Math 301 Definitions

- Cartesian product
- relation
- function
- operation and finitary operation
- universe or domain
- arity of relation, function, or operation (e.g., nullary, unary, binary, ternary, n-ary)
- n-ary relation on a set X (notation:  $\rho \subseteq X^n$ )
- n-ary function from set X to set Y (notation:  $f: X^n \to Y$ )
- *n*-ary operation on a set X (notation:  $f: X^n \to X$ )
- properties binary relations might satisfy: reflexive, (anti)symmetric, transitive
- properties of functions (e.g., onto, one-to-one, bijective)
- properties of binary operations (e.g., commutative, associative, idempotent)
- equivalence relation, equivalence class
- partition
- ullet congruence modulo n
- partial order, total order, well-order
- greatest common divisor, least common multiple
- relatively prime,
- prime number,
- prime factorization
- power set
- algebraic structure,  $\langle A, \mathcal{F} \rangle$ , with universe A and operations  $\mathcal{F}$
- algebraic structure types and examples:
  - magma,
  - semigroup,
  - monoid,
  - group
- relational structure,  $\langle A, \mathcal{R} \rangle$ , with universe A and relations  $\mathcal{R}$

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- relational structure examples:
  - partially ordered set (poset),
  - graph

(Many more examples at www.math.chapman.edu/jipsen/structures/doku.php/index.html)

- subuniverse generated by a set S, denoted by  $\langle S \rangle$ .
- subalgebra
- identity element
- inverse element and inverse operation
- abelian group
- Cayley table
- finite group
- subgroup, proper subgroup, trivial subgroup
- order (of a group or subgroup)
- order (of a group element)
- $\bullet$   $g^n$  and  $g^{-n}$  (for g an element of a multiplicative group)
- ng and -ng (for g an element of an additive group)
- cyclic group
- generators (of a group), generator (of a cyclic group)
- symmetry, rigid motion
- permutation (and two ways to write them)
- cycle, length of a cycle
- transposition
- parity of a permutation (even/odd)
- examples of groups:  $\mathbb{Z}_n$ , U(n),  $S_n$ ,  $A_n$ ,  $D_4$
- distinguished elements of partial orders:
  - upper bound (of a subset of a poset, lattice, or join semilattice)
  - least upper bound or supremum or join
  - lower bound

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- greatest lower bound or infimum or meet
- lattice,  $\langle L, \wedge, \vee \rangle$
- semilattice,  $\langle S, \cdot \rangle$
- joins and meets:
  - join (of elements),  $a \vee b$
  - meet (of elements),  $a \wedge b$
  - join (of a subset),  $\bigvee T$
  - meet (of a subset),  $\bigwedge T$
  - largest element (of a poset; need not exist)
  - smallest element (of a poset; need not exist)
- order-preserving function
- lattice homomorphism
- coset, coset representative
- index of a subgroup
- conjugate elements of a group
- Hasse diagram
- types of homomorphisms:
  - homomorphism
  - monomorphism
  - epimorphism
  - isomorphism
  - endomorphism
  - automorphism
- kernel of a function (an equivalence relation)
- kernel of a group homomorphism (a normal subgroup)
- kernel of a homomorphism (a congruence relation)
- quotient group
- quotient algebra
- First Isomorphism theorem for groups
- First Isomorphism theorem (general)
- direct product and direct power