Math 301 **Homework 10** **Fall 2014**

Exercises: Chapter 10: 1abe, 5, 10, 11, 13acd
Due date: Wednesday, 11/05

1. For each of the following groups G, determine whether H is a normal subgroup of G. If H is a normal subgroup, write out a Cayley table for the factor group G/H.

(a) $G = S_4$ and $H = A_4$
(b) $G = A_5$ and $H = \{(1), (123), (132)\}$
(e) $G = \mathbb{Z}$ and $H = 5\mathbb{Z}$

5. Show that the intersection of two normal subgroups is a normal subgroup.

10. Let H be a subgroup of index 2 of a group G. Prove that H must be a normal subgroup of G. Conclude that S_n is not simple for $n \geq 3$.

11. If a group G has exactly one subgroup H of order k, prove that H is normal in G.

13. Recall that the **center** of a group G is the set

$$Z(G) = \{x \in G : xg = gx \text{ for all } g \in G\}.$$

(a) Calculate the center of S_3.
(c) Show that the center of any group G is a normal subgroup of G.
(d) If $G/Z(G)$ is cyclic, show that G is abelian.