Programming Languages and Compilers (CS 421)

Reza Zamani

http://www.cs.illinois.edu/class/cs421/

Based in part on slides by Mattox Beckman, as updated by Vikram Adve, Gul Agha and Munawar Hafiz

Semantics

- Expresses the meaning of syntax
- Static semantics
  - Meaning based only on the form of the expression without executing it
  - Usually restricted to type checking / type inference
  - Could include Data Flow Analysis.

Dynamic semantics

- Method of describing meaning of executing a program
- Several different types:
  - Operational Semantics
  - Axiomatic Semantics
    - Algebraic Semantics
  - Denotational Semantics (Scott–Strachey semantics)

Dynamic Semantics

- Different languages better suited to different types of semantics
- Different types of semantics serve different purposes

Operational Semantics

- Start with a simple notion of machine
- Describe how to execute (implement) programs of language on virtual machine, by describing how to execute each program statement (ie, following the *structure* of the program)
- Meaning of program is how its execution changes the state of the machine
- Useful as basis for implementations

Axiomatic Semantics

- Also called Floyd-Hoare Logic
- Based on formal logic (first order predicate calculus)
- Axiomatic Semantics is a logical system built from *axioms* and *inference rules*
- Mainly suited to simple imperative programming languages

Axiomatic Semantics

- Used to formally prove a property (*post-condition*) of the *state* (the values of the program variables) after the execution of program, assuming another property (*pre-condition*) of the state before execution
- Written :

  {Precondition} Program {Postcondition}
- Source of idea of *loop invariant*

Denotational Semantics

- Construct a function *M* assigning a mathematical meaning to each program construct
- Lambda calculus often used as the range of the meaning function
- Meaning function is compositional: meaning of construct built from meaning of parts
- Useful for proving properties of programs

Denotational Semantics

- Construct a function *M* assigning a mathematical meaning to each program construct
- Meaning function is compositional: meaning of construct built from meaning of parts
- Useful for proving properties of programs

Natural Semantics

- Aka Structural Operational Semantics, aka "Big Step Semantics"
- Provide value for a program by rules and derivations, similar to type derivations
- Rule conclusions look like

$$(C, m) \Downarrow m'$$

or

$$(E, m) \Downarrow v$$

Simple Imperative Programming Language

- $I \in$ *Identifiers*
- $N \in$ *Numerals*
- B ::= true | false | B & B | B or B | not B
  | $E < E$ | $E = E$
- $E ::= N \mid I \mid E + E \mid E * E \mid E - E \mid - E$
- C::= skip | C;C | I := E
  | if B then C else C fi | while B do C od

Natural Semantics of Atomic Expressions

- Identifiers: $(I,m) \Downarrow m(I)$
- Numerals are values: $(N,m) \Downarrow N$
- Booleans: $(\text{true},m) \Downarrow \text{true}$

  $(\text{false},m) \Downarrow \text{false}$

Booleans:

$$\frac{(B,\ m) \Downarrow \text{false}}{(B\ \&\ B',\ m) \Downarrow \text{false}} \qquad \frac{(B,\ m) \Downarrow \text{true} \quad (B',\ m) \Downarrow b}{(B\ \&\ B',\ m) \Downarrow b}$$

$$\frac{(B,\ m) \Downarrow \text{true}}{(B \text{ or } B',\ m) \Downarrow \text{true}} \qquad \frac{(B,\ m) \Downarrow \text{false} \quad (B',\ m) \Downarrow b}{(B \text{ or } B',\ m) \Downarrow b}$$

$$\frac{(B,\ m) \Downarrow \text{true}}{(\text{not } B,\ m) \Downarrow \text{false}} \qquad \frac{(B,\ m) \Downarrow \text{false}}{(\text{not } B,\ m) \Downarrow \text{true}}$$

Relations

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U \sim V = b}{(E \sim E', m) \Downarrow b}$$

- By $U \sim V = b$, we mean does (the meaning of) the relation $\sim$ hold on the meaning of U and V
- May be specified by a mathematical expression/equation or rules matching U and V

Arithmetic Expressions

$$\frac{(E, m) \Downarrow U \qquad (E', m) \Downarrow V \quad U \ op\ V = N}{(E \ op\ E', m) \Downarrow N}$$

where N is the specified value for $U \ op\ V$

Commands

Skip: $(\text{skip}, m) \Downarrow m$

Assignment:
$$\frac{(E,m) \Downarrow V}{(I::=E,m) \Downarrow m[I \texttt{<--} V]}$$

Sequencing:
$$\frac{(C,m) \Downarrow m' \quad (C',m') \Downarrow m''}{(C;C', m) \Downarrow m''}$$

If Then Else Command

$$\frac{(B,m) \Downarrow \text{true} \quad (C,m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$

$$\frac{(B,m) \Downarrow \text{false} \quad (C',m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$

While Command

$$\frac{(B,m) \Downarrow \text{false}}{(\text{while } B \text{ do } C \text{ od}, m) \Downarrow m}$$

$$\frac{(B,m)\Downarrow\text{true} \quad (C,m)\Downarrow m' \quad (\text{while } B \text{ do } C \text{ od}, m')\Downarrow m''}{(\text{while } B \text{ do C od}, m) \Downarrow m''}$$

Example

$$\frac{}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi}, \; \{x \to 7\}) \Downarrow \ ?}$$

Example

$$\dfrac{\dfrac{}{(x > 5, \{x \text{ -> } 7\}) \Downarrow ?}}{(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi}, \{x \text{ -> } 7\}) \Downarrow ?}$$

Example

$$? > ? = ?$$

$$\cfrac{\cfrac{(x,\{x\text{->}7\})\Downarrow ? \qquad (5,\{x\text{->}7\})\Downarrow ?}{(x > 5,\ \{x \text{ -> } 7\})\Downarrow ?}}{\begin{array}{c}(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi},\\ \{x \text{ -> } 7\})\Downarrow ?\end{array}}$$

Example

$$7 > 5 = \text{true}$$

$$\dfrac{\dfrac{(x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5}{(x > 5,\ \{x\ \text{->}\ 7\})\Downarrow ?}}{(\text{if } x > 5 \text{ then } y:= 2 + 3 \text{ else } y:=3 + 4 \text{ fi},\ \{x\ \text{->}\ 7\}) \Downarrow ?}$$

Example

$$\dfrac{\dfrac{7 > 5 = \text{true}}{(x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5}}{\dfrac{(x > 5,\ \{x \text{->} 7\})\Downarrow \text{true}}{(\text{if } x > 5 \text{ then } y{:}= 2 + 3 \text{ else } y{:}=3 + 4 \text{ fi},\ \{x \text{->} 7\}) \Downarrow\ ?}}$$

Example

$$\frac{\dfrac{\begin{array}{c}7 > 5 = \text{true} \\ (x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5\end{array}}{(x > 5,\ \{x \text{->} 7\})\Downarrow \text{true}} \qquad \dfrac{}{(y{:}= 2 + 3,\ \{x\text{->}\ 7\})\Downarrow\ ?}}{(\text{if } x > 5 \text{ then } y{:}= 2 + 3 \text{ else } y{:}=3 + 4 \text{ fi},\ \{x \text{->} 7\}) \Downarrow\ ?}$$

Example

$$
\dfrac{\dfrac{(x,\{x\text{->}7\})\Downarrow 7 \quad 7 > 5 = \text{true} \quad (5,\{x\text{->}7\})\Downarrow 5}{(x > 5,\ \{x \text{->} 7\})\Downarrow \text{true}} \qquad \dfrac{\dfrac{}{(2+3,\ \{x\text{->}7\})\Downarrow ?}}{(y{:=}\ 2 + 3,\ \{x\text{->}\ 7\})\Downarrow ?}}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi},\ \{x \text{->} 7\}) \Downarrow ?}
$$

Example

$$\dfrac{\dfrac{7 > 5 = \text{true} \qquad (\text{x},\{\text{x->7}\})\Downarrow 7 \quad (5,\{\text{x->7}\})\Downarrow 5}{(\text{x} > 5,\ \{\text{x -> 7}\})\Downarrow \text{true}} \qquad \dfrac{\dfrac{? + ? = ? \qquad (2,\{\text{x->7}\})\Downarrow ? \quad (3,\{\text{x->7}\})\Downarrow ?}{(2+3,\ \{\text{x->7}\})\Downarrow ?}}{(\text{y:= } 2 + 3,\ \{\text{x-> 7}\})\Downarrow ?}}{(\text{if x > 5 then y:= 2 + 3 else y:=3 + 4 fi},\ \{\text{x -> 7}\}) \Downarrow ?}$$

Example

$$\dfrac{\dfrac{(x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5 \quad 7 > 5 = true}{(x > 5, \{x \text{->} 7\})\Downarrow true} \qquad \dfrac{\dfrac{(2,\{x\text{->}7\})\Downarrow 2 \quad (3,\{x\text{->}7\})\Downarrow 3 \quad 2 + 3 = 5}{(2+3, \{x\text{->}7\})\Downarrow ?}}{(y := 2 + 3, \{x\text{->} 7\}) \Downarrow ?}}{(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi}, \{x \text{->} 7\}) \Downarrow ?}$$

Example

2 + 3 = 5

(2,{x->7})⇓2 (3,{x->7}) ⇓3

7 > 5 = true (2+3, {x->7})⇓5

(x,{x->7})⇓7 (5,{x->7})⇓5 (y:= 2 + 3, {x-> 7}

(x > 5, {x -> 7})⇓true ⇓?

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,

{x -> 7}) ⇓ ?

Example

2 + 3 = 5

(2,{x->7})⇓2 (3,{x->7}) ⇓3

7 > 5 = true (2+3, {x->7})⇓5

(x,{x->7})⇓7 (5,{x->7})⇓5 (y:= 2 + 3, {x-> 7})

(x > 5, {x -> 7})⇓true ⇓ {y->5,x->7}

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,

{x -> 7}) ⇓?

Example

$$
\dfrac{\dfrac{7 > 5 = \text{true} \quad \dfrac{}{(x,\{x\text{->}7\})\Downarrow 7} \quad \dfrac{}{(5,\{x\text{->}7\})\Downarrow 5}}{(x > 5,\ \{x \text{->} 7\})\Downarrow \text{true}} \qquad \dfrac{\dfrac{2 + 3 = 5 \quad \dfrac{}{(2,\{x\text{->}7\})\Downarrow 2} \quad \dfrac{}{(3,\{x\text{->}7\})\Downarrow 3}}{(2+3,\ \{x\text{->}7\})\Downarrow 5}}{(y{:}=2 + 3,\ \{x\text{->}\ 7\})\Downarrow \{y\text{->}5, x\text{->}7\}}}{(\text{if } x > 5 \text{ then } y{:}= 2 + 3 \text{ else } y{:}=3 + 4 \text{ fi},\ \{x \text{->} 7\}) \Downarrow \{y\text{->}5,x\text{->}7\}}
$$

Let in Command

$$\frac{(E,m) \Downarrow \mathrm{v} \quad (C,m[I\text{<-}v]) \Downarrow m'}{(\mathrm{let}\ I = E\ \mathrm{in}\ C,\ m) \Downarrow m''}$$

Where $m''(x) = m'(x)$ for $x \neq I$ and $m''(I) = m(I)$ if $m(I)$ is defined, otherwise $m''(I)$ is undefined

Example

$$
\dfrac{(5,\{x\text{->}17\}) \Downarrow 5 \qquad \dfrac{\dfrac{(x,\{x\text{->}5\}) \Downarrow 5 \quad (3,\{x\text{->}5\}) \Downarrow 3}{(x+3,\{x\text{->}5\}) \Downarrow 8}}{(x{:=}x+3,\{x\text{->}5\}) \Downarrow \{x\text{->}8\}}}{(\text{let } x = 5 \text{ in } (x{:=}x+3),\ \{x \text{ -> } 17\}) \Downarrow ?}
$$

Example

$$\frac{(5,\{x\text{->}17\})\Downarrow 5 \qquad \dfrac{\dfrac{(x,\{x\text{->}5\})\Downarrow 5 \quad (3,\{x\text{->}5\})\Downarrow 3}{(x+3,\{x\text{->}5\})\Downarrow 8}}{(x:=x+3,\{x\text{->}5\})\Downarrow \{x\text{->}8\}}}{(\text{let } x = 5 \text{ in } (x:=x+3),\ \{x \text{ -> } 17\})\Downarrow 17}$$

Comment

- Simple Imperative Programming Language introduces variables *implicitly* through assignment
- The let-in command introduces scoped variables *explictly*
- Clash of constructs apparent in awkward semantics

Interpretation Versus Compilation

- A **compiler** from language L1 to language L2 is a program that takes an L1 program and for each piece of code in L1 generates a piece of code in L2 of same meaning
- An **interpreter** of L1 in L2 is an L2 program that executes the meaning of a given L1 program
- Compiler would examine the body of a loop once; an interpreter would examine it every time the loop was executed

Interpreter

- An *Interpreter* represents the operational semantics of a language L1 (source language) in the language of implementation L2 (target language)
- Built incrementally
  - Start with literals
  - Variables
  - Primitive operations
  - Evaluation of expressions
  - Evaluation of commands/declarations

Interpreter

- Takes abstract syntax trees as input
  - In simple cases could be just strings
- One procedure for each syntactic category (nonterminal)
  - eg one for expressions, another for commands
- If Natural semantics used, tells how to compute final value from code
- If Transition semantics used, tells how to compute next “state”
  - To get final value, put in a loop

Natural Semantics Example

- compute_exp (Var(v), m) = look_up v m
- compute_exp (Int(n), _) = Num (n)
- ...
- compute_com(IfExp(b,c1,c2),m) =

  if compute_exp (b,m) = Bool(true)

  then compute_com (c1,m)

  else compute_com (c2,m)

Natural Semantics Example

- compute_com(While(b,c), m) =
  if compute_exp (b,m) = Bool(false)
  then m
  else compute_com
  (While(b,c), compute_com(c,m))

- May fail to terminate - exceed stack limits
- Returns no useful information then