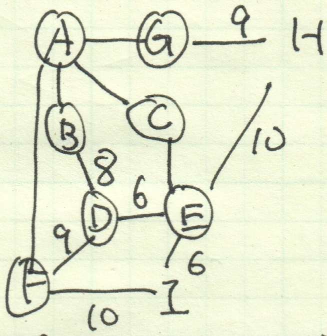


- step 5: add B

{AF, AG, AC, CE, AB}

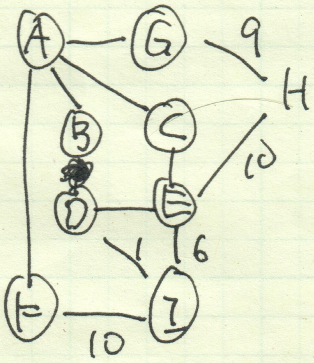


- Step 6: add D



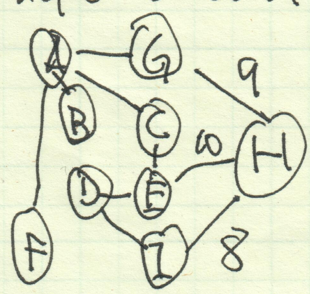
{AF, AG, AC, CE, AB, DE}

- step 7: add I



{AF, AG, AC, CE, AB, DE, DI}

- step 8: add H

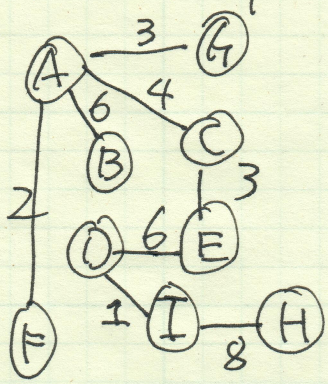


{AF, AG, AC, CE, AB, DE, DI, IH}

9 verticies

<<terminate>>

- Final Graph:

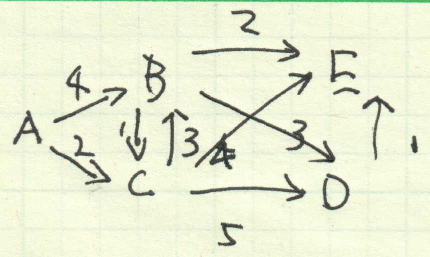


- total weight:

1+2+3+3+4+6+6+8

= 33.

(5) Dijkstra's

graph: 

Step 1: AB > AC

path: { A, C } ; not visited { B, D, E }

~~{ AB = 4~~
~~AC = 2 }~~

Distance:
A: 0
B: 4
C: 2
D: ∞
E: ∞

Step 2: ~~BA~~ path { AC, AB }

unvisited { D, E }

Distance:
A: 0
B: 4 (vs 5 ACB)
C: 2
D: 7 (ACD, ~~ABD~~)
E: 6 (ACE)

Step 3: path { AC, AB, CE }

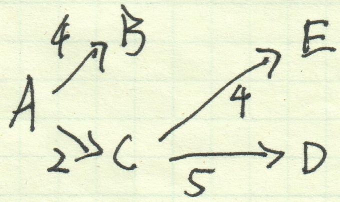
unvisited { D }

Distance:
A: 0
B: 4
C: 2
D: 7 (ACD, ABD)
E: 6 (ACE, ABE)

Step : path { AC, AB, CE, CD }

unvisited { }

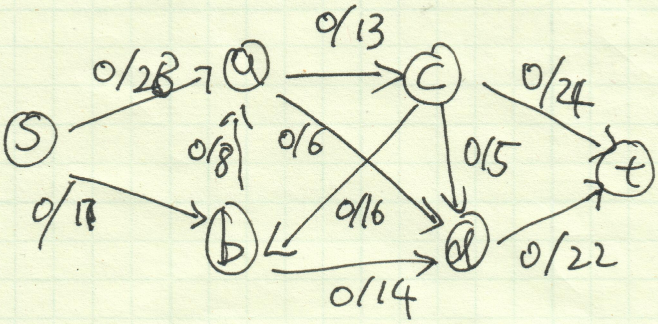
Distance:
A: 0
B: 4
C: 2
D: 7
E: 6.

Final graph: 

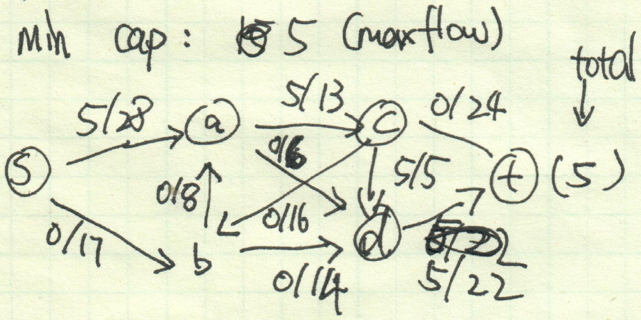
(6) Ford-Fulkerson.

Answer: 33 max flow.

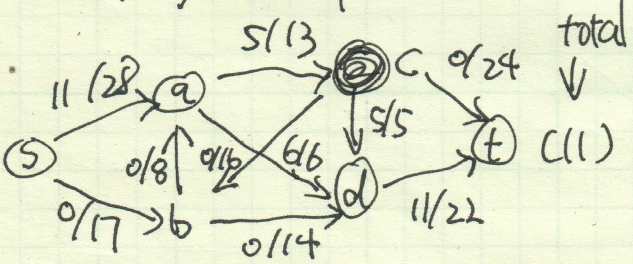
Initialization:



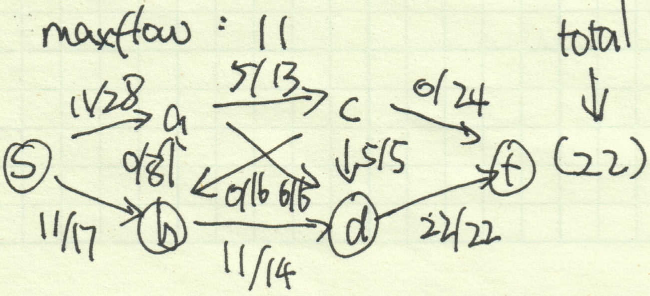
Step1:
path s a c d t.
min cap: ~~8~~ 5 (maxflow)



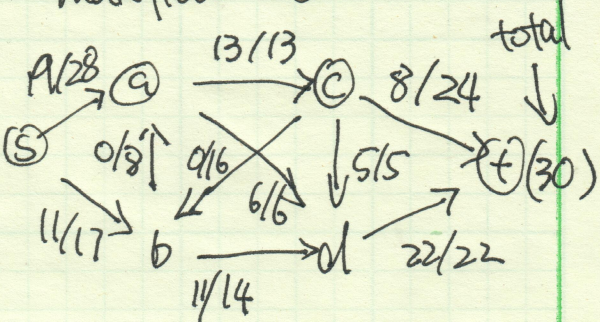
Step 2:
path s a d t
maxflow/min cap: 6



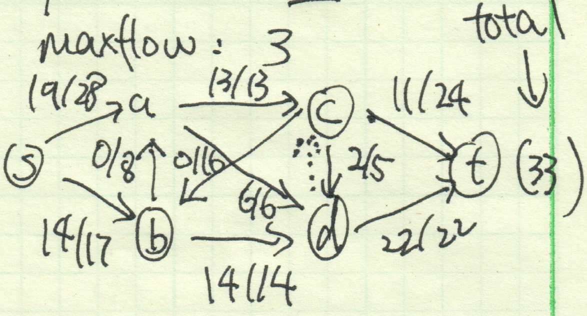
Step 3:
path s b d t
maxflow: 11



Step 4:
path s a c t
maxflow: 8



Step 5:
path s b d c t
maxflow: 3



Can't get to t after this
<< terminated >>

Max value: s→t $\boxed{33}$
- no avaible non-empty backward path from s→t
- no aviable non-full forward path from s→t

Thus according to the algorithm, $\boxed{33}$ is max flow value.