

Be sure to show all your work; your partial credit might depend on it.

No credit will be given without supporting work.

The exam is closed book and closed notes.

You are allowed to use a calculator and one 8.5" x 11" sheet with notes on it.

1. Suppose $P(A) = 0.45$, $P(B) = 0.40$, $P(C) = 0.50$, $P(A \cap B) = 0.15$, $P(A \cap C) = 0.20$, $P(B \cap C) = 0.25$, $P(A \cap B \cap C) = 0.05$.

- Find $P(A \cup C)$.
- Find $P(A \cup (B \cap C))$.
- Find $P(B \cup C|A')$.

2. Tom's lecture at UIUC often finishes late. Suppose that the lecture finishes W minutes late, where W is a discrete random variable with the following probability mass function:

w	0	1	2	3	4	5
$f_W(w) = P(W = w)$	0.4	a	b	0.10	0.05	0.05

Here a and b are missing values to be calculated. Tom calculates that $E(W) = 1.3$.

- Find the values of a and b .
 - Find the standard deviation of W , σ_W .
3. The probability that a circuit board coming off an assembly line needs rework is 0.15. Suppose that 12 boards are tested and all boards are independent of each other.

- What is the probability that exactly 4 will need rework?
- What is the probability that at least one needs rework?
- What is the probability that at most two needs rework?

4. Sixty-five percent (65%) of all women who submit to pregnancy tests are actually pregnant. A certain pregnancy test gives a *false positive* result with probability 0.02 and a *valid positive* result with probability 0.99. We say that this test has sensitivity 0.99 and specificity $1 - 0.02 = 0.98$. In other words, a pregnant woman gives a positive test value with probability 0.99. A non-pregnant woman gives a negative test result with probability 0.98.

- Among women who submit to a pregnancy test, what fraction of the tests are positive?
- If a particular woman's test is indeed positive, what is the probability that she is actually pregnant?
- If a particular woman's test is negative, what is the probability that she is actually pregnant?

5. The probability density function of a random variable X is given by

$$f(x) = c(x - 1)(2 - x) \text{ if } 1 < x < 2.$$

- a) Calculate the value of c .
- b) Find the cumulative distribution function of X .

6. According to an airline industry report, roughly 1 piece of luggage out of every 200 that are checked is lost. Suppose that a frequent-flying businesswoman will be checking 120 bags over the course of the next year. Approximate the probability that she will lose 2 or more pieces of luggage.