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Q1) Normal Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{x}\left(\frac{(x-\mu)^2}{2\sigma^2}\right)}$$

mean(μ) = θ_1, variance = θ_2

size = n

$$L(\mu) = -n\log(\sigma\sqrt{2\pi}) - \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{2\sigma^2}$$

$$\frac{\partial \log L(\mu)}{\partial \theta_1} = \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu)$$

$$= \frac{n\bar{x} - n\theta_1}{\sigma^2} = 0$$

or $\theta = \bar{x}$

$$\frac{\partial \log L(\theta_1)}{\partial \theta_1} = \frac{n}{\sigma^2}(\bar{x}-\mu)$$

$$\frac{\partial^2 \log L(\theta_1)}{\partial \theta_1^2} = \frac{-n}{\sigma^2}$$

$$\theta_2 = \frac{\sigma^2}{n}$$ → variance of sample mean

Q2) $B(m, \theta)$ $\theta \in (0, 1)$

$$\text{pmf} = P(X = k) = {}^mC_k \, \theta^k (1-\theta)^{m-k}$$

$k \to$ no. of success in m trials

$\theta \to$ prob. of success

$1-\theta \to$ prob. of failure

Likelihood $fx^n \to$ $L(\theta) = \prod_{i=1}^{n} {}^mC_{x_i} \, \theta^{x_i} (1-\theta)^{m-x_i}$

$$\ln(L(\theta)) = \sum_{i=1}^{n} \ln({}^mC_{x_i}) + \sum_{i=1}^{n} x_i \ln\theta + \sum_{i=1}^{n} (m-x_i)\ln(1-\theta)$$

$$\frac{\partial}{\partial\theta} \ln(L(\theta)) = \sum_{i=1}^{n} \frac{x_i}{\theta} - \frac{m-x_i}{1-\theta} = 0$$

$$\sum_{i=1}^{n} \frac{x_i}{\theta} = \sum_{i=1}^{n} \frac{m-x_i}{1-\theta}$$

$$\frac{\sum_{i=1}^{n} x_i}{\theta} = \frac{m \cdot n - \sum_{i=1}^{n} x_i}{1-\theta}$$

$$\theta\left(m \cdot n - \sum_{i=1}^{n} x_i\right) = (1-\theta) \sum_{i=1}^{n} x_i$$

$$\theta \cdot m \cdot n = \sum x_i$$

$$\boxed{\theta = \frac{\sum x_i}{mn} = \frac{\bar{X}}{m}}$$