

**Chapter 5:** 1bd, 3bd, 4, 6, 17, 18, 27.

Additional suggested exercises: 29, 31, 32, 33.

**Due date:** Friday, 10/10

(Exercise numbers correspond to the printed textbook, generated from 2013/08/16 source files.)

1. Write the following permutations in cycle notation.

(b)

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 1 & 3 \end{pmatrix}$$

(d)

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 3 & 2 & 5 \end{pmatrix}$$

3. Express the following permutations as products of transpositions and identify them as even or odd.

(b)  $(156)(234)$

(d)  $(17254)(1423)(154632)$

4. Find  $(a_1, a_2, \dots, a_n)^{-1}$ .

**6.** Find all of the subgroups in  $A_4$ . What is the order of each subgroup?

**17.** Prove that  $S_n$  is nonabelian for  $n \geq 3$ .

**18.** Prove that  $A_n$  is nonabelian for  $n \geq 4$ .

**27.** Let  $G$  be a group and define a map  $\lambda_g : G \rightarrow G$  by  $\lambda_g(a) = ga$ . Prove that  $\lambda_g$  is a permutation of  $G$ .

**Additional suggested exercises:** 29, 31, 32, 33.

**29.** Recall that the *center* of a group  $G$  is

$$Z(G) = \{g \in G : gx = xg \text{ for all } x \in G\}.$$

Find the center of  $D_8$ . What about the center of  $D_{10}$ ? What is the center of  $D_n$ ?

**31.** For  $\alpha$  and  $\beta$  in  $S_n$ , define  $\alpha \sim \beta$  if there exists an  $\sigma \in S_n$  such that  $\sigma\alpha\sigma^{-1} = \beta$ . Show that  $\sim$  is an equivalence relation on  $S_n$ .

**32.** Let  $\sigma \in S_X$ . If  $\sigma^n(x) = y$ , we will say that  $x \sim y$ .

(a) Show that  $\sim$  is an equivalence relation on  $X$ .

(b) If  $\sigma \in A_n$  and  $\tau \in S_n$ , show that  $\tau^{-1}\sigma\tau \in A_n$ .

(c) Define the *orbit* of  $x \in X$  under  $\sigma \in S_X$  to be the set

$$\mathcal{O}_{x,\sigma} = \{y : x \sim y\}.$$

Compute the orbits of  $\alpha, \beta, \gamma$  where

$$\alpha = (1254)$$

$$\beta = (123)(45)$$

$$\gamma = (13)(25).$$

(d) If  $\mathcal{O}_{x,\sigma} \cap \mathcal{O}_{y,\sigma} \neq \emptyset$ , prove that  $\mathcal{O}_{x,\sigma} = \mathcal{O}_{y,\sigma}$ . The orbits under a permutation  $\sigma$  are the equivalence classes corresponding to the equivalence relation  $\sim$ .

(e) A subgroup  $H$  of  $S_X$  is *transitive* if for every  $x, y \in X$ , there exists a  $\sigma \in H$  such that  $\sigma(x) = y$ . Prove that  $\langle \sigma \rangle$  is transitive if and only if  $\mathcal{O}_{x,\sigma} = X$  for some  $x \in X$ .

**33.** Let  $\alpha \in S_n$  for  $n \geq 3$ . If  $\alpha\beta = \beta\alpha$  for all  $\beta \in S_n$ , prove that  $\alpha$  must be the identity permutation; hence, the center of  $S_n$  is the trivial subgroup.