

Wizards

Standard Code Library

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# 1. 数论

## 1.1 $O(m^2 \log n)$ 线性递推

Given  $a_0, a_1, \dots, a_{m-1}$   
 $a_n = c_0 \times a_{n-m} + \dots + c_{m-1} \times a_{n-1}$   
 Solve for  $a_n = v_0 \times a_0 + v_1 \times a_1 + \dots + v_{m-1} \times a_{m-1}$

```
1 void linear_recurrence(long long n, int m, int a[], int
  ↳ c[], int p) {
2   long long v[M] = {1 % p}; u[M << 1], msk = !n;
3   for(long long i(n); i > 1; i >= 1) {
4     msk <= 1;
5   }
6   for(long long x(0); msk; msk >>= 1, x <= 1) {
7     fill_n(u, m << 1, 0);
8     int b(!n & msk);
9     x |= b;
10    if(x < m) {
11      u[x] = 1 % p;
12    } else {
13      for(int i(0); i < m; i++) {
14        for(int j(0), t(i + b); j < m; j++, t++) {
15          u[t] = (u[t] + v[i] * v[j]) % p;
16        }
17      }
18      for(int i((m << 1) - 1); i >= m; i--) {
19        for(int j(0), t(i - m); j < m; j++, t++) {
20          u[t] = (u[t] + c[j] * u[i]) % p;
21        }
22      }
23    }
24    copy(u, u + m, v);
25  }
26  //a[n] = v[0] * a[0] + v[1] * a[1] + ... + v[m - 1] *
  ↳ a[m - 1].
27  for(int i(m); i < 2 * m; i++) {
28    a[i] = 0;
29    for(int j(0); j < m; j++) {
30      a[i] = (a[i] + (long long)c[j] * a[i + j - m]) % p;
31    }
32  }
33  for(int j(0); j < m; j++) {
34    b[j] = 0;
35    for(int i(0); i < m; i++) {
36      b[j] = (b[j] + v[i] * a[i + j]) % p;
37    }
38  }
39  for(int j(0); j < m; j++) {
40    a[j] = b[j];
41  }
42 }
```

## 1.2 求逆元

```
1 void ex_gcd(long long a, long long b, long long &x, long
  ↳ long &y) {
2   if (b == 0) {
3     x = 1;
4     y = 0;
5     return;
6   }
7   long long xx, yy;
8   ex_gcd(b, a % b, xx, yy);
9   y = xx - a / b * yy;
10  x = yy;
11 }
12 long long inv(long long x, long long MODN) {
13   long long inv_x, y;
14   ex_gcd(x, MODN, inv_x, y);
15   return (inv_x % MODN + MODN) % MODN;
16 }
```

## 1.3 中国剩余定理

```
1 //返回 (ans, M), 其中 ans 是模 M 意义下的解
2 std::pair<long long, long long> CRT(const std::vector<long
  ↳ long>& m, const std::vector<long long>& a) {
3   long long M = 1, ans = 0;
4   int n = m.size();
5   for (int i = 0; i < n; i++) M *= m[i];
6   for (int i = 0; i < n; i++) {
7     ans = (ans + (M / m[i]) * a[i] % M * inv(M / m[i],
  ↳ m[i])) % M; // 可能需要大整数相乘取模
8   }
9   return std::make_pair(ans, M);
10 }
```

## 1.4 素性测试

```
1 int strong_pseudo_primetest(long long n, int base) {
2   long long n2=n-1, res;
3   int s=0;
4   while(n%2==0) n2>>=1, s++;
5   res=powmod(base, n2, n);
6   if((res==1) || (res==n-1)) return 1;
7   s--;
8   while(s>0) {
9     res=mulmod(res, res, n);
10    if(res==n-1) return 1;
11    s--;
12  }
13  return 0; // n is not a strong pseudo prime
14 }
15 int isprime(long long n) {
16   static LL testNum[]={2,3,5,7,11,13,17,19,23,29,31,37};
17   static LL lim[]={4,0,1373653LL,25326001LL,25000000000LL,
18     ↳ 3474749660383LL,341550071728321LL,0,0,0,0};
19   if(n<2 || n==3215031751LL) return 0;
20   for(int i=0; i<12; ++i){
21     if(n<lim[i]) return 1;
22     if(strong_pseudo_primetest(n, testNum[i])==0) return 0;
23   }
24   return 1;
25 }
```

## 1.5 质因数分解

```
1 int ansn; LL ans[1000];
2 LL func(LL x, LL n){ return(mod_mul(x,x,n)+1)%n; }
3 LL Pollard(LL n){
4   LL i,x,y,p;
5   if(Rabin_Miller(n)) return n;
6   if(!(n&1)) return 2;
7   for(i=1; i<20; i++){
8     x=i; y=func(x,n); p=gcd(y-x,n);
9     while(p==1) {x=func(x,n); y=func(y,n,n);
10      ↳ p=gcd((y-x+n)%n,n)%n;}
11     if(p==0 || p==n) continue;
12     return p;
13   }
14 void factor(LL n){
15   LL x;
16   x=Pollard(n);
17   if(x==n){ ans[ansn++]=x; return; }
18   factor(x), factor(n/x);
19 }
```

## 1.6 佩尔方程

```
1 import java.math.BigInteger;
2 import java.util.Scanner;
```

```

3 public class Main {
4     public static BigInteger p, q;
5     public static void solve(int n) {
6         BigInteger N, p1, p2, q1, q2, a0, a1, a2, g1, g2,
            ↪ h1, h2;
7         g1 = q2 = p1 = BigInteger.ZERO;
8         h1 = q1 = p2 = BigInteger.ONE;
9         a0 = a1 =
            ↪ BigInteger.valueOf((long)Math.sqrt(1.0*n));
10        N = BigInteger.valueOf(n);
11        while (true) {
12            g2 = a1.multiply(h1).subtract(g1);
13            h2 = N.subtract(g2.pow(2)).divide(h1);
14            a2 = g2.add(a0).divide(h2);
15            p = a1.multiply(p2).add(p1);
16            q = a1.multiply(q2).add(q1);
17            if (p.pow(2).subtract(N.multiply(q.pow(2)))
18                .compareTo(BigInteger.ONE) == 0) return;
19            g1 = g2; h1 = h2; a1 = a2;
20            p1 = p2; p2 = p;
21            q1 = q2; q2 = q;
22        }
23    }
24    public static void main(String[] args) {
25        Scanner cin = new Scanner(System.in);
26        int t=cin.nextInt();
27        while (t--!=0) {
28            solve(cin.nextInt());
29            System.out.println(p + " " + q);
30        }
31    }
32 }

```

## 1.7 二次剩余

```

1 // x^2 = a (mod p), 0 <= a < p, 返回 true or false 代表
    ↪ 是否存在解
2 // p 必须是质数, 若是多个单次质数的乘积, 可以分别
    ↪ 求解再用 CRT 合并
3 // 复杂度为 O(log n)
4 void multiply(ll &c, ll &d, ll a, ll b, ll w) {
5     int cc = (a * c + b * d % MOD * w) % MOD;
6     int dd = (a * d + b * c) % MOD;
7     c = cc, d = dd;
8 }
9
10 bool solve(int n, int &x) {
11     if (MOD == 2) return x = 1, true;
12     if (power(n, MOD / 2, MOD) == MOD - 1) return false;
13     ll c = 1, d = 0, b = 1, a, w;
14     // finding a such that a^2 - n is not a square
15     do { a = rand() % MOD;
16         w = (a * a - n + MOD) % MOD;
17         if (w == 0) return x = a, true;
18     } while (power(w, MOD / 2, MOD) != MOD - 1);
19     for (int times = (MOD + 1) / 2; times; times >= 1) {
20         if (times & 1) multiply(c, d, a, b, w);
21         multiply(a, b, a, b, w);
22     }
23     // x = (a + sqrt(w)) ^ ((p + 1) / 2)
24     return x = c, true;
25 }

```

## 1.8 一元三次方程

```

1 double a(p[3]), b(p[2]), c(p[1]), d(p[0]);
2 double k(b / a), m(c / a), n(d / a);
3 double p(-k * k / 3. + m);

```

```

4 double q(2. * k * k * k / 27 - k * m / 3. + n);
5 Complex omega[3] = {Complex(1, 0), Complex(-0.5, 0.5 *
    ↪ sqrt(3)), Complex(-0.5, -0.5 * sqrt(3))};
6 Complex r1, r2;
7 double delta(q * q / 4 + p * p * p / 27);
8 if (delta > 0) {
9     r1 = cubrt(-q / 2. + sqrt(delta));
10    r2 = cubrt(-q / 2. - sqrt(delta));
11 } else {
12     r1 = pow(-q / 2. + pow(Complex(delta), 0.5), 1. / 3);
13     r2 = pow(-q / 2. - pow(Complex(delta), 0.5), 1. / 3);
14 }
15 for(int _ (0); _ < 3; _++) {
16     Complex x = -k / 3. + r1 * omega[_ * 1] + r2 * omega[_
    ↪ * 2 % 3];
17 }

```

## 1.9 线下整点

```

1 //  $\sum_{i=0}^{n-1} \lfloor \frac{a+bi}{m} \rfloor$ ,  $n, m, a, b > 0$ 
2 LL solve(LL n, LL a, LL b, LL m) {
3     if (b==0) return n*(a/m);
4     if (a>=m) return n*(a/m)+solve(n, a%m, b, m);
5     if (b>=m) return (n-1)*n/2*(b/m)+solve(n, a, b%m, m);
6     return solve((a+b*n)/m, (a+b*n)%m, m, b);
7 }

```

## 1.10 线性同余不等式

```

1 // Find the minimal non-negative solutions for
    ↪  $l \leq d \cdot x \bmod m \leq r$ 
2 //  $0 \leq d, l, r < m; l \leq r, O(\log n)$ 
3 ll cal(ll m, ll d, ll l, ll r) {
4     if (l == 0) return 0;
5     if (d == 0) return MXL; // 无解
6     if (d * 2 > m) return cal(m, m - d, m - r, m - l);
7     if ((l - 1) / d < r / d) return (l - 1) / d + 1;
8     ll k = cal(d, (-m % d + d) % d, l % d, r % d);
9     return k == MXL ? MXL : (k * m + l - 1) / d + 1; // 无
    ↪ 解 2
10 }

```

## 1.11 组合数取模

```

1 LL prod=1,P;
2 pair<LL,LL> comput(LL n,LL p,LL k){
3     if(n<=1)return make_pair(0,1);
4     LL ans=1,cnt=0;
5     ans=pow(prod,n/P,P);
6     cnt=n/p;
7     pair<LL,LL> res=comput(n/p,p,k);
8     cnt+=res.first;
9     ans=ans*res.second%P;
10    for(int i=n-n%P+1;i<=n;i++)if(i%p){
11        ans=ans*i%P;
12    }
13    return make_pair(cnt,ans);
14 }
15
16 pair<LL,LL> calc(LL n,LL p,LL k){
17     prod=1;P=pow(p,k,1e18);
18     for(int i=1;i<P;i++)if(i%p)prod=prod*i%P;
19     pair<LL,LL> res=comput(n,p,k);
20     return res;
21 }
22 LL calc(LL n,LL m,LL p,LL k){
23     pair<LL,LL> A,B,C;
24     LL P=pow(p,k,1e18);
25     A=calc(n,p,k);
26     B=calc(m,p,k);

```

```

27 C=calc(n-m,p,k);
28 LL ans=1;
29 ans=pow(p,A.first-B.first-C.first,P);
30
    ↪ ans=ans*A.second%P*inv(B.second,P)%P*inv(C.second,P)%P;
31 return ans;
32 }

```

## 2. 代数

### 2.1 快速傅里叶变换

```

1 void fft(Complex a[], int n, int f) {
2     for (int i = 0; i < n; ++i)
3         if (R[i] < i) swap(a[i], a[R[i]]);
4     for (int i = 1, h = 0; i < n; i <= 1, h++) {
5         Complex wn = Complex(cos(pi / i), f * sin(pi / i));
6         Complex w = Complex(1, 0);
7         for (int k = 0; k < i; ++k, w = w * wn) tmp[k] = w;
8         for (int p = i < 1, j = 0; j < n; j += p) {
9             for (int k = 0; k < i; ++k) {
10                Complex x = a[j + k], y = a[j + k + i] * tmp[k];
11                a[j + k] = x + y; a[j + k + i] = x - y;
12            }
13        }
14    }
15 }

```

### 2.2 分治卷积

```

1 // n 必须是 2 的次幂
2 void fft(Complex a[], int n, int f) {
3     for (int i = 0; i < n; ++i)
4         if (R[i] < i) swap(a[i], a[R[i]]);
5     for (int i = 1, h = 0; i < n; i <= 1, h++) {
6         Complex wn = Complex(cos(pi / i), f * sin(pi / i));
7         Complex w = Complex(1, 0);
8         for (int k = 0; k < i; ++k, w = w * wn) tmp[k] = w;
9         for (int p = i < 1, j = 0; j < n; j += p) {
10            for (int k = 0; k < i; ++k) {
11                Complex x = a[j + k], y = a[j + k + i] * tmp[k];
12                a[j + k] = x + y; a[j + k + i] = x - y;
13            }
14        }
15    }
16 }

```

### 2.3 快速数论变换

```

1 // n 必须是 2 的次幂
2 void fft(Complex a[], int n, int f) {
3     for (int i = 0; i < n; ++i)
4         if (R[i] < i) swap(a[i], a[R[i]]);
5     for (int i = 1, h = 0; i < n; i <= 1, h++) {
6         Complex wn = Complex(cos(pi / i), f * sin(pi / i));
7         Complex w = Complex(1, 0);
8         for (int k = 0; k < i; ++k, w = w * wn) tmp[k] = w;
9         for (int p = i < 1, j = 0; j < n; j += p) {
10            for (int k = 0; k < i; ++k) {
11                Complex x = a[j + k], y = a[j + k + i] * tmp[k];
12                a[j + k] = x + y; a[j + k + i] = x - y;
13            }
14        }
15    }
16 }

```

### 2.4 快速沃尔什变换

```

1 void FWT(LL a[],int n,int ty){ //the length is 2^n
2     for(int d=1;d<n;d<=1){

```

```

3         for(int m=(d<<1),i=0;i<n;i+=m){
4             if(ty==1){
5                 for(int j=0;j<d;j++){
6                     LL x=a[i+j],y=a[i+j+d];
7                     a[i+j]=x+y;
8                     a[i+j+d]=x-y;
9                     //and:a[i+j]=x+y; or:a[i+j+d]=x+y;
10                }
11            }else{
12                for(int j=0;j<d;j++){
13                    LL x=a[i+j],y=a[i+j+d];
14                    a[i+j]=(x+y)/2;
15                    a[i+j+d]=(x-y)/2;
16                    //and:a[i+j]=x-y; or:a[i+j+d]=y-x;
17                }
18            }
19        }
20    }
21 }

```

### 2.5 自适应辛普森积分

```

1 namespace adaptive_simpson {
2     template<typename function>
3     inline double area(function f, const double &left, const
4         ↪ double &right) {
5         double mid = (left + right) / 2;
6         return (right - left) * (f(left) + 4 * f(mid) +
7             ↪ f(right)) / 6;
8     }
9     template<typename function>
10    inline double simpson(function f, const double &left,
11        ↪ const double &right, const double &eps, const
12        ↪ double &area_sum) {
13        double mid = (left + right) / 2;
14        double area_left = area(f, left, mid);
15        double area_right = area(f, mid, right);
16        double area_total = area_left + area_right;
17        if (fabs(area_total - area_sum) <= 15 * eps) {
18            return area_total + (area_total - area_sum) / 15;
19        }
20        return simpson(f, left, right, eps / 2, area_left) +
21            ↪ simpson(f, mid, right, eps / 2, area_right);
22    }
23    template<typename function>
24    inline double simpson(function f, const double &left,
25        ↪ const double &right, const double &eps) {
26        return simpson(f, left, right, eps, area(f, left,
27            ↪ right));
28    }
29 }

```

### 2.6 单纯形

```

1 const double eps = 1e-8;
2 // max{c * x | Ax <= b, x >= 0} 的解, 无解返回空的
3 ↪ vector, 否则就是解.
4 vector<double> simplex(vector<vector<double>> &A,
5     ↪ vector<double> b, vector<double> c) {
6     int n = A.size(), m = A[0].size() + 1, r = n, s = m - 1;
7     vector<vector<double>> D(n + 2, vector<double>(m + 1));
8     vector<int> ix(n + m);
9     for(int i = 0; i < n + m; i++) {
10         ix[i] = i;
11     }
12     for(int i = 0; i < n; i++) {
13         for(int j = 0; j < m - 1; j++) {
14             D[i][j] = -A[i][j];
15         }
16     }
17     D[i][m - 1] = 1;
18     D[i][m] = b[i];

```

```

16     if (D[r][m] > D[i][m]) {
17         r = i;
18     }
19 }
20 for(int j = 0; j < m - 1; j++) {
21     D[n][j] = c[j];
22 }
23 D[n + 1][m - 1] = -1;
24 for(double d; ;) {
25     if (r < n) {
26         swap(ix[s], ix[r + m]);
27         D[r][s] = 1. / D[r][s];
28         for(int j = 0; j <= m; j++) {
29             if (j != s) {
30                 D[r][j] *= -D[r][s];
31             }
32         }
33         for(int i = 0; i <= n + 1; i++) {
34             if (i != r) {
35                 for(int j = 0; j <= m; j++) {
36                     if (j != s) {
37                         D[i][j] += D[r][j] * D[i][s];
38                     }
39                 }
40                 D[i][s] *= D[r][s];
41             }
42         }
43     }
44     r = -1, s = -1;
45     for(int j = 0; j < m; j++) {
46         if (s < 0 || ix[s] > ix[j]) {
47             if (D[n + 1][j] > eps || D[n + 1][j] > -eps &&
48                 ↪ D[n][j] > eps) {
49                 s = j;
50             }
51         }
52     }
53     if (s < 0) break;
54     for(int i = 0; i < n; i++) {
55         if (D[i][s] < -eps) {
56             if (r < 0 || (d = D[r][m] / D[r][s] - D[i][m] /
57                 ↪ D[i][s]) < -eps
58                 || d < eps && ix[r + m] > ix[i + m]) {
59                 r = i;
60             }
61         }
62     }
63     if (r < 0) return vector<double> ();
64 }
65 if (D[n + 1][m] < -eps) return vector<double> ();
66 vector<double> x(m - 1);
67 for(int i = m; i < n + m; i++) {
68     if (ix[i] < m - 1) {
69         x[ix[i]] = D[i - m][m];
70     }
71 }
72 return x;
73 }

```

```

9     Point rotate(DB ang) const { // 逆时针旋转 ang 弧度
10         return Point(cos(ang) * x - sin(ang) * y, cos(ang) * y
11             ↪ + sin(ang) * x);
12     }
13     Point turn90() const { // 逆时针旋转 90 度
14         return Point(-y, x);
15     }
16     Point unit() const {
17         return *this / len();
18     }
19 };
20 DB dot(const Point& a, const Point& b) {
21     return a.x * b.x + a.y * b.y;
22 }
23 DB det(const Point& a, const Point& b) {
24     return a.x * b.y - a.y * b.x;
25 }
26 #define cross(p1,p2,p3)
27     ↪ ((p2.x-p1.x)*(p3.y-p1.y)-(p3.x-p1.x)*(p2.y-p1.y))
28 #define crossOp(p1,p2,p3) sign(cross(p1,p2,p3))
29 bool isLL(const Line& l1, const Line& l2, Point& p) { //
30     ↪ 直线与直线交点
31     DB s1 = det(l2.b - l2.a, l1.a - l2.a),
32         s2 = -det(l2.b - l2.a, l1.b - l2.a);
33     if (!sign(s1 + s2)) return false;
34     p = (l1.a * s2 + l1.b * s1) / (s1 + s2);
35     return true;
36 }
37 bool onSeg(const Line& l, const Point& p) { // 点在线段
38     ↪ 上
39     return sign(det(p - l.a, l.b - l.a)) == 0 && sign(dot(p
40         ↪ - l.a, p - l.b)) <= 0;
41 }
42 Point projection(const Line & l, const Point& p) {
43     return l.a + (l.b - l.a) * (dot(p - l.a, l.b - l.a) /
44         ↪ (l.b - l.a).len2());
45 }
46 DB disToLine(const Line& l, const Point& p) { // 点到 *
47     ↪ 直线 * 距离
48     return fabs(det(p - l.a, l.b - l.a) / (l.b -
49         ↪ l.a).len());
50 }
51 DB disToSeg(const Line& l, const Point& p) { // 点到线段
52     ↪ 距离
53     return sign(dot(p - l.a, l.b - l.a)) * sign(dot(p - l.b,
54         ↪ l.a - l.b)) == 1 ? disToLine(l, p) : std::min((p -
55         ↪ l.a).len(), (p - l.b).len());
56 }
57 // 圆与直线交点
58 bool isCL(Circle a, Line l, Point& p1, Point& p2) {
59     DB x = dot(l.a - a.o, l.b - l.a),
60         y = (l.b - l.a).len2(),
61         d = x * x - y * ((l.a - a.o).len2() - a.r * a.r);
62     if (sign(d) < 0) return false;
63     Point p = l.a - ((l.b - l.a) * (x / y)), delta = (l.b -
64         ↪ l.a) * (msqrt(d) / y);
65     p1 = p + delta; p2 = p - delta;
66     return true;
67 }
68 // 圆与圆的交面积
69 DB areaCC(const Circle& c1, const Circle& c2) {
70     DB d = (c1.o - c2.o).len();
71     if (sign(d - (c1.r + c2.r)) >= 0) return 0;
72     if (sign(d - std::abs(c1.r - c2.r)) <= 0) {
73         DB r = std::min(c1.r, c2.r);
74         return r * r * PI;
75     }
76     DB x = (d * d + c1.r * c1.r - c2.r * c2.r) / (2 * d),
77         t1 = acos(x / c1.r), t2 = acos((d - x) / c2.r);
78     return c1.r * c1.r * t1 + c2.r * c2.r * t2 - d * c1.r *
79         ↪ sin(t1);
80 }

```

## 3. 计算几何

### 3.1 二维

#### 3.1.1 点类

```

1 int sign(DB x) {
2     return (x > eps) - (x < -eps);
3 }
4 DB msqrt(DB x) {
5     return sign(x) > 0 ? sqrt(x) : 0;
6 }
7 struct Point {
8     DB x, y;

```

```

68 // 圆与圆交点
69 bool isCC(Circle a, Circle b, P& p1, P& p2) {
70     DB s1 = (a.o - b.o).len();
71     if (sign(s1 - a.r - b.r) > 0 || sign(s1 - std::abs(a.r -
72         ↪ b.r)) < 0) return false;
73     DB s2 = (a.r * a.r - b.r * b.r) / s1;
74     DB aa = (s1 + s2) * 0.5, bb = (s1 - s2) * 0.5;
75     P o = (b.o - a.o) * (aa / (aa + bb)) + a.o;
76     P delta = (b.o - a.o).unit().turn90() * msqrt(a.r * a.r
77         ↪ - aa * aa);
78     p1 = o + delta, p2 = o - delta;
79     return true;
80 }
81 // 求点到圆的切点, 按关于点的顺时针方向返回两个点
82 bool tanCP(const Circle &c, const Point &p0, Point &p1,
83     ↪ Point &p2) {
84     double x = (p0 - c.o).len2(), d = x - c.r * c.r;
85     if (d < eps) return false; // 点在圆上认为没有切点
86     Point p = (p0 - c.o) * (c.r * c.r / x);
87     Point delta = ((p0 - c.o) * (-c.r * sqrt(d) /
88         ↪ x)).turn90();
89     p1 = c.o + p + delta;
90     p2 = c.o + p - delta;
91     return true;
92 }
93 // 求圆到圆的外共切线, 按关于 c1.o 的顺时针方向返
94     ↪ 回两条线
95 vector<Line> extanCC(const Circle &c1, const Circle &c2) {
96     vector<Line> ret;
97     if (sign(c1.r - c2.r) == 0) {
98         Point dir = c2.o - c1.o;
99         dir = (dir * (c1.r / dir.len())).turn90();
100         ret.push_back(Line(c1.o + dir, c2.o + dir));
101         ret.push_back(Line(c1.o - dir, c2.o - dir));
102     } else {
103         Point p = (c1.o * -c2.r + c2.o * c1.r) / (c1.r -
104             ↪ c2.r);
105         Point p1, p2, q1, q2;
106         if (tanCP(c1, p, p1, p2) && tanCP(c2, p, q1, q2)) {
107             if (c1.r < c2.r) swap(p1, p2), swap(q1, q2);
108             ret.push_back(Line(p1, q1));
109             ret.push_back(Line(p2, q2));
110         }
111     }
112     return ret;
113 }
114 // 求圆到圆的内共切线, 按关于 c1.o 的顺时针方向返
115     ↪ 回两条线
116 std::vector<Line> intanCC(const Circle &c1, const Circle
117     ↪ &c2) {
118     std::vector<Line> ret;
119     Point p = (c1.o * c2.r + c2.o * c1.r) / (c1.r + c2.r);
120     Point p1, p2, q1, q2;
121     if (tanCP(c1, p, p1, p2) && tanCP(c2, p, q1, q2)) { //
122         ↪ 两圆相切认为没有切线
123         ret.push_back(Line(p1, q1));
124         ret.push_back(Line(p2, q2));
125     }
126     return ret;
127 }
128 bool contain(vector<Point> polygon, Point p) { // 判断点
129     ↪ p 是否被多边形包含, 包括落在边界上
130     int ret = 0, n = polygon.size();
131     for (int i = 0; i < n; ++i) {
132         Point u = polygon[i], v = polygon[(i + 1) % n];
133         if (onSeg(Line(u, v), p)) return true; // Here I
134             ↪ guess.
135         if (sign(u.y - v.y) <= 0) swap(u, v);
136         if (sign(p.y - u.y) > 0 || sign(p.y - v.y) <= 0)
137             ↪ continue;
138         ret += sign(det(p, v, u)) > 0;
139     }
140 }

```

```

128     return ret & 1;
129 }
130 // 用半平面 (q1,q2) 的逆时针方向去切凸多边形
131 std::vector<Point> convexCut(const std::vector<Point>&ps,
132     ↪ Point q1, Point q2) {
133     std::vector<Point> qs; int n = ps.size();
134     for (int i = 0; i < n; ++i) {
135         Point p1 = ps[i], p2 = ps[(i + 1) % n];
136         int d1 = crossOp(q1, q2, p1), d2 = crossOp(q1, q2, p2);
137         if (d1 >= 0) qs.push_back(p1);
138         if (d1 * d2 < 0) qs.push_back(isSS(p1, p2, q1, q2));
139     }
140     return qs;
141 }
142 // 求凸包
143 std::vector<Point> convexHull(std::vector<Point> ps) {
144     int n = ps.size(); if (n <= 1) return ps;
145     std::sort(ps.begin(), ps.end());
146     std::vector<Point> qs;
147     for (int i = 0; i < n; qs.push_back(ps[i ++]))
148         while (qs.size() > 1 && sign(det(qs[qs.size() - 2],
149             ↪ qs.back(), ps[i])) <= 0)
150             qs.pop_back();
151     for (int i = n - 2, t = qs.size(); i >= 0;
152         ↪ qs.push_back(ps[i --]))
153         while ((int)qs.size() > t && sign(det(qs[qs.size() -
154             ↪ 2], qs.back(), ps[i])) <= 0)
155             qs.pop_back();
156     return qs;
157 }

```

### 3.1.2 凸包

```

1 // 凸包中的点按逆时针方向
2 struct Convex {
3     int n;
4     std::vector<Point> a, upper, lower;
5     void make_shell(const std::vector<Point>& p,
6         ↪ std::vector<Point>& shell) { // p needs to be
7         ↪ sorted.
8         clear(shell); int n = p.size();
9         for (int i = 0, j = 0; i < n; i++, j++) {
10             for (; j >= 2 && sign(det(shell[j-1] - shell[j-2],
11                 ↪ p[i] - shell[j-2])) <= 0; --j)
12                 shell.pop_back();
13             shell.push_back(p[i]);
14         }
15     }
16     void make_convex() {
17         std::sort(a.begin(), a.end());
18         make_shell(a, lower);
19         std::reverse(a.begin(), a.end());
20         make_shell(a, upper);
21         a = lower; a.pop_back();
22         a.insert(a.end(), upper.begin(), upper.end());
23         if ((int)a.size() >= 2) a.pop_back();
24         n = a.size();
25     }
26     void init(const std::vector<Point>& _a) {
27         clear(a); a = _a; n = a.size();
28         make_convex();
29     }
30     void read(int _n) { // Won't make convex.
31         clear(a); n = _n; a.resize(n);
32         for (int i = 0; i < n; i++)
33             a[i].read();
34     }
35     std::pair<DB, int> get_tangent(
36         ↪ const std::vector<Point>& convex, const Point& vec)
37         ↪ {
38         int l = 0, r = (int)convex.size() - 2;
39         assert(r >= 0);

```



```

37     for (; l + 1 < r; ) {
38         int mid = (l + r) / 2;
39         if (sign(det(convex[mid + 1] - convex[mid], vec)) >
40             ↪ 0)
41             r = mid;
42         else l = mid;
43     }
44     return std::max(std::make_pair(det(vec, convex[r]),
45         ↪ r),
46         std::make_pair(det(vec, convex[0]), 0));
47 }
48 int binary_search(Point u, Point v, int l, int r) {
49     int s1 = sign(det(v - u, a[l % n] - u));
50     for (; l + 1 < r; ) {
51         int mid = (l + r) / 2;
52         int smid = sign(det(v - u, a[mid % n] - u));
53         if (smid == s1) l = mid;
54         else r = mid;
55     }
56     return l % n;
57 }
58 // 求凸包上和向量 vec 叉积最大的点, 返回编号, 共
59 ↪ 线的多个切点返回任意一个
60 int get_tangent(Point vec) {
61     std::pair<DB, int> ret = get_tangent(upper, vec);
62     ret.second = (ret.second + (int)lower.size() - 1) % n;
63     ret = std::max(ret, get_tangent(lower, vec));
64     return ret.second;
65 }
66 // 求凸包和直线 u, v 的交点, 如果不相交返回 false,
67 ↪ 如果有则是和 (i, next(i)) 的交点, 交在点上不
68 ↪ 确定返回前后两条边其中之一
69 bool get_intersection(Point u, Point v, int &i0, int
70     ↪ &i1) {
71     int p0 = get_tangent(u - v), p1 = get_tangent(v - u);
72     if (sign(det(v - u, a[p0] - u)) * sign(det(v - u,
73         ↪ a[p1] - u)) <= 0) {
74         if (p0 > p1) std::swap(p0, p1);
75         i0 = binary_search(u, v, p0, p1);
76         i1 = binary_search(u, v, p1, p0 + n);
77         return true;
78     }
79     else return false;
80 }
81 };

```

### 3.1.3 凸包最近点对

```

1 //判断点是否是在多边形内
2 int isPointInPolygon(point p, point *a, int n) {
3     int cnt = 0;
4     for(int i=0; i<n; ++i) {
5         if(OnSegment(p, a[i], a[(i+1)%n])) return 1;
6         double k = cross(a[(i+1)%n]-a[i], p-a[i]);
7         double d1 = a[i].y - p.y;
8         double d2 = a[(i+1)].y - p.y;
9         if(k>0 &&d1<=0 &&d2>0) cnt++;
10        if(k<0 &&d2<=0 &&d1>0) cnt++;
11        //k==0, 点和线段共线的情况不考虑
12    }
13    if(cnt&1)return 1;
14    return 0;
15 }
16 //判断凸包是否相离
17 bool two_getaway_ConvexHull(point *cha, int n1, point
18     ↪ *chb, int m1) {
19     if(n1==1 && m1==1) {
20         if(cha[0]==chb[0])
21             return false;
22     } else if(n1==1 && m1==2) {
23         if(OnSegment(cha[0], chb[0], chb[1]))
24             return false;

```

```

25     } else if(n1==2 && m1==1) {
26         if(OnSegment(chb[0], cha[0], cha[1]))
27             return false;
28     } else if(n1==2 && m1==2) {
29         if(SegmentIntersection(cha[0], cha[1], chb[0],
30             ↪ chb[1]))
31             return false;
32     } else if(n1==2) {
33         for(int i=0; i<n1; ++i)
34             if(isPointInPolygon(cha[i], chb, m1))
35                 return false;
36     } else if(m1==2) {
37         for(int i=0; i<m1; ++i)
38             if(isPointInPolygon(chb[i], cha, n1))
39                 return false;
40     } else {
41         for(int i=0; i<n1; ++i) {
42             for(int j=0; j<m1; ++j) {
43                 if(SegmentIntersection(cha[i],
44                     ↪ cha[(i+1)%n1], chb[j],
45                     ↪ chb[(j+1)%m1]))
46                     return false;
47             }
48         }
49         for(int i=0; i<n1; ++i)
50             if(isPointInPolygon(cha[i], chb, m1))
51                 return false;
52         for(int i=0; i<m1; ++i)
53             if(isPointInPolygon(chb[i], cha, n1))
54                 return false;
55     }
56     return true;
57 }
58 //旋转卡壳求两个凸包最近距离
59 double solve(point *P, point *Q, int n, int m) {
60     if(n==1 && m==1) {
61         return length(P[0] - Q[0]);
62     } else if(n==1 && m==2) {
63         return DistanceToSegment(P[0], Q[0], Q[1]);
64     } else if(n==2 && m==1) {
65         return DistanceToSegment(Q[0], P[0], P[1]);
66     } else if(n==2 && m==2) {
67         return SegmentToSegment(P[0], P[1], Q[0], Q[1]);
68     }
69 }
70 int yminP = 0, ymaxQ = 0;
71 for(int i=0; i<n; ++i) if(P[i].y < P[yminP].y) yminP =
72     ↪ i;
73 for(int i=0; i<m; ++i) if(Q[i].y > Q[ymaxQ].y) ymaxQ =
74     ↪ i;
75 P[n] = P[0];
76 Q[m] = Q[0];
77 double INF2 = 1e100;
78 double arg, ans = INF2;
79 for(int i=0; i<n; ++i) {
80     //当叉积负转正时, 说明点 ymaxQ 就是对踵点
81     while((arg=cross(P[yminP] - P[yminP+1], Q[ymaxQ+1]
82         ↪ - Q[ymaxQ])) < -eps)
83         ymaxQ = (ymaxQ+1)%m;
84     double ret;
85     if(arg > eps) { //卡住第二个凸包上的点。
86         ret = DistanceToSegment(Q[ymaxQ], P[yminP],
87             ↪ P[yminP+1]);
88         ans = min(ans, ret);
89     } else { //arg==0, 卡住第二个凸包的边
90         ret =
91             ↪ SegmentToSegment(P[yminP], P[yminP+1], Q[ymaxQ], Q[
92                 ↪ ymaxQ+1]);
93         ans = min(ans, ret);
94     }
95     yminP = (yminP+1)%n;
96 }
97 return ans;
98 }

```



```

89 double mindis_twotubao(point *P, point *Q, int n, int m){
90     //return min(solve(P, Q, n, m),solve(Q,P,m,n));
91     if(two_getaway_ConvexHull(P,n,Q,m)==true) return
        ↪ min(solve(P, Q, n, m),solve(Q,P,m,n));
92     else return 0.0;
93 }

```

### 3.1.4 三角形的心

```

1 Point inCenter(const Point &A, const Point &B, const Point
    ↪ &C) { // 内心
2     double a = (B - C).len(), b = (C - A).len(), c = (A -
        ↪ B).len(),
3     s = fabs(det(B - A, C - A)),
4     r = s / p;
5     return (A * a + B * b + C * c) / (a + b + c);
6 }
7 Point circumCenter(const Point &a, const Point &b, const
    ↪ Point &c) { // 外心
8     Point bb = b - a, cc = c - a;
9     double db = bb.len(), dc = cc.len2(), d = 2 * det(bb,
        ↪ cc);
10    return a - Point(bb.y * dc - cc.y * db, cc.x * db - bb.x
        ↪ * dc) / d;
11 }
12 Point othroCenter(const Point &a, const Point &b, const
    ↪ Point &c) { // 垂心
13     Point ba = b - a, ca = c - a, bc = b - c;
14     double Y = ba.y * ca.y * bc.y,
15     A = ca.x * ba.y - ba.x * ca.y,
16     x0 = (Y + ca.x * ba.y * b.x - ba.x * ca.y * c.x) /
        ↪ A,
17     y0 = -ba.x * (x0 - c.x) / ba.y + ca.y;
18     return Point(x0, y0);
19 }

```

### 3.1.5 半平面交

```

1 struct Point {
2     int quad() const { return sign(y) == 1 || (sign(y) == 0
        ↪ && sign(x) >= 0);}
3 };
4 struct Line {
5     bool include(const Point &p) const { return sign(det(b -
        ↪ a, p - a)) > 0; }
6     Line push() const{ // 将半平面向外推 eps
7         const double eps = 1e-6;
8         Point delta = (b - a).turn90().norm() * eps;
9         return Line(a - delta, b - delta);
10    }
11 };
12 bool sameDir(const Line &l0, const Line &l1) { return
    ↪ parallel(l0, l1) && sign(dot(l0.b - l0.a, l1.b -
    ↪ l1.a)) == 1; }
13 bool operator < (const Point &a, const Point &b) {
14     if (a.quad() != b.quad()) {
15         return a.quad() < b.quad();
16     } else {
17         return sign(det(a, b)) > 0;
18     }
19 }
20 bool operator < (const Line &l0, const Line &l1) {
21     if (sameDir(l0, l1)) {
22         return l1.include(l0.a);
23     } else {
24         return (l0.b - l0.a) < (l1.b - l1.a);
25     }
26 }
27 bool check(const Line &u, const Line &v, const Line &w) {
    ↪ return w.include(intersect(u, v)); }
28 vector<Point> intersection(vector<Line> &l) {
    ↪ sort(l.begin(), l.end());

```

```

30 deque<Line> q;
31 for (int i = 0; i < (int)l.size(); ++i) {
32     if (i && sameDir(l[i], l[i - 1])) {
33         continue;
34     }
35     while (q.size() > 1 && !check(q[q.size() - 2],
        ↪ q[q.size() - 1], l[i])) q.pop_back();
36     while (q.size() > 1 && !check(q[1], q[0], l[i]))
        ↪ q.pop_front();
37     q.push_back(l[i]);
38 }
39 while (q.size() > 2 && !check(q[q.size() - 2],
    ↪ q[q.size() - 1], q[0])) q.pop_back();
40 while (q.size() > 2 && !check(q[1], q[0], q[q.size() -
    ↪ 1])) q.pop_front();
41 vector<Point> ret;
42 for (int i = 0; i < (int)q.size(); ++i)
    ↪ ret.push_back(intersect(q[i], q[(i + 1) %
    ↪ q.size()]));
43 return ret;
44 }

```

### 3.1.6 最大空凸包

```

1 inline double eq(double x, double y) {
2     return fabs(x-y)<eps;
3 }
4 double xmult(point a, point b, point o) {
5     return (a.x-o.x)*(o.y-b.y)-(a.y-o.y)*(o.x-b.x);
6 }
7 double dist(point a, point b) {
8     return (a.x-b.x)*(a.x-b.x)+(a.y-b.y)*(a.y-b.y);
9 }
10 point o;
11 bool cmp_angle(point a,point b) {
12     if(eq(xmult(a,b,o),0.0)) {
13         return dist(a,o)<dist(b,o);
14     }
15     return xmult(a,o,b)>0;
16 }
17 double empty_convex(point *p, int pn) {
18     double ans=0;
19     for(int i=0; i<pn; i++) {
20         for(int j=0; j<pn; j++) {
21             dp[i][j]=0;
22         }
23     }
24     for(int i=0; i<pn; i++) {
25         int j = i-1;
26         while(j>=0 && eq(xmult(p[i], p[j],
            ↪ o),0.0))j--; //coline
27         bool flag= j==i-1;
28         while(j>=0) {
29             int k = j-1;
30             while(k >= 0 && xmult(p[i],p[k],p[j])>0)k--;
31             double area = fabs(xmult(p[i],p[j],o))/2;
32             if(k >= 0)area+=dp[j][k];
33             if(flag) dp[i][j]=area;
34             ans=max(ans,area);
35             j=k;
36         }
37         if(flag) {
38             for(int j=1; j<i; j++) {
39                 dp[i][j] = max(dp[i][j],dp[i][j-1]);
40             }
41         }
42     }
43     return ans;
44 }
45 double largest_empty_convex(point *p, int pn) {
46     point data[maxn];
47     double ans=0;

```

```

48     for(int i=0; i<pn; i++) {
49         o=p[i];
50         int dn=0;
51         for(int j=0; j<pn; j++) {
52             if(p[j].y>o.y||p[j].y==o.y&&p[j].x>o.x) {
53                 data[dn++]=p[j];
54             }
55         }
56         sort(data, data+dn, cmp_angle);
57         ans=max(ans, empty_convex(data, dn));
58     }
59     return ans;
60 }

```

### 3.1.7 平面最近点对

```

1  double Dis(Point a, Point b) {
2      return sqrt((a.x-b.x)*(a.x-b.x) + (a.y-b.y)*(a.y-b.y));
3  }
4  double Closest_Pair(int left, int right) {
5      double d = INF;
6      if(left == right) return d;
7      if(left + 1 == right)
8          return Dis(p[left], p[right]);
9      int mid = (left+right)>>1;
10     double d1 = Closest_Pair(left, mid);
11     double d2 = Closest_Pair(mid, right);
12     d = min(d1, d2);
13     int k = 0;
14     for(int i = left; i <= right; i++) {
15         if(fabs(p[mid].x - p[i].x) <= d)
16             temp[k++] = p[i];
17     }
18     sort(temp, temp+k, cmpy);
19     for(int i = 0; i < k; i++) {
20         for(int j = i+1; j < k && temp[j].y - temp[i].y < d;
21             ↪ j++) {
22             double d3 = Dis(temp[i], temp[j]);
23             d = min(d, d3);
24         }
25     }
26     return d;
27 }

```

### 3.1.8 最小覆盖圆

```

1  #include<cmath>
2  #include<cstdio>
3  #include<algorithm>
4  using namespace std;
5  const double eps=1e-6;
6  struct couple {
7      double x, y;
8      couple(){}
9      couple(const double &xx, const double &yy) {
10         x = xx; y = yy;
11     }
12 } a[100001];
13 int n;
14 //dis means distance, dis2 means square of it
15 struct circle {
16     double r; couple c;
17 } cir;
18 inline bool inside(const couple &x) {
19     return di2(x, cir.c) < cir.r*cir.r+eps;
20 }
21 inline void p2c(int x, int y) {
22     cir.c.x = (a[x].x+a[y].x)/2;
23     cir.c.y = (a[x].y+a[y].y)/2;
24     cir.r = dis(cir.c, a[x]);
25 }
26 inline void p3c(int i, int j, int k) {

```

```

27     couple x = a[i], y = a[j], z = a[k];
28     cir.r =
29         ↪ sqrt(di2(x,y)*di2(y,z)*di2(z,x))/fabs(x*y+y*z+z*x)/2;
30     couple t1((x-y).x, (y-z).x), t2((x-y).y, (y-z).y),
31         ↪ t3((len(x)-len(y))/2, (len(y)-len(z))/2);
32     cir.c = couple(t3*t2, t1*t3)/(t1*t2);
33 }
34 inline circle mi() {
35     sort(a + 1, a + 1 + n);
36     n = unique(a + 1, a + 1 + n) - a - 1;
37     if(n == 1) {
38         cir.c = a[1];
39         cir.r = 0;
40         return cir;
41     }
42     random_shuffle(a + 1, a + 1 + n);
43     p2c(1, 2);
44     for(int i = 3; i <= n; i++)
45         if(!inside(a[i])) {
46             p2c(1, i);
47             for(int j = 2; j < i; j++)
48                 if(!inside(a[j])) {
49                     p2c(i, j);
50                     for(int k = 1; k < j; k++)
51                         if(!inside(a[k]))
52                             p3c(i, j, k);
53                 }
54             return cir;
55 }

```

### 3.1.9 多边形内部可视

```

1  int C(const Point &P, const Point &A, const Point &Q,
2      ↪ const Point &B) {
3      Point C = GetIntersection(P, A - P, Q, Q - B);
4      return OnLine(Q, C, B);
5  }
6  int Onleft(const Point &a, const Point &b, const Point &
7      ↪ c) {
8      return dcmp(Cross(b - c, a - c)) > 0;
9  }
10 int visible(int x, int y) {
11     int P = (x + n - 1) % n, Q = (x + 1) % n;
12     Point u = p[y] - p[x], v = p[x] - p[P], w = p[x] - p[Q];
13     if (Onleft(p[Q], p[x], p[P])) {
14         return dcmp(Cross(v, u)) > 0 && dcmp(Cross(w, u)) < 0;
15     } else {
16         return !(dcmp(Cross(v, u)) < 0 && dcmp(Cross(w, u)) >
17             ↪ 0);
18     }
19 }
20 int solve(int x, int y) {
21     if (vis[x][y] == dfn) return g[x][y];
22     vis[x][y] = dfn;
23     if (x == y || y == x + 1) return g[x][y] = 1;
24     for (int i = x; i + 1 <= y; i++) {
25         if (C(p[x], p[y], p[i], p[i + 1])) return g[x][y] = 0;
26     }
27     for (int i = x + 1; i < y; i++) {
28         if (OnLine(p[x], p[i], p[y])) {
29             return g[x][y] = solve(x, i) && solve(i, y);
30         }
31     }
32     if (!visible(x, y) || !visible(y, x)) return g[x][y] =
33         ↪ 0;
34     return g[x][y] = 1;
35 }

```

## 3.2 三维

### 3.2.1 三维点类

```

1 // 三维绕轴旋转, 大拇指指向 axis 向量方向, 四指弯曲
  ↳ 方向转 w 弧度
2 Point rotate(const Point& s, const Point& axis, DB w) {
3     DB x = axis.x, y = axis.y, z = axis.z;
4     DB s1 = x * x + y * y + z * z, ss1 = msqrt(s1),
5         cosw = cos(w), sinw = sin(w);
6     DB a[4][4];
7     memset(a, 0, sizeof a);
8     a[3][3] = 1;
9     a[0][0] = ((y * y + z * z) * cosw + x * x) / s1;
10    a[0][1] = x * y * (1 - cosw) / s1 + z * sinw / ss1;
11    a[0][2] = x * z * (1 - cosw) / s1 - y * sinw / ss1;
12    a[1][0] = x * y * (1 - cosw) / s1 - z * sinw / ss1;
13    a[1][1] = ((x * x + z * z) * cosw + y * y) / s1;
14    a[1][2] = y * z * (1 - cosw) / s1 + x * sinw / ss1;
15    a[2][0] = x * z * (1 - cosw) / s1 + y * sinw / ss1;
16    a[2][1] = y * z * (1 - cosw) / s1 - x * sinw / ss1;
17    a[2][2] = ((x * x + y * y) * cosw + z * z) / s1;
18    DB ans[4] = {0, 0, 0, 0}, c[4] = {s.x, s.y, s.z, 1};
19    for (int i = 0; i < 4; ++i)
20        for (int j = 0; j < 4; ++j)
21            ans[i] += a[j][i] * c[j];
22    return Point(ans[0], ans[1], ans[2]);
23 }

```

### 3.2.2 凸包

```

1 __inline P cross(const P& a, const P& b) {
2     return P(
3         a.y * b.z - a.z * b.y,
4         a.z * b.x - a.x * b.z,
5         a.x * b.y - a.y * b.x
6     );
7 }
8 __inline DB mix(const P& a, const P& b, const P& c) {
9     return dot(cross(a, b), c);
10 }
11 __inline DB volume(const P& a, const P& b, const P& c,
12     ↳ const P& d) {
13     return mix(b - a, c - a, d - a);
14 }
15 struct Face {
16     int a, b, c;
17     __inline Face() {}
18     __inline Face(int _a, int _b, int _c):
19         a(_a), b(_b), c(_c) {}
20     __inline DB area() const {
21         return 0.5 * cross(p[b] - p[a], p[c] - p[a]).len();
22     }
23     __inline P normal() const {
24         return cross(p[b] - p[a], p[c] - p[a]).unit();
25     }
26     __inline DB dis(const P& p0) const {
27         return dot(normal(), p0 - p[a]);
28     };
29 std::vector<Face> face, tmp; // Should be O(n).
30 int mark[N][N], Time, n;
31 __inline void add(int v) {
32     ++ Time;
33     clear(tmp);
34     for (int i = 0; i < (int)face.size(); ++i) {
35         int a = face[i].a, b = face[i].b, c = face[i].c;
36         if (sign(volume(p[v], p[a], p[b], p[c])) > 0) {
37             mark[a][b] = mark[b][a] = mark[a][c] =
38             mark[c][a] = mark[b][c] = mark[c][b] = Time;
39         } else {
40             tmp.push_back(face[i]);
41         }
42     }
43 }

```

```

42 }
43 clear(face); face = tmp;
44 for (int i = 0; i < (int)tmp.size(); ++i) {
45     int a = face[i].a, b = face[i].b, c = face[i].c;
46     if (mark[a][b] == Time) face.emplace_back(v, b, a);
47     if (mark[b][c] == Time) face.emplace_back(v, c, b);
48     if (mark[c][a] == Time) face.emplace_back(v, a, c);
49     assert(face.size() < 500u);
50 }
51 }
52 void reorder() {
53     for (int i = 2; i < n; ++i) {
54         P tmp = cross(p[i] - p[0], p[i] - p[1]);
55         if (sign(tmp.len())) {
56             std::swap(p[i], p[2]);
57             for (int j = 3; j < n; ++j)
58                 if (sign(volume(p[0], p[1], p[2], p[j]))) {
59                     std::swap(p[j], p[3]);
60                     return;
61                 }
62         }
63     }
64 }
65 void build_convex() {
66     reorder();
67     clear(face);
68     face.emplace_back(0, 1, 2);
69     face.emplace_back(0, 2, 1);
70     for (int i = 3; i < n; ++i)
71         add(i);
72 }

```

### 3.2.3 最小覆盖球

```

1 const int eps = 1e-8;
2 struct Tpoint {
3     double x, y, z;
4 };
5 int npoint, nouter;
6 Tpoint pt[200000], outer[4], res;
7 double radius, tmp;
8 inline double dist(Tpoint p1, Tpoint p2) {
9     double dx=p1.x-p2.x, dy=p1.y-p2.y, dz=p1.z-p2.z;
10    return ( dx*dx + dy*dy + dz*dz );
11 }
12 inline double dot(Tpoint p1, Tpoint p2) {
13    return p1.x*p2.x + p1.y*p2.y + p1.z*p2.z;
14 }
15 void ball() {
16     Tpoint q[3]; double m[3][3], sol[3], L[3], det;
17     int i, j;
18     res.x = res.y = res.z = radius = 0;
19     switch ( nouter ) {
20         case 1: res=outer[0]; break;
21         case 2:
22             res.x=(outer[0].x+outer[1].x)/2;
23             res.y=(outer[0].y+outer[1].y)/2;
24             res.z=(outer[0].z+outer[1].z)/2;
25             radius=dist(res, outer[0]);
26             break;
27         case 3:
28             for (i=0; i<2; ++i) {
29                 q[i].x=outer[i+1].x-outer[0].x;
30                 q[i].y=outer[i+1].y-outer[0].y;
31                 q[i].z=outer[i+1].z-outer[0].z;
32             }
33             for (i=0; i<2; ++i) for (j=0; j<2; ++j)
34                 m[i][j]=dot(q[i], q[j])*2;
35             for (i=0; i<2; ++i) sol[i]=dot(q[i], q[i]);
36             if (fabs(det=m[0][0]*m[1][1]-m[0][1]*m[1][0])<eps)
37                 return;
38             L[0]=(sol[0]*m[1][1]-sol[1]*m[0][1])/det;

```

```

39     L[i]=(sol[1]*m[0][0]-sol[0]*m[1][0])/det;
40     res.x=outer[0].x+q[0].x*L[0]+q[1].x*L[1];
41     res.y=outer[0].y+q[0].y*L[0]+q[1].y*L[1];
42     res.z=outer[0].z+q[0].z*L[0]+q[1].z*L[1];
43     radius=dist(res, outer[0]);
44     break;
45 case 4:
46     for (i=0; i<3; ++i) {
47         q[i].x=outer[i+1].x-outer[0].x;
48         q[i].y=outer[i+1].y-outer[0].y;
49         q[i].z=outer[i+1].z-outer[0].z;
50         sol[i]=dot(q[i], q[i]);
51     }
52     for (i=0; i<3; ++i)
53         for (j=0; j<3; ++j) m[i][j]=dot(q[i], q[j])*2;
54     det= m[0][0]*m[1][1]*m[2][2]
55         + m[0][1]*m[1][2]*m[2][0]
56         + m[0][2]*m[1][0]*m[2][1]
57         - m[0][2]*m[1][1]*m[2][0]
58         - m[0][1]*m[1][0]*m[2][2]
59         - m[0][0]*m[1][2]*m[2][1];
60     if ( fabs(det)<eps ) return;
61     for (j=0; j<3; ++j) {
62         for (i=0; i<3; ++i) m[i][j]=sol[i];
63         L[j]=( m[0][0]*m[1][1]*m[2][2]
64             + m[0][1]*m[1][2]*m[2][0]
65             + m[0][2]*m[1][0]*m[2][1]
66             - m[0][2]*m[1][1]*m[2][0]
67             - m[0][1]*m[1][0]*m[2][2]
68             - m[0][0]*m[1][2]*m[2][1]
69             ) / det;
70         for (i=0; i<3; ++i)
71             m[i][j]=dot(q[i], q[j])*2;
72     }
73     res=outer[0];
74     for (i=0; i<3; ++i) {
75         res.x += q[i].x * L[i];
76         res.y += q[i].y * L[i];
77         res.z += q[i].z * L[i];
78     }
79     radius=dist(res, outer[0]);
80 }
81 }
82 void minball(int n) {
83     ball();
84     if ( nouter<4 )
85         for (int i=0; i<n; ++i)
86             if (dist(res, pt[i])-radius>eps) {
87                 outer[nouter]=pt[i];
88                 ++nouter;
89                 minball(i);
90                 --nouter;
91                 if (i>0) {
92                     Tpoint Tt = pt[i];
93                     memmove(&pt[1], &pt[0], sizeof(Tpoint)*i);
94                     pt[0]=Tt;
95                 }
96             }
97 }
98 void solve() {
99     for (int i=0; i<npoint; i++)
100         scanf("%lf%lf%lf", &pt[i].x, &pt[i].y, &pt[i].z);
101     random_shuffle(pt, pt + npoint);
102     radius=-1;
103     for (int i=0; i<npoint; i++){
104         if (dist(res, pt[i])-radius>eps){
105             nouter=1;
106             outer[0]=pt[i];
107             minball(i);
108         }
109     }
110     printf("%.5f\n", sqrt(radius));
111 }

```

```

111 int main(){
112     for( ; cin >> npoint && npoint; )
113         solve();
114     return 0;
115 }

```

## 4. 字符串

### 4.1 AC 自动机

```

1 int newnode() {
2     ++tot;
3     memset(ch[tot], 0, sizeof(ch[tot]));
4     fail[tot] = 0;
5     dep[tot] = 0;
6     par[tot] = 0;
7     return tot;
8 }
9 void insert(char *s, int x) {
10     if(*s == '\0') return;
11     else {
12         int &y = ch[x][*s - 'a'];
13         if(y == 0) {
14             y = newnode();
15             par[y] = x;
16             dep[y] = dep[x] + 1;
17         }
18         insert(s + 1, y);
19     }
20 }
21 void build() {
22     int line[maxn];
23     int f = 0, r = 0;
24     fail[root] = root;
25     for(int i = 0; i < alpha; i++) {
26         if(ch[root][i]) {
27             fail[ch[root][i]] = root;
28             line[r++] = ch[root][i];
29         } else {
30             ch[root][i] = root;
31         }
32     }
33     while(f != r) {
34         int x = line[f++];
35         for(int i = 0; i < alpha; i++) {
36             if(ch[x][i]) {
37                 fail[ch[x][i]] = ch[fail[x]][i];
38                 line[r++] = ch[x][i];
39             } else {
40                 ch[x][i] = ch[fail[x]][i];
41             }
42         }
43     }
44 }

```

### 4.2 后缀数组

```

1 const int MAXN = MAXL * 2 + 1;
2 int a[MAXN], x[MAXN], y[MAXN], c[MAXN], sa[MAXN],
3     ↪ rank[MAXN], height[MAXN];
4 void calc_sa(int n) {
5     int m = alphabet, k = 1;
6     memset(c, 0, sizeof(*c) * (m + 1));
7     for (int i = 1; i <= n; ++i) c[x[i] = a[i]]++;
8     for (int i = 1; i <= m; ++i) c[i] += c[i - 1];
9     for (int i = n; i; --i) sa[c[x[i]]--] = i;
10    for (; k <= n; k <= 1) {
11        int tot = k;
12        for (int i = n - k + 1; i <= n; ++i) y[i - n + k] = i;
13        for (int i = 1; i <= n; ++i)
14            if (sa[i] > k) y[++tot] = sa[i] - k;
15        memset(c, 0, sizeof(*c) * (m + 1));

```

```

15     for (int i = 1; i <= n; ++i) c[x[i]]++;
16     for (int i = 1; i <= m; ++i) c[i] += c[i - 1];
17     for (int i = n; i; --i) sa[c[x[y[i]]]--] = y[i];
18     for (int i = 1; i <= n; ++i) y[i] = x[i];
19     tot = 1; x[sa[1]] = 1;
20     for (int i = 2; i <= n; ++i) {
21         if (max(sa[i], sa[i - 1]] + k > n || y[sa[i]] !=
            ↳ y[sa[i - 1]] || y[sa[i] + k] != y[sa[i - 1] +
            ↳ k]) ++tot;
22         x[sa[i]] = tot;
23     }
24     if (tot == n) break; else m = tot;
25 }
26 }
27 void calc_height(int n) {
28     for (int i = 1; i <= n; ++i) rank[sa[i]] = i;
29     for (int i = 1; i <= n; ++i) {
30         height[rank[i]] = max(0, height[rank[i - 1]] - 1);
31         if (rank[i] == 1) continue;
32         int j = sa[rank[i] - 1];
33         while (max(i, j) + height[rank[i]] <= n && a[i +
            ↳ height[rank[i]]] == a[j + height[rank[i]]])
            ↳ ++height[rank[i]];
34     }
35 }

```

### 4.3 后缀自动机

```

1 static const int MAXL = MAXN * 2; // MAXN is original
  ↳ length
2 static const int alphabet = 26; // sometimes need
  ↳ changing
3 int l, last, cnt, trans[MAXL][alphabet], par[MAXL],
  ↳ sum[MAXL], seq[MAXL], mxl[MAXL], size[MAXL]; // mxl
  ↳ is maxlength, size is the size of right
4 char str[MAXL];
5 inline void init() {
6     l = strlen(str + 1); cnt = last = 1;
7     for (int i = 0; i <= l * 2; ++i) memset(trans[i], 0,
            ↳ sizeof(trans[i]));
8     memset(par, 0, sizeof(*par) * (l * 2 + 1));
9     memset(mxl, 0, sizeof(*mxl) * (l * 2 + 1));
10    memset(size, 0, sizeof(*size) * (l * 2 + 1));
11 }
12 inline void extend(int pos, int c) {
13     int p = last, np = last = ++cnt;
14     mxl[np] = mxl[p] + 1; size[np] = 1;
15     for (; p && !trans[p][c]; p = par[p]) trans[p][c] = np;
16     if (!p) par[np] = 1;
17     else {
18         int q = trans[p][c];
19         if (mxl[p] + 1 == mxl[q]) par[np] = q;
20         else {
21             int nq = ++cnt;
22             mxl[nq] = mxl[p] + 1;
23             memcpy(trans[nq], trans[q], sizeof(trans[nq]));
24             par[nq] = par[q];
25             par[np] = par[q] = nq;
26             for (; trans[p][c] == q; p = par[p]) trans[p][c] =
                ↳ nq;
27         }
28     }
29 }
30 inline void buildsam() {
31     for (int i = 1; i <= l; ++i) extend(i, str[i] - 'a');
32     memset(sum, 0, sizeof(*sum) * (l * 2 + 1));
33     for (int i = 1; i <= cnt; ++i) sum[mxl[i]]++;
34     for (int i = 1; i <= l; ++i) sum[i] += sum[i - 1];
35     for (int i = cnt; i; --i) seq[sum[mxl[i]]--] = i;
36     for (int i = cnt; i; --i) size[par[seq[i]]] +=
        ↳ size[seq[i]];
37 }

```

### 4.4 广义后缀自动机

```

1 inline void add_node(int x, int &last) {
2     int lastnode = last;
3     if (c[lastnode][x]) {
4         int nownode = c[lastnode][x];
5         if (l[nownode] == l[lastnode] + 1) last = nownode;
6         else {
7             int auxnode = ++cnt; l[auxnode] = l[lastnode] + 1;
8             for (int i = 0; i < alphabet; ++i) c[auxnode][i] =
                ↳ c[nownode][i];
9             par[auxnode] = par[nownode]; par[nownode] = auxnode;
10            for (; lastnode && c[lastnode][x] == nownode;
                ↳ lastnode = par[lastnode]) {
11                c[lastnode][x] = auxnode;
12            }
13            last = auxnode;
14        }
15    } else {
16        int newnode = ++cnt; l[newnode] = l[lastnode] + 1;
17        for (; lastnode && !c[lastnode][x]; lastnode =
            ↳ par[lastnode]) c[lastnode][x] = newnode;
18        if (!lastnode) par[newnode] = 1;
19        else {
20            int nownode = c[lastnode][x];
21            if (l[lastnode] + 1 == l[nownode]) par[newnode] =
                ↳ nownode;
22            else {
23                int auxnode = ++cnt; l[auxnode] = l[lastnode] + 1;
24                for (int i = 0; i < alphabet; ++i) c[auxnode][i] =
                    ↳ c[nownode][i];
25                par[auxnode] = par[nownode]; par[nownode] =
                    ↳ par[newnode] = auxnode;
26                for (; lastnode && c[lastnode][x] == nownode;
                    ↳ lastnode = par[lastnode]) {
27                    c[lastnode][x] = auxnode;
28                }
29            }
30        }
31        last = newnode;
32    }
33 }

```

### 4.5 manacher

```

1 void Manacher(std::string s, int p[]) {
2     string t = "$#";
3     for (int i = 0; i < s.size(); i++) {
4         t += s[i];
5         t += "#";
6     }
7     std::vector<int> p(t.size(), 0);
8     int mx = 0, id = 0;
9     for (int i = 1; i < t.size(); i++) {
10        p[i] = mx > i ? min(p[2 * id - i], mx - i) : 1;
11        while (t[i + p[i]] == t[i - p[i]]) ++p[i];
12        if (mx < i + p[i]) {
13            mx = i + p[i];
14            id = i;
15        }
16    }
17 }

```

### 4.6 回文自动机

```

1 int nT, nStr, last, c[MAXT][26], fail[MAXT], r[MAXN],
  ↳ l[MAXN], s[MAXN];
2 int allocate(int len) {
3     l[nT] = len;
4     r[nT] = 0;

```

```

5   fail[nT] = 0;
6   memset(c[nT], 0, sizeof(c[nT]));
7   return nT++;
8 }
9 void init() {
10  nT = nStr = 0;
11  int newE = allocate(0);
12  int newO = allocate(-1);
13  last = newE;
14  fail[newE] = newO;
15  fail[newO] = newE;
16  s[0] = -1;
17 }
18 void add(int x) {
19  s[++nStr] = x;
20  int now = last;
21  while (s[nStr - 1[now] - 1] != s[nStr]) now = fail[now];
22  if (!c[now][x]) {
23      int newnode = allocate(1[now] + 2), &newfail =
24          ↪ fail[newnode];
25      newfail = fail[now];
26      while (s[nStr - 1[newfail] - 1] != s[nStr]) newfail =
27          ↪ fail[newfail];
28      newfail = c[newfail][x];
29      c[now][x] = newnode;
30  }
31  last = c[now][x];
32  r[last]++;
33 }
34 void count() {
35     for (int i = nT - 1; i >= 0; i--) {
36         r[fail[i]] += r[i];
37     }
38 }

```

## 4.7 循环串的最小表示

```

1 // n 必须是 2 的次幂
2 void fft(Complex a[], int n, int f) {
3     for (int i = 0; i < n; ++i)
4         if (R[i] < i) swap(a[i], a[R[i]]);
5     for (int i = 1, h = 0; i < n; i <= 1, h++) {
6         Complex wn = Complex(cos(pi / i), f * sin(pi / i));
7         Complex w = Complex(1, 0);
8         for (int k = 0; k < i; ++k, w = w * wn) tmp[k] = w;
9         for (int p = i < 1, j = 0; j < n; j += p) {
10             for (int k = 0; k < i; ++k) {
11                 Complex x = a[j + k], y = a[j + k + i] * tmp[k];
12                 a[j + k] = x + y; a[j + k + i] = x - y;
13             }
14         }
15     }
16 }

```

# 5. 数据结构

## 5.1 可并堆

```

1 int merge(int x, int y) {
2     //p[i] 结点 i 的权值, 这里是维护大根堆
3     //a[i] 在 i 的子树中, i 到右叶子结点的最远距离.
4     if (!x) return y;
5     if (!y) return x;
6     if (p[x] < p[y]) std::swap(x, y);
7     r[x] = merge(r[x], y);
8     if (r[x]) fa[r[x]] = x;
9     if (d[l[x]] < d[r[x]]) std::swap(l[x], r[x]); //调整树
10    ↪ 的结构, 使其满足左偏性质
11    d[x] = d[r[x]] + 1;
12    return x;
13 }

```

## 5.2 KD-Tree

```

1 long long norm(const long long &x) {
2     return std::abs(x);
3     return x * x;
4 }
5 struct Point {
6     int x, y, id;
7     const int& operator [] (int index) const {
8         if (index == 0) {
9             return x;
10        } else {
11            return y;
12        }
13    }
14    friend long long dist(const Point &a, const Point &b)
15    ↪ {
16        long long result = 0;
17        for (int i = 0; i < 2; ++i) {
18            result += norm(a[i] - b[i]);
19        }
20        return result;
21    }
22 } point[N];
23 struct Rectangle {
24     int min[2], max[2];
25     Rectangle() {
26         min[0] = min[1] = INT_MAX; // sometimes int is
27         ↪ not enough
28         max[0] = max[1] = INT_MIN;
29     }
30     void add(const Point &p) {
31         for (int i = 0; i < 2; ++i) {
32             min[i] = std::min(min[i], p[i]);
33             max[i] = std::max(max[i], p[i]);
34         }
35     }
36     long long dist(const Point &p) {
37         long long result = 0;
38         for (int i = 0; i < 2; ++i) {
39             result += norm(std::min(std::max(p[i],
40             ↪ min[i]), max[i]) - p[i]);
41             result += std::max(norm(max[i] - p[i]),
42             ↪ norm(min[i] - p[i]));
43         }
44         return result;
45     }
46 };
47 struct Node {
48     Point separator;
49     Rectangle rectangle;
50     int child[2];
51     void reset(const Point &p) {
52         separator = p;
53         rectangle = Rectangle();
54         rectangle.add(p);
55         child[0] = child[1] = 0;
56     }
57 } tree[N << 1];
58 int size, pivot;
59 bool compare(const Point &a, const Point &b) {
60     if (a[pivot] != b[pivot]) {
61         return a[pivot] < b[pivot];
62     }
63     return a.id < b.id;
64 }
65 // 左閉右開: build(1, n + 1)
66 int build(int l, int r, int type = 1) {
67     pivot = type;
68     if (l >= r) {
69         return 0;
70     }

```



```

66     }
67     int x = ++size;
68     int mid = l + r >> 1;
69     std::nth_element(point + l, point + mid, point + r,
70         ⇨ compare);
71     tree[x].reset(point[mid]);
72     for (int i = l; i < r; ++i) {
73         tree[x].rectangle.add(point[i]);
74     }
75     tree[x].child[0] = build(l, mid, type ^ 1);
76     tree[x].child[1] = build(mid + 1, r, type ^ 1);
77     return x;
78 }
79 int insert(int x, const Point &p, int type = 1) {
80     pivot = type;
81     if (x == 0) {
82         tree[++size].reset(p);
83         return size;
84     }
85     tree[x].rectangle.add(p);
86     if (compare(p, tree[x].separator)) {
87         tree[x].child[0] = insert(tree[x].child[0], p,
88             ⇨ type ^ 1);
89     } else {
90         tree[x].child[1] = insert(tree[x].child[1], p,
91             ⇨ type ^ 1);
92     }
93     return x;
94 }
95 // For minimum distance
96 // For maximum: 下面递归 query 时 0, 1 换顺序;< and
97 ⇨ >; min and max
98 void query(int x, const Point &p, std::pair<long long,
99 ⇨ int> &answer, int type = 1) {
100     pivot = type;
101     if (x == 0 || tree[x].rectangle.dist(p) >
102         ⇨ answer.first) {
103         return;
104     }
105     answer = std::min(answer,
106         std::make_pair(dist(tree[x].separator, p),
107             ⇨ tree[x].separator.id));
108     if (compare(p, tree[x].separator)) {
109         query(tree[x].child[0], p, answer, type ^ 1);
110         query(tree[x].child[1], p, answer, type ^ 1);
111     } else {
112         query(tree[x].child[1], p, answer, type ^ 1);
113         query(tree[x].child[0], p, answer, type ^ 1);
114     }
115 }
116 std::priority_queue<std::pair<long long, int> > answer;
117 void query(int x, const Point &p, int k, int type = 1) {
118     pivot = type;
119     if (x == 0 || (int)answer.size() == k &&
120         ⇨ tree[x].rectangle.dist(p) > answer.top().first) {
121         return;
122     }
123     answer.push(std::make_pair(dist(tree[x].separator, p),
124         ⇨ tree[x].separator.id));
125     if ((int)answer.size() > k) {
126         answer.pop();
127     }
128     if (compare(p, tree[x].separator)) {
129         query(tree[x].child[0], p, k, type ^ 1);
130         query(tree[x].child[1], p, k, type ^ 1);
131     } else {
132         query(tree[x].child[1], p, k, type ^ 1);
133         query(tree[x].child[0], p, k, type ^ 1);
134     }
135 }

```

## 5.3 Treap

```

1 struct Node{
2     int mn, key, size, tag;
3     bool rev;
4     Node* ch[2];
5     Node(int mn, int key, int size): mn(mn), key(key),
6         ⇨ size(size), rev(0), tag(0){}
7     void downtag();
8     Node* update(){
9         mn = min(ch[0] -> mn, min(key, ch[1] -> mn));
10        size = ch[0] -> size + 1 + ch[1] -> size;
11        return this;
12    }
13 };
14 typedef pair<Node*, Node*> Pair;
15 Node *null, *root;
16 void Node::downtag(){
17     if(rev){
18         for(int i = 0; i < 2; i++)
19             if(ch[i] != null){
20                 ch[i] -> rev ^= 1;
21                 swap(ch[i] -> ch[0], ch[i] -> ch[1]);
22             }
23         rev = 0;
24     }
25     if(tag){
26         for(int i = 0; i < 2; i++)
27             if(ch[i] != null){
28                 ch[i] -> key += tag;
29                 ch[i] -> mn += tag;
30                 ch[i] -> tag += tag;
31             }
32         tag = 0;
33     }
34 }
35 int r(){
36     static int s = 3023192386;
37     return (s += (s << 3) + 1) & (~0u >> 1);
38 }
39 bool random(int x, int y){
40     return r() % (x + y) < x;
41 }
42 Node* merge(Node *p, Node *q){
43     if(p == null) return q;
44     if(q == null) return p;
45     p -> downtag();
46     q -> downtag();
47     if(random(p -> size, q -> size)){
48         p -> ch[1] = merge(p -> ch[1], q);
49         return p -> update();
50     }else{
51         q -> ch[0] = merge(p, q -> ch[0]);
52         return q -> update();
53     }
54 }
55 Pair split(Node *x, int n){
56     if(x == null) return make_pair(null, null);
57     x -> downtag();
58     if(n <= x -> ch[0] -> size){
59         Pair ret = split(x -> ch[0], n);
60         x -> ch[0] = ret.second;
61         return make_pair(ret.first, x -> update());
62     }
63     Pair ret = split(x -> ch[1], n - x -> ch[0] -> size -
64         ⇨ 1);
65     x -> ch[1] = ret.first;
66     return make_pair(x -> update(), ret.second);
67 }
68 pair<Node*, Pair> get_segment(int l, int r){
69     Pair ret = split(root, l - 1);
70     return make_pair(ret.first, split(ret.second, r - l +
71         ⇨ 1));

```



```

69 }
70 int main(){
71     null = new Node(INF, INF, 0);
72     null -> ch[0] = null -> ch[1] = null;
73     root = null;
74 }

```

## 5.4 Splay

```

1  template<class T>void checkmin(T &x,T y) {
2      if(y < x) x = y;
3  }
4  struct Node {
5      Node *c[2], *fa;
6      int size, rev;
7      LL val, add, min;
8      Node *init(LL v) {
9          val = min = v;
10         add = rev = 0;
11         c[0] = c[1] = fa = NULL;
12         size = 1;
13         return this;
14     }
15     void rvs() {
16         std::swap(c[0], c[1]);
17         rev ^= 1;
18     }
19     void inc(LL x) {
20         val += x;
21         add += x;
22         min += x;
23     }
24     void pushdown() {
25         if(rev) {
26             if(c[0]) c[0]->rvs();
27             if(c[1]) c[1]->rvs();
28             rev = 0;
29         }
30         if(add) {
31             if(c[0]) c[0]->inc(add);
32             if(c[1]) c[1]->inc(add);
33             add = 0;
34         }
35     }
36     void update() {
37         min = val;
38         if(c[0]) checkmin(min, c[0]->min);
39         if(c[1]) checkmin(min, c[1]->min);
40         size = 1;
41         if(c[0]) size += c[0]->size;
42         if(c[1]) size += c[1]->size;
43     }
44 } *root;
45 Node* newnode(LL x) {
46     static Node pool[maxs], *p = pool;
47     return (++p)->init(x);
48 }
49 void setc(Node *x,int t,Node *y) {
50     x->c[t] = y;
51     if(y) y->fa = x;
52 }
53 Node *find(int k) {
54     Node *now = root;
55     while(true) {
56         now->pushdown();
57         int t = (now->c[0] ? now->c[0]->size : 0) + 1;
58         if(t == k) break;
59         if(t > k) now = now->c[0];
60         else now = now->c[1], k -= t;
61     }
62     return now;
63 }
64 void rotate(Node *x,Node* &k) {

```

```

65     Node *y = x->fa, *z = y->fa;
66     if(y != k) z->c[z->c[1] == y] = x;
67     else k = x;
68     x->fa = z;
69     int i = (y->c[1] == x);
70     setc(y, i, x->c[i ^ 1]);
71     setc(x, i ^ 1, y);
72     y->update(), x->update();
73 }
74 void spaly(Node *x,Node* &k) {
75     static Node *st[maxs];
76     int top = 0;
77     Node *y, *z;
78     y = x;
79     while(y != k) st[++top] = y, y = y->fa;
80     st[++top] = y;
81     while(top) st[top]->pushdown(), top--;
82     while(x != k) {
83         y = x->fa, z = y->fa;
84         if(y != k) {
85             if((y == z->c[1]) ^ (x == y->c[1])) rotate(x, k);
86             else rotate(y, k);
87         }
88         rotate(x, k);
89     }
90 }
91 Node *subtree(int l,int r) {
92     assert((++l) <= (++r));
93     spaly(find(l - 1), root);
94     spaly(find(r + 1), root->c[1]);
95     return root->c[1]->c[0];
96 }
97 void ins(int pos,int v) {
98     pos++;
99     spaly(find(pos), root);
100    spaly(find(pos + 1), root->c[1]);
101    setc(root->c[1], 0, newnode(v));
102    root->c[1]->update();
103    root->update();
104 }
105 void del(int pos) {
106     pos++;
107     spaly(find(pos - 1), root);
108     spaly(find(pos + 1), root->c[1]);
109     root->c[1]->c[0] = NULL;
110     root->c[1]->update();
111     root->update();
112 }
113 void init() {
114     root = newnode(0);
115     setc(root, 1, newnode(0));
116     root->update();
117 }

```

## 5.5 Link cut Tree

```

1  inline void reverse(int x) {
2      tr[x].rev ^= 1; swap(tr[x].c[0], tr[x].c[1]);
3  }
4  inline void rotate(int x, int k) {
5      int y = tr[x].fa, z = tr[y].fa;
6      tr[x].fa = z; tr[z].c[tr[z].c[1] == y] = x;
7      tr[tr[x].c[k ^ 1]].fa = y; tr[y].c[k] = tr[x].c[k ^
8      ↪ 1];
9      tr[x].c[k ^ 1] = y; tr[y].fa = x;
10 }
11 inline void splay(int x, int w) {
12     int z = x; pushdown(x);
13     while (tr[x].fa != w) {
14         int y = tr[x].fa; z = tr[y].fa;
15         if (z == w) {

```

```

16     rotate(x, tr[y].c[1] == x);
17     update(y); update(x);
18 } else {
19     pushdown(z); pushdown(y); pushdown(x);
20     int t1 = tr[y].c[1] == x, t2 = tr[z].c[1] == y;
21     if (t1 == t2) rotate(y, t2), rotate(x, t1);
22     else rotate(x, t1), rotate(x, t2);
23     update(z); update(y); update(x);
24 }
25 }
26 update(x);
27 if (x != z) par[x] = par[z], par[z] = 0;
28 }
29 inline void access(int x) {
30     for (int y = 0; x; y = x, x = par[x]) {
31         splay(x, 0);
32         if (tr[x].c[1]) par[tr[x].c[1]] = x, tr[tr[x].c[1]].fa
            ↪ = 0;
33         tr[x].c[1] = y; par[y] = 0; tr[y].fa = x; update(x);
34     }
35 }
36 inline void makeroot(int x) {
37     access(x); splay(x, 0); reverse(x);
38 }
39 inline void link(int x, int y) {
40     makeroot(x); par[x] = y;
41 }
42 inline void cut(int x, int y) {
43     access(x); splay(y, 0);
44     if (par[y] != x) swap(x, y), access(x), splay(y, 0);
45     par[y] = 0;
46 }
47 inline void split(int x, int y) { // x will be the root
            ↪ of the tree
48     makeroot(y); access(x); splay(x, 0);
49 }

```

```

35 } else {
36     cnt[val[u]]++;
37     used[u] = true;
38 }
39 now += (cnt[val[u]] > 0);
40 }
41 void move(int &x, int y, int z) {
42     int fwd = y;
43     rev(getlca(x, z));
44     rev(getlca(y, z));
45     while(x != y) {
46         if(dep[x] < dep[y]) std::swap(x, y);
47         rev(x), x = fa[x][0];
48     }
49     x = fwd;
50 }
51 void solve() {
52     std::sort(query + 1, query + m + 1);
53     int L = 1, R = 1;
54     rev(1);
55     for(int i = 1; i <= m; i++) {
56         int l = query[i].u;
57         int r = query[i].v;
58         move(L, l, R);
59         move(R, r, L);
60         ans[query[i].t] = now;
61     }
62 }

```

## 5.6 树上莫队

```

1 void dfs(int u) {
2     dep[u] = dep[fa[u][0]] + 1;
3     for(int i = 1; i < logn; i++)
4         fa[u][i] = fa[fa[u][i - 1]][i - 1];
5     stk.push(u);
6     for(int i = 0; i < vec[u].size(); i++) {
7         int v = vec[u][i];
8         if(v == fa[u][0]) continue;
9         fa[v][0] = u, dfs(v);
10        size[u] += size[v];
11        if(size[u] >= bufsize) {
12            ++bcnt;
13            while(stk.top() != u) {
14                block[stk.top()] = bcnt;
15                stk.pop();
16            }
17            size[u] = 0;
18        }
19    }
20    size[u]++;
21 }
22 void prework() {
23     dfs(1);
24     ++bcnt;
25     while(!stk.empty()) {
26         block[stk.top()] = bcnt;
27         stk.pop();
28     }
29 }
30 void rev(int u) {
31     now -= (cnt[val[u]] > 0);
32     if(used[u]) {
33         cnt[val[u]]--;
34         used[u] = false;

```

## 5.7 CDQ 分治

```

1 struct Node {
2     int x, y, z, idx;
3     friend bool operator == (const Node &a, const Node &b) {
4         return a.x == b.x && a.y == b.y && a.z == b.z;
5     }
6     friend bool operator < (const Node &a, const Node &b) {
7         return a.y < b.y;
8     }
9 } triple[maxn];
10 bool cmpx(const Node &a, const Node &b) {
11     if(a.x != b.x) return a.x < b.x;
12     if(a.y != b.y) return a.y < b.y;
13     return a.z < b.z;
14 }
15 void solve(int l, int r) {
16     if(l == r) return;
17     int mid = (l + r) >> 1;
18     solve(l, mid);
19     static std::pair<Node, int> Lt[maxn], Rt[maxn];
20     int Ls = 0, Rs = 0;
21     for(int i = l; i <= mid; i++)
22         Lt[++Ls] = std::make_pair(triple[i], i);
23     for(int i = mid + 1; i <= r; i++)
24         Rt[++Rs] = std::make_pair(triple[i], i);
25     int pos = 1;
26     std::sort(Lt + 1, Lt + Ls + 1);
27     std::sort(Rt + 1, Rt + Rs + 1);
28     backup.clear();
29     for(int i = 1; i <= Rs; i++) {
30         while(pos <= Ls && !Rt[i].first < Lt[pos].first) {
31             insert(Lt[pos].first.z, 1);
32             pos++;
33         }
34         f[Rt[i].second] += query(Rt[i].first.z);
35     }
36     for(int i = 0; i < backup.size(); i++) pre[backup[i]] =
            ↪ 0;
37     solve(mid + 1, r);
38 }

```

## 5.8 整体二分

```

1 void solve(int l, int r, std::vector<int> q) {
2     if(l == r || q.empty()) {
3         for(int i = 0; i < q.size(); i++) {
4             ans[q[i]] = 1;
5         }
6     } else {
7         int mid = (l + r) >> 1;
8         backup.clear();
9         for(int i = 1; i <= mid; i++) {
10             Event e = event[i];
11             if(e.l <= e.r) {
12                 add(e.l, e.v);
13                 add(e.r + 1, -e.v);
14             } else {
15                 add(1, e.v);
16                 add(e.r + 1, -e.v);
17                 add(e.l, e.v);
18             }
19         }
20         std::vector<int> qL, qR;
21         for(int i = 0; i < q.size(); i++) {
22             LL val = 0;
23             for(int j = 0; j < vec[q[i]].size(); j++) {
24                 val += count(vec[q[i]][j]);
25                 if(val >= p[q[i]]) break;
26             }
27             if(cnt[q[i]] + val >= p[q[i]]) {
28                 qL.push_back(q[i]);
29             } else {
30                 cnt[q[i]] += val;
31                 qR.push_back(q[i]);
32             }
33         }
34         for(int i = 0; i < backup.size(); i++) sum[backup[i]]
35             ↪ = 0;
36         solve(l, mid, qL);
37         solve(mid + 1, r, qR);
38     }
39 }

```

```

28 }
29 }
30 bool solve() {
31     for(int i = 1; i <= 2 * n; i++)
32         if(!dfn[i]) tarjan(i);
33     bool ans = true;
34     for(int i = 1; i <= n; i++)
35         if(block[2 * i] == block[2 * i - 1]) {
36             ans = false;
37             break;
38         }
39     return ans;
40 }

```

## 6.2 KM

```

1 struct KM {
2     // Truly  $O(n^3)$ 
3     // 邻接矩阵, 不能连的边设为 -INF, 求最小权匹配时
4     ↪ 边权取负, 但不能连的还是 -INF, 使用时先对 1
5     ↪  $\rightarrow n$  调用 hungary(), 再 get_ans() 求值
6
7     int w[N][N];
8     int lx[N], ly[N], match[N], way[N], slack[N];
9     bool used[N];
10    void init() {
11        for (int i = 1; i <= n; i++) {
12            match[i] = 0;
13            lx[i] = 0;
14            ly[i] = 0;
15            way[i] = 0;
16        }
17    }
18    void hungary(int x) {
19        match[0] = x;
20        int j0 = 0;
21        for (int j = 0; j <= n; j++) {
22            slack[j] = INF;
23            used[j] = false;
24        }
25        do {
26            used[j0] = true;
27            int i0 = match[j0], delta = INF, j1 = 0;
28            for (int j = 1; j <= n; j++) {
29                if (used[j] == false) {
30                    int cur = -w[i0][j] - lx[i0] - ly[j];
31                    if (cur < slack[j]) {
32                        slack[j] = cur;
33                        way[j] = j0;
34                    }
35                    if (slack[j] < delta) {
36                        delta = slack[j];
37                        j1 = j;
38                    }
39                }
40            }
41            for (int j = 0; j <= n; j++) {
42                if (used[j]) {
43                    lx[match[j]] += delta;
44                    ly[j] -= delta;
45                }
46                else slack[j] -= delta;
47            }
48            j0 = j1;
49        } while (match[j0] != 0);
50        do {
51            int j1 = way[j0];
52            match[j0] = match[j1];
53            j0 = j1;
54        } while (j0);
55    }
56    int get_ans() {
57        int sum = 0;
58    }
59 }

```

# 6. 图论

## 6.1 2-SAT tarjan

```

1 template<class TAT>void checkmin(TAT &x, TAT y) {
2     if(y < x) x = y;
3 }
4 void tarjan(int u) {
5     dfn[u] = low[u] = ++dt;
6     flag[u] = true;
7     stk.push(u);
8     for(int i = 0; i < vec[u].size(); i++) {
9         int v = vec[u][i];
10        if(!dfn[v]) {
11            tarjan(v);
12            checkmin(low[u], low[v]);
13        }
14        else if(flag[v]) {
15            checkmin(low[u], dfn[v]);
16        }
17    }
18    if(low[u] == dfn[u]) {
19        ++bcnt;
20        while(stk.top() != u) {
21            block[stk.top()] = bcnt;
22            flag[stk.top()] = false;
23            stk.pop();
24        }
25        block[u] = bcnt;
26        flag[u] = false;
27        stk.pop();
28    }
29 }

```

```

55     for(int i = 1; i <= n; i++) {
56         if (w[match[i]][i] == -INF) ; // 无解
57         if (match[i] > 0) sum += w[match[i]][i];
58     }
59     return sum;
60 }
61 } km;

```

### 6.3 点双连通分量

```

1  const bool BCC_VERTEX = 0, BCC_EDGE = 1;
2  struct BCC { // N = NO + MO. Remember to call
    ↳ init(&raw_graph).
3  Graph *g, forest; // g is raw graph ptr.
4  int dfn[N], DFN, low[N];
5  int stack[N], top;
6  int expand_to[N]; // Where edge i is expanded to in
    ↳ expanded graph.
7  // Vertex i expanded to i.
8  int compress_to[N]; // Where vertex i is compressed to.
9  bool vertex_type[N], cut[N], compress_cut[N], branch[M];
10 //std::vector<int> BCC_component[N]; // Cut vertex
    ↳ belongs to none.
11 __inline void init(Graph *raw_graph) {
12     g = raw_graph;
13 }
14 void DFS(int u, int pe) {
15     dfn[u] = low[u] = ++DFN; cut[u] = false;
16     if (!~g->adj[u]) {
17         cut[u] = 1;
18         compress_to[u] = forest.new_node();
19         compress_cut[compress_to[u]] = 1;
20     }
21     for (int e = g->adj[u]; ~e; e = g->nxt[e]) {
22         int v = g->v[e];
23         if ((e ^ pe) > 1 && dfn[v] > 0 && dfn[v] < dfn[u]) {
24             stack[top++] = e;
25             low[u] = std::min(low[u], dfn[v]);
26         }
27         else if (!dfn[v]) {
28             stack[top++] = e; branch[e] = 1;
29             DFS(v, e);
30             low[u] = std::min(low[v], low[u]);
31             if (low[v] >= dfn[u]) {
32                 if (!cut[u]) {
33                     cut[u] = 1;
34                     compress_to[u] = forest.new_node();
35                     compress_cut[compress_to[u]] = 1;
36                 }
37                 int cc = forest.new_node();
38                 forest.bi_ins(compress_to[u], cc);
39                 compress_cut[cc] = 0;
40                 //BCC_component[cc].clear();
41                 do {
42                     int cur_e = stack[--top];
43                     compress_to[expand_to[cur_e]] = cc;
44                     compress_to[expand_to[cur_e^1]] = cc;
45                     if (branch[cur_e]) {
46                         int v = g->v[cur_e];
47                         if (cut[v])
48                             forest.bi_ins(cc, compress_to[v]);
49                         else {
50                             //BCC_component[cc].push_back(v);
51                             compress_to[v] = cc;
52                         }
53                     }
54                 } while (stack[top] != e);
55             }
56         }
57     }
58 }
59 void solve() {
60     forest.init(g->base);

```

```

61     int n = g->n;
62     for (int i = 0; i < g->e; i++) {
63         expand_to[i] = g->new_node();
64     }
65     memset(branch, 0, sizeof(*branch) * g->e);
66     memset(dfn + g->base, 0, sizeof(*dfn) * n); DFN = 0;
67     for (int i = 0; i < n; i++)
68         if (!dfn[i + g->base]) {
69             top = 0;
70             DFS(i + g->base, -1);
71         }
72 }
73 } bcc;
74
75 bcc.init(&raw_graph);
76 bcc.solve();
77 // Do something with bcc.forest ...

```

### 6.4 边双连通分量

```

1  struct BCC {
2  Graph *g, forest;
3  int dfn[N], low[N], stack[N], tot[N], belong[N], vis[N],
    ↳ top, dfs_clock;
4  // tot[] is the size of each BCC, belong[] is the BCC
    ↳ that each node belongs to
5  pair<int, int> ori[M]; // bridge in raw_graph(raw node)
6  bool is_bridge[M];
7  __inline void init(Graph *raw_graph) {
8  g = raw_graph;
9  memset(is_bridge, false, sizeof(*is_bridge) * g->e);
10     memset(vis + g->base, 0, sizeof(*vis) * g->n);
11 }
12 void tarjan(int u, int from) {
13     dfn[u] = low[u] = ++dfs_clock; vis[u] = 1;
14     ↳ stack[++top] = u;
15     for (int p = g->adj[u]; ~p; p = g->nxt[p]) {
16         if ((p ^ 1) == from) continue;
17         int v = g->v[p];
18         if (vis[v]) {
19             if (vis[v] == 1) low[u] = min(low[u], dfn[v]);
20         } else {
21             tarjan(v, p);
22             low[u] = min(low[u], low[v]);
23             if (low[v] > dfn[u]) is_bridge[p / 2] = true;
24         }
25     }
26     if (dfn[u] != low[u]) return;
27     tot[forest.new_node()] = 0;
28     do {
29         belong[stack[top]] = forest.n;
30         vis[stack[top]] = 2;
31         tot[forest.n]++;
32         --top;
33     } while (stack[top + 1] != u);
34 }
35 void solve() {
36     forest.init(g->base);
37     int n = g->n;
38     for (int i = 0; i < n; ++i)
39         if (!vis[i + g->base]) {
40             top = dfs_clock = 0;
41             tarjan(i + g->base, -1);
42         }
43     for (int i = 0; i < g->e / 2; ++i)
44         if (is_bridge[i]) {
45             int e = forest.e;
46             forest.bi_ins(belong[g->v[i * 2]], belong[g->v[i * 2 + 1]], g->w[i * 2]);
47             ori[e] = make_pair(g->v[i * 2 + 1], g->v[i * 2]);

```

```

47     ori[e + 1] = make_pair(g -> v[i * 2], g -> v[i * 2
48         ↪ + 1]);
49     }
50 } bcc;

```

## 6.5 最小树形图

```

1  const int MAXN, INF; // INF >= sum( W_ij )
2  int from[MAXN + 10][MAXN * 2 + 10], n, m, edge[MAXN +
3  ↪ 10][MAXN * 2 + 10];
4  int sel[MAXN * 2 + 10], fa[MAXN * 2 + 10], vis[MAXN * 2 +
5  ↪ 10];
6  int getfa(int x){if(x == fa[x]) return x; return fa[x] =
7  ↪ getfa(fa[x]);}
8  void liuzhu(){ // 1-base: root is 1, answer = (sel[i], i)
9  ↪ for i in [2..n]
10     fa[1] = 1;
11     for(int i = 2; i <= n; ++i){
12         sel[i] = 1; fa[i] = i;
13         for(int j = 1; j <= n; ++j) if(fa[j] != i)
14             if(from[j][i] = i, edge[sel[i]][i] > edge[j][i])
15                 ↪ sel[i] = j;
16     }
17     int limit = n;
18     while(1){
19         int prelimit = limit; memset(vis, 0, sizeof(vis));
20         ↪ vis[1] = 1;
21         for(int i = 2; i <= prelimit; ++i) if(fa[i] == i &&
22             ↪ !vis[i]){
23             int j = i; while(!vis[j]) vis[j] = i, j =
24                 ↪ getfa(sel[j]);
25             if(j == 1 || vis[j] != i) continue; vector<int> C;
26             ↪ int k = j;
27             do C.push_back(k), k = getfa(sel[k]); while(k != j);
28             ++limit;
29             for(int i = 1; i <= n; ++i){
30                 edge[i][limit] = INF, from[i][limit] = limit;
31             }
32             fa[limit] = vis[limit] = limit;
33             for(int i = 0; i < int(C.size()); ++i){
34                 int x = C[i], fa[x] = limit;
35                 for(int j = 1; j <= n; ++j)
36                     if(edge[j][x] != INF && edge[j][limit] >
37                         ↪ edge[j][x] - edge[sel[x]][x]){
38                         edge[j][limit] = edge[j][x] - edge[sel[x]][x];
39                         from[j][limit] = x;
40                     }
41             }
42             for(int j=1; j<=n; ++j) if(getfa(j)==limit)
43                 ↪ edge[j][limit] = INF;
44             sel[limit] = 1;
45             for(int j = 1; j <= n; ++j)
46                 if(edge[sel[limit]][limit] > edge[j][limit])
47                     ↪ sel[limit] = j;
48         }
49         if(prelimit == limit) break;
50     }
51     for(int i = limit; i > 1; --i) sel[from[sel[i]][i]] =
52         ↪ sel[i];
53 }

```

## 6.6 带花树

```

1  vector<int> link[maxn];
2  int n, match[maxn], Queue[maxn], head, tail;
3  int pred[maxn], base[maxn], start, finish, newbase;
4  bool InQueue[maxn], InBlossom[maxn];
5  void push(int u){ Queue[tail++] = u; InQueue[u] = true; }
6  int pop(){ return Queue[head++]; }
7  int FindCommonAncestor(int u, int v){
8  ↪ bool InPath[maxn];

```

```

9  for(int i=0; i<n; i++) InPath[i] = 0;
10 while(true){ u = base[u]; InPath[u] = true; if(u == start)
11     ↪ break; u = pred[match[u]]; }
12 while(true){ v = base[v]; if(InPath[v])
13     ↪ break; v = pred[match[v]]; }
14 return v;
15 }
16 void ResetTrace(int u){
17     int v;
18     while(base[u] != newbase){
19         v = match[u];
20         InBlossom[base[u]] = InBlossom[base[v]] = true;
21         u = pred[v];
22         if(base[u] != newbase) pred[u] = v;
23     }
24 }
25 void BlossomContract(int u, int v){
26     newbase = FindCommonAncestor(u, v);
27     for (int i=0; i<n; i++){
28         InBlossom[i] = 0;
29         ResetTrace(u); ResetTrace(v);
30         if(base[u] != newbase) pred[u] = v;
31         if(base[v] != newbase) pred[v] = u;
32         for(int i=0; i<n; i++){
33             if(InBlossom[base[i]]){
34                 base[i] = newbase;
35                 if(!InQueue[i]) push(i);
36             }
37         }
38     }
39 }
40 bool FindAugmentingPath(int u){
41     bool found = false;
42     for(int i=0; i<n; i++) pred[i] = -1, base[i] = i;
43     for (int i=0; i<n; i++) InQueue[i] = 0;
44     start = u; finish = -1; head = tail = 0; push(start);
45     while(head < tail){
46         int u = pop();
47         for(int i = link[u].size() - 1; i >= 0; i--){
48             int v = link[u][i];
49             if(base[u] != base[v] && match[u] != v)
50                 if(v == start || (match[v] >= 0 && pred[match[v]] >= 0))
51                     BlossomContract(u, v);
52             else if(pred[v] == -1){
53                 pred[v] = u;
54                 if(match[v] >= 0) push(match[v]);
55                 else{ finish = v; return true; }
56             }
57         }
58     }
59     return found;
60 }
61 void AugmentPath(){
62     int u = finish, v = w;
63     while(u >= 0){
64         ↪ v = pred[u]; w = match[v]; match[v] = u; match[u] = v; u = w; }
65 }
66 void FindMaxMatching(){
67     for(int i=0; i<n; i++) match[i] = -1;
68     for(int i=0; i<n; i++) if(match[i] == -1)
69         ↪ if(FindAugmentingPath(i)) AugmentPath();
70 }

```

## 6.7 支配树

```

1  vector<int> prec[N], succ[N];
2  vector<int> ord;
3  int stamp, vis[N];
4  int num[N];
5  int fa[N];
6  void dfs(int u) {
7  ↪ vis[u] = stamp;
8  ↪ num[u] = ord.size();
9  ↪ ord.push_back(u);

```

```

10 for (int i = 0; i < (int)succ[u].size(); ++i) {
11     int v = succ[u][i];
12     if (vis[v] != stamp) {
13         fa[v] = u;
14         dfs(v);
15     }
16 }
17 }
18 int fs[N], mins[N], dom[N], sem[N];
19 int find(int u) {
20     if (u != fs[u]) {
21         int v = fs[u];
22         fs[u] = find(fs[u]);
23         if (mins[v] != -1 && num[sem[mins[v]]] <
24             ↳ num[sem[mins[u]]]) {
25             mins[u] = mins[v];
26         }
27     }
28     return fs[u];
29 }
30 void merge(int u, int v) { fs[u] = v; }
31 vector<int> buf[N];
32 int buf2[N];
33 void mark(int source) {
34     ord.clear();
35     ++stamp;
36     dfs(source);
37     for (int i = 0; i < (int)ord.size(); ++i) {
38         int u = ord[i];
39         fs[u] = u, mins[u] = -1, buf2[u] = -1;
40     }
41     for (int i = (int)ord.size() - 1; i > 0; --i) {
42         int u = ord[i], p = fa[u];
43         sem[u] = p;
44         for (int j = 0; j < (int)prec[u].size(); ++j) {
45             int v = prec[u][j];
46             if (use[v] != stamp) continue;
47             if (num[v] > num[u]) {
48                 find(v); v = sem[mins[v]];
49             }
50             if (num[v] < num[sem[u]]) {
51                 sem[u] = v;
52             }
53         }
54         buf[sem[u]].push_back(u);
55         mins[u] = u;
56         merge(u, p);
57         while (buf[p].size()) {
58             int v = buf[p].back();
59             buf[p].pop_back();
60             find(v);
61             if (sem[v] == sem[mins[v]]) {
62                 dom[v] = sem[v];
63             } else {
64                 buf2[v] = mins[v];
65             }
66         }
67     }
68     dom[ord[0]] = ord[0];
69     for (int i = 0; i < (int)ord.size(); ++i) {
70         int u = ord[i];
71         if (~buf2[u]) {
72             dom[u] = dom[buf2[u]];
73         }
74     }
75 }

```

## 6.8 无向图最小割

```

1 int cost[maxn][maxn], seq[maxn], len[maxn], n, m, pop, ans;
2 bool used[maxn];
3 void Init() {
4     int i, j, a, b, c;

```

```

5     for(i=0;i<n;i++) for(j=0;j<n;j++) cost[i][j]=0;
6     for(i=0;i<m;i++){
7         scanf("%d %d %d",&a,&b,&c); cost[a][b]+=c;
8         ↳ cost[b][a]+=c;
9     }
10    pop=n; for(i=0;i<n;i++) seq[i]=i;
11 }
12 void Work(){
13     ans=inf; int i,j,k,l,mm,sum,pk;
14     while(pop > 1){
15         for(i=1;i<pop;i++) used[seq[i]]=0; used[seq[0]]=1;
16         for(i=1;i<pop;i++) len[seq[i]]=cost[seq[0]][seq[i]];
17         pk=0; mm=-inf; k=-1;
18         for(i=1;i<pop;i++) if(len[seq[i]] > mm){
19             ↳ mm=len[seq[i]]; k=i; }
20         for(i=1;i<pop;i++){
21             used[seq[l=k]]=1;
22             if(i==pop-2) pk=k;
23             if(i==pop-1) break;
24             mm=-inf;
25             for(j=1;j<pop;j++) if(!used[seq[j]])
26                 if((len[seq[j]]+cost[seq[l]][seq[j]]) > mm)
27                     mm=len[seq[j]], k=j;
28         }
29         sum=0;
30         for(i=0;i<pop;i++) if(i != k)
31             ↳ sum+=cost[seq[k]][seq[i]];
32         ans=min(ans,sum);
33         for(i=0;i<pop;i++)
34             cost[seq[k]][seq[i]]=cost[seq[i]][seq[k]]+=cost[seq[pk]][seq[i]];
35         seq[pk]=seq[--pop];
36     }
37     printf("%d\n",ans);
38 }

```

## 6.9 最大团搜索

```

1 const int N = 1000 + 7;
2 vector<vector<bool>> adj;
3 class MaxClique {
4     const vector<vector<bool>> adj;
5     const int n;
6     vector<int> result, cur_res;
7     vector<vector<int>> color_set;
8     const double t_limit; // MAGIC
9     int para, level;
10    vector<pair<int, int>> steps;
11 public:
12     class Vertex {
13     public:
14         int i, d;
15         Vertex(int i, int d = 0) : i(i), d(d) {}
16     };
17     void reorder(vector<Vertex> &p) {
18         for (auto &u : p) {
19             u.d = 0;
20             for (auto v : p) u.d += adj[v.i][u.i];
21         }
22         sort(p.begin(), p.end(), [&](const Vertex &a,
23             ↳ const Vertex &b) { return a.d > b.d; });
24     }
25     // reuse p[i].d to denote the maximum possible clique
26     ↳ for first i vertices.
27     void init_color(vector<Vertex> &p) {
28         int maxd = p[0].d;
29         for (int i = 0; i < p.size(); i++) p[i].d = min(i,
30             ↳ maxd) + 1;
31     }
32     bool bridge(const vector<int> &s, int x) {
33         for (auto v : s) if (adj[v][x]) return true;
34         return false;
35     }

```

```

33 // approximate estimate the p[i].d
34 // Do not care about first mink color class (For better
    ↳ result, we must get some vertex in some color class
    ↳ larger than mink )
35 void color_sort(vector<Vertex> &cur) {
36     int totc = 0, ptr = 0, mink =
        ↳ max((int)result.size() - (int)cur_res.size(),
        ↳ 0);
37     for (int i = 0; i < cur.size(); i++) {
38         int x = cur[i].i, k = 0;
39         while (k < totc && bridge(color_set[k], x))
            ↳ k++;
40         if (k == totc) color_set[totc++].clear();
41         color_set[k].push_back(x);
42         if (k < mink) cur[ptr++].i = x;
43     }
44     if (ptr) cur[ptr - 1].d = 0;
45     for (int i = mink; i < totc; i++) {
46         for (auto v : color_set[i]) {
47             cur[ptr++] = Vertex(v, i + 1);
48         }
49     }
50 }
51 void expand(vector<Vertex> &cur) {
52     steps[level].second = steps[level].second -
        ↳ steps[level].first + steps[level - 1].first;
53     steps[level].first = steps[level - 1].second;
54     while (cur.size()) {
55         if (cur_res.size() + cur.back().d <=
            ↳ result.size()) return ;
56         int x = cur.back().i;
57         cur_res.push_back(x); cur.pop_back();
58         vector<Vertex> remain;
59         for (auto v : cur) {
60             if (adj[v.i][x]) remain.push_back(v.i);
61         }
62         if (remain.size() == 0) {
63             if (cur_res.size() > result.size()) result
                ↳ = cur_res;
64             } else {
65                 // Magic ballance.
66                 if (1. * steps[level].second / ++para < t_limit)
                    ↳ reorder(remain);
67                 color_sort(remain);
68                 steps[level++].second++;
69                 expand(remain);
70                 level--;
71             }
72             cur_res.pop_back();
73         }
74     }
75 public:
76     MaxClique(const vector<vector<bool> > &adj, int n,
        ↳ double tt = 0.025) : adj(_adj), n(n), t_limit(tt)
        ↳ {
77         result.clear();
78         cur_res.clear();
79         color_set.resize(n);
80         steps.resize(n + 1);
81         fill(steps.begin(), steps.end(), make_pair(0, 0));
82         level = 1;
83         para = 0;
84     }
85     vector<int> solve() {
86         vector<Vertex> p;
87         for (int i = 0; i < n; i++)
            ↳ p.push_back(Vertex(i));
88         reorder(p);
89         init_color(p);
90         expand(p);
91         return result;
92     }
93 };

```

## 6.10 斯坦纳树

```

1 void SPFA(int *dist) {
2     static int line[maxn + 5];
3     static bool hash[maxn + 5];
4     int f = 0, r = 0;
5     for(int i = 1; i <= N; i++)
6         if(dist[i] < inf) {
7             line[r] = i;
8             hash[i] = true;
9             r = (r + 1) % (N + 1);
10        }
11    while(f != r) {
12        int t = line[f];
13        hash[t] = false;
14        f = (f + 1) % (N + 1);
15        for(int i = head[t]; i ; i = edge[i].next) {
16            int v = edge[i].v, dt = dist[t] + edge[i].w;
17            if(dt < dist[v]) {
18                dist[v] = dt;
19                if(!hash[v]) {
20                    if(dist[v] < dist[line[f]]) {
21                        f = (f + N) % (N + 1);
22                        line[f] = v;
23                    }
24                    else {
25                        line[r] = v;
26                        r = (r + 1) % (N + 1);
27                    }
28                    hash[v] = true;
29                }
30            }
31        }
32    }
33 }
34 void solve() {
35     for(int i = 1; i <= S; i++) {
36         for(int j = 1; j <= N; j++)
37             for(int k = (i - 1) & i; k ; k = (k - 1) & i)
38                 G[i][j] = std::min(G[i][j], G[k][j] + G[k
                    ↳ ^ i][j]);
39         SPFA(G[i]);
40     }
41 }

```

## 6.11 虚树

```

1 bool cmp(const int lhs, const int rhs) {
2     return dfn[lhs] < dfn[rhs];
3 }
4 void build() {
5     std::sort(h + 1, h + 1 + m, cmp);
6     int top = 0;
7     for (int i = 1; i <= m; i++) {
8         if (!top) father[st[++top]] = h[i]] = 0;
9         else {
10             int p = h[i], lca = LCA(h[i], st[top]);
11             while(d[st[top]] > d[lca]) {
12                 if (d[st[top - 1]] <= d[lca])
13                     father[st[top]] = lca;
14                 top--;
15             }
16             if (st[top] != lca) {
17                 t[++tot] = lca;
18                 father[lca] = st[top];
19                 st[++top] = lca;
20             }
21             father[p] = lca;
22             st[++top] = p;
23         }
24     }
25 }

```



```
24 }
25 }
```

## 6.12 点分治

```
1 template<class TAT>void checkmax(TAT &x,TAT y) {
2     if(x < y) x = y;
3 }
4 template<class TAT>void checkmin(TAT &x,TAT y) {
5     if(y < x) x = y;
6 }
7 void getsize(int u,int fa) {
8     size[u] = 1;
9     smax[u] = 0;
10    for(int i = 0; i < G[u].size(); i++) {
11        int v = G[u][i];
12        if(v == fa || ban[v]) continue;
13        getsize(v, u);
14        size[u] += size[v];
15        checkmax(smax[u], size[v]);
16    }
17 }
18 int getroot(int u,int ts,int fa) {
19     checkmax(smax[u], ts - size[u]);
20     int res = u;
21     for(int i = 0; i < G[u].size(); i++) {
22         int v = G[u][i];
23         if(v == fa || ban[v]) continue;
24         int w = getroot(v, ts, u);
25         if(smax[w] < smax[res]) res = w;
26     }
27     return res;
28 }
29 void solve() {
30     static int line[maxn];
31     static std::vector<int> vec;
32     int f = 0, r = 0;
33     line[r++] = 1;
34     while(f != r) {
35         int u = line[f++];
36         getsize(u, 0);
37         u = getroot(u, size[u], 0);
38         ban[u] = true;
39         vec.clear();
40         for(int i = 0; i < G[u].size(); i++)
41             if(!ban[G[u][i]]) vec.push_back(G[u][i]);
42         for(int i = 0; i < vec.size(); i++)
43             line[r++] = vec[i];
44     }
45 }
```

## 6.13 最小割最大流

```
1 bool BFS() {
2     for(int i = 1; i <= ind; i++) dep[i] = 0;
3     dep[S] = 1, line.push(S);
4     while(!line.empty()) {
5         int now = line.front();
6         line.pop();
7         for(int i = head[now], p; i; i = edge[i].next)
8             if(edge[i].cap && !dep[p = edge[i].v])
9                 dep[p] = dep[now] + 1, line.push(p);
10    }
11    if(dep[T]) {
12        for(int i = 1; i <= ind; i++)
13            cur[i] = head[i];
14        return true;
15    } else return false;
16 }
17 int DFS(int a,int flow) {
18     if(a == T) return flow;
19     int ret = 0;
```

```
20     for(int &i = cur[a], p; i; i = edge[i].next)
21         if(dep[p = edge[i].v] == dep[a] + 1 &&
22             ↳ edge[i].cap) {
23             int ff = DFS(p, std::min(flow, edge[i].cap));
24             flow -= ff, edge[i].cap -= ff;
25             ret += ff, edge[i ^ 1].cap += ff;
26             if(!flow) break;
27         }
28     return ret;
29 }
30 int solve() {
31     int totflow = 0;
32     while(BFS())
33         totflow += DFS(S, INF);
34     return totflow;
35 }
```

## 6.14 最小费用流

```
1 bool SPFA() {
2     static int line[maxv];
3     static bool hash[maxv];
4     register int f = 0, r = 0;
5     for(int i = 1; i <= ind; i++) {
6         dist[i] = inf;
7         from[i] = 0;
8     }
9     dist[S] = 0, line[r] = S, r = (r + 1) % maxv;
10    hash[S] = true;
11    while(f != r) {
12        int x = line[f];
13        line[f] = 0, f = (f + 1) % maxv;
14        hash[x] = false;
15        for(int i = head[x]; i; i = edge[i].next)
16            if(edge[i].cap) {
17                int v = edge[i].v;
18                int w = dist[x] + edge[i].cost;
19                if(w < dist[v]) {
20                    dist[v] = w;
21                    from[v] = i;
22                    if(!hash[v]) {
23                        if(f != r && dist[v] <=
24                            ↳ dist[line[f]])
25                            f = (f - 1 + maxv) % maxv,
26                            ↳ line[f] = v;
27                        else line[r] = v, r = (r + 1) %
28                            ↳ maxv;
29                        hash[v] = true;
30                    }
31                }
32            }
33    }
34    return from[T];
35 }
36 int back(int x,int flow) {
37     if(from[x]) {
38         flow = back(edge[from[x] ^ 1].v, std::min(flow,
39             ↳ edge[from[x]].cap));
40         edge[from[x]].cap -= flow;
41         edge[from[x] ^ 1].cap += flow;
42     }
43     return flow;
44 }
45 int solve() {
46     int mincost = 0, maxflow = 0;
47     while(SPFA()) {
48         int flow = back(T, inf);
49         mincost += dist[T] * flow;
50         maxflow += flow;
51     }
52 }
```

```

48     return mincost;
49 }

```

## 6.15 zkw 费用流

```

1  int S, T, totFlow, totCost;
2  int dis[N], slack[N], visit[N];
3  int modlable () {
4      int delta = INF;
5      for (int i = 1; i <= T; i++) {
6          if (!visit[i] && slack[i] < delta) delta =
            ↪ slack[i];
7          slack[i] = INF;
8      }
9      if (delta == INF) return 1;
10     for (int i = 1; i <= T; i++)
11         if (visit[i]) dis[i] += delta;
12     return 0;
13 }
14 int dfs (int x, int flow) {
15     if (x == T) {
16         totFlow += flow;
17         totCost += flow * (dis[S] - dis[T]);
18         return flow;
19     }
20     visit[x] = 1;
21     int left = flow;
22     for (int i = e.last[x]; ~i; i = e.succ[i])
23         if (e.cap[i] > 0 && !visit[e.other[i]]) {
24             int y = e.other[i];
25             if (dis[y] + e.cost[i] == dis[x]) {
26                 int delta = dfs (y, min (left, e.cap[i]));
27                 e.cap[i] -= delta;
28                 e.cap[i ^ 1] += delta;
29                 left -= delta;
30                 if (!left) { visit[x] = 0; return flow; }
31             } else {
32                 slack[y] = min (slack[y], dis[y] +
                    ↪ e.cost[i] - dis[x]);
33             }
34         }
35     return flow - left;
36 }
37 pair <int, int> minCost () {
38     totFlow = 0; totCost = 0;
39     fill (dis + 1, dis + T + 1, 0);
40     do {
41         do {
42             fill (visit + 1, visit + T + 1, 0);
43         } while (dfs (S, INF));
44     } while (!modlable ());
45     return make_pair (totFlow, totCost);
46 }

```

```

16     int u=q.front();
17     q.pop();
18     for (int i=last[u];i;i=e[i].next)
19         if (e[i].c&&!dis[e[i].to]) {
20             dis[e[i].to]=dis[u]+1;
21             if (e[i].to==t) return 1;
22             q.push(e[i].to);
23         }
24     }
25     return 0;
26 }
27 int dfs(int x,int maxf) {
28     if (x==t||!maxf) return maxf;
29     int ret=0;
30     for (int i=last[x];i;i=e[i].next)
31         if (e[i].c&&dis[e[i].to]==dis[x]+1) {
32             int f=dfs(e[i].to,min(e[i].c,maxf-ret));
33             e[i].c-=f;
34             e[i^1].c+=f;
35             ret+=f;
36             if (ret==maxf) break;
37         }
38     if (!ret) dis[x]=0;
39     return ret;
40 }
41 void dfs(int x) {
42     mark[x]=1;
43     for (int i=last[x];i;i=e[i].next)
44         if (e[i].c&&!mark[e[i].to]) dfs(e[i].to);
45 }
46 void solve(int l,int r) {
47     if (l==r) return;
48     s=a[l];t=a[r];
49     for (int i=2;i<=cnt;i+=2)
50         e[i].c=e[i^1].c=(e[i].c+e[i^1].c)/2;
51     int flow=0;
52     while (bfs()) flow+=dfs(s,inf);
53     memset(mark,0,sizeof(mark));
54     dfs(s);
55     for (int i=1;i<=n;i++)
56         if (mark[i])
57             for (int j=1;j<=n;j++)
58                 if (!mark[j])
59                     ans[i][j]=ans[j][i]=min(ans[i][j],flow);
60     int i=l,j=r;
61     for (int k=l;k<=r;k++)
62         if (mark[a[k]]) tmp[i++]=a[k];
63         else tmp[j--]=a[k];
64     for (int k=1;k<=r;k++)
65         a[k]=tmp[k];
66     solve(l,i-1);
67     solve(j+1,r);
68 }

```

## 6.16 最小割树

```

1  int
    ↪ cnt,n,m,dis[N],last[N],a[N],tmp[N],ans[N][N],s,t,mark[N];
2  struct edge{int to,c,next;}e[N*200];
3  queue <int> q;
4  void addedge(int u,int v,int c) {
5      e[++cnt].to=v;e[cnt].c=c;
6      e[cnt].next=last[u];last[u]=cnt;
7      e[++cnt].to=u;e[cnt].c=c;
8      e[cnt].next=last[v];last[v]=cnt;
9  }
10 bool bfs() {
11     memset(dis,0,sizeof(dis));
12     dis[s]=2;
13     while (!q.empty()) q.pop();
14     q.push(s);
15     while (!q.empty()) {

```

## 6.17 上下界网络流建图

$B(u, v)$  表示边  $(u, v)$  流量的下界,  $C(u, v)$  表示边  $(u, v)$  流量的上界,  $F(u, v)$  表示边  $(u, v)$  的流量。设  $G(u, v) = F(u, v) - B(u, v)$ , 显然有

$$0 \leq G(u, v) \leq C(u, v) - B(u, v)$$

### 6.17.1 无源汇的上下界可行流

建立超级源点  $S^*$  和超级汇点  $T^*$ , 对于原图每条边  $(u, v)$  在新网络中连如下三条边:  $S^* \rightarrow v$ , 容量为  $B(u, v)$ ;  $u \rightarrow T^*$ , 容量为  $B(u, v)$ ;  $u \rightarrow v$ , 容量为  $C(u, v) - B(u, v)$ 。最后求新网络的最大流, 判断从超级源点  $S^*$  出发的边是否都满流即可, 边  $(u, v)$  的最终解中的实际流量为  $G(u, v) + B(u, v)$ 。

### 6.17.2 有源汇的上下界可行流

从汇点  $T$  到源点  $S$  连一条上界为  $\infty$ ，下界为 0 的边。按照无源汇的上下界可行流一样做即可，流量即为  $T \rightarrow S$  边上的流量。

### 6.17.3 有源汇的上下界最大流

1. 在有源汇的上下界可行流中，从汇点  $T$  到源点  $S$  的边改为连一条上界为  $\infty$ ，下界为  $x$  的边。 $x$  满足二分性质，找到最大的  $x$  使得新网络存在无源汇的上下界可行流即为原图的最大流。
2. 从汇点  $T$  到源点  $S$  连一条上界为  $\infty$ ，下界为 0 的边，变成无源汇的网络。按照无源汇的上下界可行流的方法，建立超级源点  $S^*$  和超级汇点  $T^*$ ，求一遍  $S^* \rightarrow T^*$  的最大流，再将汇点  $T$  到源点  $S$  的这条边拆掉，求一次  $S \rightarrow T$  的最大流即可。

### 6.17.4 有源汇的上下界最小流

1. 在有源汇的上下界可行流中，从汇点  $T$  到源点  $S$  的边改为连一条上界为  $x$ ，下界为 0 的边。 $x$  满足二分性质，找到最小的  $x$  使得新网络存在无源汇的上下界可行流即为原图的最小流。
2. 按照无源汇的上下界可行流的方法，建立超级源点  $S^*$  与超级汇点  $T^*$ ，求一遍  $S^* \rightarrow T^*$  的最大流，但是注意这一次不加上汇点  $T$  到源点  $S$  的这条边，即不使之改为无源汇的网络去求解。求完后，再加上那条汇点  $T$  到源点  $S$  上界  $\infty$  的边。因为这条边下界为 0，所以  $S^*, T^*$  无影响，再直接求一次  $S^* \rightarrow T^*$  的最大流。若超级源点  $S^*$  出发的边全部满流，则  $T \rightarrow S$  边上的流量即为原图的最小流，否则无解。

## 7. 其他

### 7.1 Dancing Links

#### 7.1.1 精确覆盖

```

1 #pragma comment(linker, "/STACK:1024000000,1024000000")
2 #define maxn 1000005
3 using namespace std;
4 int head,sz;
5 int U[maxn],D[maxn],L[maxn],R[maxn];
6 int H[maxn],ROW[maxn],C[maxn],S[maxn],O[maxn];
7 void remove(int c) {
8     L[R[c]]=L[c];
9     R[L[c]]=R[c];
10    for(int i=D[c]; i!=c; i=D[i])
11        for(int j=R[i]; j!=i; j=R[j]) {
12            U[D[j]]=U[j];
13            D[U[j]]=D[j];
14            --S[C[j]];
15        }
16 }
17 void resume(int c) {
18     for(int i=U[c]; i!=c; i=U[i]) {
19         for(int j=L[i]; j!=i; j=L[j]) {
20             ++S[C[j]];
21             U[D[j]]=j;
22             D[U[j]]=j;
23         }
24     }
25     L[R[c]]=c;
26     R[L[c]]=c;
27 }
28 void init(int m) {
29     head=0; //头指针为 0
30     for(int i=0; i<=m; i++) {
31         U[i]=i;
32         D[i]=i; //建立双向十字链表
33         L[i]=i-1;
34         R[i]=i+1;
35         S[i]=0;

```

```

36     }
37     R[m]=0;
38     L[0]=m;
39     S[0]=INF+1;
40     sz=m+1;
41     memset(H,0,sizeof(H));
42 }
43 void insert(int i, int j) {
44     if(H[i]) {
45         L[sz] = L[H[i]];
46         R[sz] = H[i];
47         L[R[sz]] = sz;
48         R[L[sz]] = sz;
49     }
50     else {
51         L[sz] = sz;
52         R[sz] = sz;
53         H[i] = sz;
54     }
55     U[sz] = U[j];
56     D[sz] = j;
57     U[D[sz]] = sz;
58     D[U[sz]] = sz;
59     C[sz] = j;
60     ROW[sz] = i;
61     ++S[j];
62     ++sz;
63 }
64 bool dfs(int k,int len) {
65     if(R[head]==head) return true;
66     int s=INF,c;
67     for (int t=R[head]; t!=head; t=R[t])
68         if (S[t]<s) s=S[t],c=t;
69     remove(c);
70     for(int i=D[c]; i!=c; i=D[i]) {
71         O[k]=ROW[i];
72         for(int j=R[i]; j!=i; j=R[j])
73             remove(C[j]);
74         if(dfs(k+1,len))
75             return true;
76         for(int j=L[i]; j!=i; j=L[j])
77             resume(C[j]);
78     }
79     resume(c);
80     return false;
81 }

```

#### 7.1.2 重复覆盖

```

1 int h() {
2     int i,j,k,count=0;
3     bool visit[N];
4     memset(visit,0,sizeof(visit));
5     for(i=R[0]; i!=R[i]) {
6         if(visit[i]) continue;
7         count++;
8         visit[i]=1;
9         for(j=D[i]; j!=i; j=D[j]) {
10             for(k=R[j]; k!=j; k=R[k])
11                 visit[C[k]]=1;
12         }
13     }
14     return count;
15 }
16 void Dance(int k) {
17     int i,j,c,Min,ans;
18     ans=h();
19     if(k+ans>K || k+ans>=ak) return;
20     if(!R[0]) {
21         if(k<ak) ak=k;
22         return;
23     }
24     for(Min=N, i=R[0]; i!=R[i])

```

```

25     if(S[i]<Min) Min=S[i],c=i;
26     for(i=D[c];i!=c;i=D[i]) {
27         remove(i);
28         for(j=R[i];j!=i;j=R[j])
29             remove(j);
30         Dance(k+1);
31         for(j=L[i];j!=i;j=L[j])
32             resume(j);
33         resume(i);
34     }
35     return;
36 }

```

## 7.2 蔡勒公式

```

1 int zeller(int y,int m,int d) {
2     if (m<=2) y--,m+=12; int c=y/100; y%=100;
3     int w=((c>>2)-(c<<1)+y+(y>>2)+(13*(m+1)/5)+d-1)%7;
4     if (w<0) w+=7; return(w);
5 }

```

## 7.3 五边形数定理

$$p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k-1} p(n - \frac{k(3k-1)}{2})$$

```

1 LL dp[N],fi[N];
2 LL five(LL x){ return (3*x*x-x)/2; }
3 void wbxs(){
4     dp[0]=1;
5     int t=1000; //其实可以等于 sqrt(N)
6     for(int i=-t;i<=t;++i)
7         fi[i+t]=five(i); //Q
8     for(int i=1;i<=100000;++i){
9         int flag=1;
10        for(int j=1;;++j){
11            LL a=fi[j+t],b=fi[-j+t];
12            if(a>i && b>i) break;
13            if(a<=i) dp[i]=(dp[i]+dp[i-a]*flag+MOD)%MOD;
14                ↳ //p
15            if(b<=i) dp[i]=(dp[i]+dp[i-b]*flag+MOD)%MOD;
16            flag*=-1;
17        }
18    }
19 }

```

## 7.4 凸包闵可夫斯基和

```

1 // cv[0..1] 为两个顺时针凸包，其中起点等于终点，求
2   ↳ 出的闵可夫斯基和不一定严格凸包
3 int i[2] = {0, 0}, len[2] = {(int)cv[0].size() - 1,
4   ↳ (int)cv[1].size() - 1};
5 vector<P> mnk;
6 mnk.push_back(cv[0][0] + cv[1][0]);
7 do {
8     int d((cv[0][i[0] + 1] - cv[0][i[0]]) * (cv[1][i[1] + 1]
9     ↳ - cv[1][i[1]]) >= 0);
10    mnk.push_back(cv[d][i[d] + 1] - cv[d][i[d]] +
11    ↳ mnk.back());
12    i[d] = (i[d] + 1) % len[d];
13 } while(i[0] || i[1]);

```

# 8. 技巧

## 8.1 STL 归还空间

```

1 template <typename T>
2 __inline void clear(T& container) {
3     container.clear(); // 或者删除了一堆元素
4     T(container).swap(container);
5 }

```

## 8.2 大整数取模

```

1 // 需要保证 x 和 y 非负
2 long long mult(long long x, long long y, long long MODN) {
3     long long t = (x * y - (long long)((long double)x / MODN
4     ↳ * y + 1e-3) * MODN) % MODN;
5     return t < 0 ? t + MODN : t;
6 }

```

## 8.3 读入优化

```

1 // getchar() 读入优化 << 关同步 cin << 此优化
2 // 用 isdigit() 会小幅变慢
3 // 返回 false 表示读到文件尾
4 namespace Reader {
5     const int L = (1 << 15) + 5;
6     char buffer[L], *S, *T;
7     __inline bool getchar(char &ch) {
8         if (S == T) {
9             T = (S = buffer) + fread(buffer, 1, L, stdin);
10            if (S == T) {
11                ch = EOF;
12                return false;
13            }
14        }
15        ch = *S++;
16        return true;
17    }
18    __inline bool getint(int &x) {
19        char ch; bool neg = 0;
20        for (; getchar(ch) && (ch < '0' || ch > '9'); ) neg ^=
21        ↳ ch == '-';
22        if (ch == EOF) return false;
23        x = ch - '0';
24        for (; getchar(ch), ch >= '0' && ch <= '9'; )
25            x = x * 10 + ch - '0';
26        if (neg) x = -x;
27        return true;
28    }
29 }

```

## 8.4 二次随机法

```

1 #include <random>
2
3 int main() {
4     std::mt19937 g(seed); // std::mt19937_64
5     std::cout << g() << std::endl;
6 }

```

## 8.5 vimrc

```

1 set ruler
2 set number
3 set smartindent
4 set autoindent
5 set tabstop=4
6 set softtabstop=4
7 set shiftwidth=4
8 set hlsearch
9 set incsearch
10 set autoread
11 set backspace=2
12 set mouse=a
13
14 syntax on
15
16 nmap <C-A> ggVG

```

```

17 vmap <C-C> "+y
18
19 filetype plugin indent on
20
21 autocmd FileType cpp set cindent
22 autocmd FileType cpp map <F9> :!g++ % -o %< -g -std=c++11
    ↪ -Wall -Wextra -Wconversion && size %< <CR>
23 autocmd FileType cpp map <C-F9> :!g++ % -o %< -std=c++11
    ↪ -O2 && size %< <CR>
24 autocmd FileType cpp map <F8> :!time ./%< %<.in <CR>
25 autocmd FileType cpp map <F5> :!time ./%< <CR>
26
27 map <F3> :vnew %<.in <CR>
28 map <F4> :!gedit % <CR>

```

## 8.6 控制 cout 输出实数精度

```
1 std::cout << std::fixed << std::setprecision(5);
```

## 8.7 汇编技巧

```

1 03优化
2 #define __ __attribute__ ((optimize("-O3")))
3 #define __ __inline__ __attribute__ ((__gnu_inline__,
    ↪ __always_inline__, __artificial__))
4
5 汇编开栈
6 #pragma comment(linker, "/STACK:256000000")
7
8 int __size = 256 << 20;
9 char* __p__ = (char *) malloc(__size__) + __size__;
10
11 int main() {
12     __asm__("movl %0, %%esp\n" :: "r"(__p__));
13     return 0;
14 }

```

# 9. 提示

## 9.1 线性规划转对偶

$\begin{aligned} &\text{maximize } \mathbf{c}^T \mathbf{x} \\ &\text{subject to } \mathbf{A} \mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq 0 \end{aligned} \iff \begin{aligned} &\text{minimize } \mathbf{y}^T \mathbf{b} \\ &\text{subject to } \mathbf{y}^T \mathbf{A} \geq \mathbf{c}^T, \mathbf{y} \geq 0 \end{aligned}$

## 9.2 NTT 素数及其原根

Prime	Primitive root
1053818881	7
1051721729	6
1045430273	3
1012924417	5
1007681537	3

## 9.3 积分表

- $$\int \frac{dx}{ax^2+bx+c} = \begin{cases} \frac{2}{\sqrt{4ac-b^2}} \arctan \frac{2ax+b}{\sqrt{4ac-b^2}} + C & (b^2 < 4ac) \\ \frac{1}{\sqrt{b^2-4ac}} \ln \left| \frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}} \right| + C & (b^2 > 4ac) \end{cases}$$
- $$\int \frac{x}{ax^2+bx+c} dx = \frac{1}{2a} \ln |ax^2+bx+c| - \frac{b}{2a} \int \frac{dx}{ax^2+bx+c}$$
- $$\int \frac{dx}{\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{a}} \ln |2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}| + C$$
- $$\int \sqrt{ax^2+bx+c} dx = \frac{2ax+b}{4a} \sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8\sqrt{a^3}} \ln |2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}| + C$$
- $$\int \frac{x}{\sqrt{ax^2+bx+c}} dx = \frac{1}{a} \sqrt{ax^2+bx+c} - \frac{b}{2\sqrt{a^3}} \ln |2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}| + C$$
- $$\int \frac{dx}{\sqrt{c+bx-ax^2}} = -\frac{1}{\sqrt{a}} \arcsin \frac{2ax-b}{\sqrt{b^2+4ac}} + C$$

$$5. \int \sqrt{c+bx-ax^2} dx = \frac{2ax-b}{4a} \sqrt{c+bx-ax^2} + \frac{b^2+4ac}{8\sqrt{a^3}} \arcsin \frac{2ax-b}{\sqrt{b^2+4ac}} + C$$

$$6. \int \frac{x}{\sqrt{c+bx-ax^2}} dx = -\frac{1}{a} \sqrt{c+bx-ax^2} + \frac{b}{2\sqrt{a^3}} \arcsin \frac{2ax-b}{\sqrt{b^2+4ac}} + C$$

$$1. \int \frac{dx}{\sqrt{(x-a)(b-x)}} = 2 \arcsin \sqrt{\frac{x-a}{b-x}} + C \quad (a < b)$$

2.

$$\int \sqrt{(x-a)(b-x)} dx = \frac{2x-a-b}{4} \sqrt{(x-a)(b-x)} + \frac{(b-a)^2}{4} \arcsin \sqrt{\frac{x-a}{b-x}} + C, (a < b) \quad (1)$$

$$1. \int \tan x dx = -\ln |\cos x| + C$$

$$2. \int \cot x dx = \ln |\sin x| + C$$

$$3. \int \sec x dx = \ln \left| \tan \left( \frac{\pi}{4} + \frac{x}{2} \right) \right| + C = \ln |\sec x + \tan x| + C$$

$$4. \int \csc x dx = \ln \left| \tan \frac{x}{2} \right| + C = \ln |\csc x - \cot x| + C$$

$$5. \int \sec^2 x dx = \tan x + C$$

$$6. \int \csc^2 x dx = -\cot x + C$$

$$7. \int \sec x \tan x dx = \sec x + C$$

$$8. \int \csc x \cot x dx = -\csc x + C$$

$$9. \int \sin^2 x dx = \frac{x}{2} - \frac{1}{4} \sin 2x + C$$

$$10. \int \cos^2 x dx = \frac{x}{2} + \frac{1}{4} \sin 2x + C$$

$$11. \int \sin^n x dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x dx$$

$$12. \int \cos^n x dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x dx$$

$$13. \int \frac{dx}{\sin^n x} = -\frac{1}{n-1} \frac{\cos x}{\sin^{n-1} x} + \frac{n-2}{n-1} \int \frac{dx}{\sin^{n-2} x}$$

$$14. \int \frac{dx}{\cos^n x} = \frac{1}{n-1} \frac{\sin x}{\cos^{n-1} x} + \frac{n-2}{n-1} \int \frac{dx}{\cos^{n-2} x}$$

15.

$$\int \cos^m x \sin^n x dx$$

$$= \frac{1}{m+n} \cos^{m-1} x \sin^{n+1} x + \frac{m-1}{m+n} \int \cos^{m-2} x \sin^n x dx$$

$$= -\frac{1}{m+n} \cos^{m+1} x \sin^{n-1} x + \frac{n-1}{m+1} \int \cos^m x \sin^{n-2} x dx$$

$$16. \int \frac{dx}{a+b \sin x} = \begin{cases} \frac{2}{\sqrt{a^2-b^2}} \arctan \frac{a \tan \frac{x}{2} + b}{\sqrt{a^2-b^2}} + C & (a^2 > b^2) \\ \frac{1}{\sqrt{b^2-a^2}} \ln \left| \frac{a \tan \frac{x}{2} + b - \sqrt{b^2-a^2}}{a \tan \frac{x}{2} + b + \sqrt{b^2-a^2}} \right| + C & (a^2 < b^2) \end{cases}$$

$$17. \int \frac{dx}{a+b \cos x} = \begin{cases} \frac{2}{a+b} \sqrt{\frac{a+b}{a-b}} \arctan \left( \sqrt{\frac{a-b}{a+b}} \tan \frac{x}{2} \right) + C & (a^2 > b^2) \\ \frac{1}{a+b} \sqrt{\frac{a+b}{a-b}} \ln \left| \frac{\tan \frac{x}{2} + \sqrt{\frac{a+b}{b-a}}}{\tan \frac{x}{2} - \sqrt{\frac{a+b}{b-a}}} \right| + C & (a^2 < b^2) \end{cases}$$

$$18. \int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x} = \frac{1}{ab} \arctan \left( \frac{b}{a} \tan x \right) + C$$

$$19. \int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x} = \frac{1}{2ab} \ln \left| \frac{b \tan x + a}{b \tan x - a} \right| + C$$

$$20. \int x \sin ax dx = \frac{1}{a^2} \sin ax - \frac{1}{a} x \cos ax + C$$

$$21. \int x^2 \sin ax dx = -\frac{1}{a} x^2 \cos ax + \frac{2}{a^2} x \sin ax + \frac{2}{a^3} \cos ax + C$$

$$22. \int x \cos ax dx = \frac{1}{a^2} \cos ax + \frac{1}{a} x \sin ax + C$$

$$23. \int x^2 \cos ax dx = \frac{1}{a} x^2 \sin ax + \frac{2}{a^2} x \cos ax - \frac{2}{a^3} \sin ax + C$$

$$1. \int \arcsin \frac{x}{a} dx = x \arcsin \frac{x}{a} + \sqrt{a^2 - x^2} + C$$

$$2. \int x \arcsin \frac{x}{a} dx = \left( \frac{x^2}{2} - \frac{a^2}{4} \right) \arcsin \frac{x}{a} + \frac{x}{4} \sqrt{x^2 - x^2} + C$$

$$3. \int x^2 \arcsin \frac{x}{a} dx = \frac{x^3}{3} \arcsin \frac{x}{a} + \frac{1}{9} (x^2 + 2a^2) \sqrt{a^2 - x^2} + C$$

$$4. \int \arccos \frac{x}{a} dx = x \arccos \frac{x}{a} - \sqrt{a^2 - x^2} + C$$

$$5. \int x \arccos \frac{x}{a} dx = \left( \frac{x^2}{2} - \frac{a^2}{4} \right) \arccos \frac{x}{a} - \frac{x}{4} \sqrt{a^2 - x^2} + C$$

$$6. \int x^2 \arccos \frac{x}{a} dx = \frac{x^3}{3} \arccos \frac{x}{a} - \frac{1}{9} (x^2 + 2a^2) \sqrt{a^2 - x^2} + C$$

$$7. \int \arctan \frac{x}{a} dx = x \arctan \frac{x}{a} - \frac{a}{2} \ln(a^2 + x^2) + C$$

8.  $\int x \arctan \frac{x}{a} dx = \frac{1}{2}(a^2 + x^2) \arctan \frac{x}{a} - \frac{a}{2}x + C$
9.  $\int x^2 \arctan \frac{x}{a} dx = \frac{x^3}{3} \arctan \frac{x}{a} - \frac{a}{6}x^2 + \frac{a^3}{6} \ln(a^2 + x^2) + C$
1.  $\int a^x dx = \frac{1}{\ln a} a^x + C$
2.  $\int e^{ax} dx = \frac{1}{a} e^{ax} + C$
3.  $\int x e^{ax} dx = \frac{1}{a^2} (ax - 1) e^{ax} + C$
4.  $\int x^n e^{ax} dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} dx$
5.  $\int x a^x dx = \frac{x}{\ln a} a^x - \frac{1}{(\ln a)^2} a^x + C$
6.  $\int x^n a^x dx = \frac{1}{\ln a} x^n a^x - \frac{n}{\ln a} \int x^{n-1} a^x dx$
7.  $\int e^{ax} \sin bx dx = \frac{1}{a^2 + b^2} e^{ax} (a \sin bx - b \cos bx) + C$
8.  $\int e^{ax} \cos bx dx = \frac{1}{a^2 + b^2} e^{ax} (b \sin bx + a \cos bx) + C$

9.  $\int e^{ax} \sin^n bxdx = \frac{1}{a^2 + b^2 n^2} e^{ax} \sin^{n-1} bx (a \sin bx - nb \cos bx) + \frac{n(n-1)b^2}{a^2 + b^2 n^2} \int e^{ax} \sin^{n-2} bxdx$
10.  $\int e^{ax} \cos^n bxdx = \frac{1}{a^2 + b^2 n^2} e^{ax} \cos^{n-1} bx (a \cos bx + nb \sin bx) + \frac{n(n-1)b^2}{a^2 + b^2 n^2} \int e^{ax} \cos^{n-2} bxdx$
1.  $\int \ln x dx = x \ln x - x + C$
2.  $\int \frac{dx}{x \ln x} = \ln |\ln x| + C$
3.  $\int x^n \ln x dx = \frac{1}{n+1} x^{n+1} (\ln x - \frac{1}{n+1}) + C$
4.  $\int (\ln x)^n dx = x (\ln x)^n - n \int (\ln x)^{n-1} dx$
5.  $\int x^m (\ln x)^n dx = \frac{1}{m+1} x^{m+1} (\ln x)^n - \frac{n}{m+1} \int x^m (\ln x)^{n-1} dx$