Math 301 Definitions

Students must know the precise definitions of the following terms:

- Cartesian product
- relation
- function
- operation
- universe
- arity of relation, function, or operation (e.g., nullary, unary, binary, ternary, n-ary)
- n-ary relation on a set X (notation:  $\rho \subseteq X^n$ )
- n-ary function from set X to set Y (notation:  $f: X^n \to Y$ )
- n-ary operation on a set X (notation:  $f: X^n \to X$ )
- interpretation of n-ary operation as special kind of (n+1)-ary relation
- properties binary relations might satisfy: reflexive, (anti)symmetric, transitive
- properties functions might satisfy: onto, one-to-one, bijective
- properties operations might satisfy: commutative, associative, idempotent
- equivalence relation
- equivalence class
- partition
- $\bullet$  congruence modulo n
- partial order
- partially ordered set (poset),  $\langle P, \preceq \rangle$
- total order
- totally ordered set
- well-ordered set
- common divisor
- greatest common divisor
- common multiple
- least common multiple

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- relatively prime
- prime number
- prime factorization
- power set
- relational structure,  $\langle A, \mathcal{R} \rangle$ , with universe A and relations  $\mathcal{R}$
- algebraic structure,  $\langle A, \mathcal{F} \rangle$ , with universe A and operations  $\mathcal{F}$
- examples of relational structures (e.g., poset, graph)<sup>1</sup>
- examples of algebraic structures (e.g., magma, semigroup, monoid, group)<sup>1</sup>
- identity element
- inverse operation
- abelian group
- Cayley table
- finite group
- subgroup, proper subgroup, trivial subgroup
- order (of a group or subgroup)
- order (of a group element)
- $g^n$  and  $g^{-n}$  (for g an element of a multiplicative group)
- nq and -nq (for q an element of an additive group)
- cyclic group
- generator (of a cyclic group)
- generators (of a group)
- symmetry, rigid motion
- permutation (and two ways to write them)
- cycle
- length of a cycle
- transposition

<sup>&</sup>lt;sup>1</sup> Many more examples at http://www.math.chapman.edu/~jipsen/structures/doku.php/index.html

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- even, odd (permutation)
- examples of groups:  $\mathbb{Z}_n$ , U(n),  $S_n$ ,  $A_n$ ,  $D_4$
- upper bound (of a subset of a poset, lattice, or join semilattice)
- least upper bound or supremum or join
- lower bound
- greatest lower bound or infimum or meet
- lattice,  $\langle L, \wedge, \vee \rangle$
- semilattice,  $\langle S, \cdot \rangle$
- meet semilattice,  $\langle S, \wedge \rangle$
- join semilattice,  $\langle S, \vee \rangle$
- join (of elements),  $a \vee b$
- meet (of elements),  $a \wedge b$
- join (of a subset),  $\bigvee T$
- meet (of a subset),  $\bigwedge T$
- largest element (of a poset; need not exist)
- smallest element (of a poset; need not exist)
- order-preserving map
- lattice homomorphism
- left coset
- right coset
- coset representative
- index (of a subgroup)
- Euler  $\varphi$  function
- conjugate elements of a group