Chapter 3: 16, 17, 31, 33, 44, 45, 52.
Additional suggested exercises: 35, 46, 47, 54.
Due date: Friday, 9/26

NOTE: the numbers listed above correspond to the printed version of the textbook, generated from 2013/08/16 source files.

16. Give a specific example of a group G and elements $g, h \in G$ where $(gh)^n \neq g^n h^n$.

17. Give examples of three different groups with eight elements. Why are the groups different?

31. Show that if G is a finite group of even order, then there is an $a \in G$ such that a is not the identity and $a^2 = e$.

33. Find all the subgroups of $\mathbb{Z}_3 \times \mathbb{Z}_3$. Use this information to show that $\mathbb{Z}_3 \times \mathbb{Z}_3$ is not the same group as $\mathbb{Z}_9$. (See Example 40 for a short description of the product of groups.)

44. Prove that the intersection of two subgroups of a group G is also a subgroup of G.

45. Prove or disprove: If H and K are subgroups of a group G, then $H \cup K$ is a subgroup of G.

52. Prove or disprove: Every nontrivial subgroup of an nonabelian group is nonabelian.

Additional suggested exercises.

35. Compute the subgroups of the symmetry group of a square.

46. Prove or disprove: If H and K are subgroups of a group G, then $HK = \{hk : h \in H \text{ and } k \in K\}$ is a subgroup of G. What if G is abelian?

47. Let G be a group and $g \in G$. Show that

$$Z(G) = \{x \in G : gx = xg \text{ for all } g \in G\}$$

is a subgroup of G. This subgroup is called the *center* of G.

54. Let H be a subgroup of G. If $g \in G$, show that $gHg^{-1} = \{g^{-1}hg : h \in H\}$ is also a subgroup of G.