# Introduction

In this report, our group has trained machine learning models using several supervised learning techniques in order to predict the outputs for the problem statement. These machine-learning approaches are providing good results for decision-making in academic fields. this report consists of several machine learning algorithms trained using some given data in order to predict if a tumor is benign or malignant based on the values of some of its given features. The report is divided into four parts- the first part uses the perceptron learning algorithm, the second part uses Fisher's linear discriminant analysis, the third part contains logistic regression, and the fourth part is a comparative study of all three learning algorithms based on their accuracy score.

# Problem Statement

The problem solved by this report is to make a framework to analyse and predict the kind of tumor using AI based techniques that use some given data without the need to manually evaluate it.

# The Dataset

| | id | diagnosis | radius_mean | texture_mean | perimeter_mean | area_mean | smoothness_mean | compactness_mean | concavity_mean | concave points_mean |
|---|---|---|---|---|---|---|---|---|---|---|
| **0** | 842302 | M | 17.99 | 10.38 | 122.80 | 1001.0 | 0.11840 | 0.27760 | 0.30010 | 0.14710 |
| **1** | 842517 | M | 20.57 | 17.77 | 132.90 | 1326.0 | 0.08474 | 0.07864 | 0.08690 | 0.07017 |
| **2** | 84300903 | M | 19.69 | 21.25 | 130.00 | 1203.0 | 0.10960 | 0.15990 | 0.19740 | 0.12790 |
| **3** | 84348301 | M | 11.42 | 20.38 | 77.58 | 386.1 | 0.14250 | 0.28390 | 0.24140 | 0.10520 |
| **4** | 84358402 | M | 20.29 | 14.34 | 135.10 | 1297.0 | 0.10030 | 0.13280 | 0.19800 | 0.10430 |
| **...** | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| **564** | 926424 | M | 21.56 | 22.39 | 142.00 | 1479.0 | 0.11100 | 0.11590 | 0.24390 | 0.13890 |
| **565** | 926682 | M | 20.13 | 28.25 | 131.20 | 1261.0 | 0.09780 | 0.10340 | 0.14400 | 0.09791 |
| **566** | 926954 | M | 16.60 | 28.08 | 108.30 | 858.1 | 0.08455 | 0.10230 | 0.09251 | 0.05302 |
| **567** | 927241 | M | 20.60 | 29.33 | 140.10 | 1265.0 | 0.11780 | 0.27700 | 0.35140 | 0.15200 |
| **568** | 92751 | B | 7.76 | 24.54 | 47.92 | 181.0 | 0.05263 | 0.04362 | 0.00000 | 0.00000 |

569 rows × 32 columns

| ... | radius_worst | texture_worst | perimeter_worst | area_worst | smoothness_worst | compactness_worst | concavity_worst | concave points_worst | symmetry_worst |
|---|---|---|---|---|---|---|---|---|---|
| ... | 25.380 | 17.33 | 184.60 | 2019.0 | 0.16220 | 0.66560 | 0.7119 | 0.2654 | 0.4601 |
| ... | 24.990 | 23.41 | 158.80 | 1956.0 | 0.12380 | 0.18660 | 0.2416 | 0.1860 | 0.2750 |
| ... | 23.570 | 25.53 | 152.50 | 1709.0 | 0.14440 | 0.42450 | 0.4504 | 0.2430 | 0.3613 |
| ... | 14.910 | 26.50 | 98.87 | 567.7 | 0.20980 | 0.86630 | 0.6869 | 0.2575 | 0.6638 |
| ... | 22.540 | 16.67 | 152.20 | 1575.0 | 0.13740 | 0.20500 | 0.4000 | 0.1625 | 0.2364 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| ... | 25.450 | 26.40 | 166.10 | 2027.0 | 0.14100 | 0.21130 | 0.4107 | 0.2216 | 0.2060 |
| ... | 23.690 | 38.25 | 155.00 | 1731.0 | 0.11660 | 0.19220 | 0.3215 | 0.1628 | 0.2572 |
| ... | 18.980 | 34.12 | 126.70 | 1124.0 | 0.11390 | 0.30940 | 0.3403 | 0.1418 | 0.2218 |
| ... | 25.740 | 39.42 | 184.60 | 1821.0 | 0.16500 | 0.86810 | 0.9387 | 0.2650 | 0.4087 |
| ... | 9.456 | 30.37 | 59.16 | 268.6 | 0.08996 | 0.06444 | 0.0000 | 0.0000 | 0.2871 |

# General Procedure

Although every algorithm is different from one another, there are a few steps that are common while performing any machine-learning algorithm, which are as follows:

Feature engineering (optional): Feature engineering refers to manipulation — addition, deletion, combination, mutation — of your data set to improve machine learning model training, leading to better performance and greater accuracy.

Train test split: we are a set of data, a portion of which is used to train our machine learning model, and once the model is finalized, the remaining data called the testing data, is used to test how accurate our trained machine learning model actually is.

A model with a fit and predict functions: this includes the implementation of the machine learning algorithm. The fit function is used to train the classifier, while the predict function is used to predict the output for new inputs once the model is trained.

Classifier: A classifier in machine learning is an algorithm that automatically orders or categorizes data into one or more of a set of “classes.”

Calculation of accuracy: the final step in our techniques is to calculate how accurate our model is for a new set of training data.

# Part A: Perceptron Learning Algorithm

This technique is used to classify the data if the features of the dataset are linearly separable. The perceptron model can theoretically give us 100% accuracy, provided the data is linearly separable.

We have trained 4 models using this algorithm, which resulted in the following observations:

| **Model Name** | **Epochs** | **Accuracy** (Train/Test) | **Confusion Matrix** | **Precision** | **Recall** |
|---|---|---|---|---|---|
| PM1 | 1000 | (0.9259, 0.9351) | [[38, 7] [5, 135]] | 0.8444 | 0.8837 |
| PM2 | 1000 | (0.9233, 0.9135) | [[58, 2] [14, 111]] | 0.9666 | 0.8055 |
| PM3 | 2158 (Terminated) | (1.0, 0.9251) | [[40, 11] [3, 133]] | 0.7843 | 0.9302 |
| PM4 | 1000 | (0.9259, 0.9351) | [[38, 7] [5, 135]] | 0.8444 | 0.8837 |

- PM1 and PM2 have different accuracies during test as well as training as their rows are shuffled, leading to a different random state. This will cause weights to change differently, hence leading to the same.
- PM1 is not normalized whereas PM3 is normalized (both feature engineering tasks 1 and 2 are applied). This allows PM3 to have less outliers and become centred around a Gaussian Distribution.
- In PM3, Training Accuracy = 100%. Thus, we can conclude that the data is **Linearly Separable.**
- PM4 has it's attributes shuffled, but since the order of plotting the points in the same random state does not matter, it will not show any deviation from PM1. Thus, PM1 is same as PM4.
- Perceptron Learning Algorithm does not depend on Learning Rate

# Part B: Fisher's Linear Discriminant Analysis

We have seen how the perceptron learning algorithm can help us achieve high accuracy. However, the drawbacks of using this algorithm are that it requires the points to be linearly separable and is computationally very expensive hence developing a model gets costlier. To overcome these problems, we try to reduce the number of dimensions of our training features to a single dimension so that the originality of the features is retained as much as possible. Hence, we try to project our D-dimensional data features onto a one-dimensional vector, 'w.'

Here's how we achieve it:

Say x1, x2, …, xN are 'N' points each one having 'D' dimensions, to be classified into class c1 or c2.

Any general xn = (a D-dimensional vector)

$$\begin{bmatrix} x1n \\ x2n \\ . \\ . \\ . \\ xDn \end{bmatrix}$$

Mean of all points in class c1 = m1' =

$$\frac{1}{N1}\sum_{xn\in c1} xn$$

Mean of all points in class c2 = m2' =

$$\frac{1}{N2}\sum_{xn\in c2} xn$$

Now, we can choose the w for which m2'-m1' is maximized.

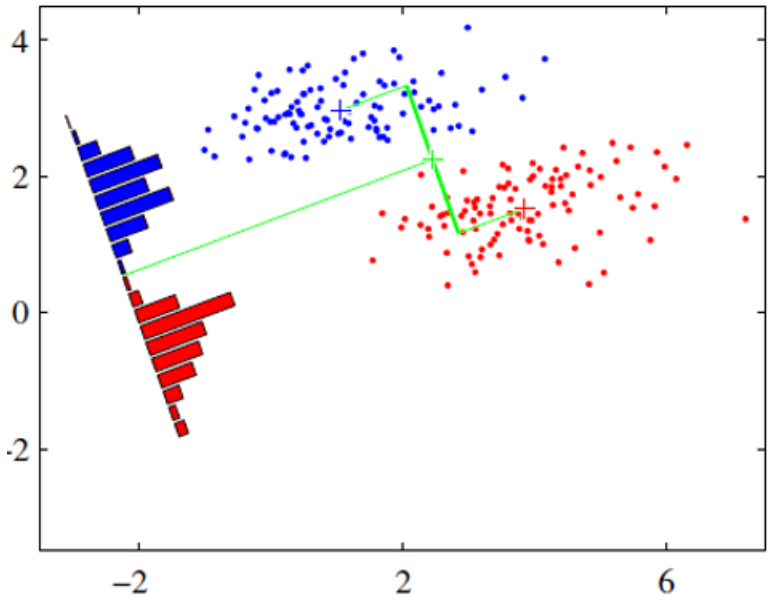
**Fisher's condition:** |m2-m1| should be maximized AND the total variance should be minimized.

Hence, our aim is to maximise $\frac{(m2-m1)^2}{(s1^2+s2^2)}$ , where,

$s1^2$ *and* $s1^2$ are variances of projected points of classes c1 and c2

m1 = mean of projected points belonging to class c1

m2 = mean of projected points belonging to class c2




Hence,

m1 = $\frac{1}{N1}\sum_{xn\in c1} w^T xn = w^T m1'$

m2 = $\frac{1}{N2}\sum_{xn\in c2} w^T xn = w^T m2'$

$s1^2 = \frac{1}{N1}\sum_{xn\in c1}(w^T xn - m1)^2 = \frac{1}{N1}\sum_{xn\in c1}(w^T xn - w^T m1')^2 = \frac{1}{N1}\sum_{xn\in c1}(w^T\ (xn - m1'))^2$

$= \frac{1}{N1}\sum_{xn\in c1}[(w^T\ (xn - m1'))(w^T(xn - m1'))^T]$

$= w^T(\frac{1}{N1}\sum_{xn\in c1}[(xn - m1')(xn - m1')^T])w$

Similarly, $s2^2 = w^T(\frac{1}{N2}\sum_{xn\in c2}[(xn - m2')(xn - m2')^T])w$

Thus, $s1^2 + s2^2 = w^T s_w w$ , where,

$s_w = (\frac{1}{N1}\sum_{xn\in c1}[(xn - m1')(xn - m1')^T]) + (\frac{1}{N2}\sum_{xn\in c2}[(xn - m2')(xn - m2')^T])$

Now, $(m2 - m1)^2 = (w^T m2' - w^T m1')^2 = w^T(m2' - m1')(m2' - m1')^T w = w^T s_B w$

Where, $s_B = (m2' - m1')(m2' - m1')^T$

Max$[\frac{(m2-m1)^2}{(s1^2+s2^2)}]$ = max$(\frac{w^T s_B w}{w^T s_w w})$

Therefore, $\frac{d}{dw}(\frac{w^T s_B w}{w^T s_w w}) = 0$

This gives: $s_B w(w^T s_w w) = s_w w(w^T s_B w)$ or $s_w w\ \alpha\ s_B w$

Now, $s_B w = (m2' - m1')(m2' - m1')^T w$

Hence, $s_B w\ \alpha\ (m2' - m1')$

Or, w $\alpha$ ${s_w}^{-1}(m2' - m1')$

This is the result we have used in the program.

After this, we have to project the points using $w^T.x$, and then find if x belongs to c1 or c2. For this, we will use the generative approach using Bayes' Theorem (assuming gaussian distribution for the projected points.

p( x $\in c1|$ $w^T x$) = p($w^T x|$ x $\in c1$ ) $p$(x $\in c1$)/$p$( $w^T x$)

p( x $\in c2|$ $w^T x$) = p($w^T x|$ x $\in c2$ ) $p$(x $\in c2$)/$p$( $w^T x$)

Hence, x $\in c1$ if p( x $\in c1|$ $w^T x$) > p( x $\in c2|$ $w^T x$) else x $\in c2$

Here, p(x $\in c1$) $and$ p(x $\in c2$) can be found out from the training data.

p($w^T x|$ x $\in c1$) and p($w^T x|$ x $\in c2$) are the prior probabilities of $w^T x$ according to the gaussian probability distributions.

We developed two models using this algorithm- FLDM1 and FLDM2

**FLDM1:** In this model, we apply the fisher's linear discriminant analysis on the raw data and thus reduce the 32-dimensional problem to univariate dimensional problem.

Confusion Matrix = [[57, 2]

[2, 125]]

Accuracy: 0.9785

Precision: 0.9661

Recall: 0.9661

**FLDM2:** In this model, we randomly change the order of the features in the dataset and apply the fisher's linear discriminant analysis on the raw data and thus reduce the 32-dimensional problem to univariate dimensional problem. **We get an accuracy of 97.849%, which is the same as the accuracy we got for FLDM1.** This is because no matter the order of the features, ultimately all the points will be the same in the 30-Dimensional space, and hence, the projections of these points will also be the same, giving us the same results.

Confusion Matrix = [[57, 2]

[2, 125]]

Accuracy: 0.9785

Precision: 0.9661

Recall: 0.9661

# Part C: Logistic Regression

In this model, we use the sigmoid function to find the posterior probability that our data point belongs to a certain class without going into the generative process. The sigmoid function is defined by

$\sigma(z) = \frac{1}{1+e^{-z}}$

hence, the probability that our point x is a positive point is given by

$p(pos \mid x) = \frac{1}{1+e^{-w.x}}$

hence, all we have to do is to find that value of w which makes our data most likely to happen. We achieve this by finding the negative log likelihood function (error function) of the data (a convex function) and minimizing it using the three gradient descent algorithms- batch, mini-batch and stochastic.

We use this approach with learning rates of 0.01, 0.001, and 0.0001 and vary the probability thresholds as 0.3, 0.4, 0.5, 0.6 and 0.7.

Also, the batch size used for mini-batch gradient descent = 20.

Number of Epochs for all the Models below = 1000

We have plotted the final values of accuracy, recall and precision in the following tables-

**LR1:** In this model, we apply the logistic regression on the raw data. Given below are the results of our model-

For **Batch Gradient Descent**

| Learning Rate | Probability Threshold | Confusion Matrix | Accuracy | Accuracy from 0.5 | Precision | Recall |
|---|---|---|---|---|---|---|
| 0.01 | 0.3 | [[67, 15] [5, 99]] | 0.8924 | No change | 0.8171 | 0.9306 |
| 0.01 | 0.4 | [[67, 15] [5, 99]] | 0.8294 | No change | 0.8171 | 0.9306 |
| 0.01 | 0.5 | [[67, 15] [5, 99]] | 0.8294 | - | 0.8171 | 0.9306 |
| 0.01 | 0.6 | [[67, 15] [5, 99]] | 0.8294 | No change | 0.8171 | 0.9306 |
| 0.01 | 0.7 | [[67, 15] [5, 99]] | 0.8294 | No change | 0.8171 | 0.9306 |
| 0.001 | 0.3 | [[67, 15] [5, 99]] | 0.8294 | No change | 0.8171 | 0.9306 |
| 0.001 | 0.4 | [[67, 15] [5, 99]] | 0.8294 | No change | 0.8171 | 0.9306 |
| 0.001 | 0.5 | [[67, 15] [5, 99]] | 0.8294 | - | 0.8171 | 0.9306 |
| 0.001 | 0.6 | [[67, 15] [5, 99]] | 0.8294 | No change | 0.8171 | 0.9306 |
| 0.001 | 0.7 | [[67, 15] [5, 99]] | 0.8294 | No change | 0.8171 | 0.9306 |
| 0.0001 | 0.3 | [[71, 25] [1, 89]] | 0.8602 | Decreases | 0.7396 | 0.9861 |
| 0.0001 | 0.4 | [[71, 23] [1, 91]] | 0.8709 | Decreases | 0.7553 | 0.9861 |
| 0.0001 | 0.5 | [[69, 20] [3, 94]] | 0.8763 | - | 0.7753 | 0.9583 |
| 0.0001 | 0.6 | [[69, 16] [3, 98]] | 0.8978 | Increases | 0.8117 | 0.9583 |
| 0.0001 | 0.7 | [[68, 15] [4, 99]] | 0.8978 | Increases | 0.8193 | 0.9444 |

For **Mini-Batch Gradient Descent**

| Learning Rate | Probability Threshold | Confusion Matrix | Accuracy | Accuracy from 0.5 | Precision | Recall |
|---|---|---|---|---|---|---|
| 0.01 | 0.3 | [[65, 7] [7, 107]] | 0.9247 | No change | 0.9027 | 0.9027 |
| 0.01 | 0.4 | [[65, 7] [7, 107]] | 0.9247 | No change | 0.9027 | 0.9027 |
| 0.01 | 0.5 | [[65, 7] [7, 107]] | 0.9247 | - | 0.9027 | 0.9027 |
| 0.01 | 0.6 | [[65, 7] [7, 107]] | 0.9247 | No change | 0.9027 | 0.9027 |
| 0.01 | 0.7 | [[65, 7] [7, 107]] | 0.9247 | No change | 0.9027 | 0.9027 |
| 0.001 | 0.3 | [[66, 7] [6, 107]] | 0.9301 | No change | 0.9041 | 0.9167 |
| 0.001 | 0.4 | [[66, 7] [6, 107]] | 0.9301 | No change | 0.9041 | 0.9167 |
| 0.001 | 0.5 | [[66, 7] [6, 107]] | 0.9301 | - | 0.9041 | 0.9167 |
| 0.001 | 0.6 | [[66, 7] [6, 107]] | 0.9301 | No change | 0.9041 | 0.9167 |
| 0.001 | 0.7 | [[65, 7] [7, 107]] | 0.9247 | Decreases | 0.9027 | 0.9027 |
| 0.0001 | 0.3 | [[68, 10] [4, 104]] | 0.9247 | Increases | 0.8718 | 0.9444 |
| 0.0001 | 0.4 | [[68, 9] [4, 105]] | 0.9301 | Increases | 0.8831 | 0.9444 |
| 0.0001 | 0.5 | [[66, 9] [6, 105]] | 0.9193 | - | 0.88 | 0.9167 |
| 0.0001 | 0.6 | [[66, 9] [6, 105]] | 0.9193 | No change | 0.88 | 0.9167 |
| 0.0001 | 0.7 | [[66, 7] [6, 107]] | 0.9301 | Increases | 0.9041 | 0.9167 |

For **Stochastic Gradient Descent**

| Learning Rate | Probability Threshold | Confusion Matrix | Accuracy | Accuracy from 0.5 | Precision | Recall |
|---|---|---|---|---|---|---|
| 0.01 | 0.3 | [[72, 50] [0, 64]] | 0.7312 | Decreases | 0.5902 | 1.0 |
| 0.01 | 0.4 | [[68, 9] [4, 105]] | 0.9301 | Decreases | 0.8831 | 0.9444 |
| 0.01 | 0.5 | [[63, 1] [9, 113]] | 0.9462 | - | 0.9844 | 0.875 |
| 0.01 | 0.6 | [[69, 13] [3, 101]] | 0.9140 | Decreases | 0.8415 | 0.9583 |
| 0.01 | 0.7 | [[71, 20] [1, 94]] | 0.8871 | Decreases | 0.7802 | 0.9861 |
| 0.001 | 0.3 | [[50, 0] [22, 114]] | 0.8817 | Decreases | 1.0 | 0.6944 |
| 0.001 | 0.4 | [[65, 2] [7, 112]] | 0.9516 | No change | 0.9701 | 0.9028 |
| 0.001 | 0.5 | [[65, 2] [7, 112]] | 0.9516 | - | 0.9701 | 0.9028 |
| 0.001 | 0.6 | [[63, 1] [9, 113]] | 0.9462 | Decreases | 0.9843 | 0.875 |
| 0.001 | 0.7 | [[66, 7] [6, 107]] | 0.9301 | Decreases | 0.9041 | 0.9167 |
| 0.0001 | 0.3 | [[63, 1] [9, 113]] | 0.9462 | Increases | 0.9843 | 0.875 |
| 0.0001 | 0.4 | [[69, 15] [3, 99]] | 0.9032 | Decreases | 0.8214 | 0.9583 |
| 0.0001 | 0.5 | [[69, 11] [3, 103]] | 0.9247 | - | 0.8625 | 0.9583 |
| 0.0001 | 0.6 | [[65, 2] [7, 112]] | 0.9516 | Increases | 0.9701 | 0.9028 |
| 0.0001 | 0.7 | [[65, 2] [7, 112]] | 0.9516 | Increases | 0.9701 | 0.9028 |

**LR2:** In this model, we apply the logistic regression on the normalised dataset (after applying feature engineering tasks 1 and 2). Given below are the results of our model-

For **Batch Gradient Descent**

| Learning Rate | Probability Threshold | Confusion Matrix | Accuracy | Accuracy from 0.5 | Precision | Recall |
|---|---|---|---|---|---|---|
| 0.01 | 0.3 | [[69, 3] [2, 114]] | 0.9734 | Decreases | 0.9583 | 0.9718 |
| 0.01 | 0.4 | [[69, 0] [2, 117]] | 0.9893 | Increases | 1.0 | 0.9718 |
| 0.01 | 0.5 | [[68, 0] [3, 117]] | 0.9840 | - | 1.0 | 0.9577 |
| 0.01 | 0.6 | [[65, 0] [6, 117]] | 0.9680 | Decreases | 1.0 | 0.9155 |
| 0.01 | 0.7 | [[61, 0] [10, 117]] | 0.9468 | Decreases | 1.0 | 0.8592 |
| 0.001 | 0.3 | [[70, 16] [1, 101]] | 0.9096 | Decreases | 0.8140 | 0.9859 |
| 0.001 | 0.4 | [[69, 7] [2, 110]] | 0.9521 | Increases | 0.9080 | 0.9718 |
| 0.001 | 0.5 | [[63, 3] [8, 116]] | 0.9512 | - | 0.9844 | 0.8873 |
| 0.001 | 0.6 | [[61, 1] [10, 116]] | 0.9145 | Decreases | 0.9839 | 0.8592 |
| 0.001 | 0.7 | [[56, 1] [15, 116]] | 0.9149 | Decreases | 0.9825 | 0.7887 |
| 0.0001 | 0.3 | [[71, 117] [0, 0]] | 0.3777 | Decreases | 0.3777 | 1.0 |
| 0.0001 | 0.4 | [[71, 86] [0, 31]] | 0.5426 | Decreases | 0.4522 | 1.0 |
| 0.0001 | 0.5 | [[63, 3] [8, 114]] | 0.9415 | - | 0.9545 | 0.8873 |
| 0.0001 | 0.6 | [[37, 1] [34, 116]] | 0.8138 | Decreases | 0.9736 | 0.5211 |
| 0.0001 | 0.7 | [[8, 0] [63, 117]] | 0.6649 | Decreases | 1.0 | 0.1127 |

For **Mini-Batch Gradient Descent**

| Learning Rate | Probability Threshold | Confusion Matrix | Accuracy | Accuracy from 0.5 | Precision | Recall |
|---|---|---|---|---|---|---|
| 0.01 | 0.3 | [[69, 1] [2, 116]] | 0.9804 | No Change | 0.9857 | 0.9718 |
| 0.01 | 0.4 | [[69, 0] [2, 117]] | 0.9894 | Increases | 1.0 | 0.9718 |
| 0.01 | 0.5 | [[68, 0] [3, 117]] | 0.9804 | - | 1.0 | 0.9577 |
| 0.01 | 0.6 | [[66, 0] [5, 117]] | 0.9734 | Decreases | 1.0 | 0.9296 |
| 0.01 | 0.7 | [[64, 0] [7, 117]] | 0.9628 | Decreases | 1.0 | 0.9014 |
| 0.001 | 0.3 | [[69, 2] [2, 115]] | 0.9787 | Decreases | 0.9718 | 0.9718 |
| 0.001 | 0.4 | [[69, 0] [2, 117]] | 0.9894 | Increases | 1.0 | 0.9718 |
| 0.001 | 0.5 | [[68, 0] [3, 117]] | 0.9804 | - | 1.0 | 0.9577 |
| 0.001 | 0.6 | [[65, 0] [6, 117]] | 0.9680 | Decreases | 1.0 | 0.9155 |
| 0.001 | 0.7 | [[63, 0] [8, 117]] | 0.9574 | Decreases | 1.0 | 0.8873 |
| 0.0001 | 0.3 | [[70, 9] [1, 108]] | 0.9468 | Decreases | 0.8861 | 0.9859 |
| 0.0001 | 0.4 | [[69, 4] [2, 113]] | 0.9681 | No change | 0.9452 | 0.9718 |
| 0.0001 | 0.5 | [[66, 1] [5, 116]] | 0.9681 | - | 0.9501 | 0.9296 |
| 0.0001 | 0.6 | [[62, 1] [9, 116]] | 0.9468 | Decreases | 0.9841 | 0.8731 |
| 0.0001 | 0.7 | [[59, 0] [12, 117]] | 0.9361 | Decreases | 1.0 | 0.8310 |

For **Stochastic Gradient Descent**

| Learning Rate | Probability Threshold | Confusion Matrix | Accuracy | Accuracy from 0.5 | Precision | Recall |
|---|---|---|---|---|---|---|
| 0.01 | 0.3 | [[68, 1] [3, 116]] | 0.9787 | Increases | 0.9855 | 0.9577 |
| 0.01 | 0.4 | [[69, 1] [2, 116]] | 0.9840 | Increases | 0.9857 | 0.9718 |
| 0.01 | 0.5 | [[66, 1] [5, 116]] | 0.9681 | - | 0.9851 | 0.9296 |
| 0.01 | 0.6 | [[66, 1] [5, 116]] | 0.9681 | No change | 0.9851 | 0.9296 |
| 0.01 | 0.7 | [[65, 1] [6, 116]] | 0.9628 | Decreases | 0.9848 | 0.9155 |
| 0.001 | 0.3 | [[68, 1] [3, 116]] | 0.9787 | No change | 0.9855 | 0.9577 |
| 0.001 | 0.4 | [[68, 0] [3, 117]] | 0.9840 | Increases | 1.0 | 0.9577 |
| 0.001 | 0.5 | [[67, 0] [4, 117]] | 0.9787 | - | 1.0 | 0.9437 |
| 0.001 | 0.6 | [[67, 0] [4, 117]] | 0.9787 | No change | 1.0 | 0.9437 |
| 0.001 | 0.7 | [[65, 0] [6, 117]] | 0.9681 | Decreases | 1.0 | 0.9155 |
| 0.0001 | 0.3 | [[69, 2] [2, 115]] | 0.9787 | Decreases | 0.9718 | 0.9718 |
| 0.0001 | 0.4 | [[69, 0] [2, 117]] | 0.9894 | Increases | 1.0 | 0.9718 |
| 0.0001 | 0.5 | [[68, 0] [3, 117]] | 0.9840 | - | 1.0 | 0.9577 |
| 0.0001 | 0.6 | [[65, 0] [6, 117]] | 0.9681 | Decreases | 1.0 | 0.9155 |
| 0.0001 | 0.7 | [[65, 0] [6, 117]] | 0.9681 | Decreases | 1.0 | 0.9155 |

# Part D: Comparative Study

| **Model Name** | **Accuracy** (Maximum in case of LR) |
|---|---|
| PM1 | 0.9351 |
| PM3 | 0.9251 |
| PM4 | 0.9351 |
| FLDM1 | 0.9785 |
| FLDM2 | 0.9785 |
| LR1 | 0.9516 |
| LR2 | 0.9894 |

As clear from the above table, LR2 is the best model out of all the given models. This is because:

1. LR2 is normalized, thus no outliers as in the case if PM1, PM4, FLDM1, FLDM2, LR1.
2. LR2 uses a sigmoid activation function instead of a unit step function, thus above a certain threshold, it gets better accuracies.
3. LR2 is also easier to realise than all the other models, such as FLDM1 or FLDM2.

Among the remaining models, FLDM1 or FLDM2, LR1, PM1 or PM4 and PM3 can be used in the given order respectively.