STAT 400 – AL1 **Examples for 08/26/2011** Fall 2011

1. The probability that a randomly selected student at Anytown College owns a bicycle is 0.55, the probability that a student owns a car is 0.30, and the probability that a student owns both is 0.10.

$P(B) = 0.55$, $P(C) = 0.30$, $P(B \cap C) = 0.10$.

	C	C′	
B	0.10	0.45	0.55
B′	0.20	0.25	0.45
	0.30	0.70	1.00

a) What is the probability that a student selected at random does not own a bicycle?

$P(B') = 1 - P(B) = 1 - 0.55 = \mathbf{0.45}$.

b) What is the probability that a student selected at random owns either a car or a bicycle, or both?

$P(B \cup C) = P(B) + P(C) - P(B \cap C) = 0.55 + 0.30 - 0.10 = \mathbf{0.75}$.

OR

$P(B \cup C) = P(B \cap C) + P(B' \cap C) + P(B \cap C') = 0.10 + 0.20 + 0.45 = \mathbf{0.75}$.

OR

$P(B \cup C) = 1 - P(B' \cap C') = 1 - 0.25 = \mathbf{0.75}$.

b½) What is the probability that a student selected at random has neither a car nor a bicycle?

$P(B' \cap C') = \mathbf{0.25}$.

The **conditional probability of A, given B** (the probability of event A, computed on the assumption that event B has happened) is

$$\mathbf{P(A \mid B) = \frac{P(A \cap B)}{P(B)}} \qquad (\text{assuming } P(B) \neq 0).$$

Similarly, the **conditional probability of B, given A** is

$$\mathbf{P(B \mid A) = \frac{P(A \cap B)}{P(A)}} \qquad (\text{assuming } P(A) \neq 0).$$

c) What is the probability that a student owns a bicycle, <u>given that he/she owns a car</u>?

$P(B \mid C) = {}^{0.10}/_{0.30} = \mathbf{{}^{1}/_{3}}$.

d) <u>Suppose a student does not have a bicycle</u>. What is the probability that he/she has a car?

$P(C \mid B') = {}^{0.20}/_{0.45} = \mathbf{{}^{4}/_{9}}$.

2. Suppose

$P(A) = 0.22$,

$P(B) = 0.25$,

$P(C) = 0.28$,

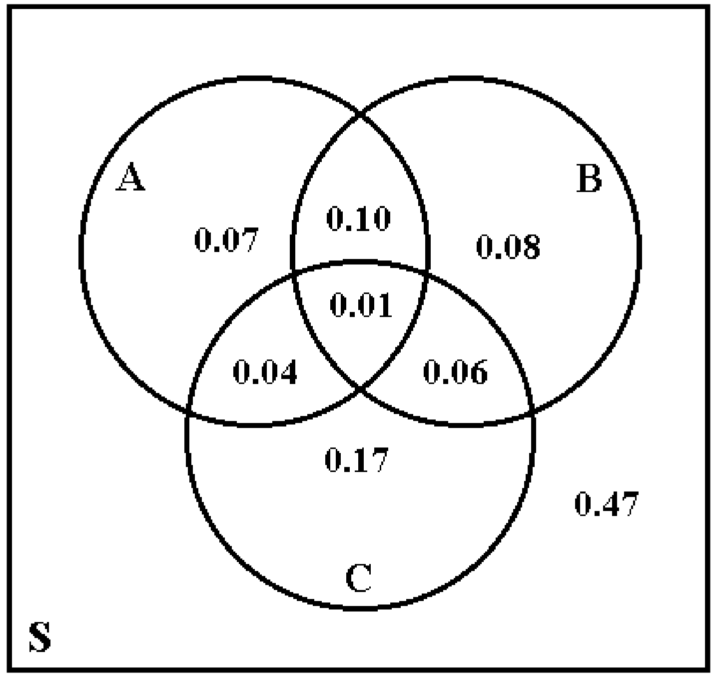
$P(A \cap B) = 0.11$,

$P(A \cap C) = 0.05$,

$P(B \cap C) = 0.07$,

$P(A \cap B \cap C) = 0.01$.

Find the following:



a) $P(A \cup B) = \mathbf{0.36}$.

b) $P(A' \cap B') = \mathbf{0.64}$.

c) $P(A \cup B \cup C) = \mathbf{0.53}$.

d) $P(A' \cap B' \cap C') = \mathbf{0.47}$.

e) $P(A' \cap B' \cap C) = \mathbf{0.17}$.

f) $P((A' \cap B') \cup C) = \mathbf{0.75}$.

g) $P((A \cup B) \cap C) = \mathbf{0.11}$.

h) $P((B \cap C') \cup A') = \mathbf{0.88}$.

i) $P(B \mid A) = {}^{0.11}/_{0.22} = \mathbf{0.50}$.

j) $P(B \mid C) = {}^{0.07}/_{0.28} = \mathbf{0.25}$.

k) $P(B \cap C \mid A) = {}^{0.15}/_{0.22} = \mathbf{{}^{15}/_{22}}$.

l) $P(B \cup C \mid A) = {}^{0.11}/_{0.36} = \mathbf{{}^{11}/_{36}}$.

m) $P(C \mid A \cup B) = {}^{0.11}/_{0.36} = \mathbf{{}^{11}/_{36}}$.

n) $P(C \mid A \cap B) = {}^{0.01}/_{0.11} = \mathbf{{}^{1}/_{11}}$.

o) $P(A \cap B \cap C \mid A \cup B \cup C) = {}^{0.01}/_{0.53} = \mathbf{{}^{1}/_{53}}$.

Multiplication Law of Probability

If A and B are any two events, then

$$\mathbf{P(A \cap B) = P(A) \cdot P(B \mid A)}$$

$$\mathbf{P(A \cap B) = P(B) \cdot P(A \mid B)}$$

3. It is known that 30% of all the students at Anytown College live off campus. Suppose also that 48% of all the students are females. <u>Of the female students</u>, 25% live off campus.

$P(\text{Off}) = 0.30$, $P(F) = 0.48$, $P(\text{Off} \mid F) = 0.25$.

a) What is the probability that a randomly selected student is a female <u>and</u> lives off campus?

$P(F \cap \text{Off}) = P(F) \times P(\text{Off} \mid F) = 0.48 \times 0.25 = \mathbf{0.12}$.

	Off	On	
F	0.12	0.36	0.48
M	0.18	0.34	0.52
	0.30	0.70	1.00

b) What is the probability that a randomly selected student either is a female <u>or</u> lives off campus, or both?

$P(F \cup \text{Off}) = P(F) + P(\text{Off}) - P(F \cap \text{Off}) = 0.48 + 0.30 - 0.12 = \mathbf{0.66}$.

OR

$P(F \cup \text{Off}) = P(F \cap \text{Off}) + P(F' \cap \text{Off}) + P(F \cap \text{Off}')$

$= 0.12 + 0.18 + 0.36 = \mathbf{0.66}$.

OR

$P(F \cup \text{Off}) = 1 - P(F' \cap \text{Off}') = 1 - 0.34 = \mathbf{0.66}$.

c) What proportion of the off-campus students are females?

$P(\text{ F} \mid \text{Off }) = {}^{0.12}\!/_{0.30} = \mathbf{0.40}$.

d) What proportion of the male students live off campus?

$P(\text{ Off} \mid \text{M }) = {}^{0.18}\!/_{0.52} = {}^{9}\!/_{26} \approx \mathbf{0.346154}$.

4. Suppose that Joe's Discount Store has received a shipment of 25 television sets, 5 of which are defective. On the following day, 2 television sets are sold.

a) Find the probability that both of the television sets are defective.

$$P(\text{ both defective }) = P(\text{ 1st D} \cap \text{2nd D }) = P(\text{ 1st D }) \times P(\text{ 2nd D} \mid \text{1st D })$$

$$= {}^{5}\!/_{25} \times {}^{4}\!/_{24} = \mathbf{{}^{1}\!/_{30}}.$$

b) Find the probability that at least one of the two television sets sold is defective.

✓	D	D	${}^{5}\!/_{25} \times {}^{4}\!/_{24} = {}^{1}\!/_{30}$.
✓	D	D′	${}^{5}\!/_{25} \times {}^{20}\!/_{24} = {}^{5}\!/_{30}$.
✓	D′	D	${}^{20}\!/_{25} \times {}^{5}\!/_{24} = {}^{5}\!/_{30}$.
×	D′	D′	

$$P(\text{ at least one D }) = {}^{1}\!/_{30} + {}^{5}\!/_{30} + {}^{5}\!/_{30} = \mathbf{{}^{11}\!/_{30}}.$$

OR

$$P(\text{ at least one D }) = 1 - P(\text{ D}'\text{D}'\text{ }) = 1 - {}^{20}\!/_{25} \times {}^{19}\!/_{24} = 1 - {}^{19}\!/_{30} = \mathbf{{}^{11}\!/_{30}}.$$

5. Cards are drawn one-by-one **without** replacement from a standard 52-card deck. What is the probability that …

a) … both the first and the second card drawn are ♥'s?

$$P(\text{ 1st } \heartsuit \cap \text{ 2nd } \heartsuit) = P(\text{ 1st } \heartsuit) \times P(\text{ 2nd } \heartsuit \mid \text{ 1st } \heartsuit) = {}^{13}/_{52} \times {}^{12}/_{51} = \mathbf{{}^{1}/_{17}}.$$

a½) … the first two cards drawn are a ♥ and a ♣ (or a ♣ and a ♥) ?

$$P(\text{ 1st } \heartsuit \cap \text{ 2nd } \clubsuit) + P(\text{ 1st } \heartsuit \cap \text{ 2nd } \clubsuit) = {}^{13}/_{52} \times {}^{13}/_{51} + {}^{13}/_{52} \times {}^{13}/_{51} .$$

b) … there are at least two ♥'s among the first three cards drawn?

♥	♥	$\cancel{\heartsuit}$	${}^{13}/_{52} \times {}^{12}/_{51} \times {}^{39}/_{50}$
	or		+
♥	$\cancel{\heartsuit}$	♥	${}^{13}/_{52} \times {}^{39}/_{51} \times {}^{12}/_{50}$
	or		+
$\cancel{\heartsuit}$	♥	♥	${}^{39}/_{52} \times {}^{13}/_{51} \times {}^{12}/_{50}$
	or		+
♥	♥	♥	${}^{13}/_{52} \times {}^{12}/_{51} \times {}^{11}/_{50}$

$${}^{19968}/_{132600} \approx \mathbf{0.150588}.$$