Answers

1. (a) Generators are g^k for $1 \le k \le 4$.
   (b) Generators are g^k for $k \in \{1, 3, 7, 9\}$.
   (c) Generators are g^{2k-1} for $1 \le k \le 8$.
   (d) Generators are g^k for $k \in \{1, 3, 7, 9, 11, 13, 17, 19\}$.

2. (a) Generators of $\mathbb{Z}_5$ are k for $1 \le k \le 4$.
   (b) Generators of $\mathbb{Z}_{10}$ are k for $k \in \{1, 3, 7, 9\}$.
   (c) Generators of $\mathbb{Z}_{16}$ are $2k-1$ for $1 \le k \le 8$.
   (d) Generators of $\mathbb{Z}_{20}$ are k for $k \in \{1, 3, 7, 9, 11, 13, 17, 19\}$.

3. (a) $U(7)$ is cyclic with generator 3.
   (b) $U(12)$ is not cyclic: every nonidentity element has order 2, but $U(12)$ has order 8.
   (c) $U(16)$ is not cyclic: every nonidentity element has order 2 or 4, but $U(16)$ has order 8.
   (d) $U(11)$ is cyclic with generator 2.

4. (a) $|g^2| = 10$ (b) $|g^8| = 5$ (c) $|g^5| = 4$ (d) $|g^3| = 20$

5. (a) Subgroups: $H_1 = \langle 1 \rangle$, $H_2 = \langle g^2 \rangle$, $H_3 = \langle g^4 \rangle$, $H_4 = G$.
   (b) Subgroups: $H_1 = \langle 1 \rangle$, $H_2 = \langle g^2 \rangle$, $H_3 = \langle g^5 \rangle$, $H_4 = G$.
   (c) Subgroups: $H_1 = \langle 1 \rangle$, $H_2 = \langle g^2 \rangle$, $H_3 = \langle g^3 \rangle$, $H_4 = \langle g^6 \rangle$, $H_5 = \langle g^9 \rangle$, $H_6 = G$.
   (d) Subgroups $H_1 = \langle 1 \rangle$, $H_2 = \langle g^p \rangle$, $H_3 = \langle g^{p^2} \rangle$, $H_4 = G$.
   (e) Subgroups $H_1 = \langle 1 \rangle$, $H_2 = \langle g^p \rangle$, $H_3 = \langle g^q \rangle$, $H_4 = G$.
   (f) Subgroups $H_1 = \langle 1 \rangle$, $H_2 = \langle g^p \rangle$, $H_3 = \langle g^{p^2} \rangle$, $H_4 = \langle g^q \rangle$, $H_5 = \langle g^{pq} \rangle$, $H_6 = G$.

6. (a) $H = \langle a \rangle$
   (b) $H = \langle a^2 \rangle$
   (c) $H = \langle a^d \rangle$
   (d) $H = G$