Let G be a group and suppose X is a nonempty set of elements of G. The **subgroup generated by** X is the smallest subgroup of G that contains X. For example, the subgroup of $\mathbb{Z}_{12}$ generated by the set $\{4, 6\}$ is $\{0, 2, 4, 6, 8, 10\}$ (explained below). For a single element $g \in G$, we often denote the subgroup generate by the set $\{g\}$ by $\langle g \rangle$ instead of $\langle \{g\} \rangle$. For small finite sets, like $\{x, y\}$, we often write, $\langle x, y \rangle$ instead of $\langle \{x, y\} \rangle$.

A **one-generated subgroup** is a subgroup generated by one element, such as $\langle g \rangle$. A one-generate subgroup is also called a **cyclic subgroup**. A **two-generate subgroup** is a subgroup $\langle x, y \rangle$ that is generated by two elements, x and y. An n**-generated subgroup** is a subgroup of the form $\langle x_1, \dots, x_n \rangle$, generated by the n elements, $x_1, \dots, x_n$.

Let G be a group and let H be a subgroup of G. It is important to note the distinction between the following two statements:

1. "The cyclic subgroup H has a two generators x and y."

2. "The subgroup H is generated by two elements x and y."

The first sentence means $H = \langle x \rangle = \langle y \rangle$. That is, you can take either x or y as the generator of H.

The second sentence above means something entirely different, namely, $H = \langle x, y \rangle$. This says that the smallest subgroup of G that contains both x and y is H. It may or may not be the case that H is cyclic in this case. The notation $H = \langle x, y \rangle$ simply means that H can be generated by two elements. It's possible that we could find an element that generates H all by itself. That is, we may have $H = \langle g \rangle = \langle x, y \rangle$.

Examples

1. Consider the subgroup $H = \{e, (1, 2, 3), (1, 3, 2)\}$ of A_4, which can be generated by either one (or both) of its non-identity elements: $H = \langle (1, 2, 3) \rangle = \langle (1, 3, 2) \rangle$.

2. Continuing with the last example, we could write $H = \langle (1, 2, 3), (1, 3, 2) \rangle$. Here we have thrown in a redundant generator, which is harmless, but not helpful because it doesn't call attention to an important feature of$H$—namely, that it is one-generated, i.e., cyclic.

3. As mentioned above, the subgroup of $\mathbb{Z}_{12}$ generated by the set $\{4, 6\}$ is $\{0, 2, 4, 6, 8, 10\}$. To see this, note that, if 4 and 6 belong to a subgroup of $\mathbb{Z}_{12}$, then so must $4 + 4 = 8$ and $4 + 6 = 10$ and $6 + 6 = 0$ and $4 + 4 + 6 = 2$.

4. Suppose $G = \langle a \rangle$ is a cyclic group, suppose $x = a^6$ and $y = a^8$. Then

$$H = \langle x, y \rangle = \langle a^6, a^8 \rangle = \langle a^2 \rangle.$$

See also the CyclicGroupSupplement.pdf document and CyclicGroupExercises.pdf, especially Exercise 6.