Programming Paradigms. Problem Set No. 1:

1

a) $\lambda s. (\lambda g. s\, g)\, s$

b) $\lambda s. (\lambda b. b)\, s$

c) $\lambda n. \lambda t. n\, t$

d) $\lambda m. m\, (\lambda a. a)$

e) $\lambda e. (\lambda b. b)\, e$

f) $(\lambda n. \lambda f. f)\, z\, x$

2

a) $(\lambda x. \lambda y. x)\, y\, z$

- After finding α-equivalent terms:

$$(\lambda x. \lambda y_0. x)\, y\, z$$

$$\lambda y_0. y\, z$$

$$\Rightarrow \boxed{y}$$

b) $(\lambda x.\ \lambda y.\ x)(\lambda z.\ y)\ z\ w$

$$\underbrace{(\lambda x.\ \lambda y.\ x)\ (\lambda z.\ y_0)}\ z_0\ w$$

$$\underbrace{\lambda y.\ \lambda z.\ y_0\ z_0}\ w$$

$$\lambda z.\ y_0\ w$$

$$\Rightarrow \boxed{y_0}$$

c) $(\lambda b.\ \lambda f.\ \lambda t.\ b\,t\,f)(\lambda f.\ \lambda t.\ t)$

$$A = \lambda f.\ \lambda t.\ t$$

$$\underbrace{(\lambda b.\ \lambda f.\ \lambda t.\ b\,t\,f)\ A}$$

$$\lambda f.\ \lambda t.\ \underbrace{A\ t}\ f$$

$$\lambda f.\ \lambda t.\ \underbrace{(\lambda f_0.\ \lambda t_0.\ t_0)\ t}\ f$$

$$\lambda f.\ \lambda t.\ \underbrace{\lambda t_0.\ t_0\ f}$$

$$\Rightarrow \boxed{\lambda f.\ \lambda t.\ f}$$

d) $(\lambda s.\ \lambda z.\ s(s\,z))\,(\lambda b.\ \lambda f.\ \lambda t.\ b\,t\,f)\,(\lambda f_0.\ \lambda t_0.\ t_0)$

$A = \lambda s.\ \lambda z.\ s(s\,z)$

$B = \lambda b.\ \lambda f.\ \lambda t.\ b\ t\ f$

$C = \lambda f.\ \lambda t.\ t$

$\Rightarrow \quad A\ B\ C$

$\lambda s.\ \lambda z.\ s(s\,z)\ B\,C$

$\lambda z.\ B\,(B\,z)\ C$

$\lambda z.\ B\,(\lambda b.\ \lambda f.\ \lambda t.\ b\ t\ f\ z)\ C$

$\lambda z.\ B(\ \lambda f.\ \lambda t.\ z\ t\ f\ )\ C$

$\lambda z.\ \lambda b.\ \lambda f.\ \lambda t.\ b\ t\ f\ (\lambda f_0.\ \lambda t_0.\ z\ t_0\ f_0)\ C$

$\lambda z.\ \ \lambda f.\ \lambda t.\ (\lambda f_0.\ \lambda t_0.\ z\ t_0\ f_0)\ t\,f\ \ C$

$\lambda z.\ \lambda f.\ \lambda t.\ (\ \lambda t_0.\,z\ t_0\ \ t\ )\,f\ \ C$

$\lambda z.\ \lambda f.\ \lambda t.\ (\ z\ f\ t\ )\ C$

$\lambda z.\ \lambda f.\ \lambda t.\ (z\ f\ t)\ \ (\ \lambda f_0.\ \lambda t_0.\ t_0)$

$\lambda f.\ \ \lambda t.\ (\ \lambda f_0.\ \lambda t_0.\ t_0)\ f\,t$

$\lambda f.\ \ \lambda t.\ (\lambda t_0.\ \ t_0)\ \ t$

$\Rightarrow \quad \boxed{\lambda f.\ \ \lambda t.\ \ t}$

e) $(\lambda s.\ \lambda z.\ s\ (s\ z))\ (\lambda s.\ \lambda z.\ s\ (s\ z))\ (\lambda b.\ \lambda f.\ \lambda t.\ b\ t\ f)$
$\to (\lambda f.\ \lambda t.\ t)$

$A = \lambda s.\ \lambda z.\ s\ (s\ z)$

$B = \lambda b.\ \lambda f.\ \lambda t.\ b\ t\ f$

$C = \lambda f.\ \lambda t.\ t$

$\Rightarrow\ A\ A\ B\ C$

$(\lambda s.\ \lambda z.\ s(s\ z))\ A\ B\ C$

$(\lambda z.\ A\ (A\ z))\ B\ C$

$\lambda z.\ A\ (\lambda s.\ \lambda z_0.\ s\ (s\ z_0)\ z)\ B\ C$

$\lambda z.\ A\ (\lambda z_0.\ z\ (z\ z_0))\ B\ C$

$\Rightarrow \boxed{\lambda f\ \lambda t\ t}$

3

a) We already know that logical AND is represented as: AND: $\lambda x. \lambda y. x\ y$ fls

Which stands for: If x is false then return false, otherwise, return y.

Similarly logical Implication stands for: If x is false then return true, otherwise, return y.

Thus, logical implication can be written as:

IMPL: $\boxed{\lambda x. \lambda y. x\ y \text{ tru}}$

- So, in terms of bare lambda calculus, it would be:

$$\boxed{\lambda x. \lambda y. x\ y\ (\lambda x. \lambda y. x)}$$

b) IMPL fls tru

$(\lambda x. \lambda y. x\ y \text{ tru})$ fls tru

$(\lambda y. \text{fls}\ y \text{ tru})$ tru

fls tru tru

$(\lambda x. \lambda y. y)\ (\lambda x. \lambda y. x)$ tru

$\lambda y. y$ tru

$\Rightarrow \boxed{\text{tru}}$

$\boxed{4}$ Defining some operations and numerals:

MUL: $\lambda n.\ \lambda m.\ \lambda s.\ n\,(m\,s)$ (multiplication)

SUCC: $\lambda n.\ \lambda s.\ \lambda z.\ s\,(n\,s\,z)$ (increment by 1)

Exp: $\lambda n.\ \lambda m.\ m\,(\text{MUL}\ n)\ C_1$ (exponentiation) n

C_0: $\lambda s.\ \lambda z.\ z$

C_1: $\lambda s.\ \lambda z.\ s\,z$

C_2: $\lambda s.\ \lambda z.\ s\,(s\,z)$

a) i. $n \longrightarrow 2n+1$

$\Leftrightarrow$ SUCC (MUL (C_2 n))

ii. $n \longrightarrow n^2+1$

$\Leftrightarrow$ SUCC (MUL (n n))

iii. $n \longrightarrow 2^n+1$

$\Leftrightarrow$ SUCC(EXP (C_2 n))

iv. $n \longrightarrow 2^{n+1}$

$\Leftrightarrow$ ~~EXP(2 SUCC(n))~~

Exp (C_2 SUCC(n))

4 Defining some operations and numerals:

MUL: $\lambda n.\ \lambda m.\ \lambda s.\ n\ (m\ s)$ (multiplication)

SUCC: $\lambda n.\ \lambda s.\ \lambda z.\ s\ (n\ s\ z)$ (increment by 1)

Exp: $\lambda n.\ \lambda m.\ m\ (MUL\ n)\ C_1$ (exponentiation) n^m

C_0: $\lambda s.\ \lambda z.\ z$

C_1: $\lambda s.\ \lambda z.\ s\ z$

C_2: $\lambda s.\ \lambda z.\ s\ (s\ z)$

a) i. $n \rightarrow 2n+1$

$\Leftrightarrow$ SUCC (MUL (C_2 n))

ii. $n \rightarrow n^2+1$

$\Leftrightarrow$ SUCC (MUL (n n))

iii. $n \rightarrow 2^n+1$

$\Leftrightarrow$ SUCC (Exp (C_2 n))

iv. $n \rightarrow 2^{n+1}$

$\Leftrightarrow$ ~~Exp (2 SUCC(n))~~

Exp (C_2 SUCC(n))

b) i. $SUCC(MUL(C_2\ C_2))$

$SUCC(C_4)$

$\Rightarrow C_5$

ii. $SUCC(MUL(C_2\ C_2))$

$SUCC(C_4)$

$\Rightarrow C_5$

iii. $SUCC(EXP(C_2\ C_2))$

$SUCC(C_4)$

$\Rightarrow C_5$

iv. $EXP(C_2\ SUCC\ C_2)$

$EXP(C_2\ C_3)$

$\Rightarrow C_8$