Study Guide: Random Walks and Markov Chains

Definition and Characteristics

* **Random Walk**: A random walk is a sequence of random steps, each step is determined by the previous step, and the probability of taking each step is based on the current state.
* **Markov Chain**: A Markov chain is a mathematical system that can be in one of several states, and it can change state according to certain rules, called transition probabilities.

Key Characteristics

* **States**: A finite set of states, where each state has a transition probability to another state.
* **Transition Probabilities**: For each pair of states x and y, there is a transition probability pxy of going from state x to state y, where for each x, $\sum$y pxy = 1.
* **Stationary Probability**: The limiting probability of being at a particular state, independent of the starting state.

Applications

* **Pagerank**: Defining the importance of pages on the World Wide Web by their stationary probability.
* **Markov Chain Monte Carlo (MCMC)**: Sampling a large space according to a probability distribution by designing a Markov chain.
* **Gambler's Assets**: Modeling a gambler's assets as a Markov chain, where the current state is the amount of money the gambler has on hand.
* **Electrical Networks**: Random walks on undirected graphs have connections to electrical networks, where each edge has a conductance parameter.

Diagram Suggestion (ASCII)

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A ---> B (0.5)

B ---> A (0.5)

C ---> D (0.8)

D ---> E (0.2)

E ---> F (0.9)

F ---> G (0.1)

**Elaboration**

* A random walk can be represented by a directed graph, where each state is a vertex, and each edge has a transition probability.

* A Markov chain can be represented by a transition probability matrix P, where P[ij] is the probability of going from state i to state k.

* The stationary probabilities are the limiting probabilities of being at a particular state, independent of the starting state.

**Summary of Key Points**

* Random walks and Markov chains are interchangeable terms.

* A Markov chain has a finite set of states, with transition probabilities between them.

* The stationary probability is the limiting probability of being at a particular state, independent of the starting state.

* Applications include pagerank, MCMC, gambler's assets, and electrical networks.

**Flashcards (Q&A)**

Q1. What is a random walk?

A: A sequence of random steps, each step determined by the previous step, with
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probability based on the current state.

Q2. What is a Markov chain?

A: A mathematical system with a finite set of states, that can change state according to transition probabilities.

Q3. What is the stationary probability?

A: The limiting probability of being at a particular state, independent of the starting state.

Q4. What is MCMC used for?

A: Sampling a large space according to a probability distribution by designing a Markov chain.

Q5. What is the significance of conductance in electrical networks?

A: Each edge has a conductance parameter, used to determine the transition probabilities in a random walk on an undirected graph.