# Virtual Synthetic Array System for Directional Channel Characterization

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**Abstract** This paper discusses intrinsic problems to realize low side-lobe beam forming by synthetic cylindrical array, and presents some indeterminate results. It is shown that the phase mode excitation based beamformer on the circular array is useful only if all impinging waves have the same elevation angle, but is applicable to the multi-path signals having arbitrary elevation angles which are usually unknown.

**Key words** Microwave, Beam-forming, Synthetic array, Angular power spectra(APS), Dolph-Chebyshev, Phase mode excitation

#### 1. Introduction

In future mobile systems, ultra high speed wireless LAN (IEEE 802.11.ay) [1] using microwave band and millimeter wave band has been studied. Radio wave propagation characteristics are very different between those frequency bands, hence a optimal multi-band operation of wireless LAN systems become important. In system operation, it is important to clarify the relationship by comparison of radio wave propagation characteristics in various environments.

In our research group, highly directivedouble directional channel characterization has been conducted by rotating highly directive horn antennas at the millimeter band of 58.5 GHz [2]. In order to compare them with those at 2.4 GHz, a virtual synthetic array which is designed to have the same angular resolution at mm-wave band can be introduced. A cylindrical array beamformer can be easily developed by stacked circular array using a rotator, but circular array has high side-lobe level in the azimuth angle domain. In [3] approach using phase mode excitation [4] has been proposed. However, since the method of phase mode excitation can be used only if the assumption that the elevation angles of all incident waves are approximately equal, which is difficult to expect in actual multipath environment.

In this study, we discusses intrinsic problems to realize low side-lobe beam forming by synthetic cylindrical array, and presents some indeterminate results.

#### 2. Problem Formulation

A synthetic array is comosed of uniform circular array (UCA) and uniform linear array (ULA). Because processing

to lower side lobe (Dolph-Chebyshev approach) is applicable only to ULA, the UCA needs to be transformed into the ULA by phase mode excitation.

For N element UCA with radius r, the i-th component of the array response  $a_i$  for a narrowband signal of wavelength  $\lambda$  k-th impinging at zenith angle  $\theta_k$  and azimuth angle  $\phi_k$ given by

$$a_{i}(\theta_{k}, \phi_{k}) = \exp\left(j\frac{2\pi}{\lambda}r\sin\theta_{k}\cos\left(\phi_{k} - \frac{2\pi(i-1)}{N}\right)\right)$$
$$= \exp\left(j\zeta_{k}\cos(\phi_{k} - \eta_{i})\right) \tag{1}$$

where  $\zeta_k = \frac{2\pi}{\lambda} r \sin \theta_k$  and  $\eta_i = \frac{2\pi(i-1)}{N}$ 

Substituting Jacobi-Anger expansion [5], given by

$$e^{j\alpha\cos\gamma} = \sum_{m=-\infty}^{\infty} j^m J_m(\alpha) e^{jm\gamma}$$
 (2)

where  $J_m$  is the Bessel function of order m, we get

$$a_i(\theta_k, \phi_k) = \exp\left(j\zeta_k \cos(\phi_k - \eta_i)\right)$$
$$= \sum_{m=1}^{\infty} j^m J_m(\zeta_k) e^{-jm\eta_i} e^{-jm\phi_k}$$
(3)

if  $N \gg \frac{2\pi}{\lambda}r$  we have an approximate expression

$$a_i(\theta_k, \phi_k) \simeq \sum_{m=-M}^{M} j^m J_m(\zeta_k) e^{-jm\eta_i} e^{-jm\phi_k}$$
 (4)

Array response vector vector  $\boldsymbol{a}_{\text{UCA}}$  can be transform to  $\boldsymbol{\hat{a}}_{\text{ULA}}(\phi_k)$  by  $\mathbf{J}(\zeta_k)$  and  $\mathbf{F}$ 

$$\hat{\boldsymbol{a}}_{\text{ULA}}(\phi_k) = \mathbf{J}(\zeta_k) \mathbf{F} \boldsymbol{a}_{\text{UCA}}(\theta_k, \phi_k)$$
 (5)

where the elements of  $\mathbf{F}$  are

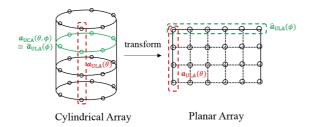


Figure 1 Transform of cylindrical array into planar array

$$F_{mn} = e^{-j2\pi n m/N} \begin{cases} n = 1, ...N \\ m = -M, ..., 0, ..., M \end{cases}$$
 (6)

and J is the diagonal matrix as

$$\mathbf{J} = \operatorname{diag}\left[\frac{1}{\sqrt{N}\mathbf{j}^{m}J_{m}(\frac{2\pi}{\lambda}r\sin\theta_{k})}\right], m = -M, ..., 0, ..., M$$
 (7)

From (7), it is assumed that  $J_m(\zeta) \neq 0$  and the design of the array should be such that  $\zeta$  will yield J as well conditioned as possible. However, as long as  $\theta_k$  is unknown in multi-path scenario, it is an impractical to guarantee (7) as a well condition.

Now, by transforming the UCA into the ULA in phase mode domain, cylindrical array can be transform into planar array as shown in Fig. 1. Planner array respose vector  $\boldsymbol{a}_{\text{planer}}$  is given by

$$\mathbf{a}_{\text{planer}}(\theta_k, \phi_k) = \hat{\mathbf{a}}_{\text{ULA}}(\phi_k) \otimes \mathbf{a}_{\text{ULA}}(\theta_k)$$
 (8)

where  $\otimes$  is kronecker product. The array weight vector at the look direction of  $\theta$  and  $\phi$  is expressed as

$$\boldsymbol{w} = \boldsymbol{G}\hat{\boldsymbol{a}}_{\text{ULA}}(\phi) \otimes \boldsymbol{G}\boldsymbol{a}_{\text{ULA}}(\theta) \tag{9}$$

where G is Dolph Chebyshev diagonal matrix [6]. We can obtain the output signal  $y(\theta, \phi)$ 

$$y(t) = \boldsymbol{w}^H \boldsymbol{x}(t) \tag{10}$$

where H and denotes complex conjugate transpose. and input signal vector  $\boldsymbol{x}(t)$ 

$$\boldsymbol{x}(t) = \sum_{k=0}^{K} s_k(t) \boldsymbol{a}_{\text{planer}}(\theta_k, \phi_k) + \boldsymbol{n}(t)$$
 (11)

where  $s_k(t)$  denotes received signal of the k-th path,  $\boldsymbol{n}(t)$  denotes the noise vector. Output power  $P_{\text{out}}(\theta, \phi)$  is given by

$$P_{\text{out}}(\theta, \phi) = \mathbb{E}\{|y(t)|^2\}$$
(12)

#### 3. Simulation Results

We considered a cylindrical array of 9 stages of 30 element UCA with  $\frac{\lambda}{4}$  element spacing. We choosed M=12 and  $\theta_k=90$  in (7), the side lobe level in Dolph-Chebyshev is set by -40dB and the results are normalized with the maximum values. In the first simulation, two waves are impinging at

the same zenith angles of 90 degree and the azimuth angles of  $\phi=60$  and 90 respectively. In Fig. 2, since zenith angle is equal, the power spectrum is well produced with sufficiently low side lobe level. two waves are impinging at different zenith angles of 90 and 120 and the azimuth angles of  $\phi=60$  and 90, respectively.

### 4. Conclusion and Remarks

The synthetic cylindrical array is easily developed by using rotator. However, to resolve the critical problem of high sidelobe level of the circular array, phase mode excitation based array transform can be used. However, the zenith angles of impinging waves should be aligned and known in advance, which make difficult to use this approach in actual multipath environments as it is. We will challenge to develop a new algorithm to solve this problem clarified in this paper.

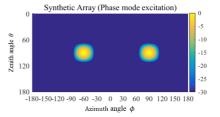


Figure 2 Same zenith angle

Synthetic Array (Phase mode excitation)

Synthetic Array (Phase mode excitation)

120

180

180-150-120 -90 -60 -30 0 30 60 90 120 150 180

Azimuth angle \$\phi\$

Figure 3 Different zenith angles

### Acknowledgment

A part of this work was supported by JSPS KAKENHI Grant Number 15H04003.

## References

- "Channel Models for IEEE 802.11ay," IEEE Document 802.11-15/1150r9, Mar. 2017.
- [2] M Kim, K Umeki, K Wangchuk, J Takada, S Sasaki, , "Polarimetric Mm-Wave Channel Measurement and Characterization in a Small Cell Office Environment," IEICE Technical Report, SR2015-20, Jul. 2015.
- [3] C. L. Dolph, "A current distribution for broadside arrays which optimizes the relationship between beam width and sidelobe level, "Proc. of the I.R.E. and Waves and Electrons, pp. 335-348, June 1946.
- [4] B. K. Lau, and Y. H. Leung, "A Dolph-Chebyshev approach to the synthesis of array patterns for uniform circular arrays", IEEE International Symposium on Circuits and Systems, vol. I, pp. 124-127, May 2000.
- Milton Abramowitz, "Handbook of Mathematical Functions", Dover Publications, 9th ed, 1965
- [6] C. A. Balanis, "Antenna Theory", John Wiley & Sons, Inc, pp 330-343, 4th ed, 2016