# Shanghai Jiao Tong University

# ACM-ICPC Team Selection Contest

# Stage I

May 21, 2012

## Statement:

1. Please answer all the problems on the Answer Sheet.
2. You may answer the problems in English or Chinese.
3. Raise your hand if you have any questions.
4. Every useful answer you write will get score although you just partial solved a problem.

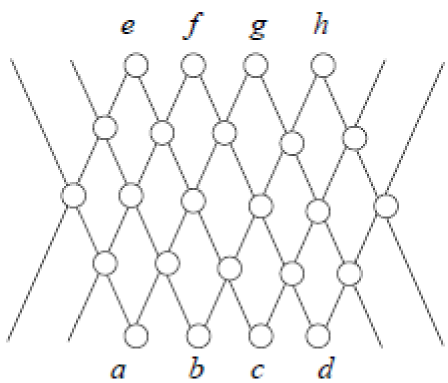
## Problems:

1. (5 Points) How many integers in range $[1,100]$ have even number of factors?

2. (5 Points) How many pairs of integers $(x,y)$ are co-prime ($gcd(x,y)=1$)? $1 \le x,y \le 30$

3. (10 Points) Construct a digraph on $\{1,2,\dots 2000\}$ where there is an edge from $i$ to $j$ if $i|j$ and $j/i$ is a prime number. In this graph, there are _____ paths from 1 to 210; there are _____ paths from 2 to 1024; and there are _____ paths from 1 to 2000.

4. (10 Points) As in the following figure, 4 people start from $a,b,c,d$, respectively. They want to escape from the bottom line to the exits on the top. There are 4 exit points $e,f,g,$ and $h$. Everyone needs to find a path with 4 steps along the lines in the picture (each step goes to a dot in the picture), and end up with one of the exit points. In addition, the paths of any two of them cannot share any common point (including the final exit). In how many ways can they do this?



5. (10 Points) Is it possible to cover an $8 \times 8$ chessboard, except the one grid on the bottom-right corner, by $1 \times 3$ dominos? Find a solution or prove it is impossible.

6. (10 Points) Given an $n \times n$ matrix $A$ with each grid assigned a number. You may ask for the sum of a sub-square of the matrix. How many times you need to ask before you can determine the sum of the numbers on the main diagonal ($\Sigma A_{ii}$)? And prove it cannot be done with less asks.

7. (10 Points) Given $n$ vertices, how many edges can you add at most without forming any triangles? And prove it.

8. (10 Points) $N$ pairs of couples sit around a table. A couple cannot sit adjacent. How many distinct arrangements? (Rotations are considered the same)

9. (10 Points) Prove $\sum_{d|n} \phi(d) = n$. $\phi(d)$ is the number of integers in $[1, d]$ that are co-prime with d.

10. (10 Points) Given a sequence of length $m \times n + 1$, prove that it must have an ascending subsequence of length $m + 1$ or a descending subsequence of length $n + 1$. And prove it doesn't hold when the sequence is of length $m \times n$.

11. (10 Points) Given $n$ different points on a plane, prove that either they are all on the same line, or there is a line that contains exactly two of the points.

12. (10 Points) Alice is gambling with Bob. Each round, the loser will lose one dollar to the winner and Alice always wins the round with probability $P$. Alice has only one dollar and Bob is so rich that will never be bankrupted. Alice will always continue gambling if she has money left. What is the probability that Alice will bankrupt?

13. (10 Points) Calculate the output of the following program.

```
n←2012
m←521
For i←1..n
    A[i][0],A[i][i]←1
    For j←1..i-1
        A[i][j]←A[i-1][j-1]+A[i-1][j]
    End For
End For
Print (A[n][m] mod 11)
```

# Answer sheet

| | | | | | |
|---|---|---|---|---|---|
| Name | : | _______________ | Class No. | : | _______________ |
| Gender | : | _______________ | Phone No. | : | _______________ |
| Student No. | : | _______________ | E-mail | : | _______________ |