Chapter 1.5 Independent Events

- What is independence?
- Pairwise independence does not imply mutual independence
- Properties of independence

Independence

Two events A and B are **independent** if the occurrence of the one does not affect (change) the occurrence of the other.

This means P(A|B) = P(A) and P(B|A) = P(B).

Example:
Grandma's age and grade on midterm are independent events.
Your attendance at STAT200 and your final grade in Bio 200 are independent events.

This is completely different from the idea of events that are ***disjoint.***

Example: Artificial pond

Artificial pond with 10 male and 10 female frogs.

- A = { the first frog is male },
- B = { the second frog is male}.

(1) If you did **not** put the first frog back to the pond,

- P(B|A): the probability of picking up another male frog on your second capture is now less than 0.5 (9/19 or 0.47)
  → successive picks are not independent.

(2) If you put the first picked frog back into the pond, the probability of picking a male the second time is just the same as the first time
→ successive picks are "independent".

- P(B|A)=P(B) and we have P(A and B)=P(B)P(A)

Examples

Let P(A)=0.4 and P(B)=0.5. Compute that

- P(A ∪ B) when A and B are independent.
- P(A ∪ B) when A and B are disjoint.
- P(A|B) when A and B are independent.
- P(A|B) when A and B are disjoint.

Properties of independence

If event A and B are independent, then

- A and B′
- A′ and B
- A′ and B’

are all independent.

Pairwise Independence

Events A, B and C are pairwise independent if

- $P(A \cap B)=P(A \text{ and } B) = P(A) * P(B)$
- $P(A \cap C)=P(A \text{ and } C) = P(A) * P(C)$
- $P(B \cap C)=P(B \text{ and } C) = P(B) * P(C)$

Example 1.5-4

Four cards numbered 1, 2,3 and 4 are on the table. One card is drawn randomly from the deck. Let events A, B, C be defined as following

- A= {The card is either 1 or 2}
- B= {The card is either 1 or 3}
- C= {The card is either 1 or 4}.

Are A, B and C pairwise independent?

Mutually Independent

Definition: Events A1, A2, ..., An are **mutually independent** if

$$P(A1 \cap A2 \cap \ldots \cap An) = P(A1)P(A2) \ldots P(An)$$

and the same multiplication rule holds for every subcollection of the events too.

Example: Events A, B and C are **mutually independent** if

1. they are pairwise independent
2. $P(A \cap B \cap C) = P(A)P(B)P(C)$

Example: Events A, B, C and D are **mutually independent** if

1.
2.
3.

Example 1.5-4 -- Revisit

Four cards numbered 1, 2,3 and 4 are on the table. One card is drawn randomly from the deck. Let events A, B, C be defined as following

- A= {The card is either 1 or 2}
- B= {The card is either 1 or 3}
- C= {The card is either 1 or 4}.

Are A, B and C mutually independent?

True or False:
If A, B, and C are mutually independent, then A' and B∩C' are independent.

Example: Hit Principle Skinner

Bart and Nelson talked Milhouse into throwing water balloons at Principal Skinner. Suppose that Bart hits his target with probability 0.80, Nelson misses 25% of the time, and Milhouse hits the target half the time. Assume that their attempts are independent of each other.

- Find the probability that all of them will hit Principal Skinner.

Solution:

1. *formulate events.*
   Let *A* =“*Bart hits PS*", *B* =“*Nelson hits PS*", *C*= “*Milhouse hits PS*”
2. *Write down all information given.*
   P(A) =_____, *P(B)* =______, *P(C)* =______.
3. *Write down what we're looking for.*

4. *Compare this to what we know*

Example: Hit Principle Skinner (Cont’d)

- Find the probability that at least one of the boys will hit Principal Skinner.

Idea: P(at least one of A_i occurs) = 1 – P(none of A_i occurs)

P (A_1 or A_2 or ... or A_n) = 1 – P ((not A_1) and (not A_2) and ... and (not A_n))

$$P(A_1 \cup A_2 \cup \dots \cup A_n) = 1 - P(A_1' \cap A_2' \cap \dots \cap A_n')$$

For independent events

P (A_1 or A_2 or ... or A_n) = 1 – P (not A_1) · P (not A_2) · ... · P (not A_n)

$$P(A_1 \cup A_2 \cup \dots \cup A_n) = 1 - P(A_1') \cdot P(A_2') \cdot \dots \cdot P(A_n')$$

- Given that exactly one shot hits PS, what is the conditional probability that is Bart's shot?

  Let *D* =“*exactly one shot hits PS". Looking for* ______________.

- Given that PS is hit, what is the conditional probability that Bart hits him?

  Let *E* =“*PS is hit". Looking for*______________ .

Example 1.5-6

A fair die is rolled six times. Let Ai be the event that side i is observed in the i-th roll, called a match. What is the probability that at least one match occurs during six rolls.

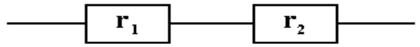
The Intel Pentium Flaw

In Oct 1994, a flaw was discovered in the pentium chip installed in many new personal computers. The chip produced an incorrect result when dividing two numbers. Intel the manufacture of the pentium chip announce that such an error would occur once in 9 billion divides or “ once in 27000 years” for a typical user; consequently, it did not immediately offer to replace the chip. However, within weeks of release of the chip, they have to call back their chip. Why?

E1={ an error occurs} E2={at least one error in 100 billion divides}

$$P(E_2) = 1 - P(E_2') = 1 - \left(1 - P(E_2)\right)^{100,000,000} \approx 1$$

Series Connection:

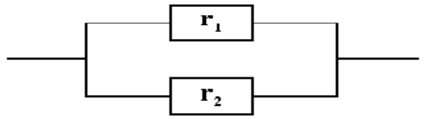


Reliability:

$r_1 \times r_2$

In general, $r_1 \times r_2 \times \ldots \times r_k$

Parallel Connection:

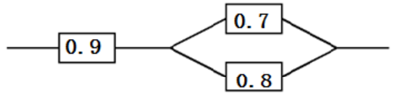


Reliability:

$1 - (1 - r_1) \times (1 - r_2)$

$r_1 + r_2 - r_1 \times r_2$

In general, $1 - (1 - r_1) \times (1 - r_2) \times \ldots \times (1 - r_k)$



What is the reliability of the left system of independent components?