MAELAS code

User manual v1.0

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WHAT IS MAGNETOSTRICTION?

Magnetostriction is a physical phenomenon in which the process of magnetization induces a change in shape or dimension of a magnetic material.

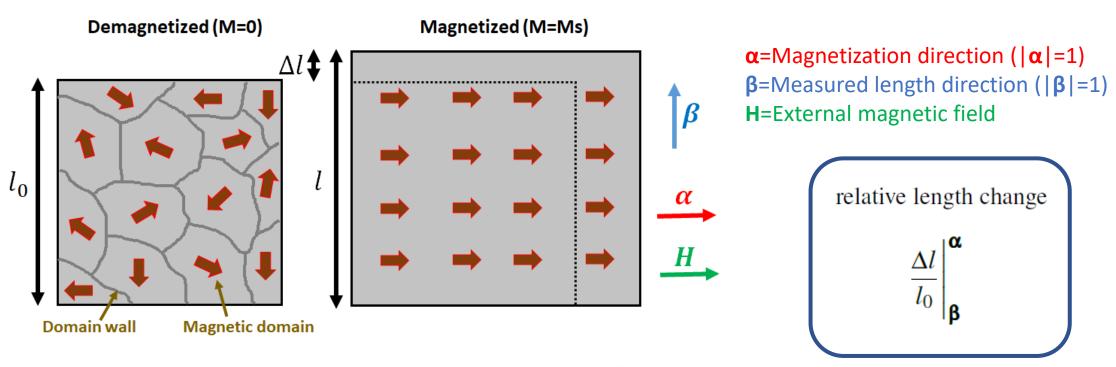


Figure 1: Magnetostriction of a single crystal under an external magnetic field ($\alpha \| H$) perpendicular to the measured length direction ($\beta \perp H$). Symbols M and M_s stand for macroscopic magnetization and saturation magnetization, respectively. Dash line on the right represents the original size of the demagnetized material. The magnetostriction effect has been magnified in order to help to visualize it easily, in real materials it is smaller ($\Delta l/l_0 \sim 10^{-3} - 10^{-6}$).

WHAT IS MAELAS CODE?

- MAELAS code is a software to calculate spin-dependent magnetostriction coefficients and magnetoelastic constants up to second order.
- ➤ It generates required input files for VASP code to perform Density Functional Theory calculations, and it deduces the value of magnetostriction coefficients from the calculated energies given by VASP.
- If the elastic tensor is provided, then it can also calculate the magnetoelastic constants.
- MAELAS can also be used with other DFT codes instead of VASP, after file conversion to VASP format files.

INSTALLATION

MAELAS code is just one python3 file "maelas.py", so it only requires to have Python3 and imported python libraries. For example, in Ubuntu Linux machine you can check the installed version of python3 by opening a terminal and typing

python3 -version

In case you need to install it in your machine, you can type

sudo apt-get update sudo apt-get install python3.8

In HPC facilities you may need to load the Python3 module. For example, in Centos 7 Linux you can check all installed Python modules by typing

ml avail Python

and load the last version of Python3 using command ml

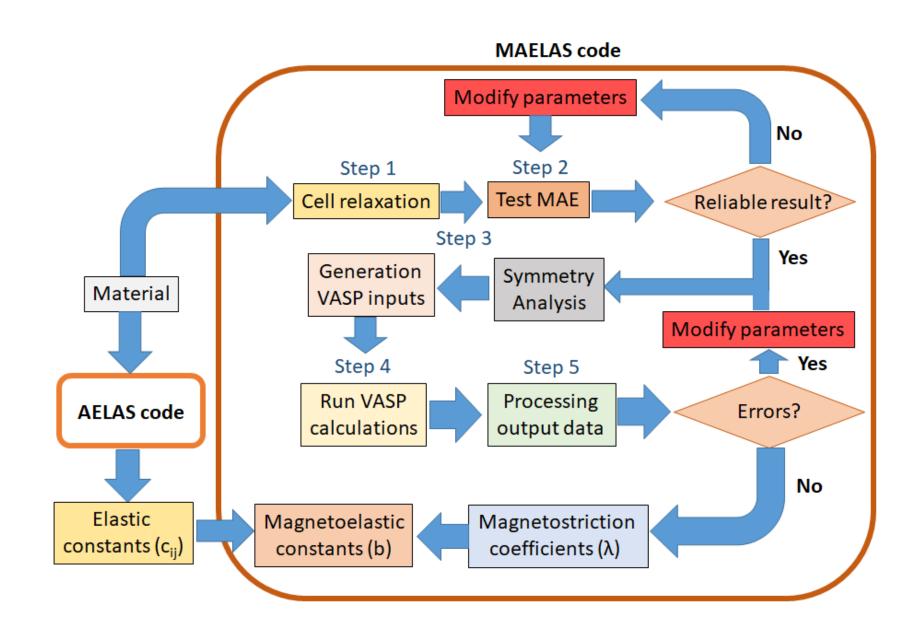
ml Python/3.8.2-GCC-8.3.0-2.32-base

Additionally, MAELAS makes use of the following python libraries: pymatgen, scikit-learn, pyfiglet, argparse, numpy, matplotlib, scipy, math, os and stat. In case you need to install them, you can do it using pip3 as:

pip3 install library name

where "library_name" is the name of the python library that you need to install. For example, you should type the following command to install numpy library:

pip3 install numpy



Step 1: Cell relaxation

If your initial POSCAR is not relaxed and you want to perform a cell relaxation before calculating the magentostriction coefficients, then you can use MAELAS code to generate INCAR and KPOINTS files to relax the structure with VASP. To do so, in the terminal you should copy your initial POSCAR and maelas.py files in the same folder where you want to generate the input files for VASP, and after going to this folder then type

python3 maelas.py -r -i POSCAR0 -k 40

where tag -r indicates that you want to generate VASP files for cell relaxation, -i POSCAR0 is the input non-relaxed POSCAR file (you can name it whatever you want) and -k 40 is the length parameter that determines a regular mesh of k-points. It will generate 4 files: POSCAR, INCAR, KPOINTS and vasp_jsub_rlx. Here, one still needs to copy manually the POTCAR file in this folder in order to have all required files for VASP run. The generated file vasp_jsub_rlx is a script to submit jobs in HPC facilities, one can specify some settings in this script by adding more tags in the command line. For instance,

python3 maelas.py -r -i POSCARO -k 40 -t 48 -c 24 -q qprod -a OPEN-00-00 -f /scratch/example_rlx

where -t 48 indicates that the number of maximum CPU hours for the VASP calculation is 48 hours,-c 24 means that the number of cores for the VASP calculation is 24, -q qprod set to production queuethe type of queue in HPC facilities, -a OPEN-00-00 is the project identification number for running jobs in HPC facilities and -f /scratch/example_rlx is the folder where you want to run VASP calculations. All these data are included in the vasp_jsub_rlx file, so one can submit this VASP job inmediately in HPC facilities by typing

qsub vasp jsub rlx

This procedure might be helpful for high-throughput routines. More options can be added in vasp_jsub_rlx file through the terminal command line, to see them just type

python3 maelas.py -h

Note that generated INCAR and KPOINTS files contain standard setting for cell relaxation. The user are free to change this setting either directly on the generated files or in the maelas.py. In case your structure is already relaxed or you do not want to perform a cell relaxation, then you can skip this step and move to step 2.

Step 2: Test MAE

Copy the relaxed POSCAR, POTCAR and maelas.py files in the same folder where you want to generate the input files for VASP jobs. In the terminal, after going to this folder then type

python3 maelas.py -m -i POSCAR_rlx -k 70 -s1 1 0 0 -s2 0 0 1

where -m indicates that you want to generate input VASP files for the calculation of MAE, -i POSCAR_rlx is the initial relaxed POSCAR file (you can name it whatever you want), -k 70 is the length parameter that determines a regular mesh of k-points, -s1 1 0 0 is the first spin direction to calculate MAE: s1x s1y s1z and -s2 0 0 1 is the second spin direction to calculate MAE: s2x s2y s2z. It will generate the following files:

POSCAR 0 0 (it is the same POSCAR as)

INCAR_0_C (non-collinear calculation where C=1,2 is the spin orientation case)

INCAR std (collinear calculation to generate the WAVECAR and CHGCAR files to run non-collinear calculations)

KPOINTS (file for the kpoint generation of VASP)

vasp_mae, vasp_mae_jsub and vasp_mae_0 (interconnected bash scripts to run VASP calculations automatically)

vasp_mae_cp_oszicar (bash script to get the calculated OSZICAR_0_0_C files after VASP calculation is finished)

The generated files vasp_mae, vasp_mae_jsub and vasp_mae_0 are interconnected scripts to submit jobs in HPC facilities. One needs only to execute the file vasp_mae in order to run all VASP jobs automatically. You can specify some job settings in these scripts by adding more tags in the command line. For instance,

python3 maelas.py -m -i POSCAR_rlx -k 70 -s1 1 0 0 -s2 0 0 1 -t 48 -c 24 -q qprod -a OPEN-00-00 -f /scratch/example_mag

where -t 48 indicates that the number of maximum CPU hours for the VASP calculation is 48 hours,-c 24 means that the number of cores for the VASP calculation is 24, -q aprod set to production queuethe type of queue in HPC facilities, -a OPEN-00-00 is the project identification number for running jobs in HPC facilities and -f /scratch/example_mag is the folder where you want to run VASP calculations. This procedure might be helpful for high-throughput routines. More options can be added in these script files through the terminal command line, to see them just type

python3 maelas.py -h

Step 3: Generation of VASP files for the calculation of anisotropic magnetostriction coefficients

Copy the relaxed POSCAR, POTCAR and maelas.py files in the same folder where you want to generate the input files for VASP run. In the terminal, after going to this folder then type

python3 maelas.py -g -i POSCAR_rlx -k 70 -n 7 -s 0.1

where -g indicates that you want to generate input VASP files for the calculation of anisotropic magnetostriction coefficients, -i POSCAR_rlx is the initial relaxed POSCAR file (you can name it whatever you want), -k 70 is the length parameter that determines a regular mesh of k-points, -n 7 means that it will generate 7 distorted states for each magentostriction mode and-s 0.1 is the maximum strain applied for distorting the structure. It will generate the following files:

POSCAR_A_B (volume-conserving distorted cell where A=magnetostriction mode, B=1,...,n distorted cell for each magentostriction mode) INCAR_A_C (non-collinear calculation where A=magnetostriction mode, C=1,2 is the spin orientation case) INCAR_std (collinear calculation to generate the WAVECAR and CHGCAR files to run non-collinear calculations) KPOINTSvasp_maelas, vasp_jsub and vasp_0 (interconnected bash scripts to run VASP calculations automatically) vasp_cp_oszicar (bash script to get the calculated OSZICAR_A_B_C files after VASP calculation is finished)

The generated files vasp_maelas, vasp_jsub and vasp_0 are interconnected scripts to submit jobs in HPC facilities, one can specify some job settings in these scripts by adding more tags in the command line. For instance,

python3 maelas.py -g -i POSCAR_rlx -k 70 -n 7 -s 0.1 -t 48 -c 24 -q qprod -a OPEN-00-00 -f /scratch/example_mag

where -t 48 indicates that the number of maximum CPU hours for the VASP calculation is 48 hours,-c 24 means that the number of cores for the VASP calculation is 24, -q qprod set to production queuethe type of queue in HPC facilities, -a OPEN-00-00 is the project identification number for running jobs in HPC facilities and -f /scratch/example_mag is the folder where you want to run VASP calculations. This procedure might be helpful for high-throughput routines. More options can be added in these script files through the terminal command line, to see them just type

python3 maelas.py -h

Step 4: Run VASP calculations

For each generated POSCAR_A_B one should run first a collinear calculation using INCAR_std and use the generated WAVECAR and CHGCAR files to run non-collinear calculations for each INCAR_A_C (C=1,2) using the same POSCAR_A_B. This procedure can be automatically done in HPC facilities just by running the generated bash script

./vasp_maelas

This will launch independent jobs for each POSCAR_A_B. Each job will run 3 VASP calculations: a collinear one to generate WAVECAR and CHGCAR files, and two non-collinear for INCAR_A_1 and INCAR_A_2. The jobs will be executed in subfolders P_A_B inside the folder indicated by tag -f in the step 2.

Once all jobs are finished, then one can easily get calculated non-collinear OSZICAR files (needed in step 4), by running the bash script

./vasp_cp_oszicar

it will copy these OSZICAR files and name them as OSZICAR_A_B_C (C=1,2) in the same folder where this script is executed.

Step 5: Derivation of anisotropic magnetostriction coefficients and magnetoelastic constants

Finally, to derive the anisotropic magnetostriction coefficients one needs to have in the same folder the following files:

maelas.py
POSCAR_rlx (the relaxed POSCAR file used as input in step 2)
POSCAR_A_B (distorted POSCAR generated in step 2)
OSZICAR_A_B_C (non-collinear OSZICAR files calculated in step 3 for each POSCAR_A_B and INCAR_A_C)

Next, in the terminal go to this folder a type

python3 maelas.py -d -i POSCAR_rlx -n 7

where -d indicates that you want to derive the spin-dependent magnetostriction coefficients from the calculated OSZICAR files, -i POSCAR_rlx is the relaxed POSCAR file used as input in step 2 (you can name it whatever you want) and -n 7 is the number of distorted states for each magentostriction mode used in step 2.

It will derive and print the calculated anisotropic magnetostriction coefficients in the terminal. If you want to print it in a file (for example, "results.out"), then you can type

python3 maelas.py -d -i POSCAR rlx -n 7 > results.out

Additionally, the energy values extracted from OSZICAR_A_B_C files are shown in generated files ene_A_C.dat and fit_ene_A_C.png. The energy difference between the two spin configurations for each magnetostriction mode are shown in Fig. dE_A.png. If the elastic tensor is provided as input, then MAELAS can also calculate the magnetoelastic constants. To do so, one needs to add tags -b and -e with the name of the file containing the elastic tensor with the same format and units (GPa) as it is written by AELAS code (file ELADAT). Hence, you could type

Format of the

elastic tensor file

python3 maelas.py -d -i POSCAR_rlx -n 7 -b -e ELADAT

where ELADAT is the name of the file (it could be whatever name you want) with the elastic tensor data.

Elastic tensor: 262.03 186.20 186.20 0.00 0.00 186.20 262.03 186.20 0.00 0.00 186.20 186.20 262.03 0.00 0.00 0.00 0.00 0.00 0.00 116.63 0.00 0.00 0.00 0.00 0.00 0.00 116.63

Full list of arguments in MAELAS code

User can see all possible optional arguments by typing

python3 maelas.py -h

The optional arguments are the following:

```
-h, --help
            Show this help message and exit
-i POS
            Name of the initial non-distorted POSCAR file (default: POSCAR)
-n NDIST
            Number of distorted states for each magnetostriction mode (default: 7)
-s STRAIN
            Maximum value of the parameter epsilon for the strain tensor to generate the distorted POSCAR files (default: 0.01)
-k KP
            VASP automatic k-point mesh generation to create the KPOINTS file (default: 60)
            Generation of required VASP files for the calculation of magnetostriction coefficients.
-g
            Derivation of magnetostriction coefficients from the energy written in the OSZICAR files.
-d
            Generation of required VASP files for the cell relaxation
-r
            Generation of required VASP files to test MAE
-m
-s1 s1x s1y s1z First spin direction to calculate MAE
-s2 s2x s2y s2z Second spin direction to calculate MAE
            Calculation of the magnetoelastic constants from the calculated magnetostriction coefficients and provided elastic tensor.
-b
-e ELAS
            File with the elastic tensor data in the same format and units (GPa) as it is written by ELAS code (file ELADAT).
-sp SYMPRE
               Tolerance for symmetry finding (default: 0.01)
-sa SYMANG
               Angle tolerance for symmetry finding (default: 5.0)
            Number of cores for the VASP calculation (default: 24)
-c CORE
-t TIME
            Number of maximum CPU hours for the VASP calculation (default: 48)
-f VASP_FOLD Folder where you will run VASP calculations (default: /scratch)
-mp MPI
             Command for mpi run of VASP (default: mpiexec.hydra)
-a P ID
           Project id for running jobs in HPC facilities (default: OPEN-X-X)
-l LOAD MODULE Module of VASP that should be loaded (default: VASP/5.4.4-intel-2017c-mkl=cluster)
```

-q QUEUE Type of gueue to be used for VASP calculations in HPC facilities (default: qprod)

Summary: In a nutshell

Step 1: Cell relaxation

python3 maelas.py -r -i POSCARO -k 40

qsub vasp_jsub_rlx

Step 2: Test MAE

python3 maelas.py -m -i POSCAR rlx -k 70 -s1 1 0 0 -s2 0 0 1

./vasp_mae

./vasp mae cp oszicar

Step 3: Generate VASP inputs for calculation of magnetostriction coefficients

python3 maelas.py -g -i POSCAR_rlx -k 70 -n 7 -s 0.1

Step 4: Run VASP calculations

./vasp_maelas

./vasp_cp_oszicar

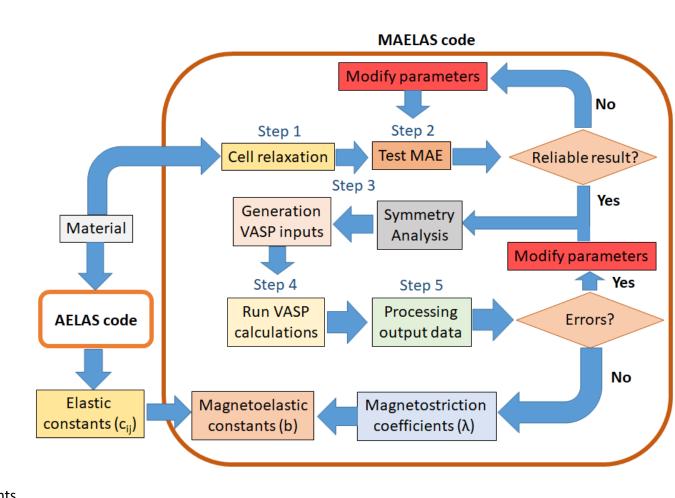
Step 5: Derivation of anisotropic magnetostriction coefficients

python3 maelas.py -d -i POSCAR_rlx -n 7

Step 5: Derivation of anisotropic magnetostriction coefficients and magnetoelastic constants

python3 maelas.py -d -i POSCAR rlx -n 7 -b -e ELADAT

See all optional arguments: python3 maelas.py -h



Using MAELAS with other DFT codes instead of VASP

MAELAS has been designed to read and write files for VASP code automatically. However, it is possible to use MAELAS with other DFT codes instead of VASP, after file conversion to VASP format files. Although, this process might require some extra work for the user. Namely, converting initial and distorted POSCAR files into the other DFT code format, reading the spin direction of each state from INCAR_A_C files (variable SAXIS) and write the calculated energies in a OSZICAR-like file (called OSZICAR_A_B_C) on the penultimate line and third column with same format as in VASP (this is the place where MAELAS reads the energy value of each OSZICAR_A_B_C file). For instance, in the following OSZICAR file, one should write the energy value at "**Energy_DFT_code**":

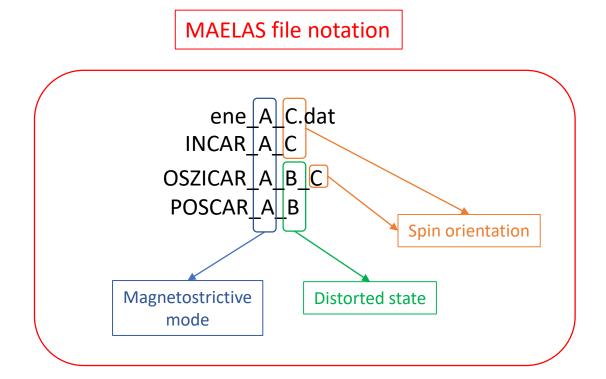
	N	E	dE	d eps ncg	rms	rms(c)	
DAV:	1	-0.219086777516E+02	-0.21909E+02	0.99185E+02*****	0.709E+00		
DAV:	2	-0.219092777733E+02	-0.60002E-03	-0.60002E-03*****	0.452E-01		
DAV:	3	-0.219092846144E+02	-0.68411E-05	-0.68405E-05*****	0.485E-02		
DAV:	4	-0.219092847670E+02	-0.15258E-06	-0.15274E-06*****	0.641E-03		
DAV:	5	-0.219092847725E+02	-0.55161E-08	-0.52995E-08*****	0.117E-03		
DAV:	6	**Energy_DFT_code**	-0.19827E-09	-0.11530E-09868760	0.143E-04		
1	F=	21909285E+02 E0=2190	99330E+02 d E	=0.135168E-03 mag=	0.0000	0.0000	2.5077

Crystal systems supported by MAELAS v1.0

Table 1: Number of independent second-order elastic constants of each crystal system. Number of independent magnetoelastic and magnetostriction coefficients up to second-order of the direction cosine polynomial in the first-order magnetoelastic energy. In the last column we specify which crystal systems are supported by the current version of MAELAS.

Crystal system	Point groups	Space groups	Elastic constants (c_{ij})	Magnetoelastic constants (b)	Magnetostriction coefficients (λ)	MAELAS
Triclinic	$1, \overline{1}$	1-2	21	36	36	No
Monoclinic	2, m, 2/m	3 - 15	13	20	20	No
Orthorhombic	222,2mm,mmm	16 - 74	9	12	12	Yes
Tetragonal (II)	$4, \bar{4}, 4/m$	75 - 88	7	10	10	No
Tetragonal (I)	$4mm, 422, \bar{4}2m, 4/mmm$	89 - 142	6	7	7	Yes
Trigonal (II)	$3,\bar{3}$	143 - 148	7	12	12	No
Trigonal (I)	$32, 3m, \bar{3}m$	149 - 167	6	8	8	Yes
Hexagonal(II)	$6,\bar{6},6/m$	168 - 176	5	8	8	No
Hexagonal (I)	$6mm, 622, \bar{6}2m, 6/mmm$	177 - 194	5	6	6	Yes
Cubic (II)	$23, m\bar{3}$	195 - 206	3	4	4	No
Cubic (I)	$432,\bar{4}3m,m\bar{3}m$	207 - 230	3	3	3	Yes

The crystal systems not supported by MAELAS might be included in the new versions of the code



Methodology

$\frac{\Delta l}{l_0} \bigg _{oldsymbol{eta}^i}^{oldsymbol{lpha}_1^i}$	$-\frac{\Delta l}{l_0}\bigg _{\boldsymbol{\beta}^i}^{\boldsymbol{\alpha}_2^i} = \eta^i \lambda^i$
	$\frac{1-l_0}{l_0} = \frac{2(l_1 - l_2)}{(l_1 + l_2) \left[1 - \frac{l_1 + l_2 - 2l_0}{l_1 + l_2}\right]}$ $\left[1 + \frac{l_1 + l_2 - 2l_0}{l_1 + l_2} + \dots\right] \approx \frac{2(l_1 - l_2)}{l_1 + l_2}$
	uadratic function $l^2 + Bl + C$
-16.540 -16.542 -16.544 -16.546	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

		Magnetostriction				
Crystal system	Eq.	coefficient	β	$\mathbf{\alpha}_{\mathrm{l}}$	α_2	η
Cubic (I)	10	λ ₀₀₁	(0,0,1)	(0,0,1)	(1,0,0)	3/2
		λ_{111}	$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$	$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$	$\left(\frac{1}{\sqrt{2}},0,\frac{-1}{\sqrt{2}}\right)$	$\frac{\frac{3}{2}}{\frac{3}{2}}$ $\frac{1}{3}$ 1
Hexagonal (I)	16	$\lambda^{\alpha 1,2}$	(1,0,0)	$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$	$(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)$	1/3
		$\lambda^{\alpha 2,2}$	(0,0,1)	(0,0,1)	(1,0,0)	1
		$\lambda^{\gamma,2}$	(1,0,0)	(1,0,0)	(0,1,0)	1
		$\lambda^{\epsilon,2}$	$\left(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)$	$\left(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)$	$\left(\frac{-1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)$	1
Trigonal (I)	20	$\lambda^{\alpha 1,2}$	(1,0,0)	(0,0,1)	$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$	1
		$\lambda^{\alpha 2,2}$	(0,0,1)	(0, 0, 1)	(1,0,0)	1
		$\lambda^{\gamma,1}$	(1,0,0)	(1,0,0)	(0,1,0)	1
		$\lambda^{\gamma,2}$	$\left(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)$	$\left(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)$	$\left(\frac{1}{\sqrt{2}},0,\frac{-1}{\sqrt{2}}\right)$	$\frac{1}{2}$
		λ_{12}	$\left(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)$	$\left(0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$	$\left(0,\frac{1}{\sqrt{2}},\frac{-1}{\sqrt{2}}\right)$	$\frac{1}{4}$
		λ_{21}	$\left(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)$	$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$	$\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right)$	$\frac{\frac{1}{4}}{\frac{1}{2}}$ $\frac{1}{3}$
Tetragonal (I)	24	$\lambda^{\alpha 1,2}$	(1,0,0)	$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$	$(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)$	1/3
		$\lambda^{\alpha 2,2}$	(0,0,1)	(0,0,1)	(1,0,0)	1
		$\lambda^{\gamma,2}$	(1,0,0)	(1,0,0)	(0,1,0)	1
		$\lambda^{\epsilon,2}$	$\left(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)$	$\left(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)$	$\left(\frac{-1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)$	1
		$\lambda^{\delta,2}$	$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$	$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$	$\left(\frac{-1}{\sqrt{2}},\frac{1}{\sqrt{2}},0\right)$	1
Orthorhombic	27	λ_1	(1,0,0)	(1,0,0)	(0,0,1)	1
		λ_2	(1,0,0)	(0, 1, 0)	(0,0,1)	1
		λ_3	(0,1,0)	(1,0,0)	(0,0,1)	1
		λ_4	(0,1,0)	(0, 1, 0)	(0,0,1)	1
		λ_5	(0,0,1)	(1,0,0)	(0,0,1)	1
		λ_6	(0,0,1)	(0,1,0)	(0, 0, 1)	1
		λ_7	$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$	$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$	(0,0,1)	1
		λ_8	$\left(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)$	$\left(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)$	(0,0,1)	1
		λ_9	$\left(0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$	$\left(0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$	(0,0,1)	1

CUBIC (I)

SG 207-230

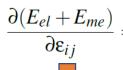
Cubic (I)

Workflow

Elastic energy

$$E_{el}^{cub} = \frac{c_{11}}{2} (\varepsilon_{xx}^2 + \varepsilon_{yy}^2 + \varepsilon_{zz}^2) + c_{12} (\varepsilon_{xx} \varepsilon_{yy} + \varepsilon_{xx} \varepsilon_{zz} + \varepsilon_{yy} \varepsilon_{zz}) + \frac{c_{44}}{2} (\varepsilon_{xy}^2 + \varepsilon_{yz}^2 + \varepsilon_{xz}^2)$$

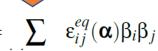












 $+b_2(\alpha_x\alpha_y\epsilon_{xy}+\alpha_x\alpha_z\epsilon_{xz}+\alpha_y\alpha_z\epsilon_{yz})$



$$E_{me}^{cub(I)} = b_0(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) + b_1(\alpha_x^2 \varepsilon_{xx} + \alpha_y^2 \varepsilon_{yy} + \alpha_z^2 \varepsilon_{zz})$$

$$\lambda^{\alpha} = \frac{-b_0 - \frac{1}{3}b_1}{c_{11} + 2c_{12}},$$

$$\lambda_{001} = \frac{-2b_1}{3(c_{11} - c_{12})},$$

$$-b_2$$

$$\lambda_{111} = \frac{-b_2}{3c_{44}}$$

 $\frac{\Delta l}{l_0} \bigg|_{\boldsymbol{\beta}}^{\boldsymbol{\alpha}} = \lambda^{\alpha} + \frac{3}{2} \lambda_{001} \left(\alpha_x^2 \beta_x^2 + \alpha_y^2 \beta_y^2 + \alpha_z^2 \beta_z^2 - \frac{1}{3} \right)$

 $+3\lambda_{111}(\alpha_x\alpha_y\beta_x\beta_y+\alpha_y\alpha_z\beta_y\beta_z+\alpha_x\alpha_z\beta_x\beta_z)$

$$\frac{\Delta l}{l_0} \Big|_{\boldsymbol{\beta}}^{\boldsymbol{\alpha}} = \lambda^{\alpha} + \frac{3}{2} \lambda_{001} \left(\alpha_x^2 \beta_x^2 + \alpha_y^2 \beta_y^2 + \alpha_z^2 \beta_z^2 - \frac{1}{3} \right) + 3 \lambda_{111} (\alpha_x \alpha_y \beta_x \beta_y + \alpha_y \alpha_z \beta_y \beta_z + \alpha_x \alpha_z \beta_x \beta_z)$$

Space Broup: 201-230

POSCAR 1 X (N POSCAR: Distortion along [0,0,1])

INCAR std (1 INCAR: collinear w/o SOC)

INCAR 1 1 (1 INCAR: non-collinear with SOC, SPIN=[0,0,1])

INCAR 1 2 (1 INCAR: non-collinear with SOC, SPIN=[1,0,0])

OSZICAR_1_X_1 OSZICAR_1_X_2

Cubic (I)

 λ_{111}

 λ_{001}

POSCAR 2 X (N POSCAR: Distortion along [1,1,1])

INCAR_std (1 INCAR: collinear w/o SOC)

INCAR 2 1 (1 INCAR: non-collinear with SOC, SPIN=[1,1,1])

INCAR 2 2 (1 INCAR: non-collinear with SOC, SPIN=[1,0,-1])

OSZICAR_2_X_1

OSZICAR_2_X_2

Symmetry **POSCAR** analysis

Fitting: $E(x)=a^*x^2+b^*x+c$ Minimum (E_{min}): x_{min} =-b/(2*a)

x=cell length along deformation

Volume-conserving transformations (determinant of transformation matrix = 1)

The maximum value of ε can be set with tag-s

$$\underline{\boldsymbol{\varepsilon}}\Big|_{\boldsymbol{\beta}=(0,0,1)}^{\lambda_{001}} = \begin{pmatrix} \frac{1}{\sqrt{1+\epsilon}} & 0 & 0\\ 0 & \frac{1}{\sqrt{1+\epsilon}} & 0\\ 0 & 0 & 1+\epsilon \end{pmatrix}, \quad \underline{\boldsymbol{\varepsilon}}\Big|_{\boldsymbol{\beta}=\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)}^{\lambda_{111}} = \zeta\begin{pmatrix} \frac{1}{\epsilon} & \frac{\epsilon}{2} & \frac{\epsilon}{2}\\ \frac{\epsilon}{2} & 1 & \frac{\epsilon}{2}\\ \frac{\epsilon}{2} & \frac{\epsilon}{2} & 1 \end{pmatrix}$$

$$\underline{\boldsymbol{\varepsilon}}\Big|_{\boldsymbol{\beta} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)}^{\lambda_{111}} = \zeta \begin{pmatrix} 1 & \frac{\varepsilon}{2} & \frac{\varepsilon}{2} \\ \frac{\varepsilon}{2} & 1 & \frac{\varepsilon}{2} \\ \frac{\varepsilon}{2} & \frac{\varepsilon}{2} & 1 \end{pmatrix}$$

$$\zeta = \sqrt[3]{4/(4-3\varepsilon^2+\varepsilon^3)}$$

$$\lambda_{001} = \frac{4}{3} \cdot \frac{x_{min1} - x_{min}}{x_{min1} + x_{min}}$$

Polycrystal:
$$\lambda_S = \frac{2}{5}\lambda_{100} + \frac{3}{5}\lambda_{11}$$

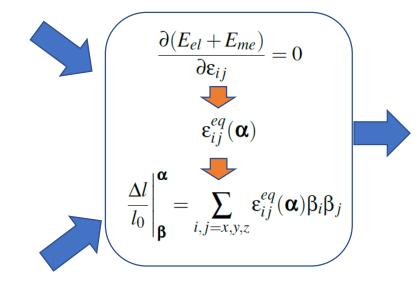
HEXAGONAL (I)

SG 177-194

Workflow

Elastic energy

$$E_{el}^{hex} = \frac{1}{2}c_{11}(\varepsilon_{xx}^2 + \varepsilon_{yy}^2) + c_{12}\varepsilon_{xx}\varepsilon_{yy} + c_{13}(\varepsilon_{xx} + \varepsilon_{yy})\varepsilon_{zz} + \frac{1}{2}c_{33}\varepsilon_{zz}^2 + \frac{1}{2}c_{44}(\varepsilon_{yz}^2 + \varepsilon_{xz}^2) + \frac{1}{4}(c_{11} - c_{12})\varepsilon_{xy}^2.$$



Magnetoelastic energy

$$\begin{split} E_{me}^{hex(I)} &= b_{11}(\varepsilon_{xx} + \varepsilon_{yy}) + b_{12}\varepsilon_{zz} + b_{21}\left(\alpha_z^2 - \frac{1}{3}\right)(\varepsilon_{xx} + \varepsilon_{yy}) + b_{22}\left(\alpha_z^2 - \frac{1}{3}\right)\varepsilon_{zz} \\ &+ \frac{1}{2}b_3[(\alpha_x^2 - \alpha_y^2)(\varepsilon_{xx} - \varepsilon_{yy}) + 2\alpha_x\alpha_y\varepsilon_{xy}] + b_4(\alpha_x\alpha_z\varepsilon_{xz} + \alpha_y\alpha_z\varepsilon_{yz}). \end{split}$$

$$\begin{split} \frac{\Delta l}{l_0} \bigg|_{\pmb{\beta}}^{\pmb{\alpha}} &= \lambda^{\alpha 1,0} (\beta_x^2 + \beta_y^2) + \lambda^{\alpha 2,0} \beta_z^2 + \lambda^{\alpha 1,2} \left(\alpha_z^2 - \frac{1}{3} \right) (\beta_x^2 + \beta_y^2) \\ &+ \lambda^{\alpha 2,2} \left(\alpha_z^2 - \frac{1}{3} \right) \beta_z^2 + \lambda^{\gamma,2} \left[\frac{1}{2} (\alpha_x^2 - \alpha_y^2) (\beta_x^2 - \beta_y^2) + 2 \alpha_x \alpha_y \beta_x \beta_y \right] \\ &+ 2 \lambda^{\varepsilon,2} (\alpha_x \alpha_z \beta_x \beta_z + \alpha_y \alpha_z \beta_y \beta_z), \end{split}$$

$$\lambda^{\alpha 1,0} = \frac{b_{11}c_{33} + b_{12}c_{13}}{c_{33}(c_{11} + c_{12}) - 2c_{13}^2},$$

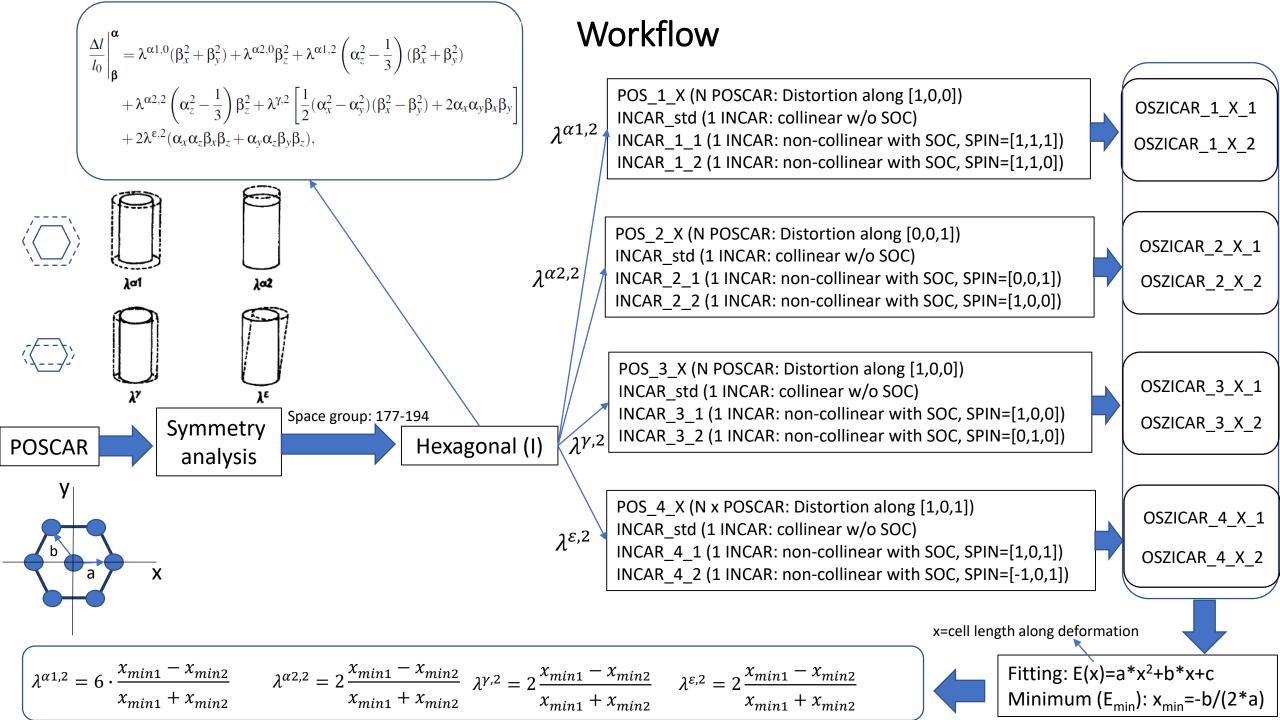
$$\lambda^{\alpha 2,0} = \frac{2b_{11}c_{33} - b_{12}(c_{11} + c_{22})}{c_{33}(c_{11} + c_{12}) - 2c_{13}^2},$$

$$\lambda^{\alpha 1,2} = \frac{-b_{21}c_{33} + b_{22}c_{13}}{c_{33}(c_{11} + c_{12}) - 2c_{13}^2},$$

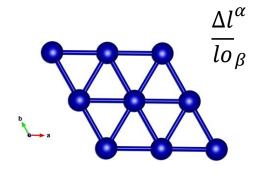
$$\lambda^{\alpha 2,2} = \frac{2b_{21}c_{33} - b_{22}(c_{11} + c_{22})}{c_{33}(c_{11} + c_{12}) - 2c_{13}^2},$$

$$\lambda^{\gamma,2} = \frac{-b_3}{c_{11} - c_{12}},$$

$$\lambda^{\epsilon,2} = \frac{-b_4}{2c_{44}}.$$



Distorted states

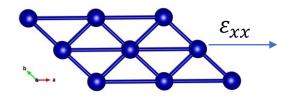


$$|\boldsymbol{\alpha}| = 1, |\boldsymbol{\beta}| = 1$$

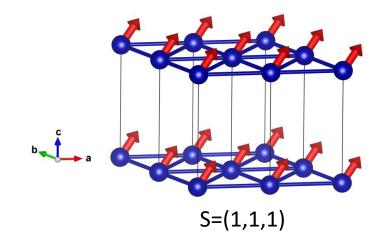
strain along x-axis: $\beta = (1,0,0)$

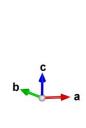
$$\underline{\underline{\mathbf{e}}}\Big|_{\mathbf{\beta}=(1,0,0)}^{\mathbf{\lambda}^{\alpha 1,2}} = \begin{pmatrix} 1+\epsilon & 0 & 0\\ 0 & \frac{1}{\sqrt{1+\epsilon}} & 0\\ 0 & 0 & \frac{1}{\sqrt{1+\epsilon}} \end{pmatrix}$$

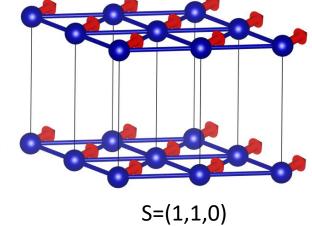
$$\lambda^{\alpha 1,2}$$



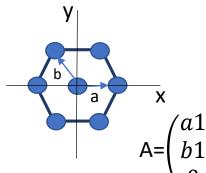
volume-conserving



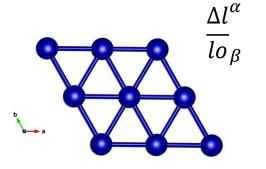




$$\lambda^{\alpha 1,2} = 6 \cdot \frac{x_{min1} - x_{min2}}{x_{min1} + x_{min2}}$$



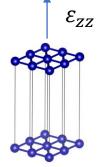
Distorted states



$$|\boldsymbol{\alpha}| = 1, |\boldsymbol{\beta}| = 1$$

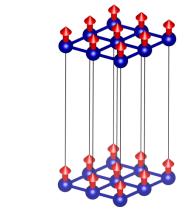
strain along z-axis: $\beta = (0,0,1)$

$$\mathbf{\underline{\varepsilon}}\Big|_{\mathbf{\beta}=(0,0,1)}^{\mathbf{\lambda}^{\alpha2,2}} = \begin{pmatrix} \frac{1}{\sqrt{1+\epsilon}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{1+\epsilon}} & 0 \\ 0 & 0 & 1+\epsilon \end{pmatrix}$$



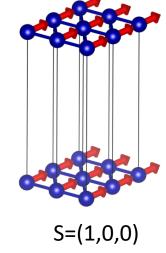
volume-conserving

 $\lambda^{\alpha 2,2}$



S=(0,0,1)

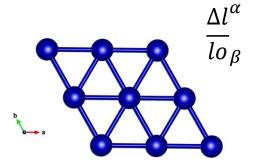
b _ _ _ c



$$A = \begin{pmatrix} a1 & 0 & 0 \\ b1 & b2 & 0 \\ 0 & 0 & c3 \end{pmatrix}$$

$$\lambda^{\alpha 2,2} = 2 \frac{x_{min1} - x_{min2}}{x_{min1} + x_{min2}}$$

Distorted states

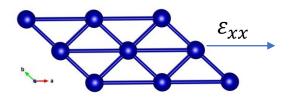


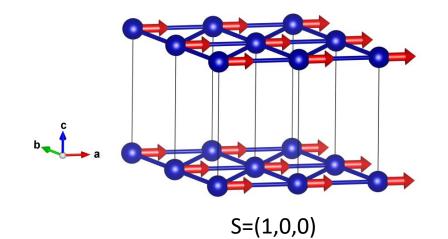
$$|\boldsymbol{\alpha}| = 1, |\boldsymbol{\beta}| = 1$$

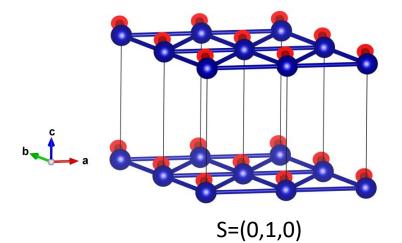
$$\underline{\boldsymbol{\varepsilon}}\Big|_{\boldsymbol{\beta}=(1,0,0)}^{\boldsymbol{\lambda}^{\boldsymbol{\gamma},2}} = \begin{pmatrix} 1+\boldsymbol{\varepsilon} & 0 & 0\\ 0 & \frac{1}{\sqrt{1+\boldsymbol{\varepsilon}}} & 0\\ 0 & 0 & \frac{1}{\sqrt{1+\boldsymbol{\varepsilon}}} \end{pmatrix}$$

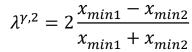
volume-conserving

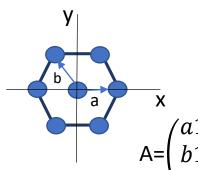
$$\lambda^{\gamma,2}$$



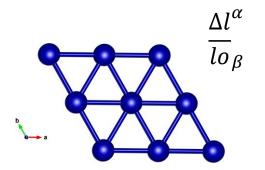








Distorted states



$$|\boldsymbol{\alpha}| = 1, |\boldsymbol{\beta}| = 1$$

shear along xz: $\beta = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

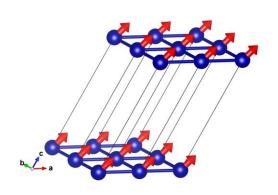
$$\underline{\boldsymbol{\varepsilon}}\Big|_{\boldsymbol{\beta}=\left(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)}^{\boldsymbol{\lambda}^{\boldsymbol{\varepsilon},2}} = \boldsymbol{\omega} \begin{pmatrix} 1 & 0 & \frac{\boldsymbol{\varepsilon}}{2} \\ 0 & 1 & 0 \\ \frac{\boldsymbol{\varepsilon}}{2} & 0 & 1 \end{pmatrix}$$

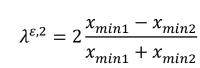


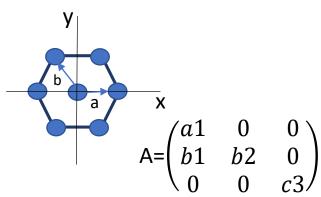
$$\omega = \sqrt[3]{4/(4-\epsilon^2)}$$

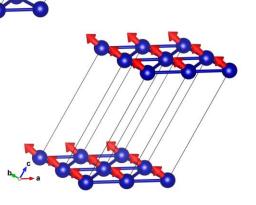
volume-conserving

$$\lambda^{\varepsilon,2}$$

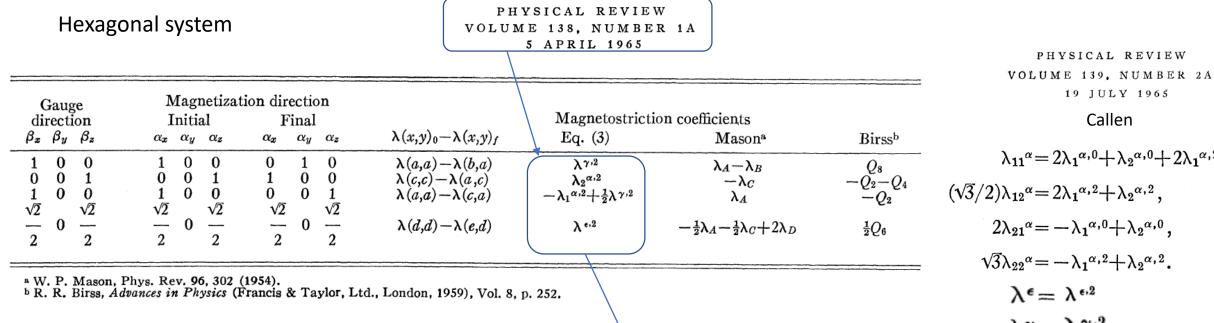








Relation between different notations



Clark

Callen $\lambda_{11}^{\alpha} = 2\lambda_1^{\alpha,0} + \lambda_2^{\alpha,0} + 2\lambda_1^{\alpha,2} + \lambda_2^{\alpha,2}$ $(\sqrt{3}/2)\lambda_{12}^{\alpha}=2\lambda_{1}^{\alpha,2}+\lambda_{2}^{\alpha,2}$ $2\lambda_{21}^{\alpha} = -\lambda_1^{\alpha,0} + \lambda_2^{\alpha,0}$ $\sqrt{3}\lambda_{22}^{\alpha} = -\lambda_1^{\alpha,2} + \lambda_2^{\alpha,2}$. $\lambda \gamma = \lambda \gamma^2$

The calculated magnetostriction coefficients are written in Clark, Mason, Birss and Callen notation in MAELAS code

MAELAS notation

TRIGONAL (I)

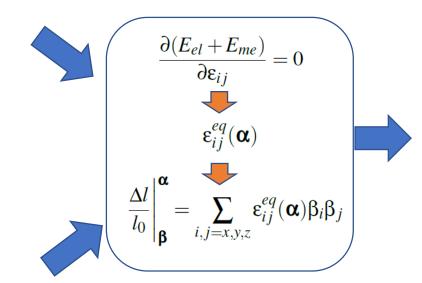
SG 149-167

Trigonal (I)

Workflow

Elastic energy

$$\begin{split} E_{el}^{trig(I)} &= \frac{1}{2}c_{11}(\varepsilon_{xx}^2 + \varepsilon_{yy}^2) + c_{12}\varepsilon_{xx}\varepsilon_{yy} + c_{13}(\varepsilon_{xx} + \varepsilon_{yy})\varepsilon_{zz} + \frac{1}{2}c_{33}\varepsilon_{zz}^2 \\ &\quad + \frac{1}{2}c_{44}(\varepsilon_{xz}^2 + \varepsilon_{yz}^2) + \frac{1}{4}(c_{11} - c_{12})\varepsilon_{xy}^2 + c_{14}(\varepsilon_{xy}\varepsilon_{xz} + \varepsilon_{xx}\varepsilon_{yz} - \varepsilon_{yy}\varepsilon_{yz}) \end{split}$$



Magnetoelastic energy

$$\begin{split} E_{me}^{trig(I)} &= b_{11}(\varepsilon_{xx} + \varepsilon_{yy}) + b_{12}\varepsilon_{zz} + b_{21}\left(\alpha_z^2 - \frac{1}{3}\right)(\varepsilon_{xx} + \varepsilon_{yy}) + b_{22}\left(\alpha_z^2 - \frac{1}{3}\right)\varepsilon_{zz} \\ &+ \frac{1}{2}b_3[(\alpha_x^2 - \alpha_y^2)(\varepsilon_{xx} - \varepsilon_{yy}) + 2\alpha_x\alpha_y] + b_4(\alpha_x\alpha_z\varepsilon_{xz} + \alpha_y\alpha_z\varepsilon_{yz}) \\ &+ b_{14}\left[\frac{1}{2}(\alpha_x^2 - \alpha_y^2)\varepsilon_{yz} + \alpha_x\alpha_y\varepsilon_{xz}\right] + b_{34}\left[\frac{1}{2}\alpha_y\alpha_z(\varepsilon_{xx} - \varepsilon_{yy}) + \alpha_x\alpha_z\varepsilon_{xy}\right]. \end{split}$$

$$\begin{split} \frac{\Delta l}{l_0} \bigg|_{\boldsymbol{\beta}}^{\boldsymbol{\alpha}} &= \lambda^{\alpha 1,0} (\beta_x^2 + \beta_y^2) + \lambda^{\alpha 2,0} \beta_z^2 + \lambda^{\alpha 1,2} \left(\alpha_z^2 - \frac{1}{3} \right) (\beta_x^2 + \beta_y^2) \\ &+ \lambda^{\alpha 2,2} \left(\alpha_z^2 - \frac{1}{3} \right) \beta_z^2 + \lambda^{\gamma,1} \left[\frac{1}{2} (\alpha_x^2 - \alpha_y^2) (\beta_x^2 - \beta_y^2) + \alpha_x \alpha_y \beta_x \beta_y \right] \\ &+ \lambda^{\gamma,2} (\alpha_x \alpha_z \beta_x \beta_z + \alpha_y \alpha_z \beta_y \beta_z) + \lambda_{12} \left[\frac{1}{2} \alpha_y \alpha_z (\beta_x^2 - \beta_y^2) + \alpha_x \alpha_z \beta_x \beta_y \right] \\ &+ \lambda_{21} \left[\frac{1}{2} (\alpha_x^2 - \alpha_y^2) \beta_y \beta_z + \alpha_x \alpha_y \beta_x \beta_z \right], \end{split}$$

$$\lambda^{\alpha 1,0} = \frac{b_{11}c_{33} + b_{12}c_{13}}{c_{33}(c_{11} + c_{12}) - 2c_{13}^2},$$

$$\lambda^{\alpha 2,0} = \frac{2b_{11}c_{33} - b_{12}(c_{11} + c_{22})}{c_{33}(c_{11} + c_{12}) - 2c_{13}^2},$$

$$\lambda^{\alpha 1,2} = \frac{-b_{21}c_{33} + b_{22}c_{13}}{c_{33}(c_{11} + c_{12}) - 2c_{13}^2},$$

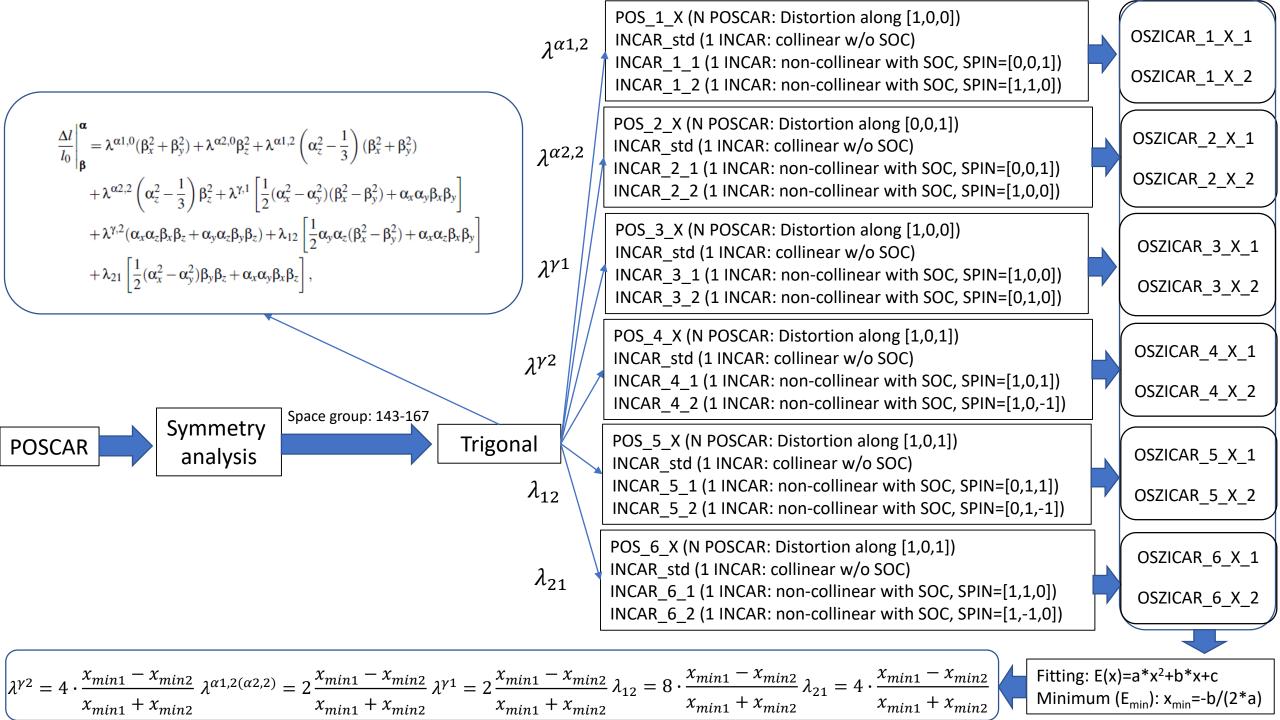
$$\lambda^{\alpha 2,2} = \frac{2b_{21}c_{33} - b_{22}(c_{11} + c_{22})}{c_{33}(c_{11} + c_{12}) - 2c_{13}^2},$$

$$\lambda^{\gamma,1} = \frac{c_{14}b_{14} - c_{44}b_{3}}{\frac{1}{2}c_{44}(c_{11} - c_{12}) - c_{14}^2},$$

$$\lambda^{\gamma,2} = \frac{\frac{1}{2}b_{4}(c_{11} - c_{12}) - b_{34}c_{14}}{\frac{1}{2}c_{44}(c_{11} - c_{12}) - c_{14}^2},$$

$$\lambda_{12} = \frac{c_{14}b_{4} - c_{44}b_{34}}{\frac{1}{2}c_{44}(c_{11} - c_{12}) - c_{14}^2},$$

$$\lambda_{21} = \frac{\frac{1}{2}b_{14}(c_{11} - c_{12}) - b_{3}c_{14}}{\frac{1}{2}c_{44}(c_{11} - c_{12}) - c_{14}^2}.$$



TETRAGONAL (I)

SG 89-142

Tetragonal (I)

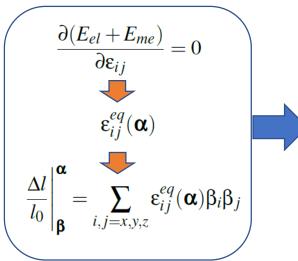
Elastic energy

$$E_{el}^{tet} = \frac{1}{2}c_{11}(\varepsilon_{xx}^2 + \varepsilon_{yy}^2) + c_{12}\varepsilon_{xx}\varepsilon_{yy} + c_{13}(\varepsilon_{xx} + \varepsilon_{yy})\varepsilon_{zz} + \frac{1}{2}c_{33}\varepsilon_{zz}^2 + \frac{1}{2}c_{44}(\varepsilon_{xz}^2 + \varepsilon_{yz}^2) + \frac{1}{2}c_{66}\varepsilon_{xy}^2.$$



Magnetoelastic energy

$$\begin{split} E^{tet(I)}_{me} &= b_{11}(\varepsilon_{xx} + \varepsilon_{yy}) + b_{12}\varepsilon_{zz} + b_{21}\left(\alpha_z^2 - \frac{1}{3}\right)(\varepsilon_{xx} + \varepsilon_{yy}) + b_{22}\left(\alpha_z^2 - \frac{1}{3}\right)\varepsilon_{zz} \\ &+ \frac{1}{2}b_3(\alpha_x^2 - \alpha_y^2)(\varepsilon_{xx} - \varepsilon_{yy}) + b_3'\alpha_x\alpha_y\varepsilon_{xy} + b_4(\alpha_x\alpha_z\varepsilon_{xz} + \alpha_y\alpha_z\varepsilon_{yz}). \end{split}$$



$$\begin{split} \frac{\Delta l}{l_0} \Bigg|_{\pmb{\beta}}^{\pmb{\alpha}} &= \lambda^{\alpha 1,0} (\beta_x^2 + \beta_y^2) + \lambda^{\alpha 2,0} \beta_z^2 + \lambda^{\alpha 1,2} \left(\alpha_z^2 - \frac{1}{3} \right) (\beta_x^2 + \beta_y^2) \\ &+ \lambda^{\alpha 2,2} \left(\alpha_z^2 - \frac{1}{3} \right) \beta_z^2 + \frac{1}{2} \lambda^{\gamma,2} (\alpha_x^2 - \alpha_y^2) (\beta_x^2 - \beta_y^2) + 2 \lambda^{\delta,2} \alpha_x \alpha_y \beta_x \beta_y \\ &+ 2 \lambda^{\varepsilon,2} (\alpha_x \alpha_z \beta_x \beta_z + \alpha_y \alpha_z \beta_y \beta_z), \end{split}$$

$$\lambda^{\alpha 1,0} = \frac{b_{11}c_{33} + b_{12}c_{13}}{c_{33}(c_{11} + c_{12}) - 2c_{13}^2},$$

$$\lambda^{\alpha 2,0} = \frac{2b_{11}c_{33} - b_{12}(c_{11} + c_{22})}{c_{33}(c_{11} + c_{12}) - 2c_{13}^2},$$

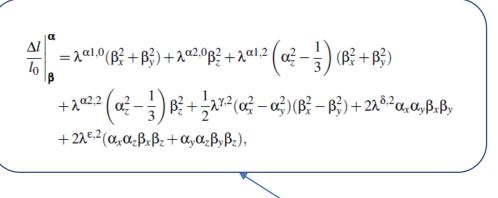
$$\lambda^{\alpha 1,2} = \frac{-b_{21}c_{33} + b_{22}c_{13}}{c_{33}(c_{11} + c_{12}) - 2c_{13}^2},$$

$$\lambda^{\alpha 2,2} = \frac{2b_{21}c_{33} - b_{22}(c_{11} + c_{22})}{c_{33}(c_{11} + c_{12}) - 2c_{13}^2},$$

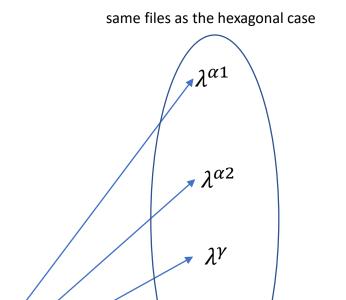
$$\lambda^{\gamma,2} = \frac{-b_3}{c_{11} - c_{12}},$$

$$\lambda^{\delta,2} = \frac{-b_3'}{2c_{66}},$$

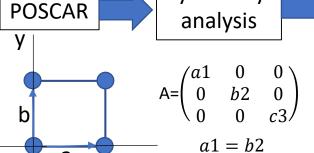
$$\lambda^{\varepsilon,2} = \frac{-b_4'}{2c_{44}}.$$



Space group: 75-142







Symmetry

 $\lambda^{\delta,2}$

Tetragonal

POS_5_X (N POSCAR: Distortion along [1,1,0])

 $\lambda^{\delta,2} = 2\frac{x_{min1} - x_{min2}}{x_{min1} + x_{min2}}$

INCAR_std (1 INCAR: collinear w/o SOC)

INCAR_5_1 (1 INCAR: non-collinear with SOC, SPIN=[1,1,0]) INCAR 5 2 (1 INCAR: non-collinear with SOC, SPIN=[-1,1,0]) OSZICAR_5_X_2

OSZICAR_5_X_1

x=cell length along deformation



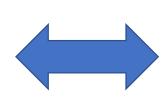
Fitting: $E(x)=a^*x^2+b^*x+c$ Minimum (E_{min}): $x_{min} = -b/(2*a)$

Relation between different notations

Tetragonal (I)

Cullen

$$\begin{split} \frac{\Delta l}{l_0} \Bigg|_{\boldsymbol{\beta}}^{\boldsymbol{\alpha}} &= \lambda^{\alpha 1,0} (\beta_x^2 + \beta_y^2) + \lambda^{\alpha 2,0} \beta_z^2 + \lambda^{\alpha 1,2} \left(\alpha_z^2 - \frac{1}{3} \right) (\beta_x^2 + \beta_y^2) \\ &+ \lambda^{\alpha 2,2} \left(\alpha_z^2 - \frac{1}{3} \right) \beta_z^2 + \frac{1}{2} \lambda^{\gamma,2} (\alpha_x^2 - \alpha_y^2) (\beta_x^2 - \beta_y^2) + 2 \lambda^{\delta,2} \alpha_x \alpha_y \beta_x \beta_y \\ &+ 2 \lambda^{\epsilon,2} (\alpha_x \alpha_z \beta_x \beta_z + \alpha_y \alpha_z \beta_y \beta_z), \end{split}$$



$$\begin{split} \frac{\Delta l}{l_0} \Bigg|_{\pmb{\beta}}^{\pmb{\alpha}} &= \lambda_{Mason}^{\alpha 1,0} (\beta_x^2 + \beta_y^2) + \lambda_{Mason}^{\alpha 2,0} \beta_z^2 + \frac{1}{2} \lambda_1 [(\alpha_x \beta_x - \alpha_y \beta_y)^2 - (\alpha_x \beta_y + \alpha_y \beta_x)^2 \\ &+ (1 - \beta_z^2) (1 - \alpha_z^2) - 2 \alpha_z \beta_z (\alpha_x \beta_x + \alpha_y \beta_y)] + 4 \lambda_2 \alpha_z \beta_z (\alpha_x \beta_x + \alpha_y \beta_y) \\ &+ 4 \lambda_3 \alpha_x \alpha_y \beta_x \beta_y + \lambda_4 [\beta_z^2 (1 - \alpha_z^2) - \alpha_z \beta_z (\alpha_x \beta_x + \alpha_y \beta_y)] \\ &+ \frac{1}{2} \lambda_5 [(\alpha_x \beta_y - \alpha_y \beta_x)^2 - (\alpha_x \beta_x + \alpha_y \beta_y)^2 + (1 - \beta_z^2) (1 - \alpha_z^2)]. \end{split}$$

$$\begin{split} \lambda_{Mason}^{\alpha 1,0} &= \lambda^{\alpha 1,0} + \frac{2}{3} \lambda^{\alpha 1,2} \\ \lambda_{Mason}^{\alpha 2,0} &= \lambda^{\alpha 2,0} + \frac{2}{3} \lambda^{\alpha 2,2} \\ \lambda_{1} &= -\lambda^{\alpha 1,2} + \frac{1}{2} \lambda^{\gamma,2} \\ \lambda_{2} &= \frac{1}{2} \lambda^{\epsilon,2} - \frac{1}{4} \lambda^{\alpha 2,2} - \frac{1}{4} \lambda^{\alpha 1,2} + \frac{1}{8} \lambda^{\gamma,2} \\ \lambda_{3} &= \frac{1}{2} \lambda^{\delta,2} - \lambda^{\alpha 1,2} \\ \lambda_{4} &= -\lambda^{\alpha 2,2} \\ \lambda_{5} &= -\lambda^{\alpha 1,2} - \frac{1}{2} \lambda^{\gamma,2}. \end{split}$$

ORTHORHOMBIC

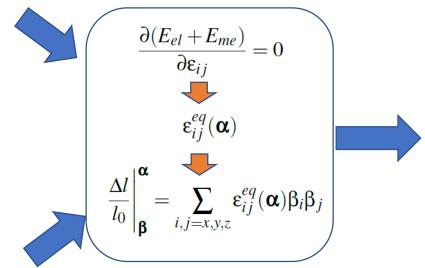
SG 16-74

Orthorhombic

Workflow

Elastic energy

$$\begin{split} E_{el}^{ortho} &= \frac{1}{2} c_{11} \varepsilon_{xx}^2 + \frac{1}{2} c_{22} \varepsilon_{yy}^2 + c_{12} \varepsilon_{xx} \varepsilon_{yy} + c_{13} \varepsilon_{xx} \varepsilon_{zz} + c_{23} \varepsilon_{yy} \varepsilon_{zz} + \frac{1}{2} c_{33} \varepsilon_{zz}^2 \\ &\quad + \frac{1}{2} c_{44} \varepsilon_{yz}^2 + \frac{1}{2} c_{55} \varepsilon_{xz}^2 + \frac{1}{2} c_{66} \varepsilon_{xy}^2. \end{split}$$



Magnetoelastic energy

$$\begin{split} E_{me}^{ortho} &= b_{01}\varepsilon_{xx} + b_{02}\varepsilon_{yy} + b_{03}\varepsilon_{zz} + b_{1}\alpha_{x}^{2}\varepsilon_{xx} + b_{2}\alpha_{y}^{2}\varepsilon_{xx} + b_{3}\alpha_{x}^{2}\varepsilon_{yy} + b_{4}\alpha_{y}^{2}\varepsilon_{yy} \\ &+ b_{5}\alpha_{x}^{2}\varepsilon_{zz} + b_{6}\alpha_{y}^{2}\varepsilon_{zz} + b_{7}\alpha_{x}\alpha_{y}\varepsilon_{xy} + b_{8}\alpha_{x}\alpha_{z}\varepsilon_{xz} + b_{9}\alpha_{y}\alpha_{z}\varepsilon_{yz}, \end{split}$$

$$\begin{split} \frac{\Delta l}{l_0} \bigg|_{\boldsymbol{\beta}}^{\boldsymbol{\alpha}} &= \lambda^{\alpha 1,0} \beta_x^2 + \lambda^{\alpha 2,0} \beta_y^2 + \lambda^{\alpha 3,0} \beta_z^2 + \lambda_1 (\alpha_x^2 \beta_x^2 - \alpha_x \alpha_y \beta_x \beta_y - \alpha_x \alpha_z \beta_x \beta_z) \\ &+ \lambda_2 (\alpha_y^2 \beta_x^2 - \alpha_x \alpha_y \beta_x \beta_y) + \lambda_3 (\alpha_x^2 \beta_y^2 - \alpha_x \alpha_y \beta_x \beta_y) \\ &+ \lambda_4 (\alpha_y^2 \beta_y^2 - \alpha_x \alpha_y \beta_x \beta_y - \alpha_y \alpha_z \beta_y \beta_z) + \lambda_5 (\alpha_x^2 \beta_z^2 - \alpha_x \alpha_z \beta_x \beta_z) \\ &+ \lambda_6 (\alpha_y^2 \beta_z^2 - \alpha_y \alpha_z \beta_y \beta_z) + 4\lambda_7 \alpha_x \alpha_y \beta_x \beta_y + 4\lambda_8 \alpha_x \alpha_z \beta_x \beta_z + 4\lambda_9 \alpha_y \alpha_z \beta_y \beta_z. \end{split}$$

$$b_{01} = -c_{11}\lambda^{\alpha 1,0} - c_{12}\lambda^{\alpha 2,0} - c_{13}\lambda^{\alpha 3,0}$$

$$b_{02} = -c_{12}\lambda^{\alpha 1,0} - c_{22}\lambda^{\alpha 2,0} - c_{23}\lambda^{\alpha 3,0}$$

$$b_{03} = -c_{13}\lambda^{\alpha 1,0} - c_{23}\lambda^{\alpha 2,0} - c_{33}\lambda^{\alpha 3,0}$$

$$b_{1} = -c_{11}\lambda_{1} - c_{12}\lambda_{3} - c_{13}\lambda_{5}$$

$$b_{2} = -c_{11}\lambda_{2} - c_{12}\lambda_{4} - c_{13}\lambda_{6}$$

$$b_{3} = -c_{12}\lambda_{1} - c_{22}\lambda_{3} - c_{23}\lambda_{5}$$

$$b_{4} = -c_{12}\lambda_{2} - c_{22}\lambda_{4} - c_{23}\lambda_{6}$$

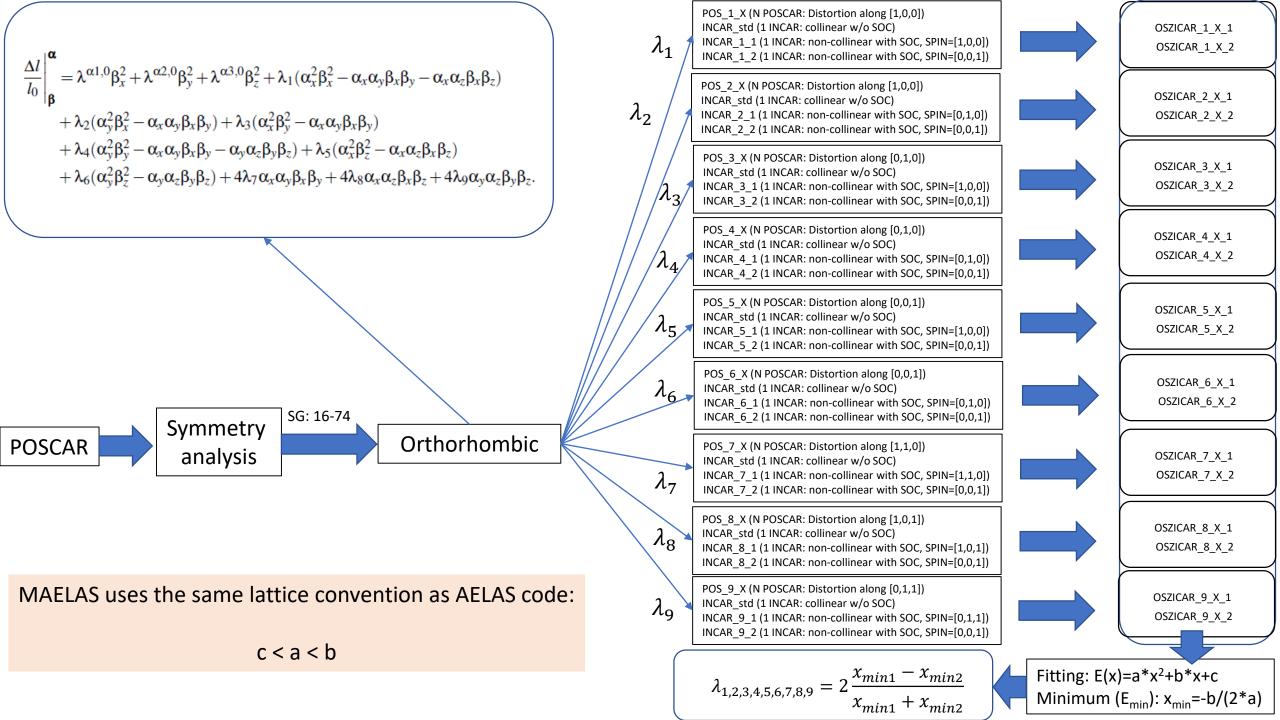
$$b_{5} = -c_{13}\lambda_{1} - c_{23}\lambda_{3} - c_{33}\lambda_{5}$$

$$b_{6} = -c_{13}\lambda_{2} - c_{23}\lambda_{4} - c_{33}\lambda_{6}$$

$$b_{7} = c_{66}(\lambda_{1} + \lambda_{2} + \lambda_{3} + \lambda_{4} - 4\lambda_{7})$$

$$b_{8} = c_{55}(\lambda_{1} + \lambda_{5} - 4\lambda_{8})$$

$$b_{9} = c_{44}(\lambda_{4} + \lambda_{6} - 4\lambda_{9}).$$

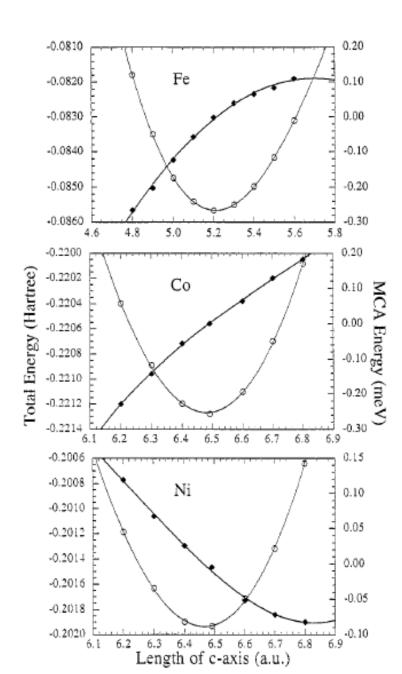


Cubic systems: Itenerant magnets

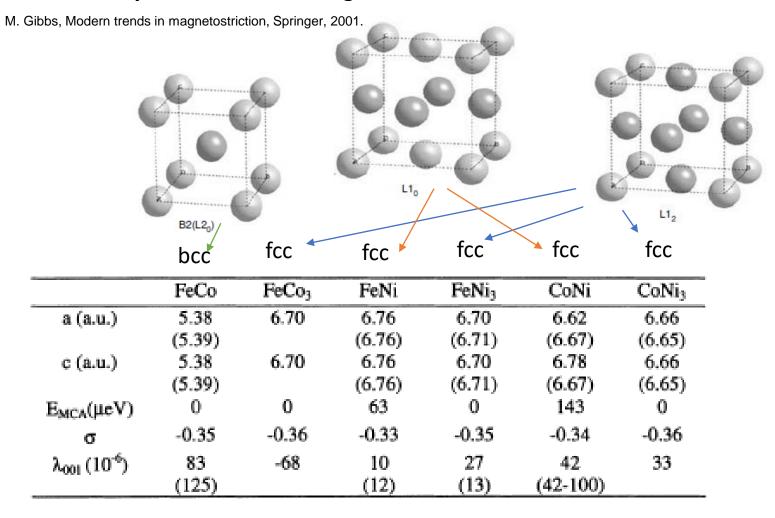
R.Q. Wu et al. | Journal of Magnetism and Magnetic Materials 177-181 (1998) 1216-1219

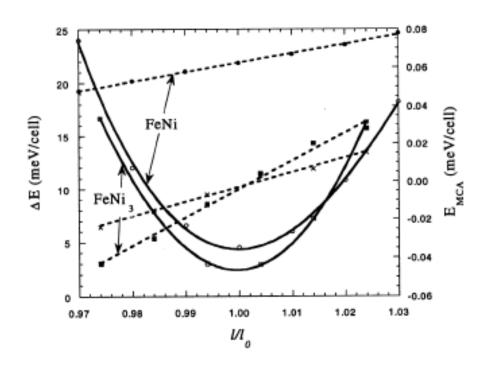
	a (a.u.)	σ	$M_s(\mu_B)$	$M_L(\mu_B)$	λ ₀₀₁ (10 ⁻⁶)
bcc Fe					
LDA	5.20	-0.409	2.05	0.048	52
GGA	5.37	-0.486	2.17	0.045	29
EXP	5.41	-0.368	2.22	0.08	21
fcc Co					
LDA	6.48	-0.374	1.59	0.076	92
GGA	6.67	-0.396	1.66	0.073	56
EXP	6.70		1.72	0.12	79
fee Ni					
LDA	6.46	-0.332	0.62	0.049	-63
GGA	6.64	-0.3376	0.66	0.050	-56
EXP	6.66	-0.376	0.57	0.05	-49

R. C. O'Handley, Modern magnetic materials, Wiley, 2000.		T = 4.	2 K	Room Temperature		
		$\lambda_{100}(\lambda^{\gamma,2})$	$\lambda_{111}(\lambda^{\epsilon,2})$	$\lambda_{100}(\lambda^{\gamma,2})$	$\lambda_{111}(\lambda^{\varepsilon,2})$	Polycrystal λ_s
			3d Metal	s		
1	BCC-Fe	26	-30	21	-21	-7
]	HCP-Co"	(-150)	(45)	(-140)	(50)	(-62)
]	FCC-Ni	-60	-35	-46	-24	-34
]	BCC-FeCo	_	_	140	30	
£	$-Fe_{80}B_{20}$	48 (isotropic)	_	_	_	+32
	a-Fe ₄₀ Ni ₄₀ B ₂₀	+20	_		_	+14
	$a-Cos_{80}B_{20}$	-4	_			-4



Cubic systems: Itenerant magnets





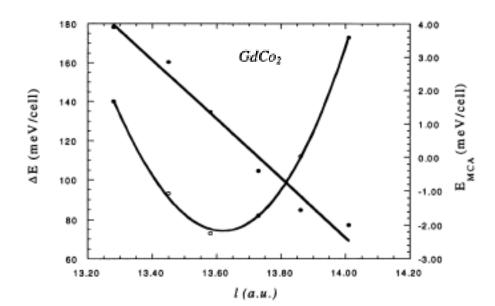
G. Engdahl, Handbook of giant magnetostrictive materials, Academic Press, 1999

Cubic systems: Rare-Earth magnets

M. Gibbs, Modern trends in magnetostriction, Springer, 2001.

C15 cubic Laves phase

	Theory	Experiment
λ ₀₀₁ (GdCo ₂)	-407	-1200
λ_{111} (GdCo ₂)	19	< 10
λ_{001} (SmCo ₂)	-290	
λ ₀₀₁ (ErCo ₂)	-516	-1000
λ_{001} (GdFe ₂)	44	39



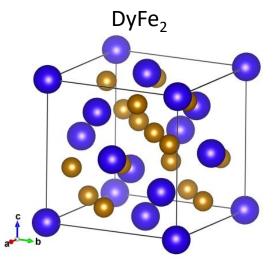


Table 16-10. Low-temperature magnetostriction constants for RCo₂ crystals [from Levitin and Markosyan (1990) unless otherwise noted].

C _c (K) ^a
409
256
159
85
36
18

Table 1.6. Magnetostriction of Cubic Laves Phase Compounds with Rare Earths at T=0

Compound	$\lambda_{111}(10^{-6})$	$\lambda_{100}(10^{-6})$	T_c (K
NdAl ₂	_	- 700	61
TbAl ₂	- 3000	_	114
DyAl ₂	-	- 1700	68
TbMn ₂	- 3000		40
TbFe ₂	4000, 4500	_	711
DyFe ₂	_	- 70	635
HoFe ₂		- 750	612
TmFe ₂	-3500, -2600	_	610
TbCo ₂	4400	_	256
DyCo ₂	_	- 2000	159
HoCo ₂	_	- 2200	85
ErCo ₂	- 2500	_	36
TbNi ₂	1500	_	45
DyNi ₂	_	- 1300	30
HoNi ₂	_	- 1000	22

Table 1.7. Magnitudes of Single-Crystal Magnetostriction in Rare Earth-Fe₂ Compounds

Compound	$\frac{3}{2}\lambda_{111} \ (10^{-6})$ (calculated at 0 K)	$\frac{3}{2}\lambda_{111}~(10^{-6})$ (measured at room temperature)	$T_{\rm e}$
SmFe ₂	- 4800	- 3150	676
TbFe ₂	6600	3690	697, 711
DyFe ₂	6300	1890	635
HoFe ₂	2400	288	606
ErFe ₂	- 2250	- 450	590, 597
TmFe ₂	- 5550	-315	560

TABLE 6
Magnetostriction coefficients at zero Kelvin in units of 10⁻³

Hexagonal Rare-Earth

Element	λ ^{α,2}	λ 2,2	λ γ,2	λ ^{ε,2}	$\lambda_{i}^{\alpha,0} - \frac{1}{3}\lambda_{i}^{\alpha,2}$	$\lambda_2^{\alpha,0} - \frac{1}{3}\lambda_2^{\alpha,2}$	λ γ.4
Gadolinium ^{a)}	0.14	-0.13	0.11	0.02	-	-	_
Terbium ^{b)}	-2.6°)	9.0°)	8.7	15.0°)	- 0.8	4.3	-2.1
Dysprosium ^{b)}	_	_	9.4	5.5	- 2.0	7.3	1.5
Holmium ^{b)}	_		2.5°)	***	-3.9	7.1	_
Erbium ^{b)}	-	-	-5.1°	-	+0.3	6.2	-

a) After Mishima et al. (1976).

R. C. O'Handley, Modern magnetic materials, Wiley, 2000.

Oxide magnets

	$T = 4.2 \mathrm{K}$		Room Temperature		
	$\lambda_{100}(\lambda^{\gamma,2})$	$\lambda_{111}(\lambda^{\varepsilon,2})$	$\lambda_{100}(\lambda^{\gamma,2})$	$\lambda_{111}(\lambda^{\varepsilon,2})$	Polycrystal λ_s
		Spinel Ferrit	es		-
Fe_3O_4	0	50	-15	56	+40
$MnFe_2O_4^u$	*****	_	(-54)	(10)	_
CoFe ₂ O ₄	_	_	-670	120	-110
		Garnets			
YIG	-0.6	-2.5	-1.4	-1.6	-2

b) After Rhyne (1972).

c) Extrapolated from paramagnetic range using single-ion theory.

MAELAS TESTS

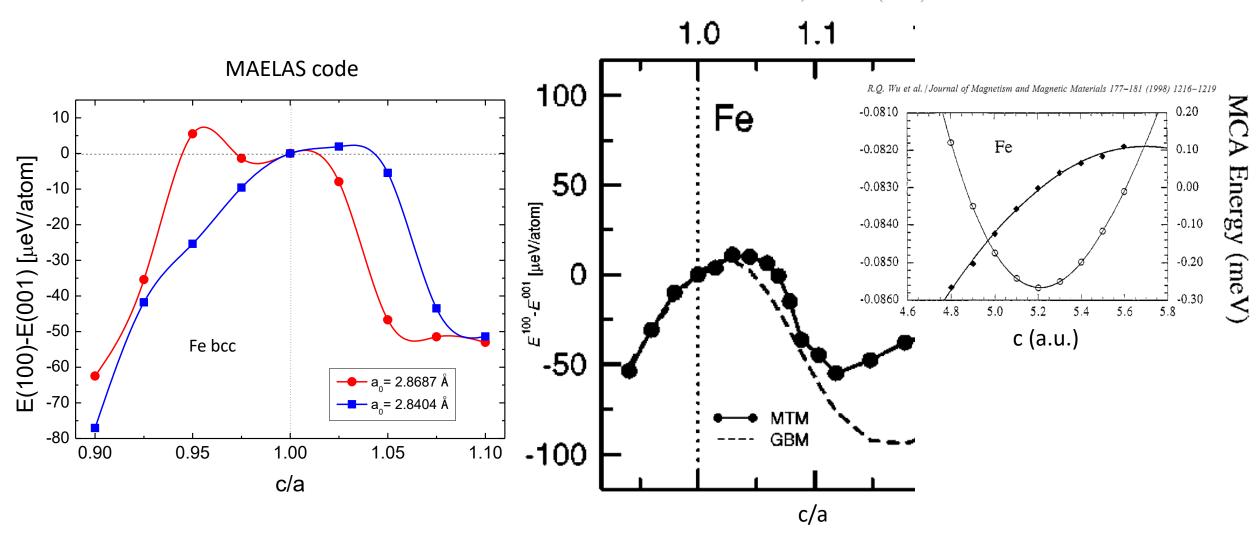
Table 3: Anisotropic magnetostriction coefficients and MAE calculated by the program MAELAS and measured in experiment ($T \approx 0 \text{ K}$) for a set of magnetic materials. In parenthesis we show the magnetostriction coefficients with Mason's definitions obtained using the relations given by Eq. A.2.

Material	Crystal system	Space group	Magnetostriction coefficient	MAELAS (×10 ⁻⁶)	Expt. (×10 ⁻⁶)	MAE	MAELAS (μeV/atom)	Expt. (µeV/atom)
Fe bcc	Cubic (I)	229	$\lambda_{001} \ \lambda_{111}$	23 19	26 [44] -30 [44]	E(110) - E(001) E(111) - E(001)	0.1 0.21	1.0 [43] 1.3 [43]
Co hcp	Hexagonal (I)	194	$\begin{array}{c c} \lambda^{\alpha 1,2} \left(\lambda_A \right) \\ \lambda^{\alpha 2,2} \left(\lambda_B \right) \\ \lambda^{\gamma,2} \left(\lambda_C \right) \\ \lambda^{\varepsilon,2} \left(\lambda_D \right) \end{array}$	85 (-78) -115 (-92) 15 (115) -17 (1)	95 (-66) [47] -126 (-123) [47] 57 (126) [47] -286 (-128) [47]	E(100) - E(001)	53	61 [43]
Gd hcp	Hexagonal (I)	194	$\lambda^{\alpha 1,2} \ \lambda^{\alpha 2,2} \ \lambda^{\gamma,2} \ \lambda^{\epsilon,2}$		163 [52] -85 [52] 105 [52] 57 [52]	E(100) - E(001)	32	35 [48]
YCo ₅	Hexagonal (I)	191	$\lambda^{\alpha 1,2} \ \lambda^{\alpha 2,2} \ \lambda^{\gamma,2} \ \lambda^{\epsilon,2}$	-90 115 73 147	$\frac{ \lambda^{\alpha 1,2} < 100 [4]}{ \lambda^{\alpha 2,2} < 100 [4]}$	E(100) - E(001)	365	567 [49]
FePd L1 ₀	Tetragonal (I)	123	$\begin{array}{c} -\frac{\lambda^{\alpha 1,2}}{3} + \frac{\lambda^{\gamma,2}}{2} \\ \lambda^{\alpha 1,2} \\ \lambda^{\alpha 2,2} \\ \lambda^{\gamma,2} \\ \lambda^{\epsilon,2} \\ \lambda^{\delta,2} \end{array}$	12 -23 83 8 25 61	100* [51]	E(100) - E(001)	109	181 [50]

^{*}The experimental measurement corresponds to $\lambda^{\alpha 1,0} + \frac{-\lambda^{\alpha 1,2}}{3} + \frac{\lambda^{\gamma,2}}{2}$ with composition Fe₄₈Pd₅₂.

MAELAS TESTS: Fe bcc

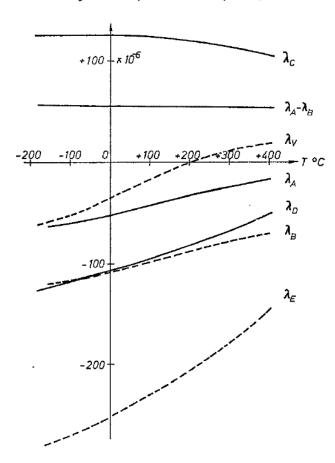
PHYSICAL REVIEW B 69, 104426 (2004)

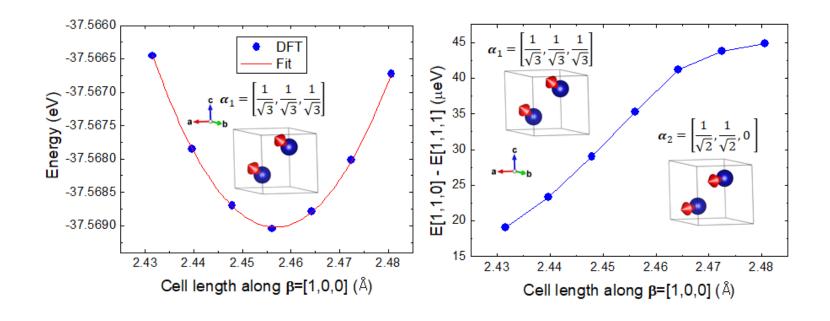


MAELAS (kp=185)
$$\lambda_{001} = 13 \cdot 10^{-6}$$
 Exp. $\lambda_{001} = 26 \cdot 10^{-6}$

MAELAS TESTS: Co hcp

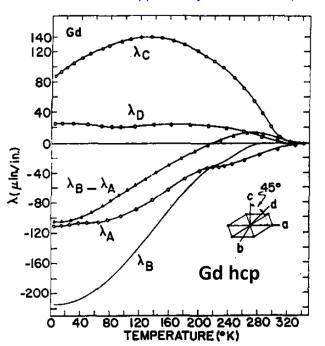
Z. Physik 224, 148-155 (1969)

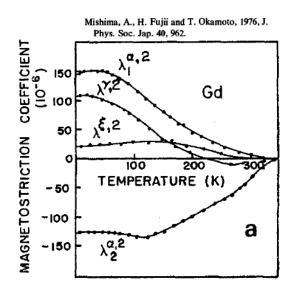




Experiments

Journal of Applied Physics 35, 1752 (1964)

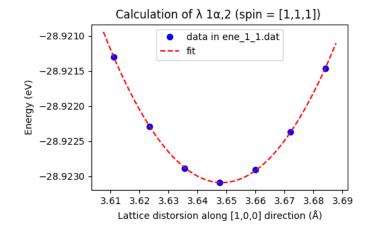


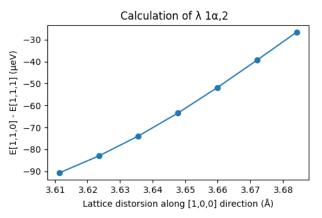


MAELAS TESTS: Gd hcp

	Experiment (x 10 ⁻⁶)	MAELAS (x 10 ⁻⁶)
λΑ	-110	-262
λВ	-215	-305
λC	85	521
λD	22	22

LDAUU=6.7 LDAUJ=0.7 kpoints 22x22x11

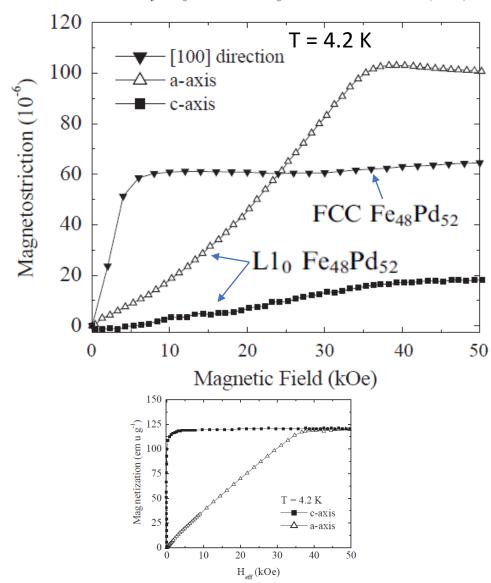


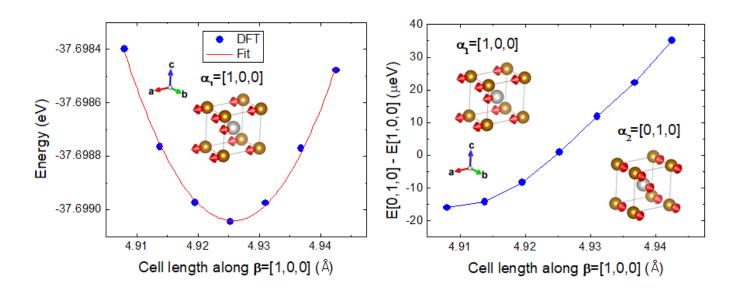


MAELAS TESTS: L1₀ FePd

Experiment

H. Shima et al. | Journal of Magnetism and Magnetic Materials 272-276 (2004) 2173-2174





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