

Homework #7
(due Friday, Oct 21st, by 3:00 p.m.)

No credit will be given without supporting work.

1. Suppose Jane has a fair 4-sided die, and Dick has a fair 6-sided die. Each day, they roll their dice (independently) until someone rolls a “1”. (Then the person who did not roll a “1” does the dishes.) Find the probability that ...
 - a) they roll the first “1” at the same time (after equal number of attempts);
 - b) it takes Dick twice as many attempts as it does Jane to roll the first “1”;
 - c) Dick rolls the first “1” before Jane does.

2. At Sam’s Butcher Shop, ground beef packages vary in weight according to a normal distribution with a mean of 3.1 pounds and a standard deviation of 0.2 pounds, and are sold for \$1 per pound. Packages of Bratwurst vary in weight according to a normal distribution with a mean of 2.6 pounds and a standard deviation of 0.3 pounds, and are sold for \$3 per pound. Anticipating nice weather during the weekend, Dick buys two packages of ground beef and one package of Bratwurst, selecting the packages at random. What is the probability that Dick would exceed the \$15 limit “suggested” by his wife Jane? (Assume independence.)

3. Suppose that 60% of all voters in a certain county favor a tax increase to pay for a better school system. A random sample of 400 voters is selected. Use Normal approximation to find the following:
 - a) What is the probability that more than 250 of the sample members favor the tax increase?
 - b) What is the probability that at least 260 of the sample members favor the tax increase?

- c) What is the probability that at most 235 of the sample members favor the tax increase?
- d) What is the probability that between 225 and 245 (both inclusive) of the sample members favor the tax increase?
- e) What is the probability that exactly 234 of the sample members favor the tax increase? Compare the answer with the exact probability.

4. Suppose the number of e-mail messages received by Mike follows Poisson distribution with the average rate of 42 messages per week. Use Normal approximation to find the following:

- a) What is the probability that Mike would receive over 50 e-mail messages in one week?
- b) What is the probability that Mike would receive exactly 50 e-mail messages in one week? Compare the answer with the exact probability.
- c) What is the probability that Mike would receive at most 45 e-mail messages in one week?
- d) What is the probability that Mike would receive at least 32 e-mail messages in one week?

5. The distribution of the baggage weights for passengers using a particular airline has a mean of 20 lbs and a standard deviation of 5 lbs. What is the probability that for (a random sample of) 100 passengers ...

- a) the total luggage weight is less than 2,100 lbs?
- b) the sample mean weight is within 0.5 lb of the overall mean? That is, what is the probability that the sample mean weight is between 19.5 and 20.5 lbs?

6. One piece of PVC pipe is to be inserted inside another piece. The length of the first piece is normally distributed with mean value 20 in. and standard deviation 0.7 in. The length of the second piece is a normal random variable with mean and standard deviation 15 in. and 0.6 in., respectively. The amount of overlap is normally distributed with mean value 1 in. and standard deviation 0.2 in. Assuming that the lengths and amount of overlap are independent of one another, what is the probability that the total length after insertion is between 32.65 in. and 35.35 in.?

7. Suppose that number of accidents at the Monstropolis power plant follows the Poisson process with the average rate of 0.60 accidents per day.

- a) Use Normal approximation to find the probability that at most 200 accidents will occur in a year (365 days).

- b) Use Normal approximation to find the probability of less than 190 accident-free days in a year (365 days).



8. A machine operation produces bearings whose diameters are normally distributed, with a mean of 0.498 inch and a standard deviation of 0.002 inch. Suppose that specifications require that the bearing diameter be 0.500 inch plus or minus 0.004 inch.

- a) What proportion of the production will be unacceptable?
- b) A quality control inspector selects bearings from the production independently and at random. What is the probability that the first unacceptable bearing will be the sixth one selected?
- c) What is the probability that the third unacceptable bearing will be the sixth one selected?
- d) Suppose ten bearings are independently and randomly selected from the production process. What is the probability that exactly 2 of the 10 will be unacceptable?

- e) What is the probability that the average diameter of 10 randomly selected bearings is within 0.003 inch of 0.500 inch?
- f) Suppose 84 bearings are independently and randomly selected from the production process. What is the probability that at most 8 of them will be unacceptable? (Use Normal approximation.)

9. Let $a > 2$. Suppose a discrete random variable X has the following probability distribution:

$$p(0) = P(X = 0) = c,$$

$$p(k) = P(X = k) = \frac{1}{a^k}, \quad k = 1, 2, 3, \dots$$

- a) Find the value of c (c will depend on a) that makes this a valid probability distribution.
- b) Find the moment-generating function of X , $M_X(t)$. For which values of t does it exist?
- c) Find $E(X)$.

From the textbook:

5.6-4 (5.4-4)

5.6-8 (5.4-8)

5.7-2 (5.5-2)

5.7-8 (5.5-8)

5.7-12 (5.5-12)

5.7-14 (5.5-14)

