Programming Languages and Compilers (CS 421)



Reza Zamani

http://www.cs.illinois.edu/class/cs421/

 $First(X) = \{t \mid X \stackrel{*}{\Rightarrow} t\alpha\} \cup \{\epsilon \mid X \stackrel{*}{\Rightarrow} \epsilon\}$

- Algorithm to compute all First sets:
 - For all terminal t, First(t)={t}.
 - For all production $X := \epsilon$, add ϵ to First(X).
 - For all production $X:=Y_1Y_2...Y_k$
 - Add terminal a to First(X) if for some i, $a \in First(Y_i)$ and $\epsilon \in First(Y_j)$ for all j < i
 - Add ϵ to First(X) if ϵ is in $First(Y_i)$ for all $1 \le i \le k$.
 - Repeat this step till no First set changes.



$$H ::= +TH \mid \epsilon$$

$$T ::= FL$$

$$L ::= *FL \mid \epsilon$$

$$F ::= (E) \mid id$$



$$H := +TH \mid \epsilon$$

$$T ::= FL$$

$$L ::= *FL \mid \epsilon$$

$$First(H) = \{\epsilon\}$$

$$First(L) = \{\epsilon\}$$



$$H := +TH \mid \epsilon$$

$$T ::= FL$$

$$L ::= *FL \mid \epsilon$$

$$F ::= (E) \mid id$$

$$First(H) = \{ \epsilon , + \}$$

$$First(F) = \{ (, id) \}$$

$$First(L) = \{ \epsilon, * \}$$



$$H := +TH \mid \epsilon$$

$$T ::= FL$$

$$L ::= *FL \mid \epsilon$$

$$First(H) = \{ \epsilon , + \}$$

$$First(F) = \{ (, id) \}$$

$$First(L) = \{ \epsilon, * \}$$



$$H := +TH \mid \epsilon$$

$$T ::= FL$$

$$L ::= *FL \mid \epsilon$$

• First(E)=
$$\{(, id)\}$$

First(T)= $\{(, id)\}$
First(H)= $\{\epsilon, +\}$
First(F)= $\{(, id)\}$
First(L)= $\{\epsilon, *\}$

• First(E)=
$$\{(, id)\}$$

First(T)= $\{(, id)\}$
First(H)= $\{\epsilon, +\}$
First(F)= $\{(, id)\}$
First(L)= $\{\epsilon, *\}$

No change! hence were are done.

Fo

Follow Sets

 $Follow(X) = \{t \mid S \stackrel{*}{\Rightarrow} \alpha X t \beta \}$

- Algorithm to compute all Follow sets:
 - For the start symbol S, put \$ in Follow(S).
 - For all productions $X := \alpha Y \beta$:
 - Add all $First(\beta) \{\epsilon\}$ to Follow(Y).
 - If $\beta = \epsilon$ or $\epsilon \in First(\beta)$, then add all Follow(X) to Follow(Y).
 - Repeat this step till now Follow set changes.

Follow Sets

$$H := +TH \mid \epsilon$$

$$T ::= FL$$

$$L ::= *FL \mid \epsilon$$

$$F ::= (E) \mid id$$

Follow(E)= {\$} Follow(T)= { } Follow(H)= { } Follow(F)= { } Follow(L)= { }

Follow Sets

$$H := +TH \mid \epsilon$$

$$T ::= FL$$

$$L ::= *FL \mid \epsilon$$

Follow(E)= {\$,)}
 Follow(T)= {+, \$}
 Follow(H)= {\$}
 Follow(F)= {*,+,\$}
 Follow(L)= { +,\$}

Follow Sets

Follow(E)= {\$,)}
 Follow(T)= {+, \$,)}
 Follow(H)= {\$,)}
 Follow(F)= {*,+,\$,)}
 Follow(L)= { +,\$,)}

Follow Sets

$$H := +TH \mid \epsilon$$

$$T ::= FL$$

$$L ::= *FL \mid \epsilon$$

Follow(E)= {\$,)}Follow(T)= {+, \$,)}

Follow(H)=
$$\{\$,\}$$

Follow(F)=
$$\{*,+,\$,\}$$

Follow(L)=
$$\{+, \$, \}$$

Start symbol is E

No change! Hence we are done.

Bottom-Up parsing

- LR(k):
 - Scan the input from Left to right.
 - Produce a Rightmost derivation (in reverse order).
- LR variants:
 - LR(0), LR(1), LR(2), ...
 - SLR
 - LALR



```
    E'::= E
    E::= E + T | T
    T::= T * F | F
    F::= (E) | id
```

```
Start symbol is E'
```

```
First(E') = First(E) = First(T)
= First(F) = { ( , id }
```

```
Follow(E')={ $ }
Follow(E)={+, ), $}
Follow(T)=Follow(F)={+,*,) , $}
```



10:

 $\mathsf{E}' ::= ullet \mathsf{E}$

6/28/2011



```
10:
```

 $\mathsf{E}' ::= ullet \mathsf{E}$

 $E := \bullet E + T$

 $\mathsf{E} := ullet \mathsf{T}$

6/28/2011



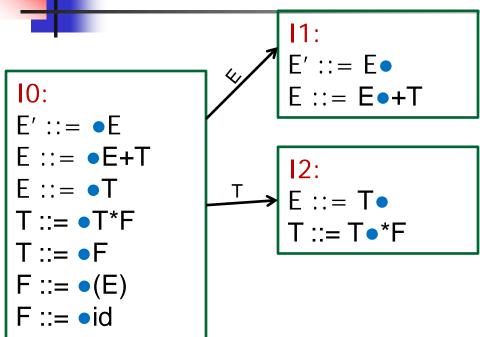
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```

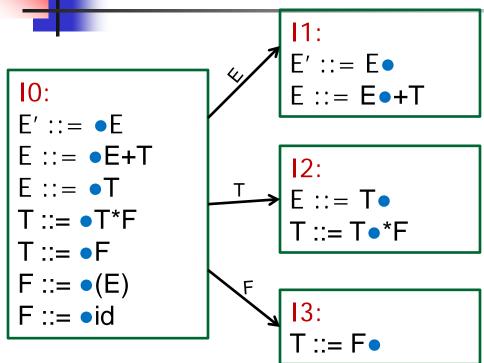
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\mathsf{E}' ::= ullet \mathsf{E}
```

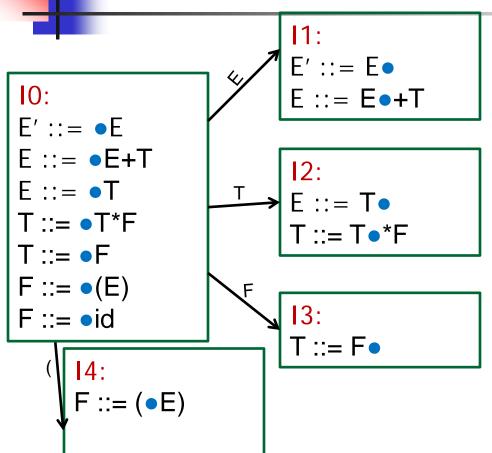
$$T ::= \bullet F$$

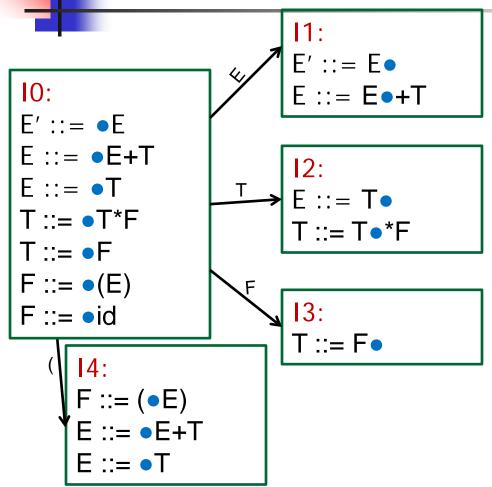
$$F := \bullet id$$

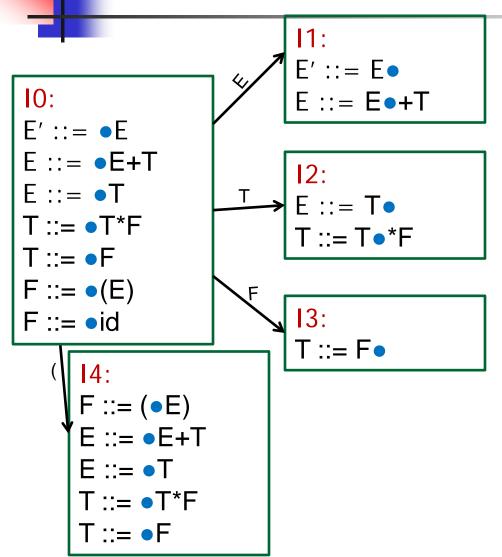
```
IO:
E' ::= • E
E ::= • E+T
E ::= • T
T ::= • T*F
T ::= • F
F ::= • (E)
F ::= • id
```

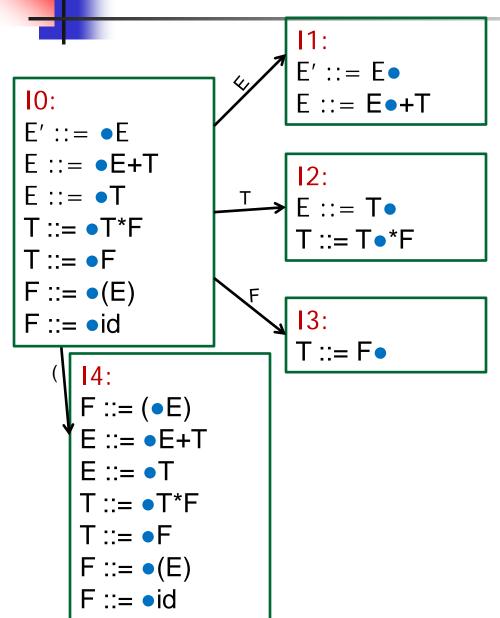




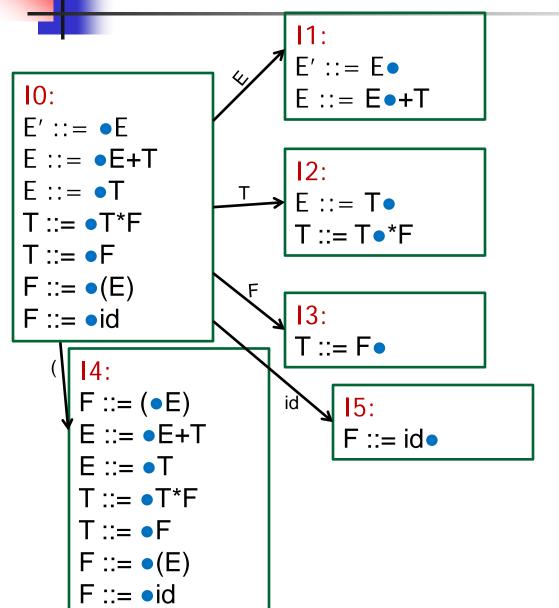


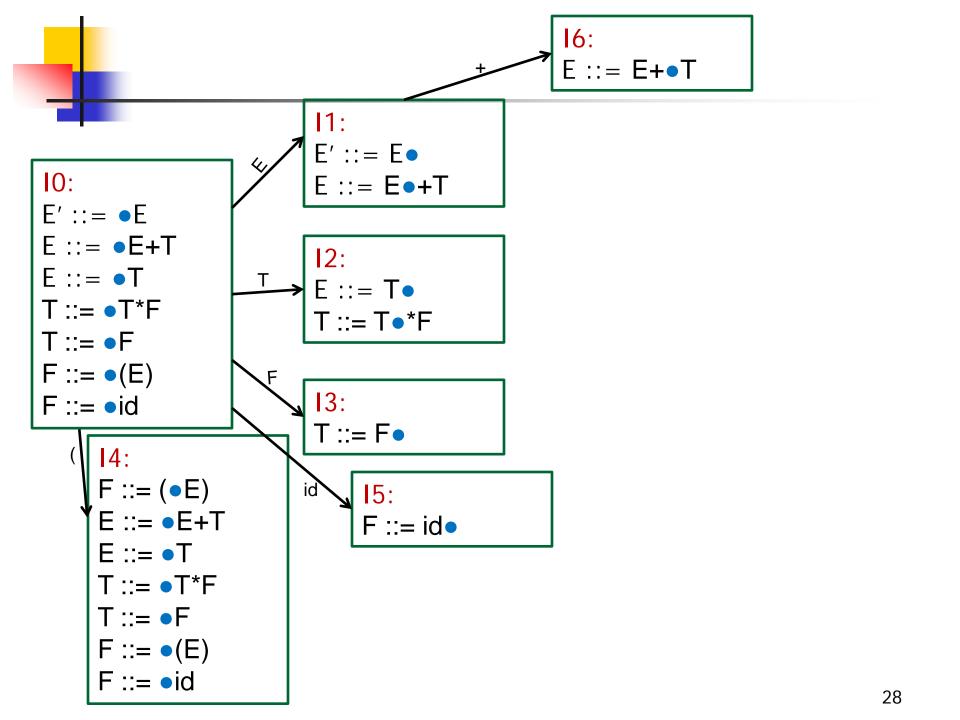


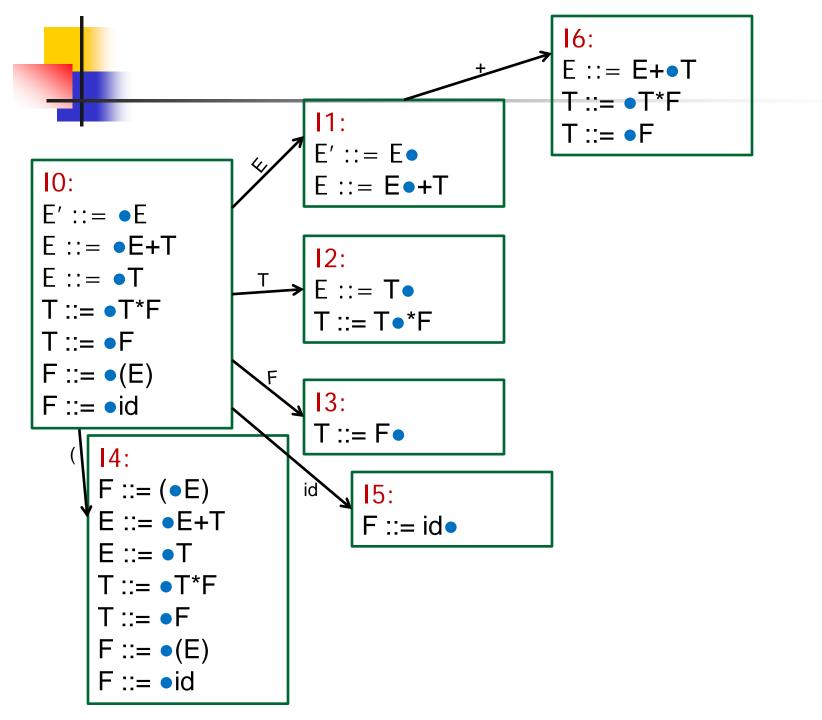


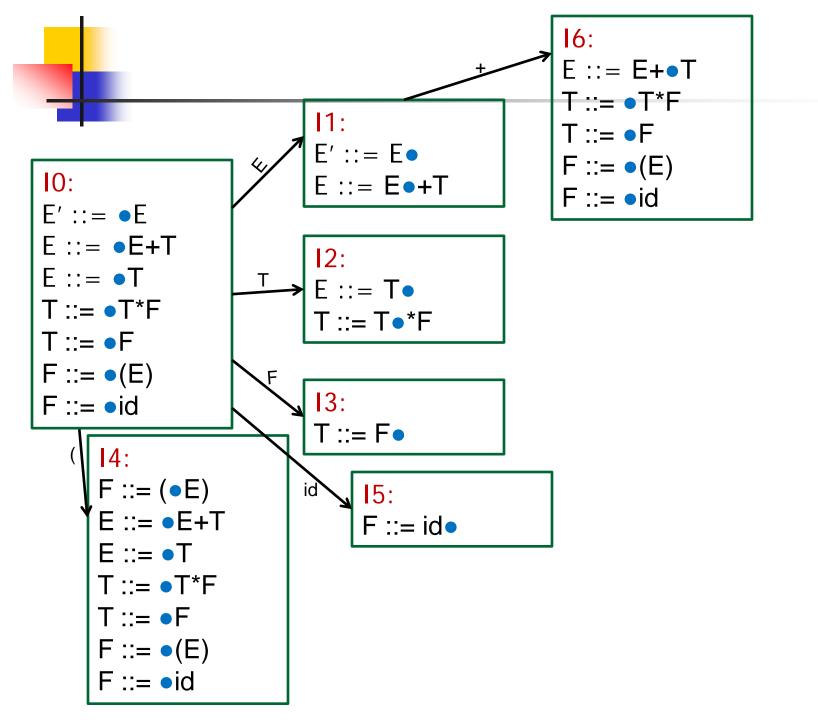


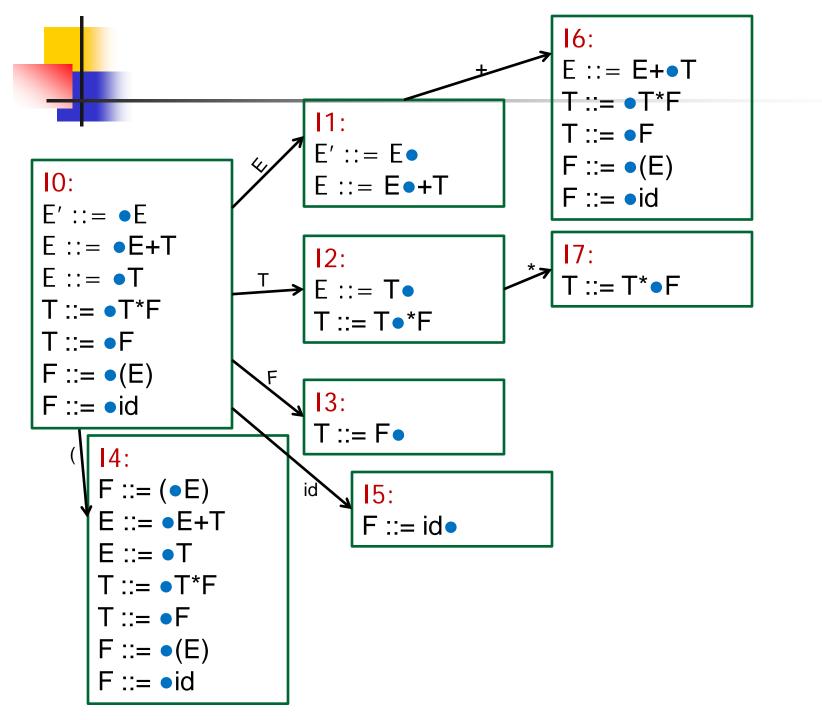
S

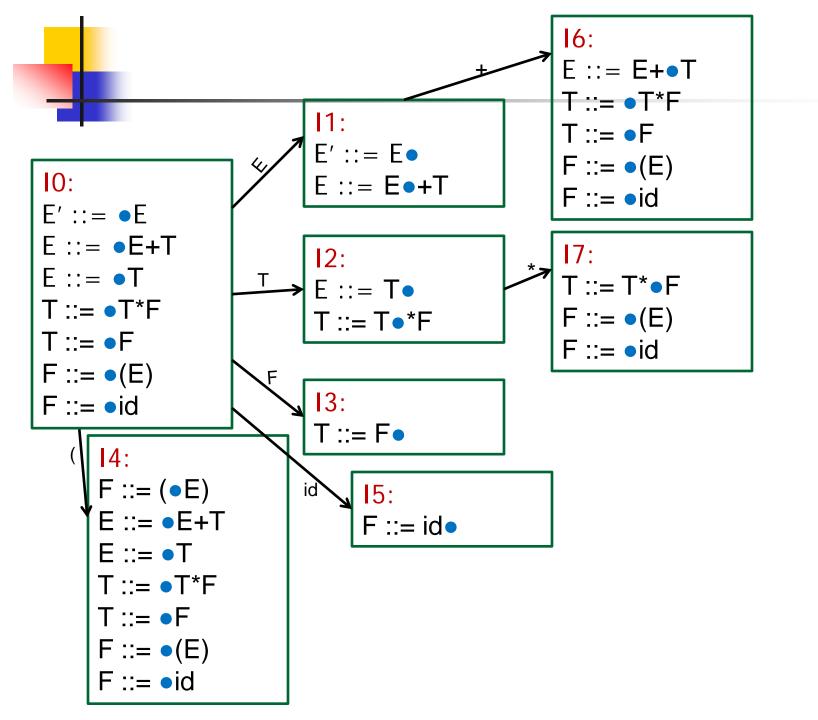


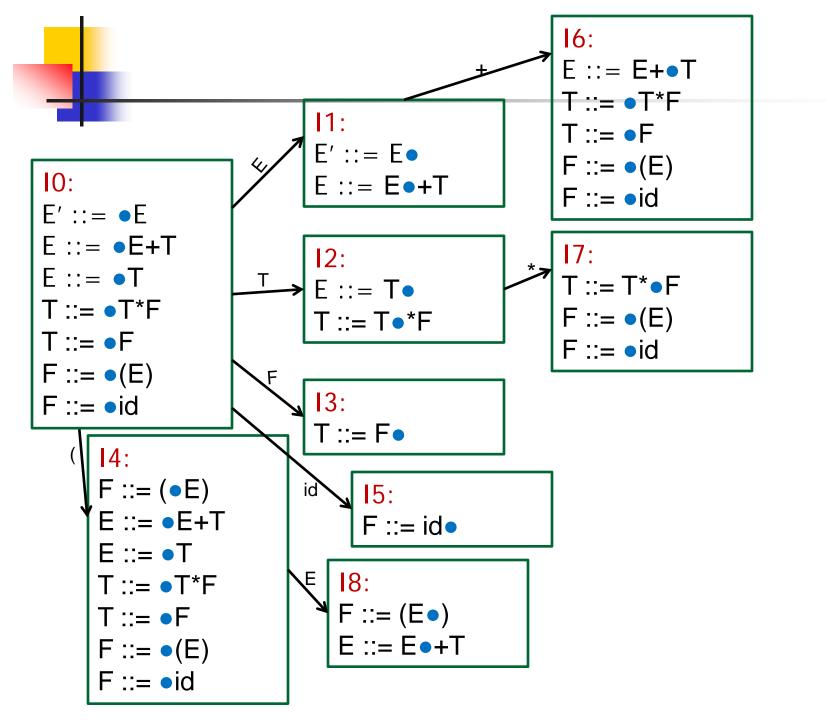


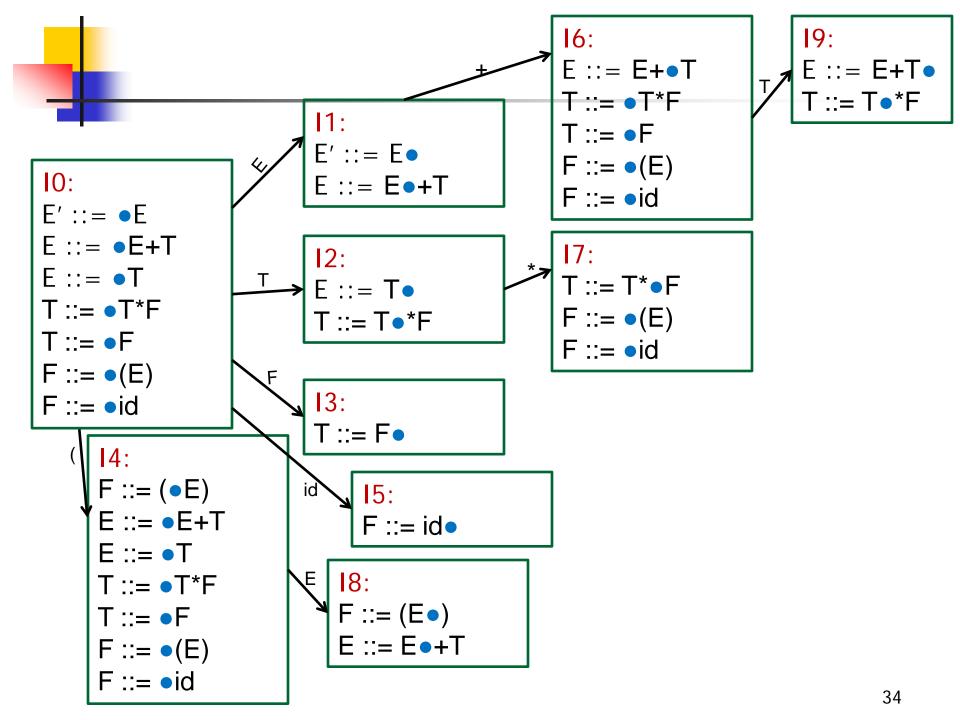


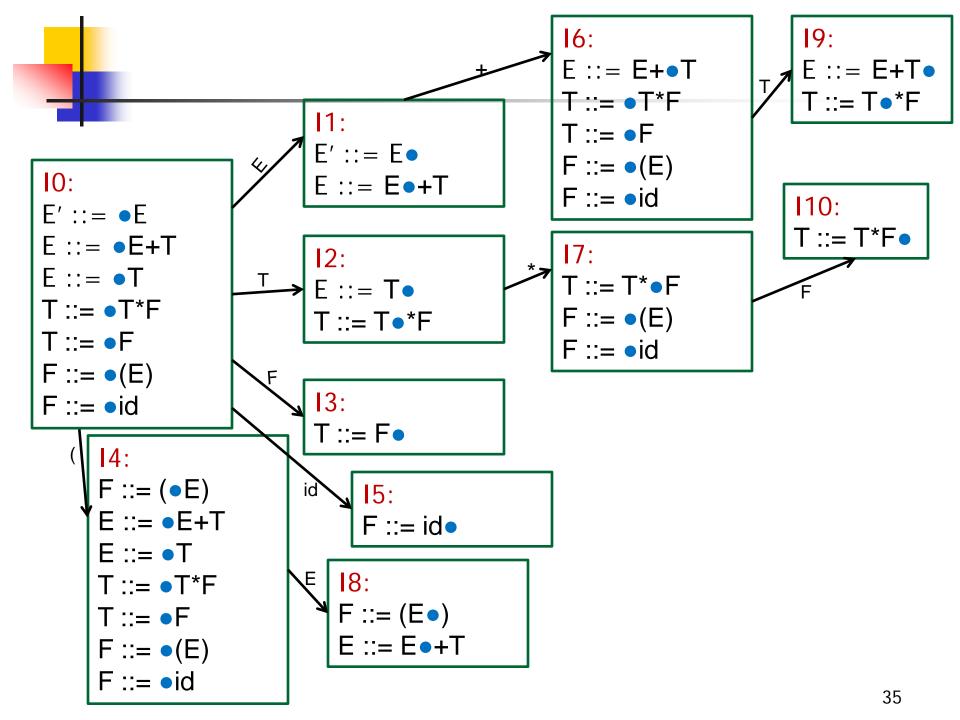


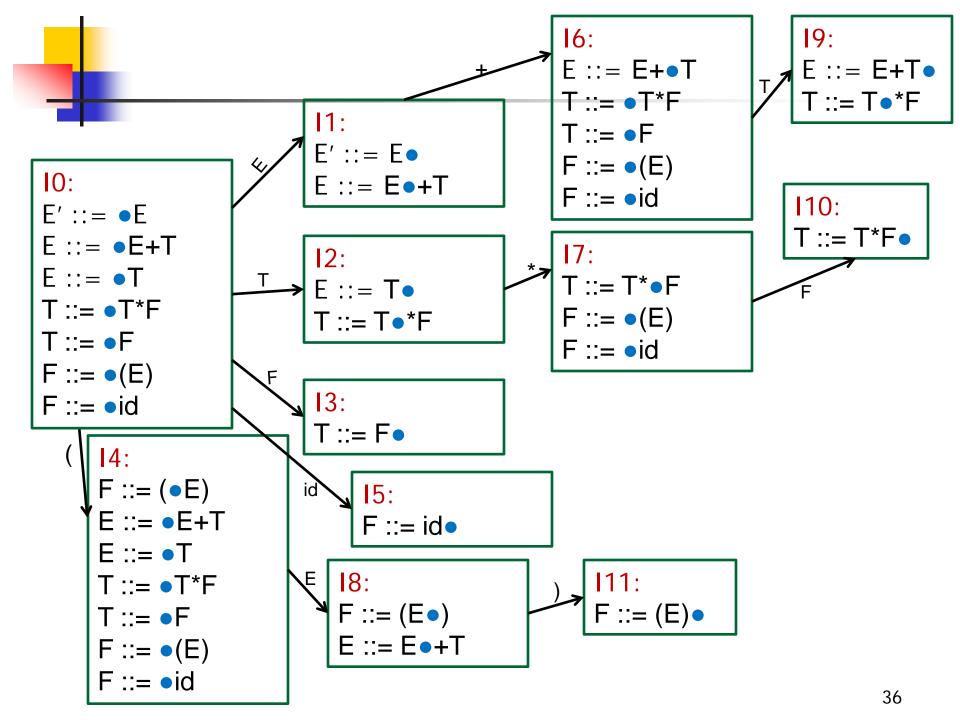


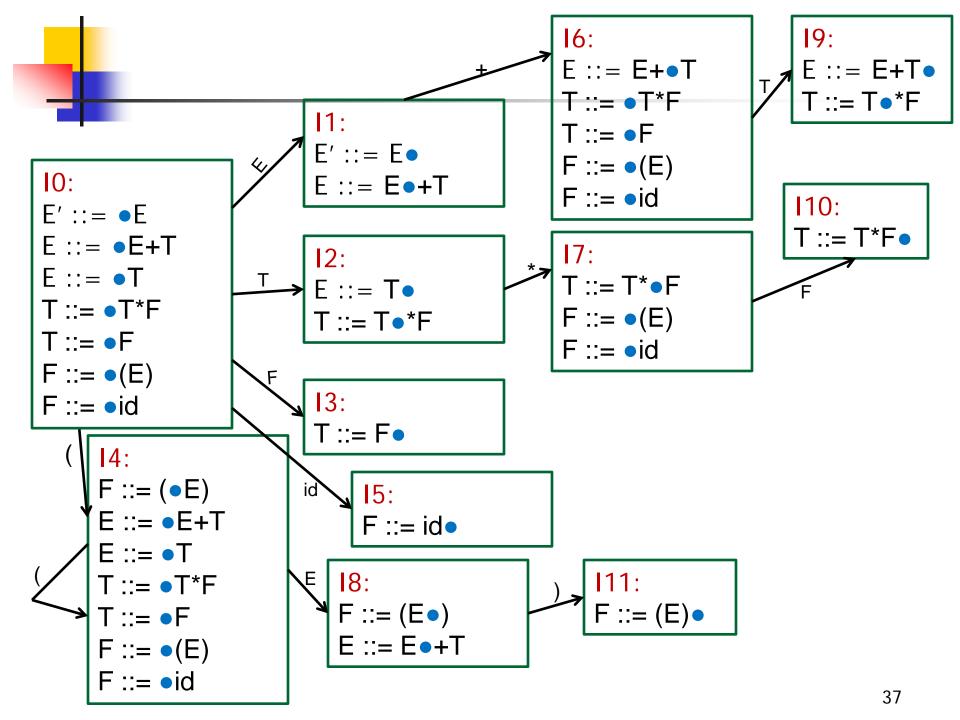


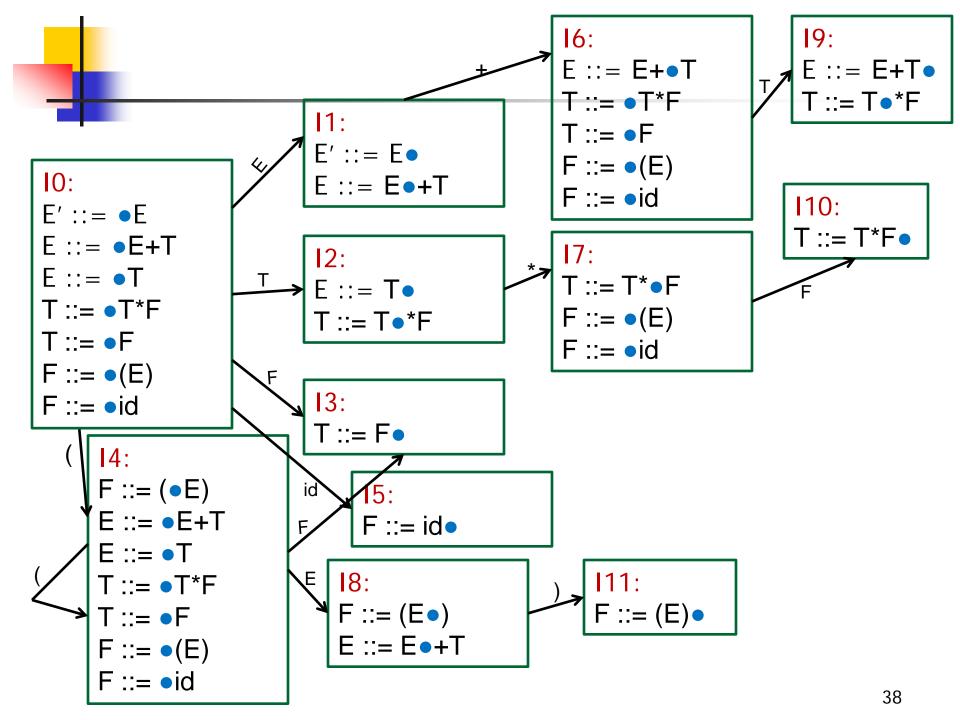


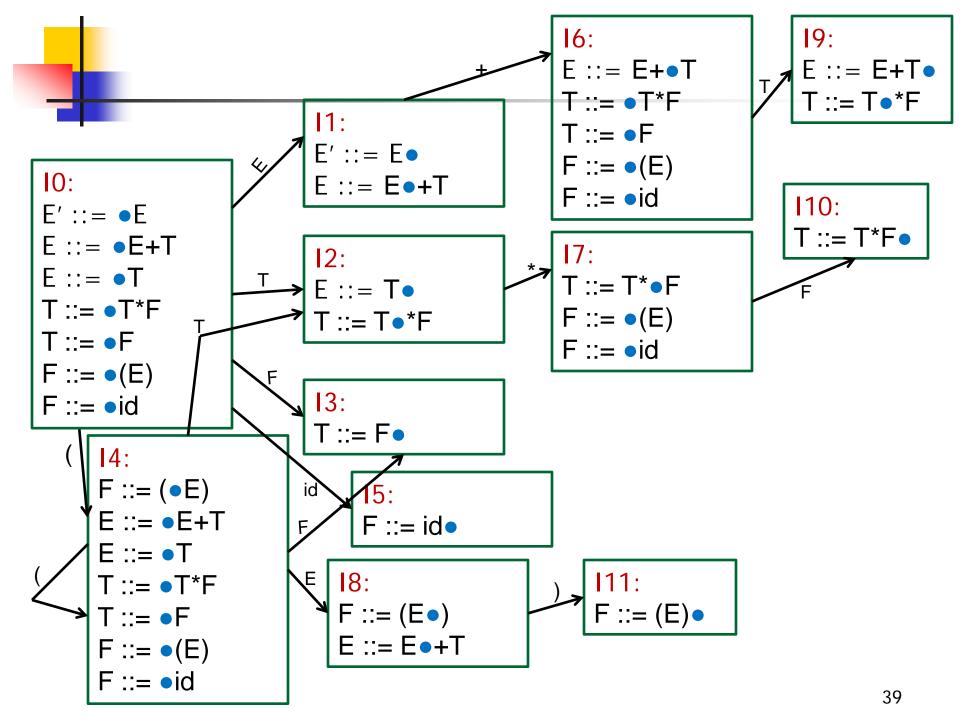


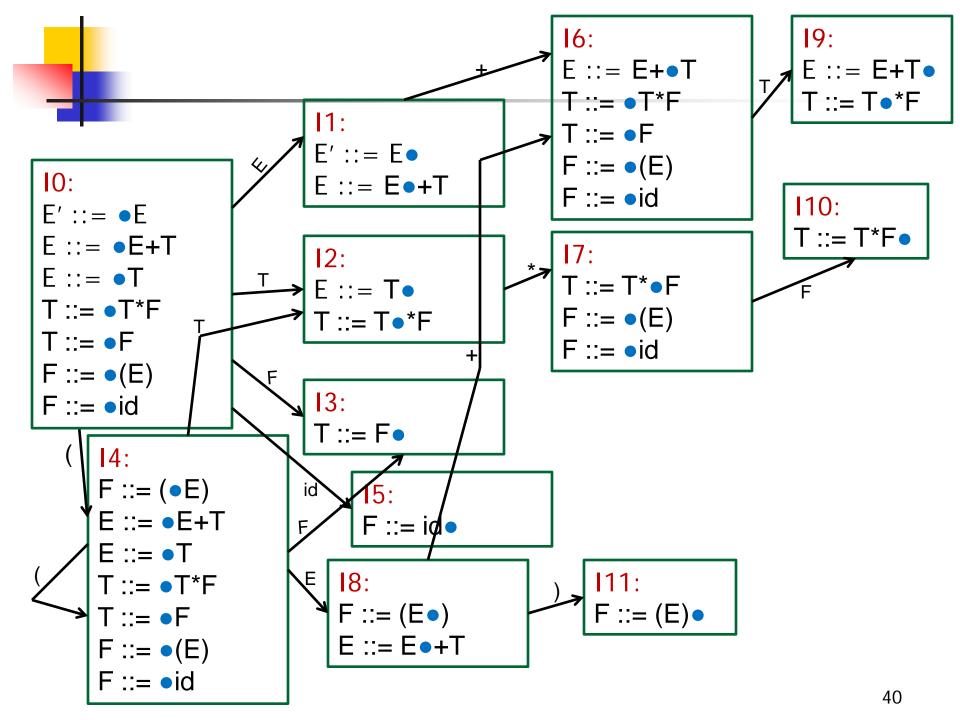


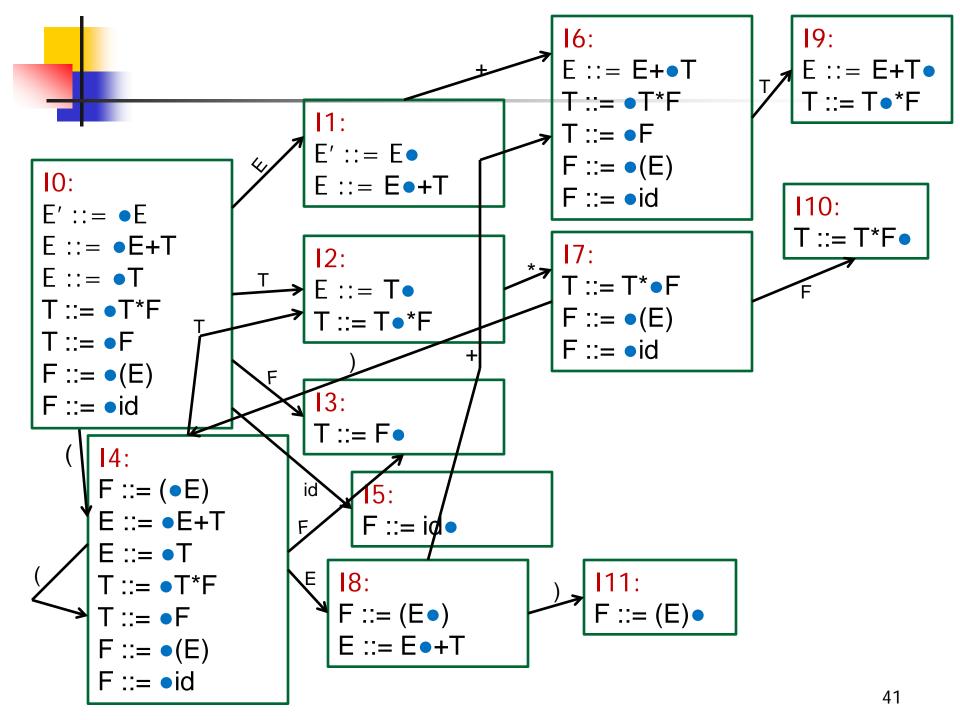


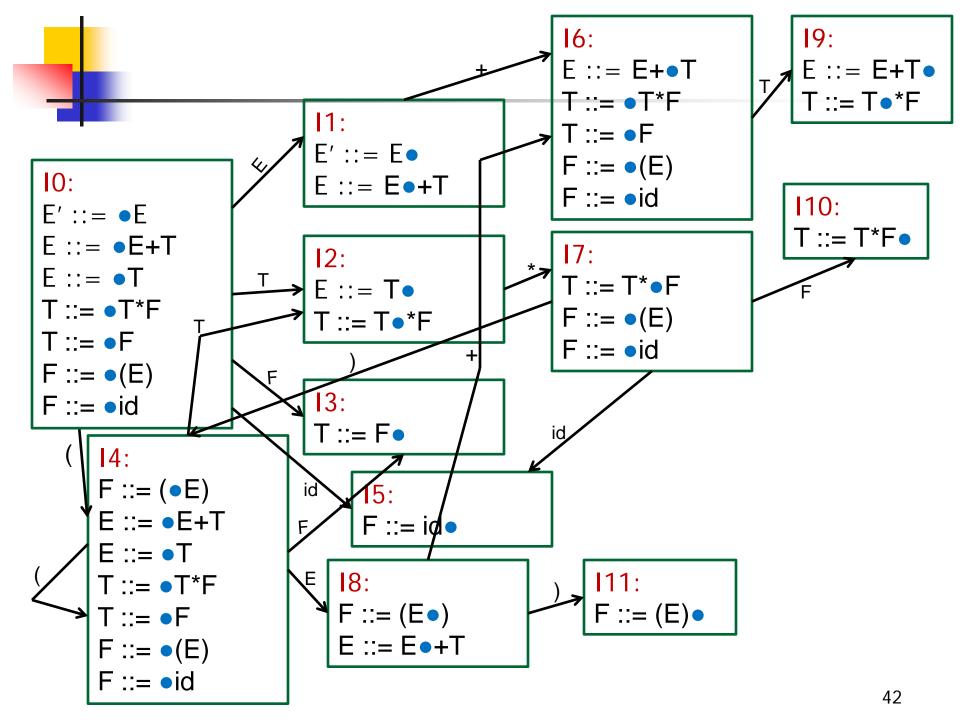


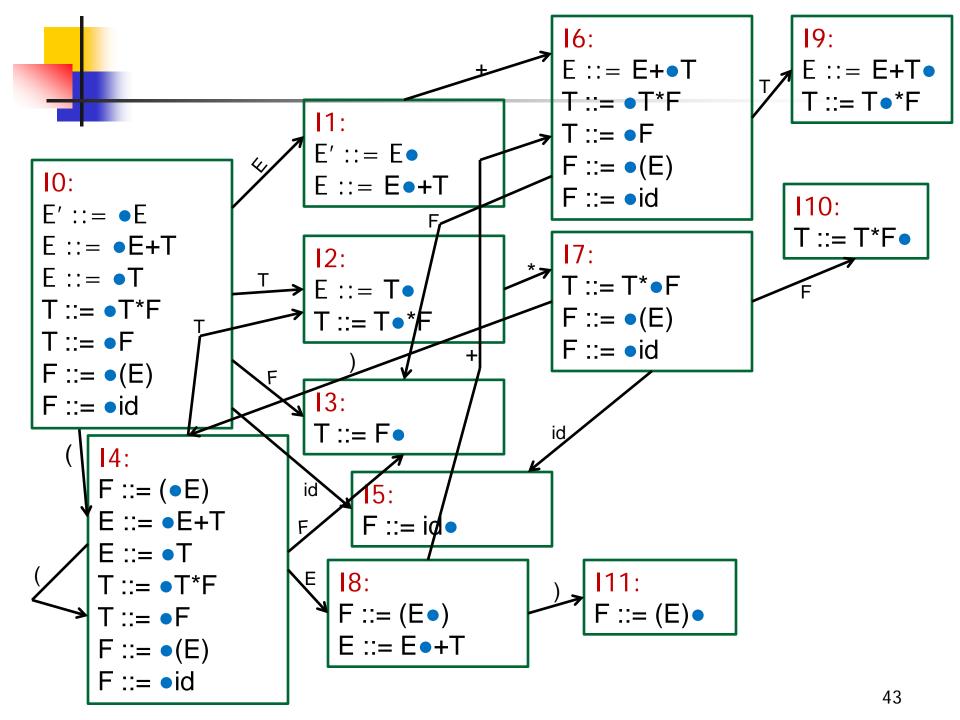


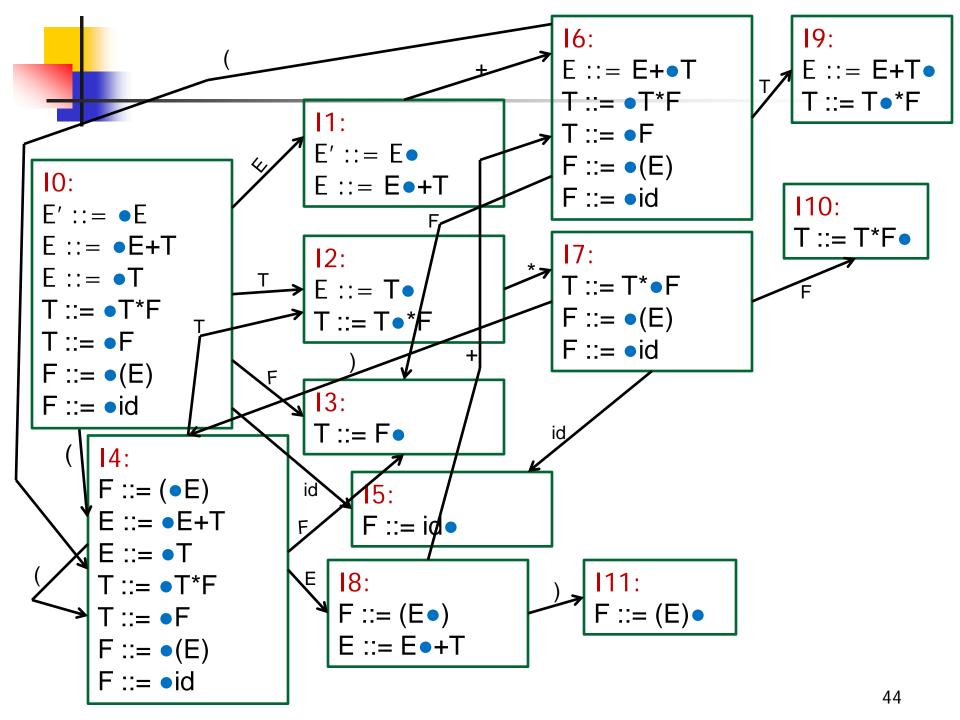


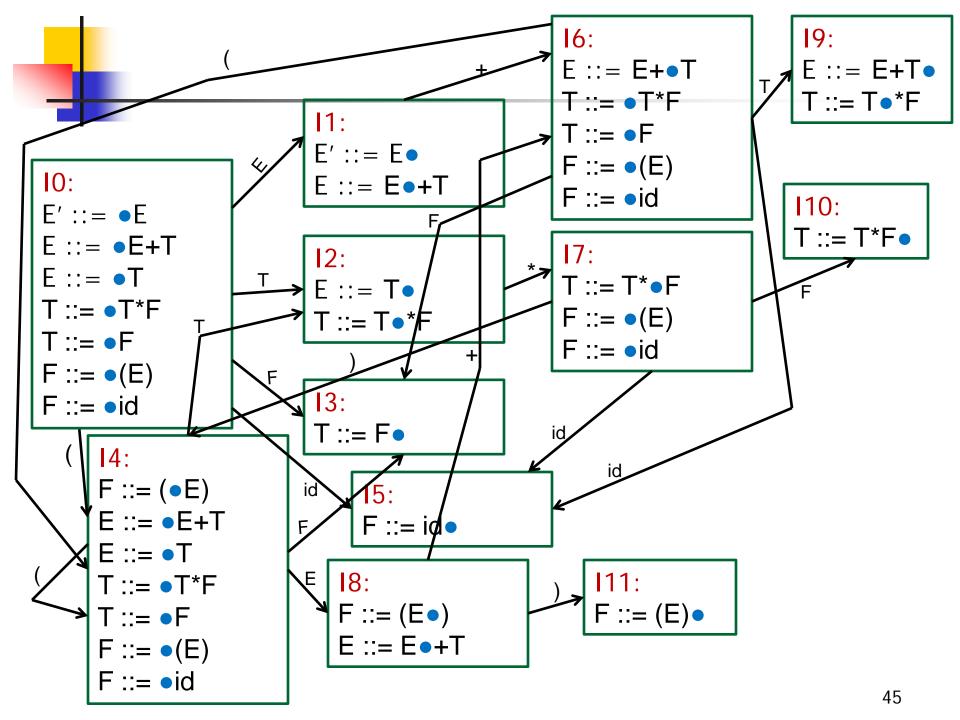


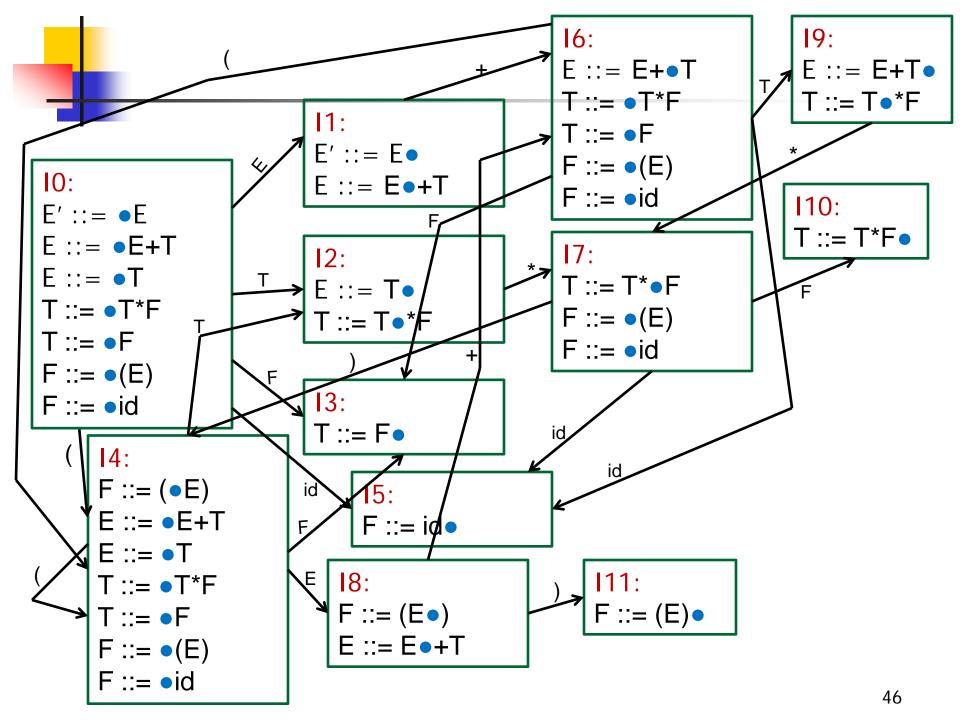


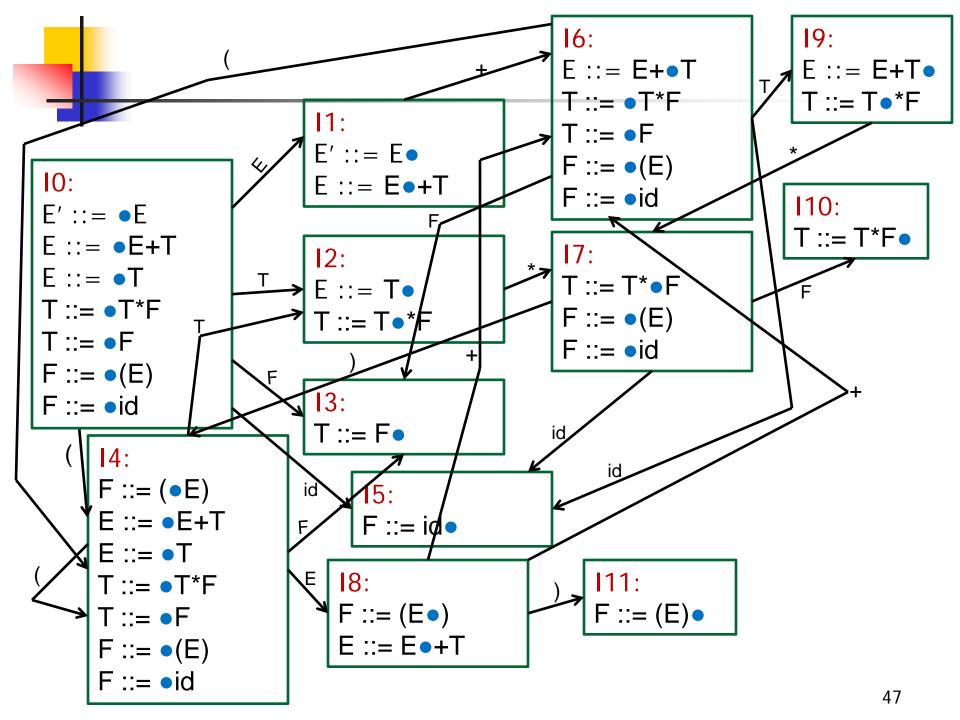


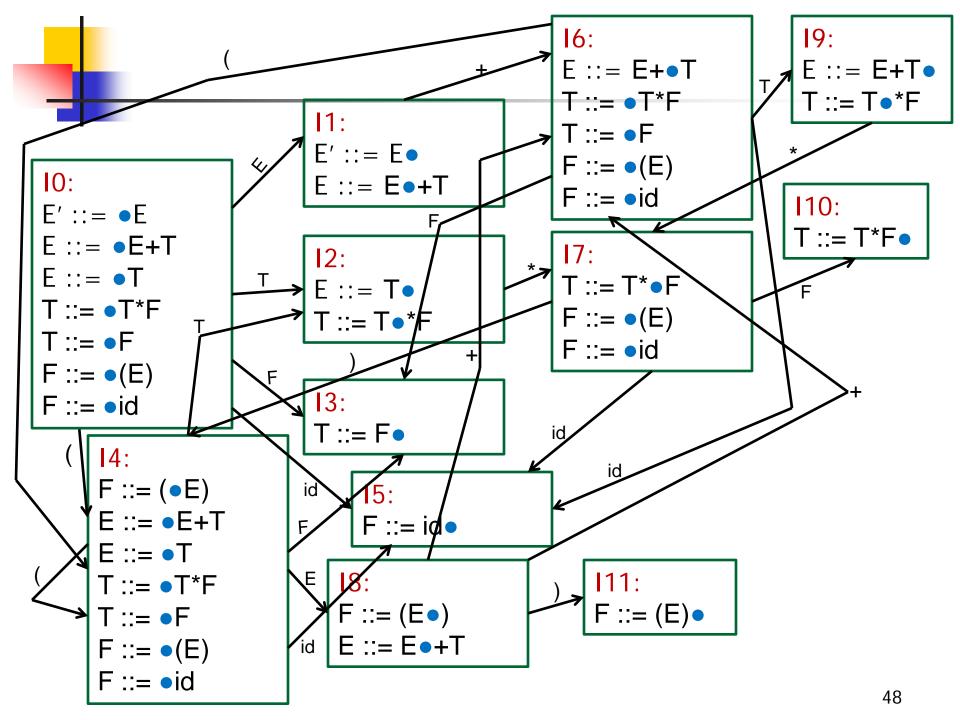












Action-Goto Table

S			Act	ion				Goto	
S T A T e	Id	+	*	()	\$	E	Т	F
0	S5			S4			1	2	3
1		S6				Acc			
2		R2	S7		R2	R2			
3		R4	R4		R4	R4			
4	S5			S4			8	2	3
5		R6	R6		R6	R6			
6	S5			S4				9	3
7	S5			S4					10
8		S6			S11				
9		R1	S7		R1	R1			
10		R3	R3		R3	R3			
11		R5	R5		R5	R5			

- (1) E := E + T
- (2) E := T
- (3) T:=T*F
- (4) T::=F
- (5) F:=(E)
- (6) F::=id

$$Follow(T) = \{+, *, \}$$

STACK	INPUT	
0	id * id + id \$	action=S5

- (1) E := E + T
- (2) E := T
- (3) T:=T*F
- (4) T::=F
- (5) F:=(E)
- (6) F::=id

STACK	INPUT	
0	id * id + id \$	action=S5
0 id 5	* id + id \$	action=R6, $goto(0,F)=3$

- (1) E := E + T
- (2) E := T
- (3) T:=T*F
- (4) T::=F
- (5) F:=(E)
- (6) F::=id

STACK	INPUT	
0	id * id + id \$	action=S5
0 id 5	* id + id \$	action=R6, goto(0,F)=3
0 F 3	* id + id \$	action=R4, goto(0,T)=2

- (1) E := E + T
- (2) E := T
- (3) T:=T*F
- (4) T::=F
- (5) F::=(E)
- (6) F::=id

STACK	INPUT	
0	id * id + id \$	action=S5
0 id 5	* id + id \$	action=R6, goto(0,F)=3
0 F 3	* id + id \$	action=R4, $goto(0,T)=2$
0 T 2	* id + id \$	action=S7

- (1) E := E + T
- (2) E := T
- (3) T:=T*F
- (4) T::=F
- (5) F::=(E)
- (6) F::=id

STACK	INPUT	
0	id * id + id \$	action=S5
0 id 5	* id + id \$	action=R6, goto(0,F)=3
0 F 3	* id + id \$	action=R4, $goto(0,T)=2$
0 T 2	* id + id \$	action=S7
0 T 2 * 7	id + id \$	action=S5

- (1) E := E + T
- (2) E := T
- (3) T:=T*F
- (4) T::=F
- (5) F:=(E)
- (6) F::=id

STACK	INPUT	
0	id * id + id \$	action=S5
0 id 5	* id + id \$	action=R6, $goto(0,F)=3$
0 F 3	* id + id \$	action=R4, $goto(0,T)=2$
0 T 2	* id + id \$	action=S7
0 T 2 * 7	id + id \$	action=S5
0 T 2 * 7 id 5	+id \$	action=R6, $goto(7,F)=10$

- (1) E := E + T
- (2) E := T
- (3) T:=T*F
- (4) T::=F
- (5) F:=(E)
- (6) F::=id

STACK	INPUT	
0	id * id + id \$	action=S5
0 id 5	* id + id \$	action=R6, goto(0,F)=3
0 F 3	* id + id \$	action=R4, $goto(0,T)=2$
0 T 2	* id + id \$	action=S7
0 T 2 * 7	id + id \$	action=S5
0 T 2 * 7 id 5	+id \$	action=R6, $goto(7,F)=10$
0 T 2 * 7 F 10	+id \$	action=R3, $goto(0,T)=2$

- (1) E := E + T
- (2) E := T
- (3) T:=T*F
- (4) T::=F
- (5) F::=(E)
- (6) F::=id

STACK	INPUT	
0	id * id + id \$	action=S5
0 id 5	* id + id \$	action=R6, goto(0,F)=3
0 F 3	* id + id \$	action=R4, $goto(0,T)=2$
0 T 2	* id + id \$	action=S7
0 T 2 * 7	id + id \$	action=S5
0 T 2 * 7 id 5	+id \$	action=R6, $goto(7,F)=10$
0 T 2 * 7 F 10	+id \$	action=R3, $goto(0,T)=2$
0 T 2	+id \$	action=R2, goto(0,E)=1

- (1) E := E + T
- (2) E := T
- (3) T:=T*F
- (4) T::=F
- (5) F::=(E)
- (6) F::=id

STACK	INPUT	
0	id * id + id \$	action=S5
0 id 5	* id + id \$	action=R6, goto(0,F)=3
0 F 3	* id + id \$	action=R4, $goto(0,T)=2$
0 T 2	* id + id \$	action=S7
0 T 2 * 7	id + id \$	action=S5
0 T 2 * 7 id 5	+id \$	action=R6, $goto(7,F)=10$
0 T 2 * 7 F 10	+id \$	action=R3, $goto(0,T)=2$
0 T 2	+id \$	action=R2, goto(0,E)=1
0 E 1	+id \$	action=S6

- (1) E := E + T
- (2) E := T
- (3) T:=T*F
- (4) T::=F
- (5) F::=(E)
- (6) F::=id

STACK	INPUT	
0	id * id + id \$	action=S5
0 id 5	* id + id \$	action=R6, $goto(0,F)=3$
0 F 3	* id + id \$	action=R4, $goto(0,T)=2$
0 T 2	* id + id \$	action=S7
0 T 2 * 7	id + id \$	action=S5
0 T 2 * 7 id 5	+id \$	action=R6, $goto(7,F)=10$
0 T 2 * 7 F 10	+id \$	action=R3, $goto(0,T)=2$
0 T 2	+id \$	action=R2, goto(0,E)=1
0 E 1	+id \$	action=S6
0 E 1 + 6	id \$	action=S5

- (1) E := E + T
- (2) E := T
- (3) T:=T*F
- (4) T::=F
- (5) F::=(E)
- (6) F::=id

STACK	INPUT	
0	id * id + id \$	action=S5
0 id 5	* id + id \$	action=R6, goto(0,F)=3
0 F 3	* id + id \$	action=R4, $goto(0,T)=2$
0 T 2	* id + id \$	action=S7
0 T 2 * 7	id + id \$	action=S5
0 T 2 * 7 id 5	+id \$	action=R6, $goto(7,F)=10$
0 T 2 * 7 F 10	+id \$	action=R3, $goto(0,T)=2$
0 T 2	+id \$	action=R2, goto(0,E)=1
0 E 1	+id \$	action=S6
0 E 1 + 6	id \$	action=S5
0 E 1 +6 id 5	\$	action=R6, $goto(6,F)=3$

- (1) E := E + T
- (2) E := T
- (3) T:=T*F
- (4) T::=F
- (5) F::=(E)
- (6) F::=id

STACK	INPUT	
0	id * id + id \$	action=S5
0 id 5	* id + id \$	action=R6, goto(0,F)=3
0 F 3	* id + id \$	action=R4, $goto(0,T)=2$
0 T 2	* id + id \$	action=S7
0 T 2 * 7	id + id \$	action=S5
0 T 2 * 7 id 5	+id \$	action=R6, $goto(7,F)=10$
0 T 2 * 7 F 10	+id \$	action=R3, $goto(0,T)=2$
0 T 2	+id \$	action=R2, goto(0,E)=1
0 E 1	+id \$	action=S6
0 E 1 + 6	id \$	action=S5
0 E 1 +6 id 5	\$	action=R6, $goto(6,F)=3$
0 E 1 +6 F 3	\$	action=R4, $goto(6,T)=9$

- (1) E := E + T
- (2) E := T
- (3) T:=T*F
- (4) T::=F
- (5) F::=(E)
- (6) F::=id

STACK	INPUT	
0	id * id + id \$	action=S5
0 id 5	* id + id \$	action=R6, $goto(0,F)=3$
0 F 3	* id + id \$	action=R4, $goto(0,T)=2$
0 T 2	* id + id \$	action=S7
0 T 2 * 7	id + id \$	action=S5
0 T 2 * 7 id 5	+id \$	action=R6, goto $(7,F)=10$
0 T 2 * 7 F 10	+id \$	action=R3, $goto(0,T)=2$
0 T 2	+id \$	action=R2, goto(0,E)=1
0 E 1	+id \$	action=S6
0 E 1 + 6	id \$	action=S5
0 E 1 +6 id 5	\$	action=R6, $goto(6,F)=3$
0 E 1 +6 F 3	\$	action=R4, $goto(6,T)=9$
0 E 1 +6 T 9	\$	action=R1, goto(0,E)=1

- (1) E := E + T
- (2) E := T
- (3) T:=T*F
- (4) T::=F
- (5) F::=(E)
- (6) F::=id

STACK	INPUT	
0	id * id + id \$	action=S5
0 id 5	* id + id \$	action=R6, goto(0,F)=3
0 F 3	* id + id \$	action=R4, $goto(0,T)=2$
0 T 2	* id + id \$	action=S7
0 T 2 * 7	id + id \$	action=S5
0 T 2 * 7 id 5	+id \$	action=R6, $goto(7,F)=10$
0 T 2 * 7 F 10	+id \$	action=R3, $goto(0,T)=2$
0 T 2	+id \$	action=R2, goto(0,E)=1
0 E 1	+id \$	action=S6
0 E 1 + 6	id \$	action=S5
0 E 1 +6 id 5	\$	action=R6, $goto(6,F)=3$
0 E 1 +6 F 3	\$	action=R4, goto(6,T)=9
0 E 1 +6 T 9	\$	action=R1, goto(0,E)=1
0 E 1	\$	action=Accept

- (1) E := E + T
- (2) E := T
- (3) T:=T*F
- (4) T::=F
- (5) F::=(E)
- (6) F::=id