

1. Find all generators of the cyclic group  $G = \langle g \rangle$  if:  
(a)  $|g| = 5$       (b)  $|g| = 10$       (c)  $|G| = 16$       (d)  $|G| = 20$
2. Find all generators of:  
(a)  $\mathbb{Z}_5$       (b)  $\mathbb{Z}_{10}$       (c)  $\mathbb{Z}_{16}$       (d)  $\mathbb{Z}_{20}$
3. In each case determine whether  $G$  is cyclic.  
(a)  $G = U(7)$       (b)  $G = U(12)$       (c)  $G = U(16)$       (d)  $G = U(11)$
4. Let  $|g| = 20$  in a group  $G$ . Compute:  
(a)  $|g^2|$       (b)  $|g^8|$       (c)  $|g^5|$       (d)  $|g^3|$
5. In each case find all the subgroups of  $G = \langle g \rangle$  and draw the lattice diagram.  
(a)  $|g| = 8$       (b)  $|g| = 10$       (c)  $|g| = 18$   
(d)  $|g| = p^3$ , where  $p$  is prime.  
(e)  $|g| = pq$ , where  $p$  and  $q$  are distinct primes.  
(f)  $|g| = p^2q$ , where  $p$  and  $q$  are distinct primes.
6. In each case, find the subgroup  $H = \langle x, y \rangle$  of  $G$ .  
(a)  $G = \langle a \rangle$  is cyclic,  $x = a^4$ ,  $y = a^3$ .  
(b)  $G = \langle a \rangle$  is cyclic,  $x = a^6$ ,  $y = a^8$ .  
(c)  $G = \langle a \rangle$  is cyclic,  $x = a^m$ ,  $y = a^k$ ,  $\gcd(m, k) = d$ .  
(d)  $G = S(3)$ ,  $x = (1, 2)$ ,  $y = (2, 3)$ .