STAT 400 Fall 2011

Homework #3

(10 points)

(due Friday, September 16, by 3:00 p.m.)

No credit will be given without supporting work.

1. Homer Simpson is going to Moe's Bar for some *Flaming Moe*'s. Let X denote the number of *Flaming Moe*'s that Homer Simpson will drink. Suppose X has the following probability distribution:

x	$f(x)$	$x\,f(x)$	$x^2 f(x)$
0	0.1	0.0	0.0
1	0.2	0.2	0.2
2	0.3	0.6	1.2
3	0.3	0.9	2.7
4	*0.1*	0.4	1.6
	1.0	2.1	5.7

a) Find the probability $f(4) = P(X = 4)$.

$f(4) = 1 - [\,0.1 + 0.2 + 0.3 + 0.3\,] =$ **0.10**.

b) Find the probability $P(X \ge 1)$.

$P(X \ge 1) =$ **0.90**.

c) Find the probability $P(X \ge 1 \mid X < 3)$.

$$P(X \ge 1 \mid X < 3) = \frac{P(X \ge 1 \cap X < 3)}{P(X < 3)} = \frac{0.5}{0.6} = \mathbf{0.8333}.$$

d) Compute the expected value of X, E(X).

$$E(X) = \sum_{\text{all } x} x \cdot f(x) = \mathbf{2.1}.$$

e) Compute the standard deviation of X, SD(X).

$$Var(X) = \sum_{\text{all } x} x^2 \cdot f(x) - [E(X)]^2 = 5.7 - (2.1)^2 = 1.29.$$

$$SD(X) = \sqrt{1.29} = \mathbf{1.1358}.$$

1. (continued)

Suppose each *Flaming Moe* costs \$1.50, and there is a cover charge of \$1.00 at the door. Let Y denote the amount of money Homer Simpson spends at the bar. Then $Y = 1.50 \cdot X + 1.00$.

f) Find the probability that Homer would spend over \$5.00.

x	y	$f(x) = f(y)$
0	\$1.00	0.10
1	\$2.50	0.20
2	\$4.00	0.30
3	\$5.50	0.30
4	\$7.00	0.10
		1.00

$P(Y > \$5.00) = P(X \geq 3) = \mathbf{0.4}$.

g) Find the expected amount of money that Homer Simpson would spend, E(Y).

$\mu_Y = E(Y) = 1.50 \cdot E(X) + 1.00 = \mathbf{\$4.15}$.

(On average, Homer drinks 2.1 bottles per day, his expected payment for the drink is \$3.15. His expected total payment is \$4.15 since he has to pay \$1.00 for the over charge.)

OR

x	y	$f(x) = f(y)$	$y \cdot f(y)$
0	\$1.00	0.10	0.10
1	\$2.50	0.20	0.50
2	\$4.00	0.30	1.20
3	\$5.50	0.30	1.65
4	\$7.00	0.10	0.70
		1.00	**4.15**

$\mu_Y = E(Y) = \sum_{\text{all } y} y \cdot f(y) = \$\mathbf{4.15}.$

h) Find the standard deviation for the amount of money that Homer Simpson would spend, SD(Y).

$\sigma_Y = SD(Y) = |\,1.50\,| \cdot SD(X) = \$\mathbf{1.7037}.$

OR

x	y	$f(x) = f(y)$	$(y - \mu_Y)^2 \cdot f(y)$	$y^2 \cdot f(y)$
0	\$1.00	0.10	0.99225	0.100
1	\$2.50	0.20	0.54450	1.250
2	\$4.00	0.30	0.00675	4.800
3	\$5.50	0.30	0.54675	9.075
4	\$7.00	0.10	0.81225	4.900
		1.00	**2.9025**	20.125

$\sigma_Y^2 = Var(Y) = \sum_{\text{all } y} (y - \mu_Y)^2 \cdot f(y) = \mathbf{2.9025}.$

OR

$\sigma_Y^2 = Var(Y) = \sum_{\text{all } y} y^2 \cdot f(y) - [\,E(Y)\,]^2 = 20.125 - (4.15)^2$

$= 20.125 - 17.2225 = \mathbf{2.9025}.$

$\sigma_Y = SD(Y) = \sqrt{\mathbf{2.9025}} = \$\cancel{\mathbf{6.6144}}$ 1.7037

2. Suppose that the probability that a duck hunter will successfully hit a duck is 0.40 on any given shot. Suppose also that the outcome of each shot is independent from the others.

a) What is the probability that the first successful hit will be on the fourth shot?

Miss Miss Miss Hit

$0.60 \times 0.60 \times 0.60 \times 0.40 = \mathbf{0.0864}$.

Geometric distribution, $p = 0.40$.

b) What is the probability that the third successful hit will be on the ninth shot?

[8 shots: 2 S's & 6 F's] S

$$\left[\binom{8}{2} \cdot (0.40)^2 \cdot (0.60)^6 \right] \cdot 0.40 \approx \mathbf{0.0836}.$$

OR

S S F F F F F F S	F S S F F F F F S	F F S F S F F F S	F F F S F F F S S
S F S F F F F F S	F S F S F F F F S	F F S F F S F F S	F F F F S S F F S
S F F S F F F F S	F S F F S F F F S	F F S F F F S F S	F F F F S F S F S
S F F F S F F F S	F S F F F S F F S	F F S F F F F S S	F F F F S F F S S
S F F F F S F F S	F S F F F F S F S	F F F S S F F F S	F F F F F S S F S
S F F F F F S F S	F S F F F F F S S	F F F S F S F F S	F F F F F S F S S
S F F F F F F S S	F F S S F F F F S	F F F S F F S F S	F F F F F F S S S

$28 \cdot (0.40)^3 \cdot (0.60)^6 \approx \mathbf{0.0836}$.

Negative Binomial distribution, $p = 0.40$, $r = 3$.

c) What is the probability that the hunter would have three successful hits in nine shots?

Let X = the number of successful hits in 9 shots.

Then X has Binomial distribution, $n = 9$, $p = 0.40$.

Need $P(X = 3) = ?$ $\qquad$ $P(X = k) = \binom{n}{k} p^{k} (1-p)^{n-k}$.

$P(X = 3) = \binom{9}{3} (0.40)^{3} (0.60)^{6} = \mathbf{0.2508}$.

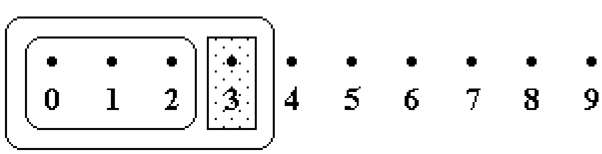
OR

S S F F F F F F S	F S S F F F F F S	F F S F S F F F S	F F F S F F F S S
S F S F F F F F S	F S F S F F F F S	F F S F F S F F S	F F F F S S F F S
S F F S F F F F S	F S F F S F F F S	F F S F F F S F S	F F F F S F S F S
S F F F S F F F S	F S F F F S F F S	F F S F F F F S S	F F F F S F F S S
S F F F F S F F S	F S F F F F S F S	F F F S S F F F S	F F F F F S S F S
S F F F F F S F S	F S F F F F F S S	F F F S F S F F S	F F F F F S F S S
S F F F F F F S S	F F S S F F F F S	F F F S F F S F S	F F F F F F S S S

S S S F F F F F F	S F F S F F F S F	F S F S F F S F F	F F S F S F F S F
S S F S F F F F F	S F F F S S F F F	F S F S F F F S F	F F S F F S S F F
S S F F S F F F F	S F F F S F S F F	F S F F S S F F F	F F S F F S F S F
S S F F F S F F F	S F F F S F F S F	F S F F S F S F F	F F S F F F S S F
S S F F F F S F F	S F F F F S S F F	F S F F S F F S F	F F F S S S F F F
S S F F F F F S F	S F F F F S F S F	F S F F F S S F F	F F F S S F S F F
S F S S F F F F F	S F F F F F S S F	F S F F F S F S F	F F F S S F F S F
S F S F S F F F F	F S S S F F F F F	F S F F F F S S F	F F F S F S S F F
S F S F F S F F F	F S S F S F F F F	F F S S S F F F F	F F F S F S F S F
S F S F F F S F F	F S S F F S F F F	F F S S F S F F F	F F F S F F S S F
S F S F F F F S F	F S S F F F S F F	F F S S F F S F F	F F F F S S S F F
S F F S S F F F F	F S S F F F F S F	F F S S F F F S F	F F F F S S F S F
S F F S F S F F F	F S F S S F F F F	F F S F S S F F F	F F F F S F S S F
S F F S F F S F F	F S F S F S F F F	F F S F S F S F F	F F F F F S S S F

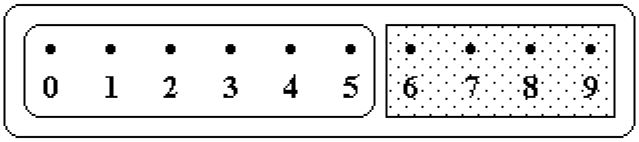
$84 \cdot (0.40)^{3} \cdot (0.60)^{6} \approx \mathbf{0.2508}$.

OR



$P(X = 3) = P(X \le 3) - P(X \le 2) = \text{CDF @ } 3 - \text{CDF @ } 2 = 0.483 - 0.232 = \mathbf{0.251}$.

d) What is the probability that the hunter would have at least six successful hits in nine shots?



P$(X \geq 6)$ = 1 – P$(X \leq 5)$ = 1 – CDF @ 5 = 1 – 0.901 = **0.099**.

3. An advertising company has 7 men and 5 women. Suppose the company has to select a team of 4 members to work on the new hybrid car, Hyper Geo Metro 2011 (☺), commercial.

a) If the members of the team are selected at random, what is the probability that 2 men and 2 women will be selected?

$$\frac{{}_7C_2 \cdot {}_5C_2}{{}_{12}C_4} = \frac{21 \cdot 10}{495} = \mathbf{0.42\overline{42}}.$$

b) What is the probability that men will constitute a majority in the team?

$$\frac{{}_7C_3 \cdot {}_5C_1}{{}_{12}C_4} + \frac{{}_7C_4 \cdot {}_5C_0}{{}_{12}C_4} = \frac{35 \cdot 5}{495} + \frac{35 \cdot 1}{495} = \mathbf{0.42\overline{42}}.$$

4. Suppose Homer Simpson has five coins: 2 nickels, 2 dimes and 1 quarter. Let X denote the amount Bart gets if he steals two coins at random.

a) Construct the probability distribution of X.

Outcomes		x	$f(x)$	
N N		0.10	$\frac{2}{5} \cdot \frac{1}{4} = \frac{2}{20}$	= 0.1
N D	D N	0.15	$\frac{2}{5} \cdot \frac{2}{4} + \frac{2}{5} \cdot \frac{2}{4} = \frac{8}{20}$	= 0.4
D D		0.20	$\frac{2}{5} \cdot \frac{1}{4} = \frac{2}{20}$	= 0.1
N Q	Q N	0.30	$\frac{2}{5} \cdot \frac{1}{4} + \frac{1}{5} \cdot \frac{2}{4} = \frac{4}{20}$	= 0.2
D Q	Q D	0.35	$\frac{2}{5} \cdot \frac{1}{4} + \frac{1}{5} \cdot \frac{2}{4} = \frac{4}{20}$	= 0.2
				1.0

OR

	N_1	N_2	D_1	D_2	Q
N_1	*	0.10	0.15	0.15	0.30
N_2	0.10	*	0.15	0.15	0.30
D_1	0.15	0.15	*	0.20	0.35
D_2	0.15	0.15	0.20	*	0.35
Q	0.30	0.30	0.35	0.35	*

* – do not steal the same coin twice.

x	$f(x)$	
0.10	$\frac{2}{20}$	= 0.1
0.15	$\frac{8}{20}$	= 0.4
0.20	$\frac{2}{20}$	= 0.1
0.30	$\frac{4}{20}$	= 0.2
0.35	$\frac{4}{20}$	= 0.2
		1.0

OR

Outcomes	x	$f(x)$	
N N	0.10	$\frac{{}_2C_2 \cdot {}_2C_0 \cdot {}_1C_0}{{}_5C_2}$	= 0.1
N D	0.15	$\frac{{}_2C_1 \cdot {}_2C_1 \cdot {}_1C_0}{{}_5C_2}$	= 0.4
D D	0.20	$\frac{{}_2C_0 \cdot {}_2C_2 \cdot {}_1C_0}{{}_5C_2}$	= 0.1
N Q	0.30	$\frac{{}_2C_1 \cdot {}_2C_0 \cdot {}_1C_1}{{}_5C_2}$	= 0.2
D Q	0.35	$\frac{{}_2C_0 \cdot {}_2C_1 \cdot {}_1C_1}{{}_5C_2}$	= 0.2
			1.0

x	$f(x)$	$x \cdot f(x)$	$(x - \mu_X)^2 \cdot f(x)$	$x^2 \cdot f(x)$
0.10	0.1	0.01	0.00144	0.0010
0.15	0.4	0.06	0.00196	0.0090
0.20	0.1	0.02	0.00004	0.0040
0.30	0.2	0.06	0.00128	0.0180
0.35	0.2	0.07	0.00338	0.0245
	1.0	**0.22**	**0.00810**	0.0565

b) Find the expected value of the amount that Bart gets, E(X).

$$\mu_X = E(X) = \sum_{\text{all } x} x \cdot f(x) = \$\mathbf{0.22}.$$

c) Find the standard deviation SD(X).

$$\sigma_X^2 = V(X) = \sum_{\text{all } x} (x - \mu_X)^2 \cdot f(x) = \mathbf{0.0081}.$$

OR

$$\sigma_X^2 = V(X) = \sum_{\text{all } x} x^2 \cdot f(x) - \mu_X^2 = 0.0565 - (0.22)^2 = 0.0565 - 0.0484 = \mathbf{0.0081}.$$

$$\sigma_X = SD(X) = \sqrt{0.0081} = \$\mathbf{0.09}.$$

5. Tom works the night shift at a gas station on the edge of Champaign. From past experience, Tom knows that 40% of all customers who purchase gas pay at the pump with a credit card. Assume that customers either pay at the pump with a credit card or not independently of each other.

a) Out of 15 customers who purchase gas, how many would you expect to pay at the pump with a credit card?

Let X be the number of customers who pay with a credit card,
Then X is Binomial, $n = 15$, $p = 0.40$.
$\mathrm{E}X = n \times p = \mathbf{6}$.

b) What is the probability that exactly 5 customers out of 15 pay at the pump with a credit card?

Let X be the number of customers who pay with a credit card,
Then X is Binomial, $n = 15$, $p = 0.40$, $x = 5$.
$\mathrm{P}(X = 5) = \binom{n}{x} p^x (1-p)^{n-x} = \binom{15}{5} 0.40^5 \times 0.60^{10} = \mathbf{0.186}.$

c) What is the probability that at least 4 customers out of 15 pay at the pump with a credit card?

Let X be the number of customers who pay with a credit card,
Then X is Binomial, $n = 15$, $p = 0.40$.
$\mathrm{P}(X \geq 4) = 1 - \mathrm{P}(X \leq 3) = 1 - \sum_{x=0}^{3} \binom{n}{x} p^x (1-p)^{n-x} = 1 - \sum_{x=0}^{3} \binom{15}{x} 0.40^x \times 0.60^{15-x} = \mathbf{0.909}.$

d) What is the probability that the fourth customer purchasing gas is the first one to pay at the pump with a credit card?

Let X be the number of customers needed to observe the first one to pay with a credit card, Then X is Geometric, $p = 0.40$, $x = 4$.
$\mathrm{P}\,(X = 4) = p(1-p)^{x-1} = 0.40 \times 0.60^3 = \mathbf{0.0864}.$

e) What is the probability that the fourth customer to pay at the pump with a credit card is the fifteenth customer purchasing gas?

Let X be the number of customers needed to observe the fourth one to pay with a credit card, Then X is Negative Binomial, r= 4, p = 0.40, x = 15.

P (X = 15) = $\binom{x-1}{r-1} p^r \ (1-p)^{x-r} = \binom{14}{3} 0.40^4 \times 0.60^{11} =$ **0.0338**.

f) What is the probability that the twentieth customer purchasing gas is the sixth one to pay at the pump with a credit card?

This is equivalent to say, *"What is the probability that the sixth customer to pay at the pump with a credit card is the twentieth customer purchasing gas?"*

Let X be the number of customers needed to observe the sixth one to pay with a credit card, Then X is Negative Binomial, r= 6, p = 0.40, x = 20.

P (X = 20) = $\binom{x-1}{r-1} p^r \ (1-p)^{x-r} = \binom{19}{5} 0.40^6 \times 0.60^{14} =$ **0.0373**.

6. Tom works the night shift at a gas station on the edge of Champaign. One night he mixed 3 stale doughnuts in with 12 fresh doughnuts. That night 6 customers bought one doughnut each, selecting them at random. What is the probability that at least 2 stale doughnuts were sold?

15 doughnuts: 3 stale, 12 fresh. 6 doughnuts sold.

P(at least 2 stale) = P(2 stale) + P(3 stale)

$$= \frac{\binom{3}{2} \cdot \binom{12}{4}}{\binom{15}{6}} + \frac{\binom{3}{3} \cdot \binom{12}{3}}{\binom{15}{6}} = \frac{3 \cdot 495}{5005} + \frac{1 \cdot 220}{5005}$$

= 0.2967 + 0.0440 = **0.3407**.

OR

P(at least 2 stale) = P(2 stale) + P(3 stale)

$$= {}_6C_2 \bullet \left[{}^{3}/_{15} \bullet {}^{2}/_{14} \right] \bullet \left[{}^{12}/_{13} \bullet {}^{11}/_{12} \bullet {}^{10}/_{11} \bullet {}^{9}/_{10} \right]$$

$$+ {}_6C_3 \bullet \left[{}^{3}/_{15} \bullet {}^{2}/_{14} \bullet {}^{1}/_{13} \right] \bullet \left[{}^{12}/_{12} \bullet {}^{11}/_{11} \bullet {}^{10}/_{10} \right]$$

$= 0.2967 + 0.0440 =$ **0.3407**.

OR

P(at least 2 stale) = 1 – [P(0 stale) + P(1 stale)]

$$= 1 - \left[\frac{\binom{3}{0} \cdot \binom{12}{6}}{\binom{15}{6}} + \frac{\binom{3}{1} \cdot \binom{12}{5}}{\binom{15}{6}} \right] = 1 - \left[\frac{1 \cdot 924}{5005} + \frac{3 \cdot 792}{5005} \right]$$

$= 1 - [\ 0.1846 + 0.4747\ ] =$ **0.3407**.

OR

P(at least 2 stale) = 1 – [P(0 stale) + P(1 stale)]

F F F F F F	${}^{12}/_{15} \bullet {}^{11}/_{14} \bullet {}^{10}/_{13} \bullet {}^{9}/_{12} \bullet {}^{8}/_{11} \bullet {}^{7}/_{10} = 0.1846.$
S F F F F F	${}^{3}/_{15} \bullet {}^{12}/_{14} \bullet {}^{11}/_{13} \bullet {}^{10}/_{12} \bullet {}^{9}/_{11} \bullet {}^{8}/_{10} = 0.0791.$
F S F F F F	${}^{12}/_{15} \cdot {}^{3}/_{14} \bullet {}^{11}/_{13} \bullet {}^{10}/_{12} \bullet {}^{9}/_{11} \bullet {}^{8}/_{10} = 0.0791.$
F F S F F F	${}^{12}/_{15} \bullet {}^{11}/_{14} \bullet {}^{3}/_{13} \bullet {}^{10}/_{12} \bullet {}^{9}/_{11} \bullet {}^{8}/_{10} = 0.0791.$
F F F S F F	${}^{12}/_{15} \bullet {}^{11}/_{14} \bullet {}^{10}/_{13} \bullet {}^{3}/_{12} \bullet {}^{9}/_{11} \bullet {}^{8}/_{10} = 0.0791.$
F F F F S F	${}^{12}/_{15} \bullet {}^{11}/_{14} \bullet {}^{10}/_{13} \bullet {}^{9}/_{12} \bullet {}^{3}/_{11} \bullet {}^{8}/_{10} = 0.0791.$
F F F F F S	${}^{12}/_{15} \bullet {}^{11}/_{14} \bullet {}^{10}/_{13} \bullet {}^{9}/_{12} \bullet {}^{8}/_{11} \bullet {}^{3}/_{10} = 0.0791.$

P(at least 2 stale) = 1 – [0.1846 + 0.0791 · 6] = **0.3408**.

7. A customer for a home insurance policy owns a $200,000 home. The probability is 0.1% that the home will be totally destroyed by fire, and the probability is 0.5% that the home will suffer a 50% loss due to fire. If we ignore all other partial losses, what premium should the insurance company charge for a policy just to break even?

Let X = the amount of compensation paid to the customer.

Then P(X = 200,000) = **0.001**, P(X = 100,000) = **0.005**,

P(X = 0) = 1 – [0.001 + 0.005] = **0.994**.

Want E(X) = $\sum x \cdot f(x)$ = ?

x	$f(x)$	$x \cdot f(x)$
0	0.994	0
100,000	0.005	500
200,000	0.001	200
	1.000	**700**

The insurance company should charge **$700** for the policy.

8. A box contains 2 green and 3 red marbles. A person draws a marble from the box at random **without replacement**. If the marble drawn is red, the game stops. If it is green, the person draws again until the red marble is drawn (note that total number of marbles drawn cannot exceed 3 since there are only 2 green marbles in the box at the start). Let the random variable X denote the number of **green** marbles drawn.

a) Find the probability distribution of X. [Hint: There are only 3 possible outcomes for this experiment. It would be helpful to list these three outcomes (three possible sequences of colors of marbles drawn). It may be helpful to make a tree diagram for this experiment. Remember that all (three) probabilities must add up to one.]

Outcomes	x	$f(x)$		$x \cdot f(x)$
R	**0**	$^3/_5$	= **0.6**	0.0
G R	**1**	$^2/_5 \cdot {^3/_4} = {^6/_{20}}$	= **0.3**	0.3
G G R	**2**	$^2/_5 \cdot {^1/_4} \cdot {^3/_3} = {^6/_{60}}$	= **0.1**	0.2
			1.0	0.5
				for part (b)

b) Suppose it costs \$5 to play the game, and the person gets \$10 for each green marble drawn. Is the game fair*? **Explain**. [Hint: Find $\mu_X = E(X)$, the average (expected) number of green marbles drawn per game.]

$$\mu_X = E(X) = \sum_{\text{all } x} x \cdot f(x) = \mathbf{0.5}.$$

That is, **on average**, you get **0.5** green marbles per game. At \$10 per green marble, you win \$5 per game, **on average**. Since it costs \$5 to play the game, **on average**, you break even. **The game is fair**.

OR

Let Y denote the net gain from playing 1 game. Then $\mathbf{Y = 10 \cdot X - 5}$.

$$\mu_Y = E(Y) = 10 \cdot E(X) - 5 = 10 \cdot 0.5 - 5 = \$\mathbf{0}.$$

The game is fair since the expected net gain is \$**0**.
(i.e. on average, in the long run, you do not gain or do not lose money).

OR

Let Y denote the net gain from playing 1 game. Then $\mathbf{Y = 10 \cdot X - 5}$.

x	y	$f(x) = f(y)$	$y \cdot f(y)$
0	-5	0.6	-3.0
1	5	0.3	1.5
2	15	0.1	1.5
		1.0	**0.0**

$$\mu_Y = E(Y) = \sum_{\text{all } y} y \cdot f(y) = \$\mathbf{0}.$$

The game is fair since the expected net gain is $**0**.

(i.e. on average, in the long run, you do not gain or do not lose money).

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* In gambling or betting, a game or situation in which the expected value of the profit for the player is zero (no net gain nor loss) is commonly called a "fair game."

From the textbook:

2.2-2 (2.2-4)

$$\mathbf{E}(\mathbf{X}) = (-1)\times\frac{4}{9} + 0\times\frac{1}{9} + 1\times\frac{4}{9} = \mathbf{0};$$

$$\mathbf{E}\left(\mathbf{X}^2\right) = (-1)^2\times\frac{4}{9} + 0^2\times\frac{1}{9} + 1^2\times\frac{4}{9} = \frac{\mathbf{8}}{\mathbf{9}};$$

$$\mathbf{E}\left(\mathbf{3X}^2 - 2\mathbf{X} + 4\right) = 3\times\frac{8}{9} - 2\times 0 + 4 = \frac{\mathbf{20}}{\mathbf{3}}.$$

2.2-4 (2.2-6)

$E(X) = \$499 \times 0.001 + \$1 \times 0.999 = \mathbf{-\$0.50}$.

2.2-8

Note that $\sum_{x=1}^{\infty} \frac{\mathbf{6}}{\pi^2 x^2} = \frac{6}{\pi^2}\sum_{x=1}^{\infty}\frac{\mathbf{1}}{x^2} = \frac{6}{\pi^2}\times\frac{\pi^2}{6} = 1,$ so this is a p.m.f.

$$E(X) = \sum_{x=1}^{\infty} x\frac{\mathbf{6}}{\pi^2 x^2} = \frac{6}{\pi^2}\sum_{x=1}^{\infty}\frac{1}{x};$$

And it is well known that the sum of this harmonic series is not finite.

2.3-4

$$\mathrm{E}\left(\frac{\mathrm{X}-\mu}{\sigma}\right) = \frac{1}{\sigma}\mathrm{E}(\mathrm{X}-\mu) = \frac{1}{\sigma}\left(\mathrm{E}(\mathrm{X}) - \mu\right) = \frac{1}{\sigma}(\mu - \mu) = \mathbf{0};$$

$$\mathbf{E}\left[\left(\frac{\mathbf{X}-\mu}{\sigma}\right)^2\right] = \frac{1}{\sigma^2}\mathbf{E}\left[(\mathbf{X}-\mu)^2\right] = \frac{1}{\sigma^2}\sigma^2 = \mathbf{1}.$$