# STAT 400: Homework #1

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# Problem 1

Suppose that $P(A) = 0.60, P(B) = 0.40, P(A \cap B) = 0.30$ what is the probability that

(a) either $A$ occurs or $B$ occurs (or both)

### Solution

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.60 + 0.40 - 0.30 = 0.70$

(b) $B$ does not occur

### Solution

$P(B') = 1 - P(B) = 1 - 0.40 = 0.60$

(c) $B$ occurs and $A$ does not occur

### Solution

$P(A' \cap B) = P(B) - P(A \cap B) = 0.40 - 0.30 = 0.10$

(d) neither $A$ nor $B$ occurs

### Solution

$P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.70 = 0.30$

# Problem 2

Suppose $P(A) = 0.60, P(B) = 0.50, P(C) = 0.40, P(A \cap B) = 0.30, P(A \cap C) = 0.20,$
$P(B \cap C) = 0.20, P(A \cap B \cap C) = 0.10.$
Find:

(a) $P(A \cup B)$

### Solution

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.60 + 0.50 - 0.30 = 0.80$

(b) $P(B \cup C)$

### Solution

$P(B \cup C) = P(B) + P(C) - P(B \cap C) = 0.50 + 0.40 - 0.20 = 0.70$

(c) $P\left((A \cup B) \cap C'\right)$

### Solution

$P((A \cup B) \cap C') = P((P(B) + P(C) - P(B \cap C)) \cap C') = P((P(B) + P(C) - P(B \cap C)) * (1 - P(C))) = 0.80 * 0.60 = 0.48$

(d) $P(A \cup (B \cap C))$

### Solution

$P(A \cup (B \cap C)) =$

## Problem 3

Suppose a baseball player steps to the plate with the intention of trying to "cosx" a base on balls by never swinging at a pitch. The umpire, of course, will necessarily call each pitch either a ball ($B$) or a Strike ($T$). What outcomes make up the event $A$, that a batter walks on the sixth pitch? Note: A batter "walks" if the fourth ball is called before the third strike.

### Solution

$S = \{BBBTTB, BBTTBB, BTTBBB, TTBBBB, BBTBTB, BTBBTB, TBBBTB, BTBTBB, TBBTBB\}$

## Problem 4

Consider a "thick coin with three possible outcomes of a toss (Heads, Tails, Edge) for which Heads and Tails are equally likely, but edge is seven times less likely than Heads. What is the probability of Heads?

### Solution

Quantifying the space with heads existing in 7 spaces, tails existing in 7 spaces and edge existing in 1 space we find the probability of flipping a head is 7/15.

## Problem 5

Suppose $S = \{0, 1, 2, 3, \dots\}$ and $P(0) = p, P(k) = \frac{1}{5^k}, k = 1, 2, 3, \dots$

(a) Find the value of $p$ that would make this a valid probability model.

### Solution

$$\sum_{1}^{\infty} \frac{1}{5^k} = 1/4 \Rightarrow P(0) = 3/4$$

(b) Find $P(odd) = P(1, 3, 5, 7, \dots)$.

Problem 5 continued on next page...

### Solution

$$\sum_{1}^{\infty} \frac{1}{5^{2x-1}} = 5/24$$

## Problem 6

Suppose $S = \{1, 2, 3, \dots\}$ and $P(k) = \frac{(\ln 2)^k}{k!}, k = 1, 2, 3, \dots$
Is this a valid probability model? *Justify your answer*

### Solution

Yes,

$$\sum_{k=1}^{\infty} \frac{(\ln 2)^k}{k!} = 1$$

## Problem 7

A bank classifies borrowers as "high risk" or "low risk," and 16% of its loans are made to those in the "high risk" category. Of all the bank's loans, 5% are in default. It is also known that 40% of the loans in default are to high-risk borrowers.

(a) What is the probability that a randomly selected loan is in default and issued to a high-risk borrower?

### Solution

$P(highRisk) * P(default) = 0.40 * 0.05 = 0.02$

(b) What is the probability that a loan will default, given that it is issued to a high-risk barrower?

### Solution

$P(default|highRisk) = 0.02/0.16 = 0.125$

(c) What is the probability that a randomly selected loan is either in default or issued to a high-risk borrower, or both?

### Solution

$P(lowRiskDefult) + P(highRisk) = 0.03 + 0.16 = 0.19$

(d) A loan is being issued to a borrower who is not high-risk. What is the probability that this loan will default?

### Solution

$P(default|lowRisk) = 0.03/0.84 \approx 0.357$

# Problem 8

A family that owns two automobiles is selected at random. Suppose that the probability that the older car is American is 0.70, the probability that the new car is American is 0.50, and the probability that both the older and the newer cars are American is 0.40.

(a) Find the probability that at least one car is American (i.e. that either the older car or the newer car, or both cars are American).

### Solution

$P(OA \cup NA) = P(OA) + P(NA) - P(OA \cap NA) = 0.70 + 0.50 - 0.40 = 0.80$

(b) Find the probability that neither car is American.

### Solution

$1 - P(OA \cup NA = 1 - 0.80 = 0.20$

(c) Suppose that the older car is American. What is the probability that the newer car is also American?

### Solution

P(OA

(d) What is the probability that the older car is American, given that the newer car is American?

### Solution

# Problem 9

$P(dinner) = 60\%$
$P(noDinner) = 40\%$
$P(happy|dinner) = 90\%$
$P(happy|noDinner) = 20\%$
Find

(a) $P(dinner|happy)$

### Solution

(b) $P(noDinner|notHappy)$

### Solution

# Problem 10

**1.2-4** A coin is tossed four times, and the sequence of heads and tails is observed.

(a) List each of the 16 sequences in the sample space $S$.

Problem 10 continued on next page...

### Solution

$S = \{HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTTH, TTHH, HTHT, THTH, THHT, HTTT, THTT, TT$

(b) Let events $A, B, C$, and $D$ be given by $A = \{H \geq 3\}, B = \{H \leq 2\}, C = \{headsOnThirdToss\}, D = \{1H, 3T\}$. If the probability set function assigns $\frac{1}{16}$ to each outcome in the sample space find

(a) $P(A)$

### Solution

$P(A) = 5/16$ Heads occurs at least 3 times 5 out of the 16 tosses.

(b) $P(A \cap B)$

### Solution

$P(A \cap B) = 0$ $A$ and $B$ are mutually exclusive.

(c) $P(B)$

### Solution

$P(B) = 11/16$ Heads occurs at most 2 times 11 out of the 16 tosses.

(d) $P(A \cap C)$

### Solution

$P(A \cap C) = 4/16$ Heads occurs at least 3 and on the third toss 4 out of the possible 16 tosses.

(e) $P(D)$

### Solution

$P(D) = 4/16$ There are 4 occurrences of one head and 3 tails in the 16 possible tosses.

(f) $P(A \cup C)$

### Solution

$P(A \cup C) = 9/16$ There are 9 occurrences where either heads occurs on the 3rd toss or 3 heads occur or both.

(g) $P(B \cap D)$

### Solution

$P(B \cap D) = 4/16$ There are 4 occurrences where both 2 or less heads and 1 head and 3 tails occur.

# Problem 11

**1.2-14** Let $x$ equal a number that is selected randomly from the closed interval from zero to on, $[0, 1]$. User your intuition to assign values to

(a) $P(\{x : 0 \leq x \leq 1/3\})$

### Solution

$P(\{x : 0 \leq x \leq 1/3\}) = 1/3$

(b) $P(\{x : 1/3 \leq x \leq 1\})$

### Solution

$P(\{x : 1/3 \leq x \leq 1\}) = 2/3$

(c) $P(\{x : x = 1/3\})$

### Solution

$P(\{x : x = 1/3\}) = 0$

(d) $P(\{x : 1/2 < x < 5\})$

### Solution

$P(\{x : 1/2 < x < 5\}) = 1/2$

# Problem 12

**1.2-16** The five numbers $1, 2, 3, 4$, and $5$ are written respectively on five disks of the same size and placed in a hat. Two disks are drawn without replacement from the hat, and the numbers written on them are observed.

(a) List the 10 possible outcomes of this experiment as unordered pairs of numbers.

### Solution

$S = \{(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5)\}$

(b) If each of the 10 outcomes has probability $1/10$, assign a value to the probability that the sum of the two numbers drawn is

(a) 3

### Solution

$P(3) = 1/10$ Each number has an equal probability and a sum of 3 occurs once

(b) between 6 and 8 inclusive.

Problem 12 continued on next page...

### Solution

$P((x, y) : 6 \leq x + y \leq 8) = 5/10$ There are 5 occurs of the ten possible outcomes where the sum of the values is greater than or equal to 6 and less than or equal to 8.

## Problem 13

**1.4-4** Two cards are drawn successively and without replacement from an ordinary deck of playing cards. Compute the probability of drawing

(a) Two hearts.

### Solution

$P(HH) = \frac{13}{52} * \frac{12}{51} = \frac{1}{7}$

(b) A heart on the first draw and a club on the second draw

### Solution

$P(HC) = \frac{13}{52} * \frac{13}{51} = \frac{13}{204}$

(c) A heart on the first draw and an ace on the second draw. Hint: In part (c), note that a heart can be drawn by getting the ace of hearts or one of the other 12 hearts

### Solution

$P(nonAceHeart, ace) + P(aceHeart, nonHeartAce) = \frac{12}{52} * \frac{4}{51} + \frac{1}{52} * \frac{3}{51}$

## Problem 14

**1.4-18** Bowl $A$ contains three red and two white chips, and bowl $B$ contains four red and three white chips. A chip is drawn at random from bowl $A$ and transferred to bowl $B$. Compute the probability of then drawing a red chip from bowl $B$.

### Solution

$\frac{3}{5} * \frac{5}{8} + \frac{2}{5} * \frac{4}{8} = \frac{23}{40}$
Solution

## Problem 15

**1.6-2** Bean seeds from supplier $A$ have an 85% germination rate and those from supplier $B$ have a 75% germination rate. A seed-packaging compony purchases 40% of its beans seeds from supplier $A$ and 60% from supplier $B$ and mixes these seeds together.

(a) Find the probability $P(G)$ that a seed selected at random from the mixed seeds will germinate.

Problem 15 continued on next page...

### Solution

$P(G) = P(A \cap G) + P(B \cap G) = P(A) * P(G|A) + P(B) * P(G|B) = 0.40 * 0.85 + 0.60 * 0.75 = 0.79$

(b) Given that a seed germinates, find the probability that the seed was purchased from supplier $A$.

### Solution

$P(A|G) = P(A \cap G)/P(G) = (0.40 * 0.85)/0.79 = 0.43$