# Programming Paradigms - Problem Set No. 1 :

**1** a) $\lambda s.(\lambda g.\ s\ g)\ s$

b) $\lambda s.(\lambda b.\ b)\ s$

c) $\lambda n.\ \lambda t.\ n\ t$

d) $\lambda m.\ m\ (\lambda a.\ a)$

e) $\lambda e.(\lambda b.\ b)\ e$

f) $(\lambda n.\ \lambda f.\ f)\ z\ x$

**2** a) $(\lambda x.\lambda y.\ x)\ y\ z$

- After finding $\alpha$-equivalent terms:

$$\underbrace{(\lambda x.\ \lambda y_0.\ x)\ y}\ z$$

$$\underbrace{\lambda y_0.\ y\ z}$$

$$\Rightarrow \boxed{y}$$

b) $(\lambda x.\ \lambda y.\ x)(\lambda z.\ y)\ z\ w$

$$\underbrace{(\lambda x.\ \lambda y.\ x)\ (\lambda z.\ y_0)}\ z_0\ w$$

$$\underbrace{\lambda y.\ \lambda z.\ y_0\ z_0}\ w$$

$$\lambda z.\ y_0\ w$$

$$\Rightarrow \boxed{y_0}$$

c) $(\lambda b.\ \lambda f.\ \lambda t.\ b\ t\ f)(\lambda f.\ \lambda t.\ t)$

$$A = \lambda f.\ \lambda t.\ t$$

$$\underbrace{(\lambda b.\ \lambda f.\ \lambda t.\ b\ t\ f)\ A}$$

$$\lambda f.\ \lambda t.\ \underbrace{A\ t}\ f$$

$$\lambda f.\ \lambda t.\ \underbrace{(\lambda f_0.\ \lambda t_0.\ t_0)\ t}\ f$$

$$\lambda f.\ \lambda t.\ \underbrace{\lambda t_0.\ t_0\ f}$$

$$\Rightarrow \boxed{\lambda f.\ \lambda t.\ f}$$

d) $(\lambda s.\ \lambda z.\ s(s z))(\lambda b.\ \lambda f.\ \lambda t.\ b t f)(\lambda f_0.\ \lambda t_0.\ t_0)$

$A = \lambda s.\ \lambda z.\ s(s z)$

$B = \lambda b.\ \lambda f.\ \lambda t.\ b\ t\ f$

$C = \lambda f.\ \lambda t.\ t$

$\Rightarrow\ A\ B\ C$

$\lambda s.\ \lambda z.\ s(s z)\ B\ C$

$\lambda z.\ B\ (B\ z)\ C$

$\lambda z.\ B\ (\lambda b.\ \lambda f.\ \lambda t.\ b\ t\ f\ z)\ C$

$\lambda z.\ B(\ \lambda f.\ \lambda t.\ z\ t\ f\ )\ C$

$\lambda z.\ \lambda b.\ \lambda f.\ \lambda t.\ b\ t\ f\ (\lambda f_0.\ \lambda t_0.\ z\ t_0\ f_0)\ C$

$\lambda z.\ \lambda f.\ \lambda t.\ (\lambda f_0.\ \lambda t_0.\ z\ t_0\ f_0)\ t f\ \ C$

$\lambda z.\ \lambda f.\ \lambda t.\ (\ \lambda t_0. z\ t_0\ \ t\ )\ f\ \ C$

$\lambda z.\ \lambda f.\ \lambda t.\ (\ z\ f\ t\ )\ C$

$\lambda z.\ \lambda f.\ \lambda t.\ (z\ f\ t)\ (\ \lambda f_0.\ \lambda t_0.\ t_0)$

$\lambda f.\ \lambda t.\ (\ \lambda f_0.\ \lambda t_0.\ t_0)\ f t$

$\lambda f.\ \lambda t.\ (\lambda t_0.\ t_0)\ t$

$\Rightarrow\ \boxed{\lambda f.\ \lambda t.\ t}$

e) $(\lambda s.\ \lambda z.\ s\,(s\,z))\ (\lambda s.\ \lambda z.\ s\,(s\,z))\ (\lambda b.\ \lambda f.\ \lambda t.\ b\ t$

$\to (\lambda f.\ \lambda t.\ t)$

$A = \lambda s.\ \lambda z.\ s\,(s\,z)$

$B = \lambda b.\ \lambda f.\ \lambda t.\ b\ t\ f$

$C = \lambda f.\ \lambda t.\ t$

$\Rightarrow A\ A\ B\ C$

$(\lambda s.\ \lambda z.\ s(s\,z))\ A\ B C$

$(\lambda z.\ A\,(A\,z))\ B C$

$\lambda z.\ A\ (\lambda s.\ \lambda z_0.\ s\,(s\,z_0)\ z)\ B\ C$

$\lambda z.\ A\ (\lambda z_0.\ z\,(z\,z_0))\ B C$

$\Rightarrow \boxed{\lambda f\ \lambda t\ t}$

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a) We already know that logical AND is represented as: AND: $\lambda x.\lambda y.\ x\ y\ fls$

Which stands for: If x is false then return false, otherwise, return y.

Similarly logical Implication stands for: If x is false then return true, otherwise, return y.

Thus, logical implication can be written as:

IMPL: $\boxed{\lambda x.\ \lambda y.\ x\ y\ tru}$

- So, in terms of bare lambda calculus, it would be:

$$\boxed{\lambda x.\ \lambda y.\ x\ y\ (\lambda x.\ \lambda y.\ x)}$$

b) IMPL fls tru

$(\lambda x.\ \lambda y.\ x\ y\ tru)\ fls\ tru$

$(\lambda y.\ fls\ y\ tru)\ tru$

$fls\ tru\ tru$

$(\lambda x.\ \lambda y.\ y)\ (\lambda x.\ \lambda y.\ x)\ tru$

$\lambda y.\ y\ tru$

$\Rightarrow \boxed{tru}$

$\boxed{4}$ Defining some operations and numerals:

$MUL: \lambda n.\ \lambda m.\ \lambda s.\ n\,(m\,s)$ (multiplication)

$SUCC: \lambda n.\ \lambda s.\ \lambda z.\ s\,(n\,s\,z)$ (increment by 1)

$Exp: \lambda n.\ \lambda m.\ m\,(MUL\ n)\ C_1$ (exponentiation)

$C_0: \lambda s.\ \lambda z.\ z$

$C_1: \lambda s.\ \lambda z.\ s\,z$

$C_2: \lambda s.\ \lambda z.\ s\,(s\,z)$

a) i. $n \longrightarrow 2n+1$

$\Leftrightarrow SUCC\ (MUL\ (C_2\ n))$

ii. $n \longrightarrow n^2+1$

$\Leftrightarrow SUCC\,(MUL\,(n\ n))$

iii. $n \longrightarrow 2^n+1$

$\Leftrightarrow SUCC(\ Exp\,(C_2\ n))$

iv. $n \longrightarrow 2^{n+1}$

$\Leftrightarrow \cancel{Exp\,(2\ succ(n))}$

$Exp\,(C_2\ succ(n))$

## $\boxed{4}$ Defining some operations and numerals:

MUL: $\lambda n.\ \lambda m.\ \lambda s.\ n\,(m\,s)$ (multiplication)

SUCC: $\lambda n.\ \lambda s.\ \lambda z.\ s\,(n\,s\,z)$ (increment by 1)

Exp: $\lambda n.\ \lambda m.\ m\,(MUL\ n)\ C_1$ (exponentiation) $n^m$

$C_0$: $\lambda s.\ \lambda z.\ z$

$C_1$: $\lambda s.\ \lambda z.\ s\ z$

$C_2$: $\lambda s.\ \lambda z.\ s\,(s\ z)$

a) i. $n \to 2n+1$

$\Leftrightarrow$ SUCC (MUL ($C_2$ n))

ii. $n \to n^2+1$

$\Leftrightarrow$ SUCC (MUL (n n))

iii. $n \to 2^n+1$

$\Leftrightarrow$ SUCC( Exp ($C_2$ n))

iv. $n \to 2^{n+1}$

$\Leftrightarrow$

Exp ($C_2$ SUCC(n))

b) i. $SUCC(MUL(C_2\ C_2))$

$SUCC(C_4)$

$\Rightarrow C_5$

ii. $SUCC(MUL(C_2\ C_2))$

$SUCC(C_4)$

$\Rightarrow C_5$

iii. $SUCC(EXP(C_2\ C_2))$

$SUCC(C_4)$

$\Rightarrow C_5$

iv. $EXP(C_2\ SUCC\ C_2)$

$EXP(C_2\ C_3)$

$\Rightarrow C_8$

e) (λs.λz.s (s z)) (λs.λz.s (s z)) (λb.λf.λt.b t f ) (λf.λt.t)
A = (λs.λz.s (s z))
B = (λb.λf.λt.b t f )
C = (λf.λt.t)
D = A B
=> A A B C
=> (λs.λz.s (s z)) A B C
=> (λz. A (A z)) B C
=> (A (A B)) C
=> (λs.λz.s (s z)) (A B)) C
=> (λs.λz.s (s z)) (D)) C
=> (λz.D (D z))) C
=> D (D C)
=> (A B) (D C)
=> ((λs.λz.s (s z)) B)(D C)
=> ((λz.B (B z)))(D C)
=> B (B (D C))
=> ((λb.λf.λt.b t f ))(B (D C))
=> λf.λt.(B (D C)) t f
=> λf.λt.((λb.λf1.λt1.b t1 f1 ) (D C)) t f
=> λf.λt.(λf1.λt1.(D C) t1 f1 ) t f
=> λf.λt.(λt1.(D C) t1 t ) f
=> λf.λt.(D C) f t
=> λf.λt.((A B) C) f t
=> λf.λt.(((λs.λz.s (s z)) B) C) f t
=> λf.λt.(((λz.B (B z)) C) f t
=> λf.λt.(B (B C)) f t
=> λf.λt.((λb.λf0.λt0.b t0 f0) (B C)) f t
=> λf.λt.(λf0.λt0.(B C) t0 f0) f t
=> λf.λt.(λt0.(B C) t0 f) t
=> λf.λt.(B C) t f
=> λf.λt.((λb.λf0.λt0.b t0 f0) C) t f
=> λf.λt.(λf0.λt0.C t0 f0) t f
=> λf.λt.(λt0.C t0 t) f
=> λf.λt.(C f t)
=> λf.λt.(λf0.λt0.t0) f t
=> λf.λt.(λt0.t0) t
=> λf.λt.t