Cse 222 Homework 2

Q1.

a) $\lim_{n\to\infty} \frac{(n^2-3n)^2}{5n^3+n} = \lim_{n\to\infty} \frac{n^4-6n^3+9n^2}{5n^3+n} = \lim_{n\to\infty} \frac{n^4(1-\frac{6}{n}+\frac{9}{n^2})}{n^3(5+\frac{1}{n^2})} = \lim_{n\to\infty} \frac{n^4}{5n^3}$

$= \lim_{n\to\infty} \frac{n}{5} = \infty \rightarrow f(n)=\Omega(g(n))$

b) $\lim_{n\to\infty} \frac{n^3}{\log_2 n^4} = \text{L'Hopital} \lim_{n\to\infty} \frac{3n^2}{\frac{1.4}{n\ln(2)}} = \lim_{n\to\infty} 3n^3.\ln(2).\frac{1}{4} = \lim_{n\to\infty} = \infty$

$\rightarrow f(n) = \Omega(g(n))$

c) $\lim_{n\to\infty} \frac{5n.\log_2(4n)}{n.\log_2(5^n)} = \lim_{n\to\infty} \frac{5\cancel{n}.\log_2(4n)}{n^{\cancel{2}}.\log_2(5)} = \text{L'Hopital} \lim_{n\to\infty} \frac{\frac{5.4}{4n.\ln 2}}{1.\log_2 5 + n.0} = \lim_{n\to\infty} \frac{5}{\ln 2.\log_2 5.n}$

$= \lim_{n\to\infty} \frac{5}{\ln 5.n} = \lim_{n\to\infty} \frac{5}{\infty} = 0 \quad f(n) = O(g(n))$

d) $\lim_{n\to\infty} \frac{n^n}{10^n} = \lim_{n\to\infty} \left(\frac{n}{10}\right)^n = \lim_{n\to\infty} \frac{\log n^n}{\log 10^n} = \lim_{n\to\infty} \frac{\cancel{n}.\log n}{\cancel{n}.\log 10}$

$= \lim_{n\to\infty} \frac{\log \infty}{\log 10} = \lim_{n\to\infty} \infty = f(n) = \Omega(g(n)).$

$n^{-2/15}$

e) $\lim_{n\to\infty} \frac{8\cancel{n}.\sqrt[5]{2n}}{\cancel{n}.\sqrt[3]{n}} = \lim_{n\to\infty} \frac{8.\sqrt[5]{2}.\sqrt[5]{n}}{\sqrt[3]{n}} = \lim_{n\to\infty} \frac{8.\sqrt[5]{2}.n^{1/5}}{n^{1/3}} = \lim_{n\to\infty} \frac{8.\sqrt[5]{2}}{n^{2/15}}$

$= \lim_{n\to\infty} \frac{8\sqrt[5]{2}}{\infty} = 0 \quad f(n) = O(g(n)).$

Q2) a) methodA for(n length)
str_array[i] = '';] $O(1)$] $O(n)$

operation takes constant time $O(1)$

The operation repeat n times $O(n * 1) = O(n)$

b) methodB for(n length)] $O(n)$ T_1
methodA(strarray);] $O(n)$
for(n length)] $O(n)$ T_2
print();] $O(1)$
this operation takes constant time $O(1)$
the operation repeat n times $O(n*1) = O(n)$

the operation repeat n times $O(n)$
the operation repeat n times $O(n) \Rightarrow O(n*n) = O(n^2)$

$T_1(n) + T_2(n) = \max(O(T_1(n)), O(T_2(n)))$
$= \max(O(n), O(n^2))$
$= O(n^2)$

methodC
c) for(n length)] $O(n)$
for(n length)] $O(n)$
methodB] $O(n^2)$
this operation takes $O(n^2)$ time
the operation repeat n times $O(n^2 . n) = O(n^3)$
the operation repeat n times $O(n^3 . n) = O(n^4)$

methodD
d) We can't solve this problem.
for(n length)
Print();] $O(1)$.
str_array[i--]; → this part decrease i and i value increase within for loop and this method infinite loop.

e) methodE
for(n length)] $O(n)$
if( ) $O(1)$] $O(1*1) = O(1)$
break;) $O(1)$
this operation takes constant time $O(1)$.
the operation repeat n times $O(n * 1) = O(n)$

Q3)

a) if array sorted

result = A[n-1] - A[0] ;

this operation takes constant time O(1)

b) method sort-array(A)

initialize variable min and max to A[0]

FOR each index of A array

IF A[index] greater than max] O(1)

THEN max equals to A[index] O(1)

ENDIF

IF A[index] smaller than min] O(1)

THEN min equals to A[index]] O(1)

ENDFOR

O(n) T_1

compute result of max minus min variable] O(1) T_2

this operation takes constant time O(1)

4 operation takes constant time O(4*1) = O(1)

the operation repeats n times O(n*1) = O(n)

$T_1(n) + T_2(n) = max(O(T_1(n)), O(T_2(n)))$

$= max(O(n), O(1))$

$= \underline{O(n)}$