STAT 400 Fall 2011

Homework #5

(10 points)

(due Friday, Oct 7th, by 3:00 p.m.)

No credit will be given without supporting work.

1. Suppose a random variable X has the following probability density function:

$$f(x) = \begin{cases} 1/x & 1 \le x \le C \\ 0 & \text{otherwise} \end{cases}$$

a) What must the value of C be so that $f(x)$ is a probability density function?

b) Find $P(X < 2)$.

c) Find $P(X < 3)$.

d) Find $\mu_X = E(X)$.

e) Find $\sigma_X^2 = Var(X)$.

2. Suppose a random variable X has the following probability density function:

$$\mathbf{f}(x) = \begin{cases} \mathbf{C}\,|\,\mathbf{x} - \mathbf{2}\,| & 0 \le \mathbf{x} \le 3 \\ 0 & \textit{otherwise} \end{cases}$$

a) What must the value of C be so that $f(x)$ is a probability density function?

b) Find the cumulative distribution function $F(x) = P(X \le x)$.

c) Find the median of the probability distribution of X.

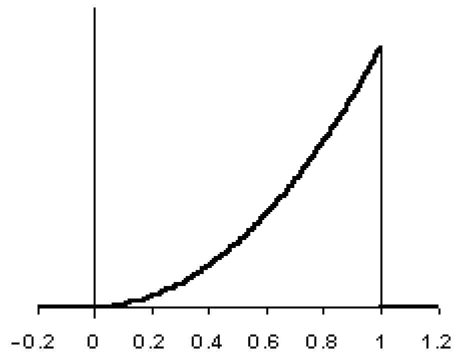
d) Find $\mu_X = E(X)$.

e) Find the moment-generating function of X, $M_X(t)$.

3. Let X be a continuous random variable with the probability density function

$$f(x) = k \cdot x^2, \qquad 0 \le x \le 1,$$

$$f(x) = 0, \qquad \text{otherwise.}$$



a) What must the value of k be so that $f(x)$ is a probability density function?

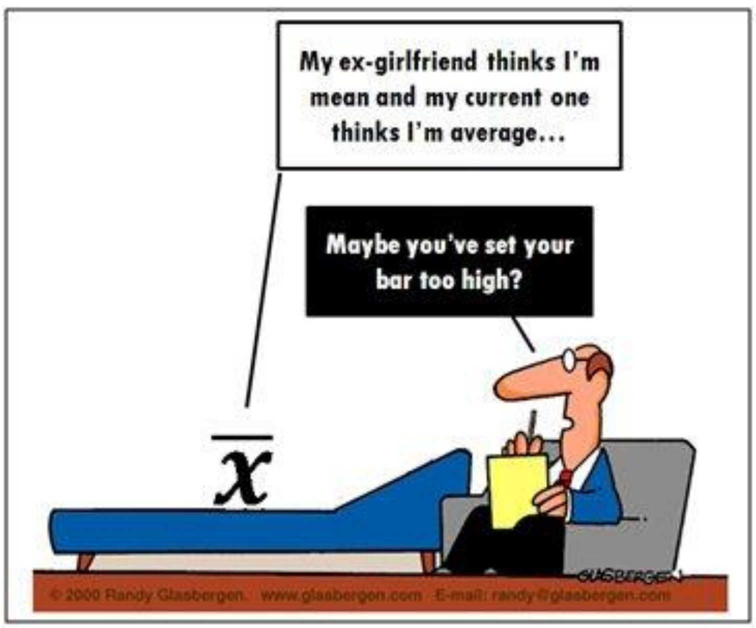
b) Find the probability $P(0.4 \le X \le 0.8)$.

c) Find the median of the distribution of X.

d) Find $\mu_X = E(X)$.

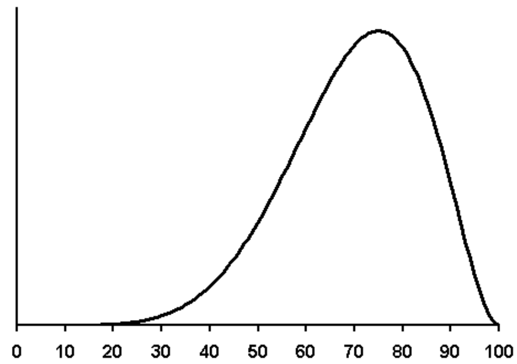
e) Find $\sigma_X = SD(X)$.

f) Find the moment-generating function of X, $M_X(t)$.



4. A simple model for describing mortality in the general population in a particular country is given by the probability density function

$$f(y) = \frac{252}{10^{18}}\, y^6 (100 - y)^2, \qquad 0 < y < 100.$$



a) Verify that $f(y)$ is a valid probability density function.

b) Based on this model, which event is more likely

A: a person dies between the ages of 70 and 80

or

B: a person lives past age 80?

c) Given that a randomly selected individual just celebrated his 60th birthday, find the probability that he will live past age 80.

d) Find the value of y that maximizes $f(y)$ (**mode**).

e) Find the (average) life expectancy.

5. An insurance policy reimburses a loss up to a benefit limit of 10. The policyholder's loss, Y, follows a distribution with density function:

$$f(y) = \begin{cases} \dfrac{2}{y^3} & \text{if } y > 1 \\ 0 & \text{otherwise} \end{cases}$$

a) What is the expected value and the variance of the policyholder's loss?

b) What is the expected value and the variance of the benefit paid under the insurance policy?

6. Suppose that number of accidents at the Monstropolis power plant follows the Poisson process with the average rate of 0.40 accidents per week. Assume all weeks are independent of each other.

a) Find the probability that at least 2 accidents will occur in one week.

b) Find the probability that 4 accidents will occur in two months (8 weeks).

c) Find the probability that there will be 5 accident-free weeks in two months (8 weeks).

d) Find the probability that the first accident would occur during the fourth week.

e) Find the probability that the third accident would occur during the fifth week.

From the textbook:

3.3-24 (a),(b) (3.2-24 (a),(b))

3.4-4	(3.3-4)	**3.4-8**	(3.3-8)
3.5-2	(3.4-2)	**3.5-8**	(3.4-8)
3.6-6	(5.2-6)	**3.6-14**	(5.2-14)