Wizards

Standard Code Library

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目录

1. 数论

1.1 $O(m^2 \log n)$ 线性递推

Given $a_0, a_1, \ldots, a_{m-1}$
$a_n = c_0 \times a_{n-m} + \cdots + c_{m-1} \times a_{n-1}$
Solve for $a_n = v_0 \times a_0 + v_1 \times a_1 + \cdots + v_{m-1} \times a_{m-1}$

```
void linear_recurrence(long long n, int m, int a[], int c[], int p) {
  long long v[M] = {1 % p}, u[M << 1], msk = !!n;
  for(long long i(n); i > 1; i >>= 1) {
    msk <<= 1;
  }
  for(long long x(0); msk; msk >>= 1, x <<= 1) {
    fill_n(u, m << 1, 0);
    int b(!!(n & msk));
    x |= b;
    if(x < m) {
      u[x] = 1 % p;
    }else {
      for(int i(0); i < m; i++) {
        for(int j(0), t(i + b); j < m; j++, t++) {
          u[t] = (u[t] + v[i] * v[j]) % p;
        }
      }
      for(int i((m << 1) - 1); i >= m; i--) {
        for(int j(0), t(i - m); j < m; j++, t++) {
          u[t] = (u[t] + c[j] * u[i]) % p;
        }
      }
    }
    copy(u, u + m, v);
  }
  //a[n] = v[0] * a[0] + v[1] * a[1] + ... + v[m - 1] * a[m - 1].
  for(int i(m); i < 2 * m; i++) {
    a[i] = 0;
    for(int j(0); j < m; j++) {
      a[i] = (a[i] + (long long)c[j] * a[i + j - m]) % p;
    }
  }
  for(int j(0); j < m; j++) {
    b[j] = 0;
    for(int i(0); i < m; i++) {
      b[j] = (b[j] + v[i] * a[i + j]) % p;
    }
  }
  for(int j(0); j < m; j++) {
    a[j] = b[j];
  }
}
```

1.2 求逆元

```
void ex_gcd(long long a, long long b, long long &x, long long &y) {
  if (b == 0) {
    x = 1;
    y = 0;
    return;
  }
  long long xx, yy;
  ex_gcd(b, a % b, xx, yy);
  y = xx - a / b * yy;
  x = yy;
}
long long inv(long long x, long long MODN) {
  long long inv_x, y;
  ex_gcd(x, MODN, inv_x, y);
  return (inv_x % MODN + MODN) % MODN;
}
```

1.3 中国剩余定理

```
//返回 (ans, M), 其中 ans 是模 M 意义下的解
std::pair<long long, long long> CRT(const std::vector<long long>& m, const std::vector<long long>& a) {
  long long M = 1, ans = 0;
  int n = m.size();
  for (int i = 0; i < n; i++) M *= m[i];
  for (int i = 0; i < n; i++) {
    ans = (ans + (M / m[i]) * a[i] % M * inv(M / m[i], m[i])) % M;  // 可能需要大整数相乘取模
  }
  return std::make_pair(ans, M);
}
```

1.4 素性测试

```
int strong_pseudo_primetest(long long n,int base) {
    long long n2=n-1,res;
    int s=0;
    while(n2%2==0) n2>>=1,s++;
    res=powmod(base,n2,n);
    if((res==1)||(res==n-1)) return 1;
    s--;
    while(s>=0) {
        res=mulmod(res,res,n);
        if(res==n-1) return 1;
        s--;
    }
    return 0; // n is not a strong pseudo prime
}
int isprime(long long n) {
  static LL testNum[]={2,3,5,7,11,13,17,19,23,29,31,37};
  static LL lim[]={4,0,1373653LL,25326001LL,25000000000LL,
  2152302898747LL, 3474749660383LL,341550071728321LL,0,0,0,0};
  if(n<2||n==3215031751LL) return 0;
  for(int i=0;i<12;++i){
    if(n<lim[i]) return 1;
    if(strong_pseudo_primetest(n,testNum[i])==0) return 0;
  }
  return 1;
}
```

1.5 质因数分解

```
int ansn; LL ans[1000];
LL func(LL x,LL n){ return(mod_mul(x,x,n)+1)%n; }
LL Pollard(LL n){
  LL i,x,y,p;
  if(Rabin_Miller(n)) return n;
  if(!(n&1)) return 2;
  for(i=1;i<20;i++){
    x=i; y=func(x,n); p=gcd(y-x,n);
    while(p==1) {x=func(x,n); y=func(func(y,n),n); p=gcd((y-x+n)%n,n)%n;}
    if(p==0||p==n) continue;
    return p;
  }
}
void factor(LL n){
  LL x;
  x=Pollard(n);
  if(x==n){ ans[ansn++]=x; return; }
  factor(x), factor(n/x);
}
```

1.6 佩尔方程

```
import java.math.BigInteger;
import java.util.Scanner;
```

```java
//a[n]=(g[n]+a[0])/h[n]
//g[n]=a[n-1]*h[n-1]-g[n-1]
//h[n]=(N-g[n]*g[n])/h[n-1]
//p[n]=a[n-1]*p[n-1]+p[n-2]
//q[n]=a[n-1]*q[n-1]+q[n-2]
//so:
//p[n]*q[n-1]-p[n-1]*q[n]=(-1)^(n+1);
//p[n]^2-N*q[n]^2=(-1)^(n+1)*h[n+1];
public class Main {
    public static BigInteger p, q;
    public static void solve(int n) {
        BigInteger N, p1, p2, q1, q2, a0, a1, a2, g1, g2,
            h1, h2;
        g1 = q2 = p1 = BigInteger.ZERO;
        h1 = q1 = p2 = BigInteger.ONE;
        a0 = a1 =
            BigInteger.valueOf((long)Math.sqrt(1.0*n));
        N = BigInteger.valueOf(n);
        while (true) {
            g2 = a1.multiply(h1).subtract(g1);
            h2 = N.subtract(g2.pow(2)).divide(h1);
            a2 = g2.add(a0).divide(h2);
            p = a1.multiply(p2).add(p1);
            q = a1.multiply(q2).add(q1);
            if
                (p.pow(2).subtract(N.multiply(q.pow(2))).compareTo(BigInteger.ONE)
                == 0) return;
          g1 = g2;h1 = h2;a1 = a2;
            p1 = p2;p2 = p;
            q1 = q2;q2 = q;
        }
    }

    public static void main(String[] args) {
        Scanner cin = new Scanner(System.in);
        int t=cin.nextInt();
        while (t--!=0) {
            solve(cin.nextInt());
            System.out.println(p + " " + q);
        }
    }
}
```

1.7 二次剩余

```cpp
// x^2 = a (mod p), 0 <= a < p, 返回 true or false 代表
    是否存在解
// p 必须是质数，若是多个单次质数的乘积，可以分别
    求解再用 CRT 合并
// 复杂度为 O(log n)
void multiply(ll &c, ll &d, ll a, ll b, ll w) {
    int cc = (a * c + b * d % MOD * w) % MOD;
    int dd = (a * d + b * c) % MOD;
    c = cc, d = dd;
}

bool solve(int n, int &x) {
    if (MOD == 2) return x = 1, true;
    if (power(n, MOD / 2, MOD) == MOD - 1) return false;
    ll c = 1, d = 0, b = 1, a, w;
    // finding a such that a^2 - n is not a square
    do { a = rand() % MOD;
        w = (a * a - n + MOD) % MOD;
        if (w == 0) return x = a, true;
    } while (power(w, MOD / 2, MOD) != MOD - 1);
    for (int times = (MOD + 1) / 2; times; times >>= 1) {
        if (times & 1) multiply(c, d, a, b, w);
        multiply(a, b, a, b, w);
    }
    // x = (a + sqrt(w)) ^ ((p + 1) / 2)
    return x = c, true;
}
```

1.8 一元三次方程

```cpp
double a(p[3]), b(p[2]), c(p[1]), d(p[0]);
double k(b / a), m(c / a), n(d / a);
double p(-k * k / 3. + m);
double q(2. * k * k * k / 27 - k * m / 3. + n);
Complex omega[3] = {Complex(1, 0), Complex(-0.5, 0.5 *
    sqrt(3)), Complex(-0.5, -0.5 * sqrt(3))};
Complex r1, r2;
double delta(q * q / 4 + p * p * p / 27);
if (delta > 0) {
    r1 = cubrt(-q / 2. + sqrt(delta));
    r2 = cubrt(-q / 2. - sqrt(delta));
} else {
    r1 = pow(-q / 2. + pow(Complex(delta), 0.5), 1. / 3);
    r2 = pow(-q / 2. - pow(Complex(delta), 0.5), 1. / 3);
}
for(int _(0); _ < 3; _++) {
    Complex x = -k / 3. + r1 * omega[_ * 1] + r2 * omega[_
        * 2 % 3];
}
```

1.9 线下整点

```cpp
// \sum_{i=0}^{n-1} \lfloor (a+bi)/m \rfloor, n,m,a,b > 0
LL solve(LL n,LL a,LL b,LL m){
  if(b==0) return n*(a/m);
  if(a>=m) return n*(a/m)+solve(n,a%m,b,m);
  if(b>=m) return (n-1)*n/2*(b/m)+solve(n,a,b%m,m);
  return solve((a+b*n)/m,(a+b*n)%m,m,b);
}
```

1.10 线性同余不等式

```cpp
// Find the minimal non-negtive solutions for
    l <= d * x mod m <= r
// 0 <= d,l,r < m; l <= r, O(log n)
ll cal(ll m, ll d, ll l, ll r) {
    if (l == 0) return 0;
    if (d == 0) return MXL; // 无解
    if (d * 2 > m) return cal(m, m - d, m - r, m - l);
    if ((l - 1) / d < r / d) return (l - 1) / d + 1;
    ll k = cal(d, (-m % d + d) % d, l % d, r % d);
    return k == MXL ? MXL : (k * m + l - 1) / d + 1; // 无
        解 2
}
```

1.11 组合数取模

```cpp
LL prod=1,P;
pair<LL,LL> comput(LL n,LL p,LL k){
    if(n<=1)return make_pair(0,1);
    LL ans=1,cnt=0;
    ans=pow(prod,n/P,P);
    cnt=n/p;
    pair<LL,LL>res=comput(n/p,p,k);
    cnt+=res.first;
    ans=ans*res.second%P;
    for(int i=n-n%P+1;i<=n;i++)if(i%p){

        ans=ans*i%P;
    }
    return make_pair(cnt,ans);
}
```

```cpp
pair<LL,LL> calc(LL n,LL p,LL k){
    prod=1;P=pow(p,k,1e18);
    for(int i=1;i<P;i++)if(i%p)prod=prod*i%P;
    pair<LL,LL> res=comput(n,p,k);
    return res;
}
LL calc(LL n,LL m,LL p,LL k){
    pair<LL,LL>A,B,C;
    LL P=pow(p,k,1e18);
    A=calc(n,p,k);
    B=calc(m,p,k);
    C=calc(n-m,p,k);
    LL ans=1;
    ans=pow(p,A.first-B.first-C.first,P);
    ans=ans*A.second%P*inv(B.second,P)%P*inv(C.second,P)%P;
    return ans;
}
```

1.12 Schreier-Sims

```cpp
struct Perm{
  vector<int> P; Perm() {} Perm(int n) { P.resize(n); }
  Perm inv()const{
    Perm ret(P.size());
    for(int i = 0; i < int(P.size()); ++i) ret.P[P[i]] = i;
    return ret;
  }
  int &operator [](const int &dn){ return P[dn]; }
  void resize(const size_t &sz){ P.resize(sz); }
  size_t size()const{ return P.size(); }
  const int &operator [](const int &dn)const{ return P[dn]; }
};
Perm operator *(const Perm &a, const Perm &b){
  Perm ret(a.size());
  for(int i = 0; i < (int)a.size(); ++i) ret[i] = b[a[i]];
  return ret;
}
typedef vector<Perm> Bucket;
typedef vector<int> Table;
typedef pair<int,int> PII;
int n, m;
vector<Bucket> buckets, bucketsInv; vector<Table> lookupTable;
int fastFilter(const Perm &g, bool addToGroup = true) {
  int n = buckets.size();
  Perm p(g);
  for(int i = 0; i < n; ++i){
    int res = lookupTable[i][p[i]];
    if(res == -1){
      if(addToGroup){
        buckets[i].push_back(p); bucketsInv[i].push_back(p.inv());
        lookupTable[i][p[i]] = (int)buckets[i].size() - 1;
      }
      return i;
    }
    p = p * bucketsInv[i][res];
  }
  return -1;
}
long long calcTotalSize(){
  long long ret = 1;
  for(int i = 0; i < n; ++i) ret *= buckets[i].size();
  return ret;
}
bool inGroup(const Perm &g){ return fastFilter(g, false) == -1; }
void solve(const Bucket &gen,int _n){// m perm[0..n - 1]s
  n = _n, m = gen.size();
  {//clear all
    vector<Bucket> _buckets(n); swap(buckets, _buckets);
    vector<Bucket> _bucketsInv(n); swap(bucketsInv, _bucketsInv);
    vector<Table> _lookupTable(n); swap(lookupTable, _lookupTable);
  }
  for(int i = 0; i < n; ++i){
    lookupTable[i].resize(n);
    fill(lookupTable[i].begin(), lookupTable[i].end(), -1);
  }
  Perm id(n);
  for(int i = 0; i < n; ++i) id[i] = i;
  for(int i = 0; i < n; ++i){
    buckets[i].push_back(id); bucketsInv[i].push_back(id);
    lookupTable[i][i] = 0;
  }
  for(int i = 0; i < m; ++i) fastFilter(gen[i]);
  queue<pair<PII,PII> > toUpdate;
  for(int i = 0; i < n; ++i)
    for(int j = i; j < n; ++j)
      for(int k = 0; k < (int)buckets[i].size(); ++k)
        for(int l = 0; l < (int)buckets[j].size(); ++l)
          toUpdate.push(make_pair(PII(i,k), PII(j,l)));
  while(!toUpdate.empty()){
    PII a = toUpdate.front().first, b = toUpdate.front().second;
    toUpdate.pop();
    int res = fastFilter(buckets[a.first][a.second] * buckets[b.first][b.second]);
    if(res==-1) continue;
    PII newPair(res, (int)buckets[res].size() - 1);
    for(int i = 0; i < n; ++i)
      for(int j = 0; j < (int)buckets[i].size(); ++j){
        if(i <= res) toUpdate.push(make_pair(PII(i, j), newPair));
        if(res <= i) toUpdate.push(make_pair(newPair, PII(i, j)));
      }
  }
}
```

2. 代数

2.1 快速傅里叶变换

```cpp
void fft(Complex a[], int n, int f) {
  for (int i = 0; i < n; ++i)
    if (R[i] < i) swap(a[i], a[R[i]]);
  for (int i = 1, h = 0; i < n; i <<= 1, h++) {
    Complex wn = Complex(cos(pi / i), f * sin(pi / i));
    Complex w = Complex(1, 0);
    for (int k = 0; k < i; ++k, w = w * wn) tmp[k] = w;
    for (int p = i << 1, j = 0; j < n; j += p) {
      for (int k = 0; k < i; ++k) {
        Complex x = a[j + k], y = a[j + k + i] * tmp[k];
        a[j + k] = x + y; a[j + k + i] = x - y;
      }
    }
  }
}
```

2.2 分治卷积

```cpp
// n 必须是 2 的次幂
void fft(Complex a[], int n, int f) {
  for (int i = 0; i < n; ++i)
    if (R[i] < i) swap(a[i], a[R[i]]);
  for (int i = 1, h = 0; i < n; i <<= 1, h++) {
    Complex wn = Complex(cos(pi / i), f * sin(pi / i));
    Complex w = Complex(1, 0);
```

```
    for (int k = 0; k < i; ++k, w = w * wn) tmp[k] = w;
    for (int p = i << 1, j = 0; j < n; j += p) {
      for (int k = 0; k < i; ++k) {
        Complex x = a[j + k], y = a[j + k + i] * tmp[k];
        a[j + k] = x + y; a[j + k + i] = x - y;
      }
    }
  }
}
```

2.3 快速数论变换

```
// n 必须是 2 的次幂
void fft(Complex a[], int n, int f) {
  for (int i = 0; i < n; ++i)
    if (R[i] < i) swap(a[i], a[R[i]]);
  for (int i = 1, h = 0; i < n; i <<= 1, h++) {
    Complex wn = Complex(cos(pi / i), f * sin(pi / i));
    Complex w = Complex(1, 0);
    for (int k = 0; k < i; ++k, w = w * wn) tmp[k] = w;
    for (int p = i << 1, j = 0; j < n; j += p) {
      for (int k = 0; k < i; ++k) {
        Complex x = a[j + k], y = a[j + k + i] * tmp[k];
        a[j + k] = x + y; a[j + k + i] = x - y;
      }
    }
  }
}
```

2.4 快速沃尔什变换

```
void FWT(LL a[],int n,int ty){ //the length is 2^n
  for(int d=1;d<n;d<<=1){
    for(int m=(d<<1),i=0;i<n;i+=m){
      if(ty==1){
        for(int j=0;j<d;j++){
          LL x=a[i+j],y=a[i+j+d];
          a[i+j]=x+y;
          a[i+j+d]=x-y;
                  //and:a[i+j]=x+y; or:a[i+j+d]=x+y;
        }
      }else{
        for(int j=0;j<d;j++){
          LL x=a[i+j],y=a[i+j+d];
          a[i+j]=(x+y)/2;
          a[i+j+d]=(x-y)/2;
                  //and:a[i+j]=x-y; or:a[i+j+d]=y-x;
        }
      }
    }
  }
}
```

2.5 自适应辛普森积分

```
namespace adaptive_simpson {
  template<typename function>
  inline double area(function f, const double &left, const
    double &right) {
    double mid = (left + right) / 2;
    return (right - left) * (f(left) + 4 * f(mid) +
      f(right)) / 6;
  }
  template<typename function>
  inline double simpson(function f, const double &left,
    const double &right, const double &eps, const
    double &area_sum) {
    double mid = (left + right) / 2;
    double area_left = area(f, left, mid);
    double area_right = area(f, mid, right);
    double area_total = area_left + area_right;
    if (fabs(area_total - area_sum) <= 15 * eps) {
      return area_total + (area_total - area_sum) / 15;
    }
    return simpson(f, left, right, eps / 2, area_left) +
      simpson(f, mid, right, eps / 2, area_right);
  }
  template<typename function>
  inline double simpson(function f, const double &left,
    const double &right, const double &eps) {
    return simpson(f, left, right, eps, area(f, left,
      right));
  }
}
```

2.6 单纯形

```
const double eps = 1e-8;
// max{c * x | Ax <= b, x >= 0} 的解，无解返回空的
  vector，否则就是解.
vector<double> simplex(vector<vector<double> > &A,
  vector<double> b, vector<double> c) {
  int n = A.size(), m = A[0].size() + 1, r = n, s = m - 1;
  vector<vector<double> > D(n + 2, vector<double>(m + 1));
  vector<int> ix(n + m);
  for(int i = 0; i < n + m; i++)  {
    ix[i] = i;
  }
  for(int i = 0; i < n; i++) {
    for(int j = 0; j < m - 1; j++) {
      D[i][j] = -A[i][j];
    }
    D[i][m - 1] = 1;
    D[i][m] = b[i];
    if (D[r][m] > D[i][m]) {
      r = i;
    }
  }
  for(int j = 0; j < m - 1; j++) {
    D[n][j] = c[j];
  }
  D[n + 1][m - 1] = -1;
  for(double d; ;) {
    if (r < n) {
      swap(ix[s], ix[r + m]);
      D[r][s] = 1. / D[r][s];
      for(int j = 0; j <= m; j++) {
        if (j != s) {
          D[r][j] *= -D[r][s];
        }
      }
      for(int i = 0; i <= n + 1; i++) {
        if (i != r) {
          for(int j = 0; j <= m; j++) {
            if (j != s) {
              D[i][j] += D[r][j] * D[i][s];
            }
          }
          D[i][s] *= D[r][s];
        }
      }
    }
    r = -1, s = -1;
    for(int j = 0; j < m; j++) {
      if (s < 0 || ix[s] > ix[j]) {
        if (D[n + 1][j] > eps || D[n + 1][j] > -eps &&
          D[n][j] > eps) {
          s = j;
        }
      }
    }
    if (s < 0) break;
    for(int i = 0; i < n; i++) {
```

```
      if (D[i][s] < -eps) {
        if (r < 0 || (d = D[r][m] / D[r][s] - D[i][m] / D[i][s]) < -eps
          || d < eps && ix[r + m] > ix[i + m]) {
          r = i;
        }
      }
    }
    if (r < 0) return vector<double> ();
  }
  if (D[n + 1][m] < -eps) return vector<double> ();
  vector<double> x(m - 1);
  for(int i = m; i < n + m; i++) {
    if (ix[i] < m - 1) {
      x[ix[i]] = D[i - m][m];
    }
  }
  return x;
}
```

3. 计算几何

3.1 二维

3.1.1 点类

```
int sign(DB x) {
  return (x > eps) - (x < -eps);
}
DB msqrt(DB x) {
  return sign(x) > 0 ? sqrt(x) : 0;
}
struct Point {
  DB x, y;
  Point rotate(DB ang) const { // 逆时针旋转 ang 弧度
    return Point(cos(ang) * x - sin(ang) * y, cos(ang) * y + sin(ang) * x);
  }
  Point turn90() const { // 逆时针旋转 90 度
    return Point(-y, x);
  }
  Point unit() const {
    return *this / len();
  }
};
DB dot(const Point& a, const Point& b) {
  return a.x * b.x + a.y * b.y;
}
DB det(const Point& a, const Point& b) {
  return a.x * b.y - a.y * b.x;
}
#define cross(p1,p2,p3) ((p2.x-p1.x)*(p3.y-p1.y)-(p3.x-p1.x)*(p2.y-p1.y))
#define crossOp(p1,p2,p3) sign(cross(p1,p2,p3))
bool isLL(const Line& l1, const Line& l2, Point& p) { // 直线与直线交点
  DB s1 = det(l2.b - l2.a, l1.a - l2.a),
     s2 = -det(l2.b - l2.a, l1.b - l2.a);
  if (!sign(s1 + s2)) return false;
  p = (l1.a * s2 + l1.b * s1) / (s1 + s2);
  return true;
}
bool onSeg(const Line& l, const Point& p) { // 点在线段上
  return sign(det(p - l.a, l.b - l.a)) == 0 && sign(dot(p - l.a, p - l.b)) <= 0;
}
Point projection(const Line & l, const Point& p) {
  return l.a + (l.b - l.a) * (dot(p - l.a, l.b - l.a) / (l.b - l.a).len2());
}
DB disToLine(const Line& l, const Point& p) { // 点到*直线*距离
  return fabs(det(p - l.a, l.b - l.a) / (l.b - l.a).len());
}
DB disToSeg(const Line& l, const Point& p) { // 点到线段距离
  return sign(dot(p - l.a, l.b - l.a)) * sign(dot(p - l.b, l.a - l.b)) == 1 ? disToLine(l, p) : std::min((p - l.a).len(), (p - l.b).len());
}
// 圆与直线交点
bool isCL(Circle a, Line l, Point& p1, Point& p2) {
  DB x = dot(l.a - a.o, l.b - l.a),
     y = (l.b - l.a).len2(),
     d = x * x - y * ((l.a - a.o).len2() - a.r * a.r);
  if (sign(d) < 0) return false;
  Point p = l.a - ((l.b - l.a) * (x / y)), delta = (l.b - l.a) * (msqrt(d) / y);
  p1 = p + delta; p2 = p - delta;
  return true;
}
//圆与圆的交面积
DB areaCC(const Circle& c1, const Circle& c2) {
  DB d = (c1.o - c2.o).len();
  if (sign(d - (c1.r + c2.r)) >= 0) return 0;
  if (sign(d - std::abs(c1.r - c2.r)) <= 0) {
    DB r = std::min(c1.r, c2.r);
    return r * r * PI;
  }
  DB x = (d * d + c1.r * c1.r - c2.r * c2.r) / (2 * d),
     t1 = acos(x / c1.r), t2 = acos((d - x) / c2.r);
  return c1.r * c1.r * t1 + c2.r * c2.r * t2 - d * c1.r * sin(t1);
}
// 圆与圆交点
bool isCC(Circle a, Circle b, P& p1, P& p2) {
  DB s1 = (a.o - b.o).len();
  if (sign(s1 - a.r - b.r) > 0 || sign(s1 - std::abs(a.r - b.r)) < 0) return false;
  DB s2 = (a.r * a.r - b.r * b.r) / s1;
  DB aa = (s1 + s2) * 0.5, bb = (s1 - s2) * 0.5;
  P o = (b.o - a.o) * (aa / (aa + bb)) + a.o;
  P delta = (b.o - a.o).unit().turn90() * msqrt(a.r * a.r - aa * aa);
  p1 = o + delta, p2 = o - delta;
  return true;
}
// 求点到圆的切点，按关于点的顺时针方向返回两个点
bool tanCP(const Circle &c, const Point &p0, Point &p1, Point &p2) {
  double x = (p0 - c.o).len2(), d = x - c.r * c.r;
  if (d < eps) return false; // 点在圆上认为没有切点
  Point p = (p0 - c.o) * (c.r * c.r / x);
  Point delta = ((p0 - c.o) * (-c.r * sqrt(d) / x)).turn90();
  p1 = c.o + p + delta;
  p2 = c.o + p - delta;
  return true;
}
// 求圆到圆的外共切线，按关于 c1.o 的顺时针方向返回两条线
vector<Line> extanCC(const Circle &c1, const Circle &c2) {
  vector<Line> ret;
  if (sign(c1.r - c2.r) == 0) {
    Point dir = c2.o - c1.o;
    dir = (dir * (c1.r / dir.len())).turn90();
    ret.push_back(Line(c1.o + dir, c2.o + dir));
    ret.push_back(Line(c1.o - dir, c2.o - dir));
  } else {
    Point p = (c1.o * -c2.r + c2.o * c1.r) / (c1.r - c2.r);
    Point p1, p2, q1, q2;
```

```
    if (tanCP(c1, p, p1, p2) && tanCP(c2, p, q1, q2)) {
      if (c1.r < c2.r) swap(p1, p2), swap(q1, q2);
      ret.push_back(Line(p1, q1));
      ret.push_back(Line(p2, q2));
    }
  }
  return ret;
}
// 求圆到圆的内共切线，按关于 c1.o 的顺时针方向返回两条线
std::vector<Line> intanCC(const Circle &c1, const Circle &c2) {
  std::vector<Line> ret;
  Point p = (c1.o * c2.r + c2.o * c1.r) / (c1.r + c2.r);
  Point p1, p2, q1, q2;
  if (tanCP(c1, p, p1, p2) && tanCP(c2, p, q1, q2)) { // 两圆相切认为没有切线
    ret.push_back(Line(p1, q1));
    ret.push_back(Line(p2, q2));
  }
  return ret;
}
bool contain(vector<Point> polygon, Point p) { // 判断点p 是否被多边形包含，包括落在边界上
  int ret = 0, n = polygon.size();
  for(int i = 0; i < n; ++ i) {
    Point u = polygon[i], v = polygon[(i + 1) % n];
    if (onSeg(Line(u, v), p)) return true;  // Here I guess.
    if (sign(u.y - v.y) <= 0) swap(u, v);
    if (sign(p.y - u.y) > 0 || sign(p.y - v.y) <= 0) continue;
    ret += sign(det(p, v, u)) > 0;
  }
  return ret & 1;
}
// 用半平面 (q1,q2) 的逆时针方向去切凸多边形
std::vector<Point> convexCut(const std::vector<Point>&ps, Point q1, Point q2) {
  std::vector<Point> qs; int n = ps.size();
  for (int i = 0; i < n; ++i) {
    Point p1 = ps[i], p2 = ps[(i + 1) % n];
    int d1 = crossOp(q1,q2,p1), d2 = crossOp(q1,q2,p2);
    if (d1 >= 0) qs.push_back(p1);
    if (d1 * d2 < 0) qs.push_back(isSS(p1, p2, q1, q2));
  }
  return qs;
}
// 求凸包
std::vector<Point> convexHull(std::vector<Point> ps) {
  int n = ps.size(); if (n <= 1) return ps;
  std::sort(ps.begin(), ps.end());
  std::vector<Point> qs;
  for (int i = 0; i < n; qs.push_back(ps[i ++]))
    while (qs.size() > 1 && sign(det(qs[qs.size() - 2], qs.back(), ps[i])) <= 0)
      qs.pop_back();
  for (int i = n - 2, t = qs.size(); i >= 0; qs.push_back(ps[i --]))
    while ((int)qs.size() > t && sign(det(qs[qs.size() - 2], qs.back(), ps[i])) <= 0)
      qs.pop_back();
  return qs;
}
```

3.1.2 凸包

```
// 凸包中的点按逆时针方向
struct Convex {
  int n;
  std::vector<Point> a, upper, lower;
  void make_shell(const std::vector<Point>& p,
      std::vector<Point>& shell) {  // p needs to be sorted.
    clear(shell); int n = p.size();
    for (int i = 0, j = 0; i < n; i++, j++) {
      for (; j >= 2 && sign(det(shell[j-1] - shell[j-2],
              p[i] - shell[j-2])) <= 0; --j) shell.pop_back();
      shell.push_back(p[i]);
    }
  }
  void make_convex() {
    std::sort(a.begin(), a.end());
    make_shell(a, lower);
    std::reverse(a.begin(), a.end());
    make_shell(a, upper);
    a = lower; a.pop_back();
    a.insert(a.end(), upper.begin(), upper.end());
    if ((int)a.size() >= 2) a.pop_back();
    n = a.size();
  }
  void init(const std::vector<Point>& _a) {
    clear(a); a = _a; n = a.size();
    make_convex();
  }
  void read(int _n) {  // Won't make convex.
    clear(a); n = _n; a.resize(n);
    for (int i = 0; i < n; i++)
      a[i].read();
  }
  std::pair<DB, int> get_tangent(
      const std::vector<Point>& convex, const Point& vec) {
    int l = 0, r = (int)convex.size() - 2;
    assert(r >= 0);
    for (; l + 1 < r; ) {
      int mid = (l + r) / 2;
      if (sign(det(convex[mid + 1] - convex[mid], vec)) > 0)
        r = mid;
      else l = mid;
    }
    return std::max(std::make_pair(det(vec, convex[r]), r),
        std::make_pair(det(vec, convex[0]), 0));
  }
  int binary_search(Point u, Point v, int l, int r) {
    int s1 = sign(det(v - u, a[l % n] - u));
    for (; l + 1 < r; ) {
      int mid = (l + r) / 2;
      int smid = sign(det(v - u, a[mid % n] - u));
      if (smid == s1) l = mid;
      else r = mid;
    }
    return l % n;
  }
  // 求凸包上和向量 vec 叉积最大的点，返回编号，共线的多个切点返回任意一个
  int get_tangent(Point vec) {
    std::pair<DB, int> ret = get_tangent(upper, vec);
    ret.second = (ret.second + (int)lower.size() - 1) % n;
    ret = std::max(ret, get_tangent(lower, vec));
    return ret.second;
  }
  // 求凸包和直线 u, v 的交点，如果不相交返回 false，如果有则是和 (i, next(i)) 的交点，交在点上不确定返回前后两条边其中之一
  bool get_intersection(Point u, Point v, int &i0, int &i1) {
    int p0 = get_tangent(u - v), p1 = get_tangent(v - u);
    if (sign(det(v - u, a[p0] - u)) * sign(det(v - u, a[p1] - u)) <= 0) {
      if (p0 > p1) std::swap(p0, p1);
```

```
        i0 = binary_search(u, v, p0, p1);
        i1 = binary_search(u, v, p1, p0 + n);
        return true;
      }
      else return false;
    }
};
```

3.1.3 凸包最近点对

```
//判断点是否在多边形内
int isPointInPolygon(point p, point *a, int n) {
    int cnt = 0;
    for(int i=0; i<n; ++i) {
        if(OnSegment(p, a[i], a[(i+1)%n])) return -1;
        double k = cross(a[(i+1)%n]-a[i], p-a[i]);
        double d1 = a[i].y - p.y;
    double  d2 = a[(i+1)].y - p.y;
        if(k>0 &&d1<=0 &&d2>0) cnt++;
        if(k<0 &&d2<=0 &&d1>0) cnt++;
        //k==0，点和线段共线的情况不考虑
    }
    if(cnt&1)return 1;
    return 0;
}
//判断凸包是否相离
bool two_getaway_ConvexHull(point *cha, int n1, point *chb, int m1) {
    if(n1==1 && m1==1) {
        if(cha[0]==chb[0])
            return false;
    } else if(n1==1 && m1==2) {
        if(OnSegment(cha[0], chb[0], chb[1]))
            return false;
    } else if(n1==2 && m1==1) {
        if(OnSegment(chb[0], cha[0], cha[1]))
            return false;
    } else if(n1==2 && m1==2) {
        if(SegmentIntersection(cha[0], cha[1], chb[0], chb[1]))
            return false;
    } else if(n1==2) {
        for(int i=0; i<n1; ++i)
            if(isPointInPolygon(cha[i], chb, m1))
                return false;
    } else if(m1==2) {
        for(int i=0; i<m1; ++i)
            if(isPointInPolygon(chb[i], cha, n1))
                return false;
    } else {
        for(int i=0; i<n1; ++i) {
            for(int j=0; j<m1; ++j) {
                if(SegmentIntersection(cha[i], cha[(i+1)%n1], chb[j], chb[(j+1)%m1]))
                  return false;
            }
        }
        for(int i=0; i<n1; ++i)
            if(isPointInPolygon(cha[i], chb, m1))
                return false;
        for(int i=0; i<m1; ++i)
            if(isPointInPolygon(chb[i], cha, n1))
                return false;
    }
    return true;
}
//旋转卡壳求两个凸包最近距离
double solve(point *P, point *Q, int n, int m) {
    if(n==1 && m==1) {
        return length(P[0] - Q[0]);
    } else if(n==1 && m==2) {
        return DistanceToSegment(P[0], Q[0], Q[1]);
    } else if(n==2 && m==1) {
        return DistanceToSegment(Q[0], P[0], P[1]);
    } else if(n==2 && m==2) {
        return SegmentToSegment(P[0], P[1], Q[0], Q[1]);
    }

    int yminP = 0, ymaxQ = 0;
    for(int i=0; i<n; ++i) if(P[i].y < P[yminP].y) yminP = i;
    for(int i=0; i<m; ++i) if(Q[i].y > Q[ymaxQ].y) ymaxQ = i;
    P[n] = P[0];
    Q[n] = Q[0];
    double INF2 = 1e100;
    double arg, ans = INF2;
    for(int i=0; i<n; ++i) {
        //当叉积负正转正时，说明点 ymaxQ 就是对踵点
        while((arg=cross(P[yminP] - P[yminP+1],Q[ymaxQ+1] - Q[ymaxQ])) < -eps)
            ymaxQ = (ymaxQ+1)%m;
        double ret;
        if(arg > eps) { //卡住第二个凸包上的点。
            ret = DistanceToSegment(Q[ymaxQ], P[yminP], P[yminP+1]);
            ans  = min(ans,ret);
        } else { //arg==0，卡住第二个凸包的边
            ret = SegmentToSegment(P[yminP],P[yminP+1],Q[ymaxQ],Q
            ans = min(ans,ret);
        }
        yminP = (yminP+1)%n;
    }
    return ans;
}
double mindis_twotubao(point *P, point *Q, int n, int m){
    //return min(solve(P, Q, n, m),solve(Q,P,m,n));
    if(two_getaway_ConvexHull(P,n,Q,m)==true) return min(solve(P, Q, n, m),solve(Q,P,m,n));
    else return 0.0;
}
```

3.1.4 三角形的心

```
Point inCenter(const Point &A, const Point &B, const Point &C) { // 内心
  double a = (B - C).len(), b = (C - A).len(), c = (A - B).len(),
    s = fabs(det(B - A, C - A)),
    r = s / p;
  return (A * a + B * b + C * c) / (a + b + c);
}
Point circumCenter(const Point &a, const Point &b, const Point &c) { // 外心
  Point bb = b - a, cc = c - a;
  double db = bb.len2(), dc = cc.len2(), d = 2 * det(bb, cc);
  return a - Point(bb.y * dc - cc.y * db, cc.x * db - bb.x * dc) / d;
}
Point othroCenter(const Point &a, const Point &b, const Point &c) { // 垂心
  Point ba = b - a, ca = c - a, bc = b - c;
  double Y = ba.y * ca.y * bc.y,
      A = ca.x * ba.y - ba.x * ca.y,
      x0 = (Y + ca.x * ba.y * b.x - ba.x * ca.y * c.x) / A,
      y0 = -ba.x * (x0 - c.x) / ba.y + ca.y;
  return Point(x0, y0);
}
```

3.1.5 半平面交

```cpp
struct Point {
  int quad() const { return sign(y) == 1 || (sign(y) == 0
    && sign(x) >= 0);}
};
struct Line {
  bool include(const Point &p) const { return sign(det(b -
    a, p - a)) > 0; }
  Line push() const{ // 将半平面向外推 eps
    const double eps = 1e-6;
    Point delta = (b - a).turn90().norm() * eps;
    return Line(a - delta, b - delta);
  }
};
bool sameDir(const Line &l0, const Line &l1) { return
  parallel(l0, l1) && sign(dot(l0.b - l0.a, l1.b -
  l1.a)) == 1; }
bool operator < (const Point &a, const Point &b) {
  if (a.quad() != b.quad()) {
    return a.quad() < b.quad();
  } else {
    return sign(det(a, b)) > 0;
  }
}
bool operator < (const Line &l0, const Line &l1) {
  if (sameDir(l0, l1)) {
    return l1.include(l0.a);
  } else {
    return (l0.b - l0.a) < (l1.b - l1.a);
  }
}
bool check(const Line &u, const Line &v, const Line &w) {
  return w.include(intersect(u, v)); }
vector<Point> intersection(vector<Line> &l) {
  sort(l.begin(), l.end());
  deque<Line> q;
  for (int i = 0; i < (int)l.size(); ++i) {
    if (i && sameDir(l[i], l[i - 1])) {
      continue;
    }
    while (q.size() > 1 && !check(q[q.size() - 2],
      q[q.size() - 1], l[i])) q.pop_back();
    while (q.size() > 1 && !check(q[1], q[0], l[i]))
      q.pop_front();
    q.push_back(l[i]);
  }
  while (q.size() > 2 && !check(q[q.size() - 2],
    q[q.size() - 1], q[0])) q.pop_back();
  while (q.size() > 2 && !check(q[1], q[0], q[q.size() -
    1])) q.pop_front();
  vector<Point> ret;
  for (int i = 0; i < (int)q.size(); ++i)
    ret.push_back(intersect(q[i], q[(i + 1) %
    q.size()]));
  return ret;
}
```

3.1.6 最大空凸包

```cpp
inline double eq(double x, double y) {
    return fabs(x-y)<eps;
}
double xmult(point a, point b, point o) {
    return (a.x-o.x)*(o.y-b.y)-(a.y-o.y)*(o.x-b.x);
}
double dist(point a, point b) {
    return (a.x-b.x)*(a.x-b.x)+(a.y-b.y)*(a.y-b.y);
}
point o;
bool cmp_angle(point a,point b) {
    if(eq(xmult(a,b,o),0.0)) {
        return dist(a,o)<dist(b,o);
    }
    return xmult(a,o,b)>0;
}
double empty_convex(point *p, int pn) {
    double ans=0;
    for(int i=0; i<pn; i++) {
        for(int j=0; j<pn; j++) {
            dp[i][j]=0;
        }
    }
    for(int i=0; i<pn; i++) {
        int j = i-1;
        while(j>=0 && eq(xmult(p[i], p[j],
          o),0.0))j--;//coline
        bool flag= j==i-1;
        while(j>=0) {
            int k = j-1;
            while(k >= 0 && xmult(p[i],p[k],p[j])>0)k--;
            double area = fabs(xmult(p[i],p[j],o))/2;
            if(k >= 0)area+=dp[j][k];
            if(flag) dp[i][j]=area;
            ans=max(ans,area);
            j=k;
        }
        if(flag) {
            for(int j=1; j<i; j++) {
                dp[i][j] = max(dp[i][j],dp[i][j-1]);
            }
        }
    }
    return ans;
}
double largest_empty_convex(point *p, int pn) {
    point data[maxn];
    double ans=0;
    for(int i=0; i<pn; i++) {
        o=p[i];
        int dn=0;
        for(int j=0; j<pn; j++) {
            if(p[j].y>o.y||(p[j].y==o.y&&p[j].x>=o.x)) {
                data[dn++]=p[j];
            }
        }
        sort(data, data+dn, cmp_angle);
        ans=max(ans, empty_convex(data, dn));
    }
    return ans;
}
```

3.1.7 平面最近点对

```cpp
double Dis(Point a, Point b) {
  return sqrt((a.x-b.x)*(a.x-b.x) + (a.y-b.y)*(a.y-b.y));
}
double Closest_Pair(int left, int right) {
  double d = INF;
  if(left == right) return d;
  if(left +1 == right)
    return Dis(p[left],p[right]);
  int mid = (left+right)>>1;
  double d1 = Closest_Pair(left,mid);
  double d2 = Closest_Pair(mid,right);
  d = min(d1,d2);
  int k = 0;
  for(int i = left; i <= right; i++) {
    if(fabs(p[mid].x - p[i].x) <= d)
      temp[k++] = p[i];
  }
  sort(temp,temp+k,cmpy);
  for(int i = 0; i < k; i++) {
    for(int j = i+1; j < k && temp[j].y - temp[i].y < d;
      j++) {
```

```
      double d3 = Dis(temp[i],temp[j]);
      d = min(d,d3);
    }
  }
  return d;
}
```

3.1.8 最小覆盖圆

```
#include<cmath>
#include<cstdio>
#include<algorithm>
using namespace std;
const double eps=1e-6;
struct couple {
  double x, y;
  couple(){}
  couple(const double &xx, const double &yy) {
    x = xx; y = yy;
  }
} a[100001];
int n;
//dis means distance, dis2 means square of it
struct circle {
  double r; couple c;
} cir;
inline bool inside(const couple & x) {
  return di2(x, cir.c) < cir.r*cir.r+eps;
}
inline void p2c(int x, int y) {
  cir.c.x = (a[x].x+a[y].x)/2;
  cir.c.y = (a[x].y+a[y].y)/2;
  cir.r = dis(cir.c, a[x]);
}
inline void p3c(int i, int j, int k) {
  couple x = a[i], y = a[j], z = a[k];
  cir.r =
    ↪ sqrt(di2(x,y)*di2(y,z)*di2(z,x))/fabs(x*y+y*z+z*x)/2;
  couple t1((x-y).x, (y-z).x), t2((x-y).y, (y-z).y),
    ↪ t3((len(x)-len(y))/2, (len(y)-len(z))/2);
  cir.c = couple(t3*t2, t1*t3)/(t1*t2);
}
inline circle mi() {
  sort(a + 1, a + 1 + n);
  n = unique(a + 1, a + 1 + n) - a - 1;
  if(n == 1) {
    cir.c = a[1];
    cir.r = 0;
    return cir;
  }
  random_shuffle(a + 1, a + 1 + n);
  p2c(1, 2);
  for(int i = 3; i <= n; i++)
    if(!inside(a[i])) {
      p2c(1, i);
      for(int j = 2; j < i; j++)
        if(!inside(a[j])) {
          p2c(i, j);
          for(int k = 1; k < j; k++)
            if(!inside(a[k]))
              p3c(i,j, k);
        }
    }
  return cir;
}
```

3.1.9 多边形内部可视

```
int C(const Point & P, const Point & A, const Point & Q,
  ↪ const Point & B) {
  Point C = GetIntersection(P, A - P, Q, Q - B);
  return OnLine(Q, C, B);
}
int Onleft(const Point & a, const Point &b, const Point &
  ↪ c) {
  return dcmp(Cross(b - c, a - c)) > 0;
}
int visible(int x, int y) {
  int P = (x + n - 1) % n, Q = (x + 1) % n;
  Point u = p[y] - p[x], v = p[x] - p[P], w = p[x] - p[Q];
  if (Onleft(p[Q], p[x], p[P])) {
    return dcmp(Cross(v, u)) > 0 && dcmp(Cross(w, u)) < 0;
  } else {
    return !(dcmp(Cross(v, u)) < 0 && dcmp(Cross(w, u)) >
      ↪ 0);
  }
}
int solve(int x, int y) {
  if (vis[x][y] == dfn) return g[x][y];
  vis[x][y] = dfn;
  if (x == y || y == x + 1) return g[x][y] = 1;
  for (int i = x; i + 1 <= y; i++) {
    if (C(p[x], p[y], p[i], p[i + 1])) return g[x][y] = 0;
  }
  for (int i = x + 1; i < y; i++) {
    if (OnLine(p[x], p[i], p[y])) {
      return g[x][y] = solve(x, i) && solve(i, y);
    }
  }
  if (!visible(x, y) || !visible(y, x)) return g[x][y] =
    ↪ 0;
  return g[x][y] = 1;
}
```

3.2 三维

3.2.1 三维点类

```
// 三维绕轴旋转，大拇指指向 axis 向量方向，四指弯曲
  ↪ 方向转 w 弧度
Point rotate(const Point& s, const Point& axis, DB w) {
  DB x = axis.x, y = axis.y, z = axis.z;
  DB s1 = x * x + y * y + z * z, ss1 = msqrt(s1),
     cosw = cos(w), sinw = sin(w);
  DB a[4][4];
  memset(a, 0, sizeof a);
  a[3][3] = 1;
  a[0][0] = ((y * y + z * z) * cosw + x * x) / s1;
  a[0][1] = x * y * (1 - cosw) / s1 + z * sinw / ss1;
  a[0][2] = x * z * (1 - cosw) / s1 - y * sinw / ss1;
  a[1][0] = x * y * (1 - cosw) / s1 - z * sinw / ss1;
  a[1][1] = ((x * x + z * z) * cosw + y * y) / s1;
  a[1][2] = y * z * (1 - cosw) / s1 + x * sinw / ss1;
  a[2][0] = x * z * (1 - cosw) / s1 + y * sinw / ss1;
  a[2][1] = y * z * (1 - cosw) / s1 - x * sinw / ss1;
  a[2][2] = ((x * x + y * y) * cos(w) + z * z) / s1;
  DB ans[4] = {0, 0, 0, 0}, c[4] = {s.x, s.y, s.z, 1};
  for (int i = 0; i < 4; ++ i)
    for (int j = 0; j < 4; ++ j)
      ans[i] += a[j][i] * c[j];
  return Point(ans[0], ans[1], ans[2]);
}
```

3.2.2 凸包

```
__inline P cross(const P& a, const P& b) {
  return P(
      a.y * b.z - a.z * b.y,
      a.z * b.x - a.x * b.z,
      a.x * b.y - a.y * b.x
        );
}
__inline DB mix(const P& a, const P& b, const P& c) {
  return dot(cross(a, b), c);
```

```
}
__inline DB volume(const P& a, const P& b, const P& c, const P& d) {
  return mix(b - a, c - a, d - a);
}
struct Face {
  int a, b, c;
  __inline Face() {}
  __inline Face(int _a, int _b, int _c):
    a(_a), b(_b), c(_c) {}
  __inline DB area() const {
    return 0.5 * cross(p[b] - p[a], p[c] - p[a]).len();
  }
  __inline P normal() const {
    return cross(p[b] - p[a], p[c] - p[a]).unit();
  }
  __inline DB dis(const P& p0) const {
    return dot(normal(), p0 - p[a]);
  }
};
std::vector<Face> face, tmp;  // Should be O(n).
int mark[N][N], Time, n;
__inline void add(int v) {
  ++ Time;
  clear(tmp);
  for (int i = 0; i < (int)face.size(); ++ i) {
    int a = face[i].a, b = face[i].b, c = face[i].c;
    if (sign(volume(p[v], p[a], p[b], p[c])) > 0) {
      mark[a][b] = mark[b][a] = mark[a][c] =
        mark[c][a] = mark[b][c] = mark[c][b] = Time;
    } else {
      tmp.push_back(face[i]);
    }
  }
  clear(face); face = tmp;
  for (int i = 0; i < (int)tmp.size(); ++ i) {
    int a = face[i].a, b = face[i].b, c = face[i].c;
    if (mark[a][b] == Time) face.emplace_back(v, b, a);
    if (mark[b][c] == Time) face.emplace_back(v, c, b);
    if (mark[c][a] == Time) face.emplace_back(v, a, c);
    assert(face.size() < 500u);
  }
}
void reorder() {
  for (int i = 2; i < n; ++ i) {
    P tmp = cross(p[i] - p[0], p[i] - p[1]);
    if (sign(tmp.len())) {
      std::swap(p[i], p[2]);
      for (int j = 3; j < n; ++ j)
        if (sign(volume(p[0], p[1], p[2], p[j]))) {
          std::swap(p[j], p[3]);
          return;
        }
    }
  }
}
void build_convex() {
  reorder();
  clear(face);
  face.emplace_back(0, 1, 2);
  face.emplace_back(0, 2, 1);
  for (int i = 3; i < n; ++ i)
    add(i);
}
```

3.2.3 最小覆盖球

```
const int eps = 1e-8;
struct Tpoint {
  double x, y, z;
};
int npoint, nouter;
Tpoint pt[200000], outer[4],res;
double radius,tmp;
inline double dist(Tpoint p1, Tpoint p2) {
  double dx=p1.x-p2.x, dy=p1.y-p2.y, dz=p1.z-p2.z;
  return ( dx*dx + dy*dy + dz*dz );
}
inline double dot(Tpoint p1, Tpoint p2) {
  return p1.x*p2.x + p1.y*p2.y + p1.z*p2.z;
}
void ball() {
  Tpoint q[3]; double m[3][3], sol[3], L[3], det;
  int i,j;
  res.x = res.y = res.z = radius = 0;
  switch ( nouter ) {
    case 1: res=outer[0]; break;
    case 2:
        res.x=(outer[0].x+outer[1].x)/2;
        res.y=(outer[0].y+outer[1].y)/2;
        res.z=(outer[0].z+outer[1].z)/2;
        radius=dist(res, outer[0]);
        break;
    case 3:
        for (i=0; i<2; ++i ) {
          q[i].x=outer[i+1].x-outer[0].x;
          q[i].y=outer[i+1].y-outer[0].y;
          q[i].z=outer[i+1].z-outer[0].z;
        }
        for (i=0; i<2; ++i) for(j=0; j<2; ++j)
          m[i][j]=dot(q[i], q[j])*2;
        for (i=0; i<2; ++i ) sol[i]=dot(q[i], q[i]);
        if (fabs(det=m[0][0]*m[1][1]-m[0][1]*m[1][0])<eps)
          return;
        L[0]=(sol[0]*m[1][1]-sol[1]*m[0][1])/det;
        L[1]=(sol[1]*m[0][0]-sol[0]*m[1][0])/det;
        res.x=outer[0].x+q[0].x*L[0]+q[1].x*L[1];
        res.y=outer[0].y+q[0].y*L[0]+q[1].y*L[1];
        res.z=outer[0].z+q[0].z*L[0]+q[1].z*L[1];
        radius=dist(res, outer[0]);
        break;
    case 4:
        for (i=0; i<3; ++i) {
          q[i].x=outer[i+1].x-outer[0].x;
          q[i].y=outer[i+1].y-outer[0].y;
          q[i].z=outer[i+1].z-outer[0].z;
          sol[i]=dot(q[i], q[i]);
        }
        for (i=0;i<3;++i)
          for(j=0;j<3;++j) m[i][j]=dot(q[i],q[j])*2;
        det= m[0][0]*m[1][1]*m[2][2]
          + m[0][1]*m[1][2]*m[2][0]
          + m[0][2]*m[2][1]*m[1][0]
          - m[0][2]*m[1][1]*m[2][0]
          - m[0][1]*m[1][0]*m[2][2]
          - m[0][0]*m[1][2]*m[2][1];
        if ( fabs(det)<eps ) return;
        for (j=0; j<3; ++j) {
          for (i=0; i<3; ++i) m[i][j]=sol[i];
          L[j]=( m[0][0]*m[1][1]*m[2][2]
              + m[0][1]*m[1][2]*m[2][0]
              + m[0][2]*m[2][1]*m[1][0]
              - m[0][2]*m[1][1]*m[2][0]
              - m[0][1]*m[1][0]*m[2][2]
              - m[0][0]*m[1][2]*m[2][1]
            ) / det;
          for (i=0; i<3; ++i)
            m[i][j]=dot(q[i], q[j])*2;
        }
        res=outer[0];
        for (i=0; i<3; ++i ) {
          res.x += q[i].x * L[i];
          res.y += q[i].y * L[i];
          res.z += q[i].z * L[i];
        }
        radius=dist(res, outer[0]);
```

```
  }
}
void minball(int n) {
  ball();
  if ( nouter<4 )
    for (int i=0; i<n; ++i)
      if (dist(res, pt[i])-radius>eps) {
        outer[nouter]=pt[i];
        ++nouter;
        minball(i);
        --nouter;
        if (i>0) {
          Tpoint Tt = pt[i];
          memmove(&pt[1], &pt[0], sizeof(Tpoint)*i);
          pt[0]=Tt;
        }
      }
}
void solve() {
  for (int i=0;i<npoint;i++)
    scanf("%lf%lf%lf",&pt[i].x,&pt[i].y,&pt[i].z);
  random_shuffle(pt, pt + npoint);
  radius=-1;
  for (int i=0;i<npoint;i++){
    if (dist(res,pt[i])-radius>eps){
      nouter=1;
      outer[0]=pt[i];
      minball(i);
    }
  }
  printf("%.5f\n",sqrt(radius));
}
int main(){
  for( ; cin >> npoint && npoint; )
    solve();
  return 0;
}
```

4. 字符串

4.1 AC 自动机

```
int newnode() {
  ++tot;
  memset(ch[tot], 0, sizeof(ch[tot]));
  fail[tot] = 0;
  dep[tot] = 0;
  par[tot] = 0;
  return tot;
}
void insert(char *s,int x) {
  if(*s == '\0') return;
  else {
    int &y = ch[x][*s - 'a'];
    if(y == 0) {
      y = newnode();
      par[y] = x;
      dep[y] = dep[x] + 1;
    }
    insert(s + 1, y);
  }
}
void build() {
  int line[maxn];
  int f = 0, r = 0;
  fail[root] = root;
  for(int i = 0; i < alpha; i++) {
    if(ch[root][i]) {
      fail[ch[root][i]] = root;
      line[r++] = ch[root][i];
    } else {
      ch[root][i] = root;
    }
  }
  while(f != r) {
    int x = line[f++];
    for(int i = 0; i < alpha; i++) {
      if(ch[x][i]) {
        fail[ch[x][i]] = ch[fail[x]][i];
        line[r++] = ch[x][i];
      } else {
        ch[x][i] = ch[fail[x]][i];
      }
    }
  }
}
```

4.2 后缀数组

```
const int MAXN = MAXL * 2 + 1;
int a[MAXN], x[MAXN], y[MAXN], c[MAXN], sa[MAXN],
  rank[MAXN], height[MAXN];
void calc_sa(int n) {
  int m = alphabet, k = 1;
  memset(c, 0, sizeof(*c) * (m + 1));
  for (int i = 1; i <= n; ++i) c[x[i] = a[i]]++;
  for (int i = 1; i <= m; ++i) c[i] += c[i - 1];
  for (int i = n; i; --i) sa[c[x[i]]--] = i;
  for (; k <= n; k <<= 1) {
    int tot = k;
    for (int i = n - k + 1; i <= n; ++i) y[i - n + k] = i;
    for (int i = 1; i <= n; ++i)
      if (sa[i] > k) y[++tot] = sa[i] - k;
    memset(c, 0, sizeof(*c) * (m + 1));
    for (int i = 1; i <= n; ++i) c[x[i]]++;
    for (int i = 1; i <= m; ++i) c[i] += c[i - 1];
    for (int i = n; i; --i) sa[c[x[y[i]]]--] = y[i];
    for (int i = 1; i <= n; ++i) y[i] = x[i];
    tot = 1; x[sa[1]] = 1;
    for (int i = 2; i <= n; ++i) {
      if (max(sa[i], sa[i - 1]) + k > n || y[sa[i]] !=
        y[sa[i - 1]] || y[sa[i] + k] != y[sa[i - 1] +
        k]) ++tot;
      x[sa[i]] = tot;
    }
    if (tot == n) break; else m = tot;
  }
}
void calc_height(int n) {
  for (int i = 1; i <= n; ++i) rank[sa[i]] = i;
  for (int i = 1; i <= n; ++i) {
    height[rank[i]] = max(0, height[rank[i - 1]] - 1);
    if (rank[i] == 1) continue;
    int j = sa[rank[i] - 1];
    while (max(i, j) + height[rank[i]] <= n && a[i +
      height[rank[i]]] == a[j + height[rank[i]]])
      ++height[rank[i]];
  }
}
```

4.3 后缀自动机

```
static const int MAXL = MAXN * 2;  // MAXN is original
  length
static const int alphabet = 26;  // sometimes need
  changing
int l, last, cnt, trans[MAXL][alphabet], par[MAXL],
  sum[MAXL], seq[MAXL], mxl[MAXL], size[MAXL];  // mxl
  is maxlength, size is the size of right
char str[MAXL];
inline void init() {
  l = strlen(str + 1); cnt = last = 1;
  for (int i = 0; i <= l * 2; ++i) memset(trans[i], 0,
    sizeof(trans[i]));
```

```
  memset(par, 0, sizeof(*par) * (l * 2 + 1));
  memset(mxl, 0, sizeof(*mxl) * (l * 2 + 1));
  memset(size, 0, sizeof(*size) * (l * 2 + 1));
}
inline void extend(int pos, int c) {
  int p = last, np = last = ++cnt;
  mxl[np] = mxl[p] + 1; size[np] = 1;
  for (; p && !trans[p][c]; p = par[p]) trans[p][c] = np;
  if (!p) par[np] = 1;
  else {
    int q = trans[p][c];
    if (mxl[p] + 1 == mxl[q]) par[np] = q;
    else {
      int nq = ++cnt;
      mxl[nq] = mxl[p] + 1;
      memcpy(trans[nq], trans[q], sizeof(trans[nq]));
      par[nq] = par[q];
      par[np] = par[q] = nq;
      for (; trans[p][c] == q; p = par[p]) trans[p][c] =
        ↪ nq;
    }
  }
}
inline void buildsam() {
  for (int i = 1; i <= l; ++i) extend(i, str[i] - 'a');
  memset(sum, 0, sizeof(*sum) * (l * 2 + 1));
  for (int i = 1; i <= cnt; ++i) sum[mxl[i]]++;
  for (int i = 1; i <= l; ++i) sum[i] += sum[i - 1];
  for (int i = cnt; i; --i) seq[sum[mxl[i]]--] = i;
  for (int i = cnt; i; --i) size[par[seq[i]]] +=
    ↪ size[seq[i]];
}
```

4.4 广义后缀自动机

```
inline void add_node(int x, int &last) {
  int lastnode = last;
  if (c[lastnode][x]) {
    int nownode = c[lastnode][x];
    if (l[nownode] == l[lastnode] + 1) last = nownode;
    else {
      int auxnode = ++cnt; l[auxnode] = l[lastnode] + 1;
      for (int i = 0; i < alphabet; ++i) c[auxnode][i] =
        ↪ c[nownode][i];
      par[auxnode] = par[nownode]; par[nownode] = auxnode;
      for (; lastnode && c[lastnode][x] == nownode;
        ↪ lastnode = par[lastnode]) {
        c[lastnode][x] = auxnode;
      }
      last = auxnode;
    }
  } else {
    int newnode = ++cnt; l[newnode] = l[lastnode] + 1;
    for (; lastnode && !c[lastnode][x]; lastnode =
      ↪ par[lastnode]) c[lastnode][x] = newnode;
    if (!lastnode) par[newnode] = 1;
    else {
      int nownode = c[lastnode][x];
      if (l[lastnode] + 1 == l[nownode]) par[newnode] =
        ↪ nownode;
      else {
        int auxnode = ++cnt; l[auxnode] = l[lastnode] + 1;
        for (int i = 0; i < alphabet; ++i) c[auxnode][i] =
          ↪ c[nownode][i];
        par[auxnode] = par[nownode]; par[nownode] =
          ↪ par[newnode] = auxnode;
        for (; lastnode && c[lastnode][x] == nownode;
          ↪ lastnode = par[lastnode]) {
          c[lastnode][x] = auxnode;
        }
      }
    }
    last = newnode;
```

```
  }
}
```

4.5 manacher

```
void Manacher(std::string s,int p[]) {
    string t = "$#";
    for (int i = 0; i < s.size(); i++) {
        t += s[i];
        t += "#";
    }
    std::vector<int> p(t.size(), 0);
    int mx = 0, id = 0;
    for (int i = 1; i < t.size(); i++) {
        p[i] = mx > i ? min(p[2 * id - i], mx - i) : 1;
        while (t[i + p[i]] == t[i - p[i]]) ++p[i];
        if (mx < i + p[i]) {
            mx = i + p[i];
            id = i;
        }
    }
}
```

4.6 回文自动机

```
int nT, nStr, last, c[MAXT][26], fail[MAXT], r[MAXN],
  ↪ l[MAXN], s[MAXN];
int allocate(int len) {
  l[nT] = len;
  r[nT] = 0;
  fail[nT] = 0;
  memset(c[nT], 0, sizeof(c[nT]));
  return nT++;
}
void init() {
  nT = nStr = 0;
  int newE = allocate(0);
  int newO = allocate(-1);
  last = newE;
  fail[newE] = newO;
  fail[newO] = newE;
  s[0] = -1;
}
void add(int x) {
  s[++nStr] = x;
  int now = last;
  while (s[nStr - l[now] - 1] != s[nStr]) now = fail[now];
  if (!c[now][x]) {
    int newnode = allocate(l[now] + 2), &newfail =
      ↪ fail[newnode];
    newfail = fail[now];
    while (s[nStr - l[newfail] - 1] != s[nStr]) newfail =
      ↪ fail[newfail];
    newfail = c[newfail][x];
    c[now][x] = newnode;
  }
  last = c[now][x];
  r[last]++;
}
void count() {
  for (int i = nT - 1; i >= 0; i--) {
    r[fail[i]] += r[i];
  }
}
```

4.7 循环串的最小表示

```
// n 必须是 2 的次幂
void fft(Complex a[], int n, int f) {
  for (int i = 0; i < n; ++i)
```

```cpp
    if (R[i] < i) swap(a[i], a[R[i]]);
  for (int i = 1, h = 0; i < n; i <<= 1, h++) {
    Complex wn = Complex(cos(pi / i), f * sin(pi / i));
    Complex w = Complex(1, 0);
    for (int k = 0; k < i; ++k, w = w * wn) tmp[k] = w;
    for (int p = i << 1, j = 0; j < n; j += p) {
      for (int k = 0; k < i; ++k) {
        Complex x = a[j + k], y = a[j + k + i] * tmp[k];
        a[j + k] = x + y; a[j + k + i] = x - y;
      }
    }
  }
}
```

5. 数据结构

5.1 可并堆

```cpp
int merge(int x,int y) {
//p[i] 结点 i 的权值，这里是维护大根堆
//d[i] 在 i 的子树中，i 到右叶子结点的最远距离.
    if(!x) return y;
    if(!y) return x;
    if(p[x] < p[y]) std::swap(x, y);
    r[x] = merge(r[x], y);
    if(r[x]) fa[r[x]] = x;
    if(d[l[x]] < d[r[x]]) std::swap(l[x], r[x]);//调整树的结构，使其满足左偏性质
    d[x] = d[r[x]] + 1;
    return x;
}
```

5.2 KD-Tree

```cpp
long long norm(const long long &x) {
    return std::abs(x);
    return x * x;
}
struct Point {
    int x, y, id;
    const int& operator [] (int index) const {
        if (index == 0) {
            return x;
        } else {
            return y;
        }
    }
    friend long long dist(const Point &a, const Point &b) {
        long long result = 0;
        for (int i = 0; i < 2; ++i) {
            result += norm(a[i] - b[i]);
        }
        return result;
    }
} point[N];
struct Rectangle {
    int min[2], max[2];
    Rectangle() {
        min[0] = min[1] = INT_MAX;  // sometimes int is not enough
        max[0] = max[1] = INT_MIN;
    }
    void add(const Point &p) {
        for (int i = 0; i < 2; ++i) {
            min[i] = std::min(min[i], p[i]);
            max[i] = std::max(max[i], p[i]);
        }
    }
    long long dist(const Point &p) {
        long long result = 0;
        for (int i = 0; i < 2; ++i) {
            result += norm(std::min(std::max(p[i], min[i]), max[i]) - p[i]);
            result += std::max(norm(max[i] - p[i]), norm(min[i] - p[i]));
        }
        return result;
    }
};
struct Node {
    Point seperator;
    Rectangle rectangle;
    int child[2];
    void reset(const Point &p) {
        seperator = p;
        rectangle = Rectangle();
        rectangle.add(p);
        child[0] = child[1] = 0;
    }
} tree[N << 1];
int size, pivot;
bool compare(const Point &a, const Point &b) {
    if (a[pivot] != b[pivot]) {
        return a[pivot] < b[pivot];
    }
    return a.id < b.id;
}
// 左閉右開: build(1, n + 1)
int build(int l, int r, int type = 1) {
    pivot = type;
    if (l >= r) {
        return 0;
    }
    int x = ++size;
    int mid = l + r >> 1;
    std::nth_element(point + l, point + mid, point + r, compare);
    tree[x].reset(point[mid]);
    for (int i = l; i < r; ++i) {
        tree[x].rectangle.add(point[i]);
    }
    tree[x].child[0] = build(l, mid, type ^ 1);
    tree[x].child[1] = build(mid + 1, r, type ^ 1);
    return x;
}
int insert(int x, const Point &p, int type = 1) {
    pivot = type;
    if (x == 0) {
        tree[++size].reset(p);
        return size;
    }
    tree[x].rectangle.add(p);
    if (compare(p, tree[x].seperator)) {
        tree[x].child[0] = insert(tree[x].child[0], p, type ^ 1);
    } else {
        tree[x].child[1] = insert(tree[x].child[1], p, type ^ 1);
    }
    return x;
}
// For minimum distance
// For maximum: 下面递归 query 时 0, 1 换顺序;< and >;min and max
void query(int x, const Point &p, std::pair<long long, int> &answer, int type = 1) {
    pivot = type;
    if (x == 0 || tree[x].rectangle.dist(p) > answer.first) {
        return;
    }
    answer = std::min(answer,
             std::make_pair(dist(tree[x].seperator, p), tree[x].seperator.id));
```

```
    if (compare(p, tree[x].seperator)) {
        query(tree[x].child[0], p, answer, type ^ 1);
        query(tree[x].child[1], p, answer, type ^ 1);
    } else {
        query(tree[x].child[1], p, answer, type ^ 1);
        query(tree[x].child[0], p, answer, type ^ 1);
    }
}
std::priority_queue<std::pair<long long, int> > answer;
void query(int x, const Point &p, int k, int type = 1) {
    pivot = type;
    if (x == 0 || (int)answer.size() == k &&
      ↪ tree[x].rectangle.dist(p) > answer.top().first) {
        return;
    }
    answer.push(std::make_pair(dist(tree[x].seperator, p),
      ↪ tree[x].seperator.id));
    if ((int)answer.size() > k) {
        answer.pop();
    }
    if (compare(p, tree[x].seperator)) {
        query(tree[x].child[0], p, k, type ^ 1);
        query(tree[x].child[1], p, k, type ^ 1);
    } else {
        query(tree[x].child[1], p, k, type ^ 1);
        query(tree[x].child[0], p, k, type ^ 1);
    }
}
```

5.3 Treap

```
struct Node{
  int mn, key, size, tag;
  bool rev;
  Node* ch[2];
  Node(int mn, int key, int size): mn(mn), key(key),
    ↪ size(size), rev(0), tag(0){}
  void downtag();
  Node* update(){
    mn = min(ch[0] -> mn, min(key, ch[1] -> mn));
    size = ch[0] -> size + 1 + ch[1] -> size;
    return this;
  }
};
typedef pair<Node*, Node*> Pair;
Node *null, *root;
void Node::downtag(){
  if(rev){
    for(int i = 0; i < 2; i++)
      if(ch[i] != null){
        ch[i] -> rev ^= 1;
        swap(ch[i] -> ch[0], ch[i] -> ch[1]);
      }
    rev = 0;
  }
  if(tag){
    for(int i = 0; i < 2; i++)
      if(ch[i] != null){
        ch[i] -> key += tag;
        ch[i] -> mn += tag;
        ch[i] -> tag += tag;
      }
    tag = 0;
  }
}
int r(){
  static int s = 3023192386;
  return (s += (s << 3) + 1) & (~0u >> 1);
}
bool random(int x, int y){
  return r() % (x + y) < x;
}
Node* merge(Node *p, Node *q){
  if(p == null) return q;
  if(q == null) return p;
  p -> downtag();
  q -> downtag();
  if(random(p -> size, q -> size)){
    p -> ch[1] = merge(p -> ch[1], q);
    return p -> update();
  }else{
    q -> ch[0] = merge(p, q -> ch[0]);
    return q -> update();
  }
}
Pair split(Node *x, int n){
  if(x == null) return make_pair(null, null);
  x -> downtag();
  if(n <= x -> ch[0] -> size){
    Pair ret = split(x -> ch[0], n);
    x -> ch[0] = ret.second;
    return make_pair(ret.first, x -> update());
  }
  Pair ret = split(x -> ch[1], n - x -> ch[0] -> size -
    ↪ 1);
  x -> ch[1] = ret.first;
  return make_pair(x -> update(), ret.second);
}
pair<Node*, Pair> get_segment(int l, int r){
  Pair ret = split(root, l - 1);
  return make_pair(ret.first, split(ret.second, r - l +
    ↪ 1));
}
int main(){
  null = new Node(INF, INF, 0);
  null -> ch[0] = null -> ch[1] = null;
  root = null;
}
```

5.4 Splay

```
template<class T>void checkmin(T &x,T y) {
  if(y < x) x = y;
}
struct Node {
  Node *c[2], *fa;
  int size, rev;
  LL val, add, min;
  Node *init(LL v) {
    val = min = v;
    add = rev = 0;
    c[0] = c[1] = fa = NULL;
    size = 1;
    return this;
  }
  void rvs() {
    std::swap(c[0], c[1]);
    rev ^= 1;
  }
  void inc(LL x) {
    val += x;
    add += x;
    min += x;
  }
  void pushdown() {
    if(rev) {
      if(c[0]) c[0]->rvs();
      if(c[1]) c[1]->rvs();
      rev = 0;
    }
    if(add) {
      if(c[0]) c[0]->inc(add);
      if(c[1]) c[1]->inc(add);
      add = 0;
    }
```

```cpp
  }
  void update() {
    min = val;
    if(c[0]) checkmin(min, c[0]->min);
    if(c[1]) checkmin(min, c[1]->min);
    size = 1;
    if(c[0]) size += c[0]->size;
    if(c[1]) size += c[1]->size;
  }
} *root;
Node* newnode(LL x) {
  static Node pool[maxs], *p = pool;
  return (++p)->init(x);
}
void setc(Node *x,int t,Node *y) {
  x->c[t] = y;
  if(y) y->fa = x;
}
Node *find(int k) {
  Node *now = root;
  while(true) {
    now->pushdown();
    int t = (now->c[0] ? now->c[0]->size : 0) + 1;
    if(t == k) break;
    if(t > k) now = now->c[0];
    else now = now->c[1], k -= t;
  }
  return now;
}
void rotate(Node *x,Node* &k) {
  Node *y = x->fa, *z = y->fa;
  if(y != k) z->c[z->c[1] == y] = x;
  else k = x;
  x->fa = z;
  int i = (y->c[1] == x);
  setc(y, i, x->c[i ^ 1]);
  setc(x, i ^ 1, y);
  y->update(), x->update();
}
void spaly(Node *x,Node* &k) {
  static Node *st[maxs];
  int top = 0;
  Node *y, *z;
  y = x;
  while(y != k) st[++top] = y, y = y->fa;
  st[++top] = y;
  while(top) st[top]->pushdown(), top--;
  while(x != k) {
    y = x->fa, z = y->fa;
    if(y != k) {
      if((y == z->c[1]) ^ (x == y->c[1])) rotate(x, k);
      else rotate(y, k);
    }
    rotate(x, k);
  }
}
Node *subtree(int l,int r) {
  assert((++l) <= (++r));
  spaly(find(l - 1), root);
  spaly(find(r + 1), root->c[1]);
  return root->c[1]->c[0];
}
void ins(int pos,int v) {
  pos++;
  spaly(find(pos), root);
  spaly(find(pos + 1), root->c[1]);
  setc(root->c[1], 0, newnode(v));
  root->c[1]->update();
  root->update();
}
void del(int pos) {
  pos++;
  spaly(find(pos - 1), root);
  spaly(find(pos + 1), root->c[1]);
  root->c[1]->c[0] = NULL;
  root->c[1]->update();
  root->update();
}
void init() {
  root = newnode(0);
  setc(root, 1, newnode(0));
  root->update();
}
```

5.5 Link cut Tree

```cpp
inline void reverse(int x) {
  tr[x].rev ^= 1; swap(tr[x].c[0], tr[x].c[1]);
}
inline void rotate(int x, int k) {
  int y = tr[x].fa, z = tr[y].fa;
    tr[x].fa = z; tr[z].c[tr[z].c[1] == y] = x;
    tr[tr[x].c[k ^ 1]].fa = y; tr[y].c[k] = tr[x].c[k ^ 1];
    tr[x].c[k ^ 1] = y; tr[y].fa = x;
}
inline void splay(int x, int w) {
  int z = x; pushdown(x);
  while (tr[x].fa != w) {
    int y = tr[x].fa; z = tr[y].fa;
    if (z == w) {
      pushdown(z = y); pushdown(x);
      rotate(x, tr[y].c[1] == x);
      update(y); update(x);
    } else {
      pushdown(z); pushdown(y); pushdown(x);
      int t1 = tr[y].c[1] == x, t2 = tr[z].c[1] == y;
      if (t1 == t2) rotate(y, t2), rotate(x, t1);
      else rotate(x, t1), rotate(x, t2);
      update(z); update(y); update(x);
    }
  }
  update(x);
  if (x != z) par[x] = par[z], par[z] = 0;
}
inline void access(int x) {
  for (int y = 0; x; y = x, x = par[x]) {
    splay(x, 0);
    if (tr[x].c[1]) par[tr[x].c[1]] = x, tr[tr[x].c[1]].fa = 0;
    tr[x].c[1] = y; par[y] = 0; tr[y].fa = x; update(x);
  }
}
inline void makeroot(int x) {
  access(x); splay(x, 0); reverse(x);
}
inline void link(int x, int y) {
  makeroot(x); par[x] = y;
}
inline void cut(int x, int y) {
  access(x); splay(y, 0);
  if (par[y] != x) swap(x, y), access(x), splay(y, 0);
  par[y] = 0;
}
inline void split(int x, int y) {  // x will be the root of the tree
  makeroot(y); access(x); splay(x, 0);
}
```

5.6 树上莫队

```cpp
void dfs(int u) {
  dep[u] = dep[fa[u][0]] + 1;
  for(int i = 1; i < logn; i++)
    fa[u][i] = fa[fa[u][i - 1]][i - 1];
```

```
  stk.push(u);
  for(int i = 0; i < vec[u].size(); i++) {
    int v = vec[u][i];
    if(v == fa[u][0]) continue;
    fa[v][0] = u, dfs(v);
    size[u] += size[v];
    if(size[u] >= bufsize) {
      ++bcnt;
      while(stk.top() != u) {
        block[stk.top()] = bcnt;
        stk.pop();
      }
      size[u] = 0;
    }
  }
  size[u]++;
}
void prework() {
  dfs(1);
  ++bcnt;
  while(!stk.empty()) {
    block[stk.top()] = bcnt;
    stk.pop();
  }
}
void rev(int u) {
  now -= (cnt[val[u]] > 0);
  if(used[u]) {
    cnt[val[u]]--;
    used[u] = false;
  } else {
    cnt[val[u]]++;
    used[u] = true;
  }
  now += (cnt[val[u]] > 0);
}
void move(int &x,int y,int z) {
  int fwd = y;
  rev(getlca(x, z));
  rev(getlca(y, z));
  while(x != y) {
    if(dep[x] < dep[y]) std::swap(x, y);
    rev(x), x = fa[x][0];
  }
  x = fwd;
}
void solve() {
  std::sort(query + 1, query + m + 1);
  int L = 1, R = 1;
  rev(1);
  for(int i = 1; i <= m; i++) {
    int l = query[i].u;
    int r = query[i].v;
    move(L, l, R);
    move(R, r, L);
    ans[query[i].t] = now;
  }
}
```

5.7 CDQ 分治

```
struct Node {
  int x, y, z, idx;
  friend bool operator == (const Node &a,const Node &b) {
    return a.x == b.x && a.y == b.y && a.z == b.z;
  }
  friend bool operator < (const Node &a,const Node &b) {
    return a.y < b.y;
  }
} triple[maxn];
bool cmpx(const Node &a,const Node &b) {
  if(a.x != b.x) return a.x < b.x;
  if(a.y != b.y) return a.y < b.y;
  return a.z < b.z;
}
void solve(int l,int r) {
  if(l == r) return;
  int mid = (l + r) >> 1;
  solve(l, mid);
  static std::pair<Node,int> Lt[maxn], Rt[maxn];
  int Ls = 0, Rs = 0;
  for(int i = l; i <= mid; i++)
    Lt[++Ls] = std::make_pair(triple[i], i);
  for(int i = mid + 1; i <= r; i++)
    Rt[++Rs] = std::make_pair(triple[i], i);
  int pos = 1;
  std::sort(Lt + 1, Lt + Ls + 1);
  std::sort(Rt + 1, Rt + Rs + 1);
  backup.clear();
  for(int i = 1; i <= Rs; i++) {
    while(pos <= Ls && !(Rt[i].first < Lt[pos].first)) {
      insert(Lt[pos].first.z, 1);
      pos++;
    }
    f[Rt[i].second] += query(Rt[i].first.z);
  }
  for(int i = 0; i < backup.size(); i++) pre[backup[i]] =
    ↪ 0;
  solve(mid + 1, r);
}
```

5.8 整体二分

```
void solve(int l,int r,std::vector<int> q) {
  if(l == r || q.empty()) {
    for(int i = 0; i < q.size(); i++) {
      ans[q[i]] = l;
    }
  } else {
    int mid = (l + r) >> 1;
    backup.clear();
    for(int i = l; i <= mid; i++) {
      Event e = event[i];
      if(e.l <= e.r) {
        add(e.l, e.v);
        add(e.r + 1, -e.v);
      } else {
        add(1, e.v);
        add(e.r + 1, -e.v);
        add(e.l, e.v);
      }
    }
    std::vector<int> qL, qR;
    for(int i = 0; i < q.size(); i++) {
      LL val = 0;
      for(int j = 0; j < vec[q[i]].size(); j++) {
        val += count(vec[q[i]][j]);
        if(val >= p[q[i]]) break;
      }
      if(cnt[q[i]] + val >= p[q[i]]) {
        qL.push_back(q[i]);
      } else {
        cnt[q[i]] += val;
        qR.push_back(q[i]);
      }
    }
    for(int i = 0; i < backup.size(); i++) sum[backup[i]]
      ↪ = 0;
    solve(l, mid, qL);
    solve(mid + 1, r, qR);
  }
}
```

6. 图论

6.1 2-SAT tarjan

```cpp
template<class TAT>void checkmin(TAT &x,TAT y) {
  if(y < x) x = y;
}
void tarjan(int u) {
  dfn[u] = low[u] = ++dt;
  flag[u] = true;
  stk.push(u);
  for(int i = 0; i < vec[u].size(); i++) {
    int v = vec[u][i];
    if(!dfn[v]) {
      tarjan(v);
      checkmin(low[u], low[v]);
    }
    else if(flag[v]) {
      checkmin(low[u], dfn[v]);
    }
  }
  if(low[u] == dfn[u]) {
    ++bcnt;
    while(stk.top() != u) {
      block[stk.top()] = bcnt;
      flag[stk.top()] = false;
      stk.pop();
    }
    block[u] = bcnt;
    flag[u] = false;
    stk.pop();
  }
}
bool solve() {
    for(int i = 1; i <= 2 * n; i++)
      if(!dfn[i]) tarjan(i);
    bool ans = true;
    for(int i = 1; i <= n; i++)
      if(block[2 * i] == block[2 * i - 1]) {
        ans = false;
        break;
      }
    return ans;
}
```

6.2 KM

```cpp
struct KM {
  // Truly O(n^3)
  // 邻接矩阵，不能连的边设为 -INF，求最小权匹配时边权取负，但不能连的还是 -INF，使用时先对 1 -> n 调用 hungary() ，再 get_ans() 求值
  int w[N][N];
  int lx[N], ly[N], match[N], way[N], slack[N];
  bool used[N];
  void init() {
    for (int i = 1; i <= n; i++) {
      match[i] = 0;
      lx[i] = 0;
      ly[i] = 0;
      way[i] = 0;
    }
  }
  void hungary(int x) {
    match[0] = x;
    int j0 = 0;
    for (int j = 0; j <= n; j++) {
      slack[j] = INF;
      used[j] = false;
    }
    do {
      used[j0] = true;
      int i0 = match[j0], delta = INF, j1 = 0;
      for (int j = 1; j <= n; j++) {
        if (used[j] == false) {
          int cur = -w[i0][j] - lx[i0] - ly[j];
          if (cur < slack[j]) {
            slack[j] = cur;
            way[j] = j0;
          }
          if (slack[j] < delta) {
            delta = slack[j];
            j1 = j;
          }
        }
      }
      for (int j = 0; j <= n; j++) {
        if (used[j]) {
          lx[match[j]] += delta;
          ly[j] -= delta;
        }
        else slack[j] -= delta;
      }
      j0 = j1;
    } while (match[j0] != 0);
    do {
      int j1 = way[j0];
      match[j0] = match[j1];
      j0 = j1;
    } while (j0);
  }
  int get_ans() {
    int sum = 0;
    for(int i = 1; i <= n; i++) {
      if (w[match[i]][i] == -INF) ; // 无解
      if (match[i] > 0) sum += w[match[i]][i];
    }
    return sum;
  }
} km;
```

6.3 点双连通分量

```cpp
const bool BCC_VERTEX = 0, BCC_EDGE = 1;
struct BCC {  // N = N0 + M0. Remember to call init(&raw_graph).
  Graph *g, forest; // g is raw graph ptr.
  int dfn[N], DFN, low[N];
  int stack[N], top;
  int expand_to[N];    // Where edge i is expanded to in expanded graph.
  // Vertex i expanded to i.
  int compress_to[N];  // Where vertex i is compressed to.
  bool vertex_type[N], cut[N], compress_cut[N], branch[M];
  //std::vector<int> BCC_component[N];  // Cut vertex belongs to none.
  __inline void init(Graph *raw_graph) {
    g = raw_graph;
  }
  void DFS(int u, int pe) {
    dfn[u] = low[u] = ++DFN; cut[u] = false;
    if (!~g->adj[u]) {
      cut[u] = 1;
      compress_to[u] = forest.new_node();
      compress_cut[compress_to[u]] = 1;
    }
    for (int e = g->adj[u]; ~e; e = g->nxt[e]) {
      int v = g->v[e];
      if ((e ^ pe) > 1 && dfn[v] > 0 && dfn[v] < dfn[u]) {
        stack[top++] = e;
        low[u] = std::min(low[u], dfn[v]);
      }
      else if (!dfn[v]) {
        stack[top++] = e; branch[e] = 1;
```

```
        DFS(v, e);
        low[u] = std::min(low[v], low[u]);
        if (low[v] >= dfn[u]) {
          if (!cut[u]) {
            cut[u] = 1;
            compress_to[u] = forest.new_node();
            compress_cut[compress_to[u]] = 1;
          }
          int cc = forest.new_node();
          forest.bi_ins(compress_to[u], cc);
          compress_cut[cc] = 0;
          //BCC_component[cc].clear();
          do {
            int cur_e = stack[--top];
            compress_to[expand_to[cur_e]] = cc;
            compress_to[expand_to[cur_e^1]] = cc;
            if (branch[cur_e]) {
              int v = g->v[cur_e];
              if (cut[v])
                forest.bi_ins(cc, compress_to[v]);
              else {
                //BCC_component[cc].push_back(v);
                compress_to[v] = cc;
              }
            }
          } while (stack[top] != e);
        }
      }
    }
  }
  void solve() {
    forest.init(g->base);
    int n = g->n;
    for (int i = 0; i < g->e; i++) {
      expand_to[i] = g->new_node();
    }
    memset(branch, 0, sizeof(*branch) * g->e);
    memset(dfn + g->base, 0, sizeof(*dfn) * n); DFN = 0;
    for (int i = 0; i < n; i++)
      if (!dfn[i + g->base]) {
        top = 0;
        DFS(i + g->base, -1);
      }
  }
} bcc;

bcc.init(&raw_graph);
bcc.solve();
// Do something with bcc.forest ...
```

6.4 边双连通分量

```
struct BCC {
  Graph *g, forest;
  int dfn[N], low[N], stack[N], tot[N], belong[N], vis[N], top, dfs_clock;
  // tot[] is the size of each BCC, belong[] is the BCC that each node belongs to
  pair<int, int > ori[M]; // bridge in raw_graph(raw node)
  bool is_bridge[M];
  __inline void init(Graph *raw_graph) {
    g = raw_graph;
    memset(is_bridge, false, sizeof(*is_bridge) * g -> e);
    memset(vis + g -> base, 0, sizeof(*vis) * g -> n);
  }
  void tarjan(int u, int from) {
    dfn[u] = low[u] = ++dfs_clock; vis[u] = 1; stack[++top] = u;
    for (int p = g -> adj[u]; ~p; p = g -> nxt[p]) {
      if ((p ^ 1) == from) continue;
      int v = g -> v[p];
      if (vis[v]) {
        if (vis[v] == 1) low[u] = min(low[u], dfn[v]);
      } else {
        tarjan(v, p);
        low[u] = min(low[u], low[v]);
        if (low[v] > dfn[u]) is_bridge[p / 2] = true;
      }
    }
    if (dfn[u] != low[u]) return;
    tot[forest.new_node()] = 0;
    do {
      belong[stack[top]] = forest.n;
      vis[stack[top]] = 2;
      tot[forest.n]++;
      --top;
    } while (stack[top + 1] != u);
  }
  void solve() {
    forest.init(g -> base);
    int n = g -> n;
    for (int i = 0; i < n; ++i)
      if (!vis[i + g -> base]) {
        top = dfs_clock = 0;
        tarjan(i + g -> base, -1);
      }
    for (int i = 0; i < g -> e / 2; ++i)
      if (is_bridge[i]) {
        int e = forest.e;
        forest.bi_ins(belong[g -> v[i * 2]], belong[g -> v[i * 2 + 1]], g -> w[i * 2]);
        ori[e] = make_pair(g -> v[i * 2 + 1], g -> v[i * 2]);
        ori[e + 1] = make_pair(g -> v[i * 2], g -> v[i * 2 + 1]);
      }
  }
} bcc;
```

6.5 最小树形图

```
const int MAXN,INF;// INF >= sum( W_ij )
int from[MAXN + 10][MAXN * 2 + 10],n,m,edge[MAXN + 10][MAXN * 2 + 10];
int sel[MAXN * 2 + 10],fa[MAXN * 2 + 10],vis[MAXN * 2 + 10];
int getfa(int x){if(x == fa[x]) return x; return fa[x] = getfa(fa[x]);}
void liuzhu(){ // 1-base: root is 1, answer = (sel[i], i) for i in [2..n]
  fa[1] = 1;
  for(int i = 2; i <= n; ++i){
    sel[i] = 1; fa[i] = i;
    for(int j = 1; j <= n; ++j) if(fa[j] != i)
      if(from[j][i] = i, edge[sel[i]][i] > edge[j][i]) sel[i] = j;
  }
  int limit = n;
  while(1){
    int prelimit = limit; memset(vis, 0, sizeof(vis)); vis[1] = 1;
    for(int i = 2; i <= prelimit; ++i) if(fa[i] == i && !vis[i]){
      int j = i; while(!vis[j]) vis[j] = i, j = getfa(sel[j]);
      if(j == 1 || vis[j] != i) continue; vector<int> C; int k = j;
      do C.push_back(k), k = getfa(sel[k]); while(k != j);
      ++limit;
      for(int i = 1; i <= n; ++i){
        edge[i][limit] = INF, from[i][limit] = limit;
      }
      fa[limit] = vis[limit] = limit;
      for(int i = 0; i < int(C.size()); ++i){
        int x = C[i], fa[x] = limit;
```

```
      for(int j = 1; j <= n; ++j)
        if(edge[j][x] != INF && edge[j][limit] >
          ↪ edge[j][x] - edge[sel[x]][x]){
          edge[j][limit] = edge[j][x] - edge[sel[x]][x];
          from[j][limit] = x;
        }
    }
    for(int j=1;j<=n;++j) if(getfa(j)==limit)
      ↪ edge[j][limit] = INF;
    sel[limit] = 1;
    for(int j = 1; j <= n; ++j)
      if(edge[sel[limit]][limit] > edge[j][limit])
        ↪ sel[limit] = j;
  }
  if(prelimit == limit) break;
 }
 for(int i = limit; i > 1; --i) sel[from[sel[i]][i]] =
   ↪ sel[i];
}
```

6.6 带花树

```
vector<int> link[maxn];
int n,match[maxn],Queue[maxn],head,tail;
int pred[maxn],base[maxn],start,finish,newbase;
bool InQueue[maxn],InBlossom[maxn];
void push(int u){ Queue[tail++]=u;InQueue[u]=true; }
int pop(){ return Queue[head++]; }
int FindCommonAncestor(int u,int v){
  bool InPath[maxn];
  for(int i=0;i<n;i++) InPath[i]=0;
  while(true){ u=base[u];InPath[u]=true;if(u==start)
    ↪ break;u=pred[match[u]]; }
  while(true){ v=base[v];if(InPath[v])
    ↪ break;v=pred[match[v]]; }
  return v;
}
void ResetTrace(int u){
  int v;
  while(base[u]!=newbase){
    v=match[u];
    InBlossom[base[u]]=InBlossom[base[v]]=true;
    u=pred[v];
    if(base[u]!=newbase) pred[u]=v;
  }
}
void BlossomContract(int u,int v){
  newbase=FindCommonAncestor(u,v);
  for (int i=0;i<n;i++)
  InBlossom[i]=0;
  ResetTrace(u);ResetTrace(v);
  if(base[u]!=newbase) pred[u]=v;
  if(base[v]!=newbase) pred[v]=u;
  for(int i=0;i<n;++i)
  if(InBlossom[base[i]]){
    base[i]=newbase;
    if(!InQueue[i]) push(i);
  }
}
bool FindAugmentingPath(int u){
  bool found=false;
  for(int i=0;i<n;++i) pred[i]=-1,base[i]=i;
  for (int i=0;i<n;i++) InQueue[i]=0;
  start=u;finish=-1; head=tail=0; push(start);
  while(head<tail){
    int u=pop();
    for(int i=link[u].size()-1;i>=0;i--){
      int v=link[u][i];
      if(base[u]!=base[v]&&match[u]!=v)
        if(v==start||(match[v]>=0&&pred[match[v]]>=0))
          BlossomContract(u,v);
        else if(pred[v]==-1){
          pred[v]=u;
          if(match[v]>=0) push(match[v]);
          else{ finish=v; return true; }
        }
    }
  }
  return found;
}
void AugmentPath(){
  int u=finish,v,w;
  while(u>=0){
    ↪ v=pred[u];w=match[v];match[v]=u;match[u]=v;u=w; }
}
void FindMaxMatching(){
  for(int i=0;i<n;++i) match[i]=-1;
  for(int i=0;i<n;++i) if(match[i]==-1)
    ↪ if(FindAugmentingPath(i)) AugmentPath();
}
```

6.7 支配树

```
vector<int> prec[N], succ[N];
vector<int> ord;
int stamp, vis[N];
int num[N];
int fa[N];
void dfs(int u) {
  vis[u] = stamp;
  num[u] = ord.size();
  ord.push_back(u);
  for (int i = 0; i < (int)succ[u].size(); ++i) {
    int v = succ[u][i];
    if (vis[v] != stamp) {
      fa[v] = u;
      dfs(v);
    }
  }
}
int fs[N], mins[N], dom[N], sem[N];
int find(int u) {
  if (u != fs[u]) {
    int v = fs[u];
    fs[u] = find(fs[u]);
    if (mins[v] != -1 && num[sem[mins[v]]] <
      ↪ num[sem[mins[u]]]) {
      mins[u] = mins[v];
    }
  }
  return fs[u];
}
void merge(int u, int v) { fs[u] = v; }
vector<int> buf[N];
int buf2[N];
void mark(int source) {
  ord.clear();
  ++stamp;
  dfs(source);
  for (int i = 0; i < (int)ord.size(); ++i) {
    int u = ord[i];
    fs[u] = u, mins[u] = -1, buf2[u] = -1;
  }
  for (int i = (int)ord.size() - 1; i > 0; --i) {
    int u = ord[i], p = fa[u];
    sem[u] = p;
    for (int j = 0; j < (int)prec[u].size(); ++j) {
      int v = prec[u][j];
      if (use[v] != stamp) continue;
      if (num[v] > num[u]) {
        find(v); v = sem[mins[v]];
      }
      if (num[v] < num[sem[u]]) {
        sem[u] = v;
      }
```

```
    }
    buf[sem[u]].push_back(u);
    mins[u] = u;
    merge(u, p);
    while (buf[p].size()) {
      int v = buf[p].back();
      buf[p].pop_back();
      find(v);
      if (sem[v] == sem[mins[v]]) {
        dom[v] = sem[v];
      } else {
        buf2[v] = mins[v];
      }
    }
  }
  dom[ord[0]] = ord[0];
  for (int i = 0; i < (int)ord.size(); ++i) {
    int u = ord[i];
    if (~buf2[u]) {
      dom[u] = dom[buf2[u]];
    }
  }
}
```

6.8 无向图最小割

```
int cost[maxn][maxn],seq[maxn],len[maxn],n,m,pop,ans;
bool used[maxn];
void Init(){
  int i,j,a,b,c;
  for(i=0;i<n;i++) for(j=0;j<n;j++) cost[i][j]=0;
  for(i=0;i<m;i++){
    scanf("%d %d %d",&a,&b,&c); cost[a][b]+=c; cost[b][a]+=c;
  }
  pop=n; for(i=0;i<n;i++) seq[i]=i;
}
void Work(){
  ans=inf; int i,j,k,l,mm,sum,pk;
  while(pop > 1){
    for(i=1;i<pop;i++) used[seq[i]]=0; used[seq[0]]=1;
    for(i=1;i<pop;i++) len[seq[i]]=cost[seq[0]][seq[i]];
    pk=0; mm=-inf; k=-1;
    for(i=1;i<pop;i++) if(len[seq[i]] > mm){ mm=len[seq[i]]; k=i; }
    for(i=1;i<pop;i++){
      used[seq[l=k]]=1;
      if(i==pop-2) pk=k;
      if(i==pop-1) break;
      mm=-inf;
      for(j=1;j<pop;j++) if(!used[seq[j]])
        if((len[seq[j]]+=cost[seq[l]][seq[j]]) > mm)
          mm=len[seq[j]], k=j;
    }
    sum=0;
    for(i=0;i<pop;i++) if(i != k) sum+=cost[seq[k]][seq[i]];
    ans=min(ans,sum);
    for(i=0;i<pop;i++)
      cost[seq[k]][seq[i]]=cost[seq[i]][seq[k]]+=cost[seq[pk]][seq[i]];
    seq[pk]=seq[--pop];
  }
  printf("%d\n",ans);
}
```

6.9 最大团搜索

```
const int N = 1000 + 7;
vector<vector<bool> > adj;
class MaxClique {
    const vector<vector<bool> > adj;
    const int n;
    vector<int> result, cur_res;
    vector<vector<int> > color_set;
    const double t_limit; // MAGIC
  int para, level;
  vector<pair<int, int> > steps;
public:
    class Vertex {
    public:
        int i, d;
        Vertex(int i, int d = 0) : i(i), d(d) {}
    };
    void reorder(vector<Vertex> &p) {
        for (auto &u : p) {
            u.d = 0;
            for (auto v : p) u.d += adj[v.i][u.i];
        }
        sort(p.begin(), p.end(), [&](const Vertex &a, const Vertex &b) { return a.d > b.d; } );
    }
  // reuse p[i].d to denote the maximum possible clique for first i vertices.
    void init_color(vector<Vertex> &p) {
        int maxd = p[0].d;
        for (int i = 0; i < p.size(); i++) p[i].d = min(i, maxd) + 1;
    }
    bool bridge(const vector<int> &s, int x) {
        for (auto v : s) if (adj[v][x]) return true;
        return false;
    }
  // approximate estimate the p[i].d
  // Do not care about first mink color class (For better result, we must get some vertex in some color class larger than mink )
    void color_sort(vector<Vertex> &cur) {
        int totc = 0, ptr = 0, mink = max((int)result.size() - (int)cur_res.size(), 0);
        for (int i = 0; i < cur.size(); i++) {
            int x = cur[i].i, k = 0;
            while (k < totc && bridge(color_set[k], x)) k++;
            if (k == totc) color_set[totc++].clear();
            color_set[k].push_back(x);
            if (k < mink) cur[ptr++].i = x;
        }
        if (ptr) cur[ptr - 1].d = 0;
        for (int i = mink; i < totc; i ++) {
            for (auto v : color_set[i]) {
                cur[ptr++] = Vertex(v, i + 1);
            }
        }
    }
    void expand(vector<Vertex> &cur) {
    steps[level].second = steps[level].second - steps[level].first + steps[level - 1].first;
    steps[level].first = steps[level - 1].second;
        while (cur.size()) {
            if (cur_res.size() + cur.back().d <= result.size()) return ;
            int x = cur.back().i;
            cur_res.push_back(x); cur.pop_back();
            vector<Vertex> remain;
            for (auto v : cur) {
                if (adj[v.i][x]) remain.push_back(v.i);
            }
            if (remain.size() == 0) {
                if (cur_res.size() > result.size()) result = cur_res;
            } else {
        // Magic ballance.
        if (1. * steps[level].second / ++para < t_limit) reorder(remain);
```

```
                color_sort(remain);
        steps[level++].second++;
                expand(remain);
        level--;
            }
            cur_res.pop_back();
        }
    }
public:
    MaxClique(const vector<vector<bool> > &_adj, int n, double tt = 0.025) : adj(_adj), n(n), t_limit(tt) {
        result.clear();
        cur_res.clear();
        color_set.resize(n);
    steps.resize(n + 1);
    fill(steps.begin(), steps.end(), make_pair(0, 0));
    level = 1;
    para = 0;
    }
    vector<int> solve() {
        vector<Vertex> p;
        for (int i = 0; i < n; i++) p.push_back(Vertex(i));
        reorder(p);
        init_color(p);
        expand(p);
        return result;
    }
};
```

6.10 斯坦纳树

```
void SPFA(int *dist) {
    static int line[maxn + 5];
    static bool hash[maxn + 5];
    int f = 0, r = 0;
    for(int i = 1; i <= N; i++)
        if(dist[i] < inf) {
            line[r] = i;
            hash[i] = true;
            r = (r + 1) % (N + 1);
        }
    while(f != r) {
        int t = line[f];
        hash[t] = false;
        f = (f + 1) % (N + 1);
        for(int i = head[t]; i ; i = edge[i].next) {
            int v = edge[i].v, dt = dist[t] + edge[i].w;
            if(dt < dist[v]) {
                dist[v] = dt;
                if(!hash[v]) {
                    if(dist[v] < dist[line[f]]) {
                        f = (f + N) % (N + 1);
                        line[f] = v;
                    }
                    else {
                        line[r] = v;
                        r = (r + 1) % (N + 1);
                    }
                    hash[v] = true;
                }
            }
        }
    }
}
void solve() {
    for(int i = 1; i <= S; i++) {
        for(int j = 1; j <= N; j++)
            for(int k = (i - 1) & i; k ; k = (k - 1) & i)
                G[i][j] = std::min(G[i][j], G[k][j] + G[k ^ i][j]);
        SPFA(G[i]);
    }
}
```

6.11 虚树

```
bool cmp(const int lhs,const int rhs) {
  return dfn[lhs] < dfn[rhs];
}
void build() {
  std::sort(h + 1, h + 1 + m, cmp);
  int top = 0;
  for (int i = 1; i <= m; i++) {
    if (!top) father[st[++top] = h[i]] = 0;
    else {
        int p = h[i], lca = LCA(h[i],st[top]);
        while(d[st[top]] > d[lca]) {
            if (d[st[top - 1]] <= d[lca])
                father[st[top]] = lca;
            top--;
        }
        if (st[top] != lca) {
            t[++tot] = lca;
            father[lca] = st[top];
            st[++top] = lca;
        }
        father[p] = lca;
        st[++top] = p;
    }
  }
}
```

6.12 点分治

```
template<class TAT>void checkmax(TAT &x,TAT y) {
  if(x < y) x = y;
}
template<class TAT>void checkmin(TAT &x,TAT y) {
  if(y < x) x = y;
}
void getsize(int u,int fa) {
  size[u] = 1;
  smax[u] = 0;
  for(int i = 0; i < G[u].size(); i++) {
    int v = G[u][i];
    if(v == fa || ban[v]) continue;
    getsize(v, u);
    size[u] += size[v];
    checkmax(smax[u], size[v]);
  }
}
int getroot(int u,int ts,int fa) {
  checkmax(smax[u], ts - size[u]);
  int res = u;
  for(int i = 0; i < G[u].size(); i++) {
    int v = G[u][i];
    if(v == fa || ban[v]) continue;
    int w = getroot(v, ts, u);
    if(smax[w] < smax[res]) res = w;
  }
  return res;
}
void solve() {
  static int line[maxn];
  static std::vector<int> vec;
  int f = 0, r = 0;
  line[r++] = 1;
  while(f != r) {
    int u = line[f++];
    getsize(u, 0);
    u = getroot(u, size[u], 0);
    ban[u] = true;
```

```
    vec.clear();
    for(int i = 0; i < G[u].size(); i++)
      if(!ban[G[u][i]]) vec.push_back(G[u][i]);
    for(int i = 0; i < vec.size(); i++)
      line[r++] = vec[i];
  }
}
```

6.13 最小割最大流

```
bool BFS() {
    for(int i = 1; i <= ind; i++) dep[i] = 0;
    dep[S] = 1, line.push(S);
    while(!line.empty()) {
        int now = line.front();
        line.pop();
        for(int i = head[now], p; i ; i = edge[i].next)
            if(edge[i].cap && !dep[p = edge[i].v])
                dep[p] = dep[now] + 1, line.push(p);
    }
    if(dep[T]) {
        for(int i = 1; i <= ind; i++)
            cur[i] = head[i];
        return true;
    } else return false;
}
int DFS(int a,int flow) {
    if(a == T) return flow;
    int ret = 0;
    for(int &i = cur[a], p; i ; i = edge[i].next)
        if(dep[p = edge[i].v] == dep[a] + 1 &&
           ↪ edge[i].cap) {
            int ff = DFS(p, std::min(flow, edge[i].cap));
            flow -= ff, edge[i].cap -= ff;
            ret += ff, edge[i ^ 1].cap += ff;
            if(!flow) break;
        }
        return ret;
}
int solve() {
    int totflow = 0;
    while(BFS())
        totflow += DFS(S, INF);
    }
    return totflow;
}
```

6.14 最小费用流

```
bool SPFA() {
    static int line[maxv];
    static bool hash[maxv];
    register int f = 0, r = 0;
  for(int i = 1; i <= ind; i++) {
      dist[i] = inf;
      from[i] = 0;
  }
    dist[S] = 0, line[r] = S, r = (r + 1) % maxv;
    hash[S] = true;
    while(f != r) {
        int x = line[f];
        line[f] = 0, f = (f + 1) % maxv;
        hash[x] = false;
        for(int i = head[x]; i; i = edge[i].next)
            if(edge[i].cap) {
                int v = edge[i].v;
                int w = dist[x] + edge[i].cost;
                if(w < dist[v]) {
                    dist[v] = w;
                    from[v] = i;
                    if(!hash[v]) {
                        if(f != r && dist[v] <=
                           ↪ dist[line[f]])
                            f = (f - 1 + maxv) % maxv,
                              ↪ line[f] = v;
                        else line[r] = v, r = (r + 1) %
                           ↪ maxv;
                        hash[v] = true;
                    }
                }
            }
    }
    return from[T];
}
int back(int x,int flow) {
  if(from[x]) {
    flow = back(edge[from[x] ^ 1].v, std::min(flow,
      ↪ edge[from[x]].cap));
    edge[from[x]].cap -= flow;
    edge[from[x] ^ 1].cap += flow;
  }
  return flow;
}
int solve() {
    int mincost = 0, maxflow = 0;
    while(SPFA()) {
        int flow = back(T, inf);
        mincost += dist[T] * flow;
        maxflow += flow;
    }
    return mincost;
}
```

6.15 zkw 费用流

```
int S, T, totFlow, totCost;
int dis[N], slack[N], visit[N];
int modlable () {
    int delta = INF;
    for (int i = 1; i <= T; i++) {
        if (!visit[i] && slack[i] < delta) delta =
           ↪ slack[i];
        slack[i] = INF;
    }
    if (delta == INF) return 1;
    for (int i = 1; i <= T; i++)
        if (visit[i]) dis[i] += delta;
    return 0;
}
int dfs (int x, int flow) {
    if (x == T) {
        totFlow += flow;
        totCost += flow * (dis[S] - dis[T]);
        return flow;
    }
    visit[x] = 1;
    int left = flow;
    for (int i = e.last[x]; ~i; i = e.succ[i])
        if (e.cap[i] > 0 && !visit[e.other[i]]) {
            int y = e.other[i];
            if (dis[y] + e.cost[i] == dis[x]) {
                int delta = dfs (y, min (left, e.cap[i]));
                e.cap[i] -= delta;
                e.cap[i ^ 1] += delta;
                left -= delta;
                if (!left) { visit[x] = 0; return flow; }
            } else {
                slack[y] = min (slack[y], dis[y] +
                   ↪ e.cost[i] - dis[x]);
            }
        }
    return flow - left;
}
pair <int, int> minCost () {
```

```
    totFlow = 0; totCost = 0;
    fill (dis + 1, dis + T + 1, 0);
    do {
        do {
            fill (visit + 1, visit + T + 1, 0);
        } while (dfs (S, INF));
    } while (!modlable ());
    return make_pair (totFlow, totCost);
}
```

6.16 最小割树

```
int
   ↪ cnt,n,m,dis[N],last[N],a[N],tmp[N],ans[N][N],s,t,mark[N];
struct edge{int to,c,next;}e[N*200];
queue <int> q;
void addedge(int u,int v,int c) {
    e[++cnt].to=v;e[cnt].c=c;
  e[cnt].next=last[u];last[u]=cnt;
    e[++cnt].to=u;e[cnt].c=c;
  e[cnt].next=last[v];last[v]=cnt;
}
bool bfs() {
    memset(dis,0,sizeof(dis));
    dis[s]=2;
    while (!q.empty()) q.pop();
    q.push(s);
    while (!q.empty()) {
        int u=q.front();
        q.pop();
        for (int i=last[u];i;i=e[i].next)
            if (e[i].c&&!dis[e[i].to]) {
                dis[e[i].to]=dis[u]+1;
                if (e[i].to==t) return 1;
                q.push(e[i].to);
            }
    }
    return 0;
}
int dfs(int x,int maxf) {
    if (x==t||!maxf) return maxf;
    int ret=0;
    for (int i=last[x];i;i=e[i].next)
        if (e[i].c&&dis[e[i].to]==dis[x]+1) {
            int f=dfs(e[i].to,min(e[i].c,maxf-ret));
            e[i].c-=f;
            e[i^1].c+=f;
            ret+=f;
            if (ret==maxf) break;
        }
    if (!ret) dis[x]=0;
    return ret;
}
void dfs(int x) {
    mark[x]=1;
    for (int i=last[x];i;i=e[i].next)
        if (e[i].c&&!mark[e[i].to]) dfs(e[i].to);
}
void solve(int l,int r) {
    if (l==r) return;
    s=a[l];t=a[r];
    for (int i=2;i<=cnt;i+=2)
        e[i].c=e[i^1].c=(e[i].c+e[i^1].c)/2;
    int flow=0;
    while (bfs()) flow+=dfs(s,inf);
    memset(mark,0,sizeof(mark));
    dfs(s);
    for (int i=1;i<=n;i++)
        if (mark[i])
            for (int j=1;j<=n;j++)
                if (!mark[j])
                    ans[i][j]=ans[j][i]=min(ans[i][j],flow);
    int i=l,j=r;
    for (int k=l;k<=r;k++)
        if (mark[a[k]]) tmp[i++]=a[k];
        else tmp[j--]=a[k];
    for (int k=l;k<=r;k++)
        a[k]=tmp[k];
    solve(l,i-1);
    solve(j+1,r);
}
```

6.17 上下界网络流建图

$B(u,v)$ 表示边 (u,v) 流量的下界，$C(u,v)$ 表示边 (u,v) 流量的上界，$F(u,v)$ 表示边 (u,v) 的流量。设 $G(u,v)=F(u,v)-B(u,v)$，显然有

$$0 \le G(u,v) \le C(u,v) - B(u,v)$$

6.17.1 无源汇的上下界可行流

建立超级源点 S^* 和超级汇点 T^*，对于原图每条边 (u,v) 在新网络中连如下三条边：$S^* \to v$，容量为 $B(u,v)$；$u \to T^*$，容量为 $B(u,v)$；$u \to v$，容量为 $C(u,v)-B(u,v)$。最后求新网络的最大流，判断从超级源点 S^* 出发的边是否都满流即可，边 (u,v) 的最终解中的实际流量为 $G(u,v)+B(u,v)$。

6.17.2 有源汇的上下界可行流

从汇点 T 到源点 S 连一条上界为 ∞，下界为 0 的边。按照**无源汇的上下界可行流**一样做即可，流量即为 $T \to S$ 边上的流量。

6.17.3 有源汇的上下界最大流

1. 在**有源汇的上下界可行流**中，从汇点 T 到源点 S 的边改为连一条上界为 ∞，下届为 x 的边。x 满足二分性质，找到最大的 x 使得新网络存在**无源汇的上下界可行流**即为原图的最大流。

2. 从汇点 T 到源点 S 连一条上界为 ∞，下界为 0 的边，变成无源汇的网络。按照**无源汇的上下界可行流**的方法，建立超级源点 S^* 和超级汇点 T^*，求一遍 $S^* \to T^*$ 的最大流，再将从汇点 T 到源点 S 的这条边拆掉，求一次 $S \to T$ 的最大流即可。

6.17.4 有源汇的上下界最小流

1. 在**有源汇的上下界可行流**中，从汇点 T 到源点 S 的边改为连一条上界为 x，下界为 0 的边。x 满足二分性质，找到最小的 x 使得新网络存在**无源汇的上下界可行流**即为原图的最小流。

2. 按照**无源汇的上下界可行流**的方法，建立超级源点 S^* 与超级汇点 T^*，求一遍 $S^* \to T^*$ 的最大流，但是注意这一次不加上汇点 T 到源点 S 的这条边，即不使之改为无源汇的网络去求解。求完后，再加上那条汇点 T 到源点 S 上界 ∞ 的边。因为这条边下界为 0，所以 S^*，T^* 无影响，再直接求一次 $S^* \to T^*$ 的最大流。若超级源点 S^* 出发的边全部满流，则 $T \to S$ 边上的流量即为原图的最小流，否则无解。

7. 其他

7.1 Dancing Links

7.1.1 精确覆盖

```
#pragma comment(linker, "/STACK:1024000000,1024000000")
#define maxn 1000005
using namespace std;
int head,sz;
int U[maxn],D[maxn],L[maxn],R[maxn];
int H[maxn],ROW[maxn],C[maxn],S[maxn],O[maxn];
void remove(int c) {
```

```
    L[R[c]]=L[c];
    R[L[c]]=R[c];
    for(int i=D[c]; i!=c; i=D[i])
        for(int j=R[i]; j!=i; j=R[j]) {
            U[D[j]]=U[j];
            D[U[j]]=D[j];
            --S[C[j]];
        }
}
void resume(int c) {
    for(int i=U[c]; i!=c; i=U[i]) {
        for(int j=L[i]; j!=i; j=L[j]) {
            ++S[C[j]];
            U[D[j]]=j;
            D[U[j]]=j;
        }
    }
    L[R[c]]=c;
    R[L[c]]=c;
}
void init(int m) {
    head=0;//头指针为 0
    for(int i=0; i<=m; i++) {
        U[i]=i;
        D[i]=i;//建立双向十字链表
        L[i]=i-1;
        R[i]=i+1;
        S[i]=0;
    }
    R[m]=0;
    L[0]=m;
    S[0]=INF+1;
    sz=m+1;
    memset(H,0,sizeof(H));
}
void insert(int i, int j) {
    if(H[i]) {
        L[sz] = L[H[i]];
        R[sz] = H[i];
        L[R[sz]] = sz;
        R[L[sz]] = sz;
    }
    else {
        L[sz] = sz;
        R[sz] = sz;
        H[i] = sz;
    }
    U[sz] = U[j];
    D[sz] = j;
    U[D[sz]] = sz;
    D[U[sz]] = sz;
    C[sz] = j;
    ROW[sz] = i;
    ++S[j];
    ++sz;
}
bool dfs(int k,int len) {
    if(R[head]==head) return true;
    int s=INF,c;
    for (int t=R[head]; t!=head; t=R[t])
        if (S[t]<s) s=S[t],c=t;
    remove(c);
    for(int i=D[c]; i!=c; i=D[i]) {
        O[k]=ROW[i];
        for(int j=R[i]; j!=i; j=R[j])
            remove(C[j]);
        if(dfs(k+1,len))
            return  true;
        for(int j=L[i]; j!=i; j=L[j])
            resume(C[j]);
    }
    resume(c);
    return  false;
}
```

7.1.2 重复覆盖

```
int h()
{
    int i,j,k,count=0;
    bool visit[N];
    memset(visit,0,sizeof(visit));
    for(i=R[0];i;i=R[i])
    {
        if(visit[i]) continue;
        count++;
        visit[i]=1;
        for(j=D[i];j!=i;j=D[j])
        {
            for(k=R[j];k!=j;k=R[k])
                visit[C[k]]=1;
        }
    }
    return count;
}
void Dance(int k)
{
    int i,j,c,Min,ans;
    ans=h();
    if(k+ans>K || k+ans>=ak) return;
    if(!R[0])
    {
        if(k<ak) ak=k;
        return;
    }
    for(Min=N,i=R[0];i;i=R[i])
        if(S[i]<Min) Min=S[i],c=i;
    for(i=D[c];i!=c;i=D[i])
    {
        remove(i);
        for(j=R[i];j!=i;j=R[j])
            remove(j);
        Dance(k+1);
        for(j=L[i];j!=i;j=L[j])
            resume(j);
        resume(i);
    }
    return;
}
```

7.2 蔡勒公式

```
int zeller(int y,int m,int d) {
  if (m<=2) y--,m+=12; int c=y/100; y%=100;
  int w=((c>>2)-(c<<1)+y+(y>>2)+(13*(m+1)/5)+d-1)%7;
  if (w<0) w+=7; return(w);
}
```

7.3 五边形数定理

$$p(n) = \sum_{k\in\mathbb{Z}\setminus\{0\}}(-1)^{k-1}p(n-\frac{k(3k-1)}{2})$$

```
LL dp[N],fi[N];
LL five(LL x){ return (3*x*x-x)/2; }
void wbxs(){
    dp[0]=1;
    int t=1000; //其实可以等于 sqrt(N)
    for(int i=-t;i<=t;++i)
        fi[i+t]=five(i);    //Q
    for(int i=1;i<=100000;++i){
        int flag=1;
        for(int j=1;;++j){
            LL a=fi[j+t],b=fi[-j+t];
            if(a>i && b>i) break;
```

```cpp
            if(a<=i) dp[i]=(dp[i]+dp[i-a]*flag+MOD)%MOD; //p
            if(b<=i) dp[i]=(dp[i]+dp[i-b]*flag+MOD)%MOD;
            flag*=-1;
        }
    }
}
```

7.4 凸包闵可夫斯基和

```cpp
// cv[0..1] 为两个顺时针凸包，其中起点等于终点，求出的闵可夫斯基和不一定是严格凸包
int i[2] = {0, 0}, len[2] = {(int)cv[0].size() - 1, (int)cv[1].size() - 1};
vector<P> mnk;
mnk.push_back(cv[0][0] + cv[1][0]);
do {
  int d((cv[0][i[0] + 1] - cv[0][i[0]]) * (cv[1][i[1] + 1] - cv[1][i[1]]) >= 0);
  mnk.push_back(cv[d][i[d] + 1] - cv[d][i[d]] + mnk.back());
  i[d] = (i[d] + 1) % len[d];
} while(i[0] || i[1]);
```

8. 技巧

8.1 STL 归还空间

```cpp
template <typename T>
__inline void clear(T& container) {
  container.clear(); // 或者删除了一堆元素
  T(container).swap(container);
}
```

8.2 大整数取模

```cpp
// 需要保证 x 和 y 非负
long long mult(long long x, long long y, long long MODN) {
  long long t = (x * y - (long long)((long double)x / MODN * y + 1e-3) * MODN) % MODN;
  return t < 0 ? t + MODN : t;
}
```

8.3 读入优化

```cpp
// getchar() 读入优化 << 关同步 cin << 此优化
// 用 isdigit() 会小幅变慢
// 返回 false 表示读到文件尾
namespace Reader {
    const int L = (1 << 15) + 5;
    char buffer[L], *S, *T;
    __inline bool getchar(char &ch) {
        if (S == T) {
            T = (S = buffer) + fread(buffer, 1, L, stdin);
            if (S == T) {
        ch = EOF;
        return false;
      }
        }
    ch = *S++;
    return true;
    }
    __inline bool getint(int &x) {
    char ch; bool neg = 0;
    for (; getchar(ch) && (ch < '0' || ch > '9'); ) neg ^= ch == '-';
    if (ch == EOF) return false;
    x = ch - '0';
    for (; getchar(ch), ch >= '0' && ch <= '9'; )
      x = x * 10 + ch - '0';
    if (neg) x = -x;
    return true;
    }
}
```

8.4 二次随机法

```cpp
#include <random>

int main() {
    std::mt19937 g(seed);  // std::mt19937_64
    std::cout << g() << std::endl;
}
```

8.5 vimrc

```vim
set ruler
set number
set smartindent
set autoindent
set tabstop=4
set softtabstop=4
set shiftwidth=4
set hlsearch
set incsearch
set autoread
set backspace=2
set mouse=a

syntax on

nmap <C-A> ggVG
vmap <C-C> "+y

filetype plugin indent on

autocmd FileType cpp set cindent
autocmd FileType cpp map <F9> :!g++ % -o %< -g -std=c++11 -Wall -Wextra -Wconversion && size %< <CR>
autocmd FileType cpp map <C-F9> :!g++ % -o %< -std=c++11 -O2 && size %< <CR>
autocmd FileType cpp map <F8> :!time ./%< < %<.in <CR>
autocmd FileType cpp map <F5> :!time ./%< <CR>

map <F3> :vnew %<.in <CR>
map <F4> :!gedit % <CR>
```

8.6 控制 cout 输出实数精度

```cpp
std::cout << std::fixed << std::setprecision(5);
```

8.7 汇编技巧

```cpp
O3优化
#define __ __attribute__ ((optimize("-O3")))
#define _ __ __inline __attribute__ ((__gnu_inline__, __always_inline__, __artificial__))

汇编开栈
#pragma comment(linker, "/STACK:256000000")

int __size = 256 << 20;
char* __p__ = (char *) malloc(__size__) + __size__;

int main() {
  __asm__("movl %0, %%esp\n" :: "r"(__p__));
  return 0;
}
```

9. 提示

9.1 线性规划转对偶

$$\begin{array}{l}\text{maximize } \mathbf{c}^T\mathbf{x}\\ \text{subject to } \mathbf{A}\mathbf{x} \le \mathbf{b}, \mathbf{x} \ge 0\end{array} \Longleftrightarrow \begin{array}{l}\text{minimize } \mathbf{y}^T\mathbf{b}\\ \text{subject to } \mathbf{y}^T\mathbf{A} \ge \mathbf{c}^T, \mathbf{y} \ge 0\end{array}$$

9.2 NTT 素数及其原根

Prime	Primitive root
1053818881	7
1051721729	6
1045430273	3
1012924417	5
1007681537	3

9.3 积分表

1. $\int \frac{\mathrm{d}x}{ax^2+bx+c} = \begin{cases} \frac{2}{\sqrt{4ac-b^2}} \arctan \frac{2ax+b}{\sqrt{4ac-b^2}} + C & (b^2 < 4ac) \\ \frac{1}{\sqrt{b^2-4ac}} \ln \left| \frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}} \right| + C & (b^2 > 4ac) \end{cases}$
2. $\int \frac{x}{ax^2+bx+c}\mathrm{d}x = \frac{1}{2a}\ln|ax^2+bx+c| - \frac{b}{2a}\int \frac{\mathrm{d}x}{ax^2+bx+c}$

1. $\int \frac{\mathrm{d}x}{\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{a}}\ln|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}| + C$
2. $\int \sqrt{ax^2+bx+c}\mathrm{d}x = \frac{2ax+b}{4a}\sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8\sqrt{a^3}}\ln|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}| + C$
3. $\int \frac{x}{\sqrt{ax^2+bx+c}}\mathrm{d}x = \frac{1}{a}\sqrt{ax^2+bx+c} - \frac{b}{2\sqrt{a^3}}\ln|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}| + C$
4. $\int \frac{\mathrm{d}x}{\sqrt{c+bx-ax^2}} = -\frac{1}{\sqrt{a}}\arcsin\frac{2ax-b}{\sqrt{b^2+4ac}} + C$
5. $\int \sqrt{c+bx-ax^2}\mathrm{d}x = \frac{2ax-b}{4a}\sqrt{c+bx-ax^2} + \frac{b^2+4ac}{8\sqrt{a^3}}\arcsin\frac{2ax-b}{\sqrt{b^2+4ac}} + C$
6. $\int \frac{x}{\sqrt{c+bx-ax^2}}\mathrm{d}x = -\frac{1}{a}\sqrt{c+bx-ax^2} + \frac{b}{2\sqrt{a^3}}\arcsin\frac{2ax-b}{\sqrt{b^2+4ac}} + C$

1. $\int \frac{\mathrm{d}x}{\sqrt{(x-a)(b-x)}} = 2\arcsin\sqrt{\frac{x-a}{b-x}} + C \ (a<b)$
2.

$$\int \sqrt{(x-a)(b-x)}\mathrm{d}x = \frac{2x-a-b}{4}\sqrt{(x-a)(b-x)} + \frac{(b-a)^2}{4}\arcsin\sqrt{\frac{x-a}{b-x}} + C, (a<b) \quad (1)$$

1. $\int \tan x\mathrm{d}x = -\ln|\cos x| + C$
2. $\int \cot x\mathrm{d}x = \ln|\sin x| + C$
3. $\int \sec x\mathrm{d}x = \ln\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| + C = \ln|\sec x + \tan x| + C$
4. $\int \csc x\mathrm{d}x = \ln\left|\tan\frac{x}{2}\right| + C = \ln|\csc x - \cot x| + C$
5. $\int \sec^2 x\mathrm{d}x = \tan x + C$
6. $\int \csc^2 x\mathrm{d}x = -\cot x + C$
7. $\int \sec x\tan x\mathrm{d}x = \sec x + C$
8. $\int \csc x\cot x\mathrm{d}x = -\csc x + C$
9. $\int \sin^2 x\mathrm{d}x = \frac{x}{2} - \frac{1}{4}\sin 2x + C$
10. $\int \cos^2 x\mathrm{d}x = \frac{x}{2} + \frac{1}{4}\sin 2x + C$
11. $\int \sin^n x\mathrm{d}x = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int\sin^{n-2}x\mathrm{d}x$
12. $\int \cos^n x\mathrm{d}x = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int\cos^{n-2}x\mathrm{d}x$
13. $\int \frac{\mathrm{d}x}{\sin^n x} = -\frac{1}{n-1}\frac{\cos x}{\sin^{n-1}x} + \frac{n-2}{n-1}\int\frac{\mathrm{d}x}{\sin^{n-2}x}$
14. $\int \frac{\mathrm{d}x}{\cos^n x} = \frac{1}{n-1}\frac{\sin x}{\cos^{n-1}x} + \frac{n-2}{n-1}\int\frac{\mathrm{d}x}{\cos^{n-2}x}$
15.

$$\begin{aligned}&\int \cos^m x\sin^n x\mathrm{d}x\\ =&\frac{1}{m+n}\cos^{m-1}x\sin^{n+1}x + \frac{m-1}{m+n}\int\cos^{m-2}x\sin^n x\mathrm{d}x\\ =&-\frac{1}{m+n}\cos^{m+1}x\sin^{n-1}x + \frac{n-1}{m+1}\int\cos^m x\sin^{n-2}x\mathrm{d}x\end{aligned}$$

16. $\int \frac{\mathrm{d}x}{a+b\sin x} = \begin{cases} \frac{2}{\sqrt{a^2-b^2}}\arctan\frac{a\tan\frac{x}{2}+b}{\sqrt{a^2-b^2}} + C & (a^2>b^2) \\ \frac{1}{\sqrt{b^2-a^2}}\ln\left|\frac{a\tan\frac{x}{2}+b-\sqrt{b^2-a^2}}{a\tan\frac{x}{2}+b+\sqrt{b^2-a^2}}\right| + C & (a^2<b^2)\end{cases}$
17. $\int \frac{\mathrm{d}x}{a+b\cos x} = \begin{cases} \frac{2}{a+b}\sqrt{\frac{a+b}{a-b}}\arctan\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right) + C & (a^2>b^2) \\ \frac{1}{a+b}\sqrt{\frac{a+b}{a-b}}\ln\left|\frac{\tan\frac{x}{2}+\sqrt{\frac{a+b}{b-a}}}{\tan\frac{x}{2}-\sqrt{\frac{a+b}{b-a}}}\right| + C & (a^2<b^2)\end{cases}$
18. $\int \frac{\mathrm{d}x}{a^2\cos^2 x+b^2\sin^2 x} = \frac{1}{ab}\arctan\left(\frac{b}{a}\tan x\right) + C$
19. $\int \frac{\mathrm{d}x}{a^2\cos^2 x-b^2\sin^2 x} = \frac{1}{2ab}\ln\left|\frac{b\tan x+a}{b\tan x-a}\right| + C$
20. $\int x\sin ax\mathrm{d}x = \frac{1}{a^2}\sin ax - \frac{1}{a}x\cos ax + C$
21. $\int x^2\sin ax\mathrm{d}x = -\frac{1}{a}x^2\cos ax + \frac{2}{a^2}x\sin ax + \frac{2}{a^3}\cos ax + C$
22. $\int x\cos ax\mathrm{d}x = \frac{1}{a^2}\cos ax + \frac{1}{a}x\sin ax + C$
23. $\int x^2\cos ax\mathrm{d}x = \frac{1}{a}x^2\sin ax + \frac{2}{a^2}x\cos ax - \frac{2}{a^3}\sin ax + C$

1. $\int \arcsin\frac{x}{a}\mathrm{d}x = x\arcsin\frac{x}{a} + \sqrt{a^2-x^2} + C$
2. $\int x\arcsin\frac{x}{a}\mathrm{d}x = (\frac{x^2}{2}-\frac{a^2}{4})\arcsin\frac{x}{a} + \frac{x}{4}\sqrt{x^2-x^2} + C$
3. $\int x^2\arcsin\frac{x}{a}\mathrm{d}x = \frac{x^3}{3}\arcsin\frac{x}{a} + \frac{1}{9}(x^2+2a^2)\sqrt{a^2-x^2} + C$
4. $\int \arccos\frac{x}{a}\mathrm{d}x = x\ arccos\frac{x}{a} - \sqrt{a^2-x^2} + C$
5. $\int x\arccos\frac{x}{a}\mathrm{d}x = (\frac{x^2}{2}-\frac{a^2}{4})\arccos\frac{x}{a} - \frac{x}{4}\sqrt{a^2-x^2} + C$
6. $\int x^2\arccos\frac{x}{a}\mathrm{d}x = \frac{x^3}{3}\arccos\frac{x}{a} - \frac{1}{9}(x^2+2a^2)\sqrt{a^2-x^2} + C$
7. $\int \arctan\frac{x}{a}\mathrm{d}x = x\arctan\frac{x}{a} - \frac{a}{2}\ln(a^2+x^2) + C$
8. $\int x\arctan\frac{x}{a}\mathrm{d}x = \frac{1}{2}(a^2+x^2)\arctan\frac{x}{a} - \frac{a}{2}x + C$
9. $\int x^2\arctan\frac{x}{a}\mathrm{d}x = \frac{x^3}{3}\arctan\frac{x}{a} - \frac{a}{6}x^2 + \frac{a^3}{6}\ln(a^2+x^2) + C$

1. $\int a^x\mathrm{d}x = \frac{1}{\ln a}a^x + C$
2. $\int \mathrm{e}^{ax}\mathrm{d}x = \frac{1}{a}a^{ax} + C$
3. $\int x\mathrm{e}^{ax}\mathrm{d}x = \frac{1}{a^2}(ax-1)a^{ax} + C$
4. $\int x^n\mathrm{e}^{ax}\mathrm{d}x = \frac{1}{a}x^n\mathrm{e}^{ax} - \frac{n}{a}\int x^{n-1}\mathrm{e}^{ax}\mathrm{d}x$
5. $\int xa^x\mathrm{d}x = \frac{x}{\ln a}a^x - \frac{1}{(\ln a)^2}a^x + C$
6. $\int x^n a^x\mathrm{d}x = \frac{1}{\ln a}x^n a^x - \frac{n}{\ln a}\int x^{n-1}a^x\mathrm{d}x$
7. $\int \mathrm{e}^{ax}\sin bx\mathrm{d}x = \frac{1}{a^2+b^2}\mathrm{e}^{ax}(a\sin bx - b\cos bx) + C$
8. $\int \mathrm{e}^{ax}\cos bx\mathrm{d}x = \frac{1}{a^2+b^2}\mathrm{e}^{ax}(b\sin bx + a\cos bx) + C$
9. $\int \mathrm{e}^{ax}\sin^n bx\mathrm{d}x = \frac{1}{a^2+b^2n^2}\mathrm{e}^{ax}\sin^{n-1}bx(a\sin bx - nb\cos bx) + \frac{n(n-1)b^2}{a^2+b^2n^2}\int\mathrm{e}^{ax}\sin^{n-2}bx\mathrm{d}x$
10. $\int \mathrm{e}^{ax}\cos^n bx\mathrm{d}x = \frac{1}{a^2+b^2n^2}\mathrm{e}^{ax}\cos^{n-1}bx(a\cos bx + nb\sin bx) + \frac{n(n-1)b^2}{a^2+b^2n^2}\int\mathrm{e}^{ax}\cos^{n-2}bx\mathrm{d}x$

1. $\int \ln x\mathrm{d}x = x\ln x - x + C$
2. $\int \frac{\mathrm{d}x}{x\ln x} = \ln\left|\ln x\right| + C$
3. $\int x^n\ln x\mathrm{d}x = \frac{1}{n+1}x^{n+1}(\ln x - \frac{1}{n+1}) + C$
4. $\int (\ln x)^n\mathrm{d}x = x(\ln x)^n - n\int(\ln x)^{n-1}\mathrm{d}x$
5. $\int x^m(\ln x)^n\mathrm{d}x = \frac{1}{m+1}x^{m+1}(\ln x)^n - \frac{n}{m+1}\int x^m(\ln x)^{n-1}\mathrm{d}x$