Math 301 **Homework 7** **Fall 2014**

Exercises: 1, 2 (below) and Judson 19.3, 19.14, 19.20.
Due date: Wednesday, 10/22

1. Let P with $\leq$ be a partially ordered set, let $S \subseteq P$ and let $u \in P$. We say that u is an *upper bound* for S iff $s \leq u$ for all $s \in S$. We say ℓ is the *least upper bound* of S iff ℓ is an upper bound of S and $\ell \leq u$ for every upper bound u of S. Prove that if ℓ is the least upper bound of the set $\{x, y\}$ and m is the least upper bound of the set $\{\ell, z\}$, then m is the least upper bound of the set $\{x, y, z\}$.

2. Let $(P, \leq)$ be a partially ordered set with the property that every pair of elements $x, y \in P$ has a greatest lower bound. For $x, y \in P$, define $x \cdot y = \mathrm{glb}(x, y)$. Prove that $(P, \cdot)$ is a semilattice.

19.3. Draw a diagram of the lattice of subgroups of $\mathbb{Z}_{12}$.

19.14. Let G be a group and X be the set of subgroups of G ordered by set-theoretic inclusion. If H and K are subgroups of G, show that the least upper bound of H and K is the subgroup generated by $H \cup K$.

19.20. Let X and Y be posets. A map $\phi : X \to Y$ is *order-preserving* if $a \preceq b$ implies that $\phi(a) \preceq \phi(b)$. Let L and M be lattices. A map $\psi : L \to M$ is a *lattice homomorphism* if $\psi(a \vee b) = \psi(a) \vee \psi(b)$ and $\psi(a \wedge b) = \psi(a) \wedge \psi(b)$. Show that every lattice homomorphism is order-preserving, but that it is not the case that every order-preserving map is a lattice homomorphism.