Chapter 5: 1bd, 3bd, 4, 6, 17, 18, 27.

Additional suggested exercises: 29, 31, 32, 33.

Due date: Friday, 10/10

(Exercise numbers correspond to the printed textbook, generated from 2013/08/16 source files.)

1. Write the following permutations in cycle notation.

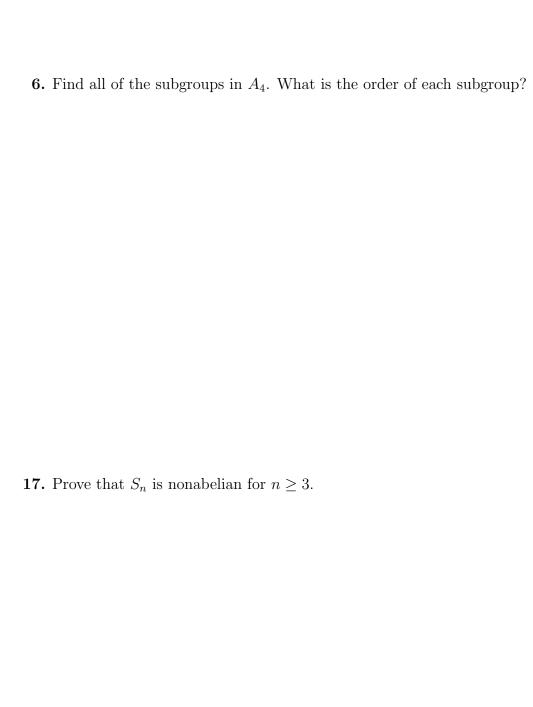
(b)

$$\begin{pmatrix}
1 & 2 & 3 & 4 & 5 \\
4 & 2 & 5 & 1 & 3
\end{pmatrix}$$

(d)

$$\begin{pmatrix}
1 & 2 & 3 & 4 & 5 \\
1 & 4 & 3 & 2 & 5
\end{pmatrix}$$

- **3.** Express the following permutations as products of transpositions and identify them as even or odd.
 - (b) (156)(234)
 - $(d) \ (17254)(1423)(154632)$
- **4.** Find $(a_1, a_2, \ldots, a_n)^{-1}$.



18. Prove that A_n is nonabelian for $n \geq 4$.

27. Let G be a group and define a map $\lambda_g: G \to G$ by $\lambda_g(a) = ga$. Prove that λ_g is a permutation of G.

Additional suggested exercises: 29, 31, 32, 33.

29. Recall that the *center* of a group G is

$$Z(G)=\{g\in G: gx=xg \text{ for all } x\in G\}.$$

Find the center of D_8 . What about the center of D_{10} ? What is the center of D_n ?

31. For α and β in S_n , define $\alpha \sim \beta$ if there exists an $\sigma \in S_n$ such that $\sigma \alpha \sigma^{-1} = \beta$. Show that \sim is an equivalence relation on S_n .

- **32.** Let $\sigma \in S_X$. If $\sigma^n(x) = y$, we will say that $x \sim y$.
 - (a) Show that \sim is an equivalence relation on X.
 - (b) If $\sigma \in A_n$ and $\tau \in S_n$, show that $\tau^{-1}\sigma\tau \in A_n$.
 - (c) Define the *orbit* of $x \in X$ under $\sigma \in S_X$ to be the set

$$\mathcal{O}_{x,\sigma} = \{y : x \sim y\}.$$

Compute the orbits of α, β, γ where

$$\alpha = (1254)$$
 $\beta = (123)(45)$
 $\gamma = (13)(25)$.

- (d) If $\mathcal{O}_{x,\sigma} \cap \mathcal{O}_{y,\sigma} \neq \emptyset$, prove that $\mathcal{O}_{x,\sigma} = \mathcal{O}_{y,\sigma}$. The orbits under a permutation σ are the equivalence classes corresponding to the equivalence relation \sim .
- (e) A subgroup H of S_X is transitive if for every $x, y \in X$, there exists a $\sigma \in H$ such that $\sigma(x) = y$. Prove that $\langle \sigma \rangle$ is transitive if and only if $\mathcal{O}_{x,\sigma} = X$ for some $x \in X$.

