Statistics 400 / Mathematics 463
Midterm Exam 1 (AL1) Version B
Sept, 28th 2011, Wednesday, 11:00-11:50am

Name: Solution Student I.D. _____

Discussion Section (circle one): AD1 AD2 AD3 AD4

1. Please **print** your name and student ID number in the above space and circle the discussion section number.
2. This is a closed book, closed-notes examination. You should have a calculator and a single two-sided page of notes that you may refer to.
3. Please provide the answers in the space provided. If you do not have enough space, please use the back of a nearby page or ask for additional blank paper. Make sure you sign any loose pages.
4. In order to receive full credit for a problem, you should show all of your work and explain your reasoning. Good work can receive substantial partial credit even if the final answer is incorrect.
5. In most cases the later parts of a question do not require the answers to earlier parts. You should try all parts of a problem even if you get stuck on an early part.

Question	Points
Question 1 (12+5 (Bonus) points)	
Question 2 (12 points)	
Question 3 (12 points)	
Question 4 (24 points)	
Question 5 (20 points)	
Question 6 (20 points)	
Total (105 points)	

1. **[12+5 (bonus) points]** Three fair dice are tossed.

   (a) **[3 points]** What is the probability that the three dice turn up all ones?

$P(\text{all ones}) = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$

   (b) **[4 points]** What is the probability that the three dice turn up all the same number?

the dice can be all ones, all twos, ..., all sixes

$P(\text{all same}) = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times 6 = \frac{1}{36}$

   (c) **[5 points]** What is the probability that the three dice turn up three different numbers?

$P(\text{all different}) = \frac{{}_6P_3}{6^3} = \frac{6 \times 5 \times 4}{6 \times 6 \times 6} = \frac{5}{9}$

Note, $P(\text{all different}) \neq 1 - P(\text{all the same})$

   (d) **[bonus 5 points]**What is the probability that two of the dice turn up one number, while the rest one of the dice turn up some other number? (For example, (1, 2, 1).)

There are 3 ways to have two 1's and one 2

i.e. (1, 1, 2) (1, 2, 1) (2, 1, 1)

and there're ${}_6P_2$ ways to pick up 2 ordered number

so, the probability is $\frac{3 \times {}_6P_2}{6^3} = \frac{3 \times 6 \times 5}{6 \times 6 \times 6} = \frac{5}{12}$

2. **[12 points]** Suppose that E and F are two events such that $P(E) = 0.6$ and $P(F) = 0.8$.

(a)**[6 points]** Prove that $P(E \cap F) \geq 0.4$.

$\because P(E \cup F) = P(E) + P(F) - P(E \cap F)$

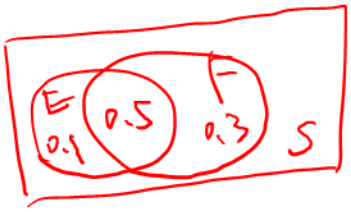
and $P(E \cup F) \leq 1$

$\therefore P(E \cap F) \geq P(E) + P(F) - 1$

$= 0.4$

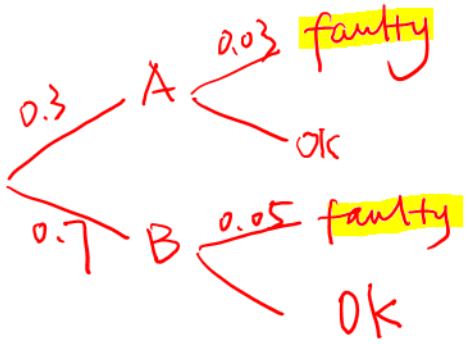
(b)**[6 points]** Suppose we know $P(E \cap F) = 0.5$, find out $P(E'|F)$.

$$P(E'|F) = \frac{P(E' \cap F)}{P(F)} = \frac{P(F) - P(E \cap F)}{P(F)} = \frac{0.3}{0.8} = \frac{3}{8}$$



3. **[12 points]** A dot.com company ships products from two different warehouses(A and B). Based on customer complaints, it appears that 3% of the shipments coming from A are somehow faulty, as are 5% of the shipments coming from B. Assume that Warehouses A and B ship 30% and 70% of the dot.com's sales, respectively.

   (a) **[6 points]** Suppose a customer is mailed an order, what is the probability that the product is faulty?



$$P(\text{faulty}) = 0.3 \times 0.03 + 0.7 \times 0.05 = 0.044$$

   (b) **[6 points]** Suppose a customer is mailed an order and calls in a complaint the next day. What is the probability the item came from Warehouse B?

$$P(\text{from B} \mid \text{faulty}) = \frac{P(\text{from B and faulty})}{P(\text{faulty})} = \frac{0.7 \times 0.05}{0.044} = \frac{35}{44} = 0.795$$

4. **[24 points]**

Suppose that 5% of all copies of a particular textbook fail a certain binding strength test. Let X denote the number among the 15 randomly selected copies that fail the test.

$X \sim b(15, 0.05)$

(a) **[6 points]** What is the probability that at most 2 book fails the test?

$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$= \binom{15}{0} 0.05^0 \cdot 0.95^{15} + \binom{15}{1} 0.05^1 \cdot 0.95^{14} + \binom{15}{2} 0.05^2 \cdot 0.95^{13}$

$= 0.964$

(b) **[6 points]** As the textbooks are finished with the binding process, they are randomly selected and their bindings are checked. What is the probability that someone has to check 5 books to get the first failure of the strength test?

Let Z be # of books checked to get 1st failure

Z is geometric (0.05)

$P(Z=5) = 0.95^4 \times 0.05 = 0.041$

(c) **[6 points]** Let Y be the minimum number of books checked to get the fifth failure of the strength test. What is the distribution of Y? (Please either give the name of the distribution with parameter values or write down the p.m.f. function)

Y is negative binomial with $r=5$, $p=0.05$

p.m.f. is $P(Y=y) = \binom{y-1}{4} 0.05^5 \, 0.95^{y-5}$

(d) **[6 points]** Suppose someone checks 120 books,can you approximate the probability that she sees less than 2 failures? Show your work.

Let X = # of failures in 120 books

then $X \sim b(120, 0.05)$

use Poisson approximation $\lambda = np = 120 \times 0.05 = 6$

$P(X<2) \approx P(\text{Pois}(6) < 2) = P(\text{Pois}(6) \le 1)$

$= e^{-6}\left[\frac{6^0}{0!} + \frac{6^1}{1!}\right] = 0.0174$

5. [**20 points**] Roger, Sam and Tony are three naughty kids. They shoot simultaneously at a bird on the tree with their BB guns. Suppose that Roger hits his target with probability 0.20, Sam has a 40% chance hitting his target, and Tony has a half chance hitting hits the target. Assume that their attempts are independent of each other.

   (a) [**4 points**] What is the chance that bird is hit by all of the three boys?

$0.2 \times 0.4 \times 0.5 = 0.04$

   (b) [**6 points**] What is the chance that the bird is hit by exactly one BB gun bullet?

$P(\text{exactly one boy hit}) = P(\text{only Roger hit}) + P(\text{only Sam hit}) + P(\text{only Tony hit})$

$= 0.2 \times 0.6 \times 0.5 + 0.8 \times 0.4 \times 0.5 + 0.8 \times 0.6 \times 0.5$

$= 0.46$

   (c) [**10 points**] What is the probability that Tony hit the bird if the bird is hit? (Hint: first find the probability that the bird is hit by at least one bullet.)

$P(\text{the bird is hit}) = 1 - P(\text{none of the boys hit})$

$= 1 - 0.8 \times 0.6 \times 0.5$

$= 0.76$

$P(\text{Tony hit} \mid \text{the bird is hit}) = \dfrac{P(\text{Tony hit})}{P(\text{the bird is hit})} = \dfrac{0.5}{0.76}$

$= 0.658$

+5 bonus

6. [**20 points**] The probability density function of a random variable X is given by

$$f(x) = c|x+1| \text{ if } 0 \le x \le 3.$$

(a) [**5 points**] Calculate the value of c.

7

$$1 = \int_0^3 f(x)dx = c\int_0^3 |x+1|\,dx = c\cdot\left(\frac{x^2}{2}+x\right)\Big|_0^3 = \frac{15}{2}c$$

$$\Rightarrow \quad c = \frac{2}{15}.$$

(b) [**5 points**] Find the cumulative distribution function of X.

7

$$F(x) = \begin{cases} 0 & \text{if } x<0 \\ \int_0^x f(u)du = \frac{2}{15}\int_0^x |u+1|\,du = \frac{1}{15}(x^2+2x) & 0\le x\le 3 \\ 1 & \text{if } x>3 \end{cases}$$

(c) [**5 points**] What is the mean of X?

6

$$EX = \int_0^3 f(x)dx = \frac{2}{15}\int_0^3 x(x+1)\,dx$$

$$= \frac{2}{15}\cdot\left(\frac{x^3}{3}+\frac{x^2}{2}\right)\Big|_0^3 = \frac{9}{5} = 1.8$$

Bonus

(d) [**5 points**] What is the median of X?

median m satisfies $F(m) = \frac{1}{2}$ and $m\in[0,3]$

from (b) $\frac{1}{15}(m^2+2m) = \frac{1}{2}$

$$\Rightarrow m^2+2m-\frac{15}{2}=0 \Rightarrow m = \frac{-2\pm\sqrt{34}}{2}$$

since m is positive, we choose $m = \frac{-2+\sqrt{34}}{2} = 1.915.$