Vorlesungsmitschrieb Theoretische Teilchenphysik I

von

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Vorlesung gehalten von

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SS 17

Contents

1. Introduction

1.1 Quarks and Leptons

Particles of matter:

- electrons (e^-) and other leptons are elementary particles.
- protons and neutrons ($|p\rangle = |uud\rangle$, $|n\rangle = |udd\rangle$) are combinations of elementary quarks and gluons. The binding energy of the quarks is very large in comparison to the absolute energy of the proton and neutron ($m_p c^2 = 938$ MeV) if you compare this to the binding energies of Atoms (~ 1 Ry) and their absolute energies ($\sim 10^9$ Ry). Because the proton and the neutron are similar/symmetric in the strong interaction (not in the electroweak interaction though) one can combine them into a isospin dublett $\begin{pmatrix} p \\ n \end{pmatrix}$.

There are many more particles/boundstates of quarks and gluons for different combination of quarks. Another example are the Δ-Baryons. These are Spin-$\frac{3}{2}$ particles and have masses of $m_\Delta c^2 \approx 1230$ MeV:

- Δ^- : $|ddd\rangle$
- Δ^0 : $|ddu\rangle$
- Δ^+ : $|duu\rangle$
- Δ^{++} : $|uuu\rangle$

Because the Δ-Baryons are spin-$\frac{3}{2}$ particles all of the quarks spins must be aligned, so the spin wavefunction is symmetric. Also the orbital wavefunction is symmetric for the Δ^{++}-Baryon because it consists of thrice the same quark. However, the total wavefunction of the Baryons must be antisymmetric because it is a fermion.
This is the reason a color charge was introduced to interpret the dirac statistics correctly and characterize the strong interaction with a new quantum number.
Alltogether one can describe the four Δ-Baryons in an isospin quartet with $I = \frac{3}{2}$.

Another group of particles are the mesons, they consist of one quark and an anti quark. The lightest examples are the Pions:

- π^+ : $|u\bar{d}\rangle$
- π^0 : $\frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle - |d\bar{d}\rangle\right)$
- π^- : $|d\bar{d}\rangle$

They have masses of $m_\pi c^2 \approx 140$ MeV and are Spin-0 particles. Together they form the isospin triplet $I = 1$.

Another group of mesons with Spin-0 are the kaons. These have another type of quark, the strange quark. For this new quark a new quantum number (next to isospin) was introduced, the strangeness.
One can summarize the kaons and the pions in a meson octett depicted in figure 1.1. The kaons have masses of $m_K c^2 \approx 495$ MeV. Additionally to the four kaons and the three pions there is a η meson in the same place as the π^0. It is like the π^0 but has additional strange quarks: $|\eta\rangle = \frac{1}{6}\left(|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle\right)$.

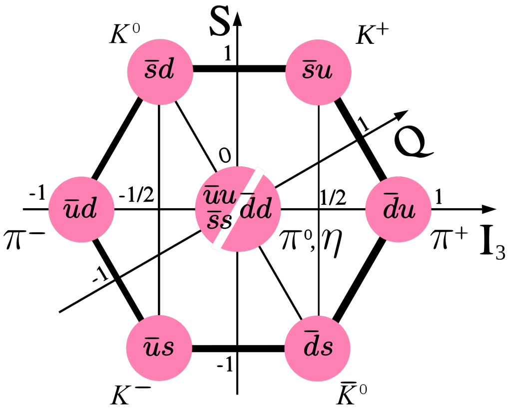

Figure 1.1: meson octett for spin-0

The quarks and leptons are probably the fundamental layer of particles; mesons and baryons are complex bound states described through nuclear physics. The quarks and leptons are described by the dirac equation

$$\left(i\not{\partial} - \frac{mc}{\hbar}\right)\psi = 0 + \text{interactions}$$

One interaction is for example the electromagnetism: $\partial_\mu \to \partial_\mu + iqA_\mu$

In table 1.1 and 1.2 all quarks and leptons are summarized with their electric charge and mass. The charge is given in units of elementary charge as $q = Q \cdot e$.

Quark	mc^2 [MeV]	Q
u	$2.2^{+0.6}_{-0.7}$	$+\frac{2}{3}$
d	4.7	$-\frac{1}{3}$
c	1270	$+\frac{2}{3}$
s	96	$-\frac{1}{3}$
t	173200	$+\frac{2}{3}$
b	4180	$-\frac{1}{3}$

Table 1.1: quarks

Lepton	mc^2 [MeV]	Q
e^-	0.511	-1
μ^-	105.66	-1
τ^-	1777	-1
ν_e		0
ν_μ		0
ν_τ		0

Table 1.2: leptons

It is important, that the down quark is slightly heavier than the up quark; because then the neutron is slightly lighter than the proton and the neutron decays more easily into a proton than the other way around. So atoms are mostly stable and remain charged.

These leptons and quarks are all known matter fields save the bosons:

- Higgs boson H

- γ, $W^{\pm}$, Z which are carriers of the electromagnetic and weak force
- gluon g which is the carrier of the strong force

Additionaly it is known, from observing the higgs coupling, that there are no more generations of quarks which behave similarly to the three existing generations. Additional generations might exist but must behave fundamentally different.

1.2 Course Contents

In this course of theoretical particle physics the following topics will be discussed:

- theoretical description of interactions of quarks and leptons
  $\rightarrow$ gauge theories (*Eichtheorien*)
- pair production of particles and γ, $W^{\pm}$, Z, g emission
  $\rightarrow$ changing particle number and content
  $\rightarrow$ quantum field theory (QFT) which is relativistic for particle physics
- development of pertubation theory for QFT
- calculation of cross sections and decay rates
- symmetries: lorentz invariance and internal symmetries like isospin and color

1.3 Natural Units

In particle physics it is not practical to use the usual units. It is much more practicable to factor out constants like ϵ_0, $\hbar$, c and k_B such that the remaining quantities have dimensions of energy to a power.
The unit of energy will be electron volts (eV). In table 1.3 some important quantities and their dimensions in natural units are shown.

quantity	MKSA units	natural units	dimension
velocity	$\tilde{v}$	$v \cdot c$	$[v] = 1$
length	$\tilde{L}$	$L \cdot \hbar c$	$[L] = 1/\text{MeV}$
time	$\tilde{t}$	$t \cdot \hbar$	$[t] = 1/\text{MeV}$
electric field	$\tilde{E}$	$\frac{1}{\sqrt{\epsilon_0(\hbar c)^3}}\vec{E}$	$[\vec{E}] = \text{MeV}^2$
magnetic field	$\tilde{B}$	$\frac{1}{\sqrt{\epsilon_0 c^2(\hbar c)^3}}\vec{B}$	$[\vec{B}] = \text{MeV}^2$

Table 1.3: natural units

An example of the simplification is the hamiltionan for radiation:

$$H_{rad} = \frac{\epsilon_0}{2}\int \mathrm{d}^3\tilde{\vec{x}}\left[\tilde{\vec{B}}^2 + c^2\tilde{\vec{E}}^2\right] \rightarrow \frac{1}{2}\int \mathrm{d}^3\vec{x}\left[\vec{B}^2 + \vec{E}^2\right]$$

Another useful thing are translations from the normal system to the natural units and vice versa. For example:

- $\hbar c = 197$ MeVfm
- $\frac{1}{\text{GeV}^2} = \frac{3.89\cdot 10^{-4}\ \text{b}}{(\hbar c)^2}$ where a barn is 10^{-28} m^2
- $\tilde{e} = 1.6\cdot 10^{-19}$ C $\quad\rightarrow\quad e = \frac{\tilde{e}}{\sqrt{\epsilon_0 \hbar c}} = \sqrt{4\pi\alpha} = 0.3028$

1.4 Lagrangedichte und Bewegungsgleichungen

2. Symmetrien und Gruppen

2.1 Darstellungstheorie

2.2 Lie Gruppen und Lie Algebra

2.3 Relativistische Invarianz und die Lorentzgruppe

2.4 Feldtransformationen: Darstellungen der Lorentzgruppe

3. Klassiche Feldtheorie: Lagrangians

3.1 Bewegungsgleichungen

3.2 Symmetrien (Noether's Theorem)

3.3 Eichsymmetrie, Eichfelder

4. Kanonische (zweite) Quantisierung von Spin 0, 1/2, 1 Feldern

4.1 Erzeugungs- und Vernichtungsoperatoren

4.2 Fockraum

4.3 Propagatoren

4.4 Gupta Bleuler Quantisierung des Photons

5. S-Matrix, LSZ Reduktionsformel

6. Störungstheorie

6.1 Feynman Regeln der QED

6.2 Wirkungsquerschnitte und Zerfallsraten

6.3 radiative Korrekturen