(*) Introduction

Digital design is concerned with the design of DIGITAL electronic circuits.
A DIGITAL COMPUTER can follow a sequence of INSTRUCTIONS, called a PROGRAM, that operates on given DATA. The USER can change programs and/or data according to the specific need. Examples of digital systems include digital computers, electronic calculators, etc.

Even though DATA and INFORMATION are slightly different terms, they are often used interchangeably.

^ The main characteristic of a digital system is its manipulation of DISCRETE elements of information, for eg., electric impulses, the decimal digits, the letters of an alphabet, etc.

The JUXTAPOSITION of discrete elements of INFORMATION represents a QUANTITY of information.
For eg., the letters d, o and g placed one after another form the word dog.
Thus, a SEQUENCE of discrete elements forms a LANGUAGE.

Discrete elements of information are represented in a digital system by physical quantities called SIGNALS, such as ELECTRICAL signals (for eg., voltages, currents, etc.).

The signals in all present-day electronic digital systems have only TWO discrete values and are said to be BINARY. This is because MANY-VALUED electronic circuits, for eg., a circuit with ten states (one discrete voltage value for each state), have low RELIABILITY of operation.

Discrete quantities of information arise either from the NATURE of a process or may be QUANTIZED from a continuous process.

For eg., An employee's paycheck is processed using DISCRETE data values such as letters (for name), digits (for salary), and special symbols such as $.

On the other hand, a scientist may observe a CONTINUOUS phenomenon but record only specific quantities in tabular form.

An ANALOG computer performs a direct SIMULATION of a continuous physical system. However, to simulate a continuous physical process in a DIGITAL computer, the quantities must be QUANTIZED.

The terms DIGITAL SIGNAL & ANALOG SIGNAL are sometimes substituted for DISCRETE SIGNAL & CONTINUOUS SIGNAL, respectively.

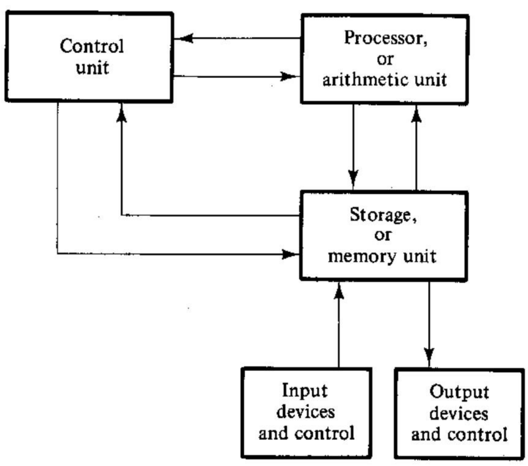

FIGURE 1-1
Block diagram of a digital computer

The MEMORY unit stores PROGRAMS as well as INPUT, OUTPUT and INTERMEDIATE data.

The PROCESSING unit performs data-processing tasks as specified by a PROGRAM.

The CONTROL unit retrieves the INTSTRUCTIONS of a program one by one and for each instruction, it informs the PROCESSOR to execute the OPERATION specified by the instruction.

The PROGRAM and DATA prepared by the USER are transferred into the MEMORY unit by means of INPUT devices, and OUTPUT devices receive the RESULTS of the computations.

(*) Number Systems (Unsigned)

An UNSIGNED (non-negative) number in any BASE (or RADIX) is represented by a series of COEFFICIENTS as $...a_3 a_2 a_1 a_0 . a_{-1} a_{-2} a_{-3} ...$ (this dot is known as the RADIX POINT). The common bases are 10, 2, 8 & 16, and a similar logic works for ANY base r other than these.

TABLE 1-1
Numbers with Different Bases

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

For base 10, the radix point is also known as the DECIMAL point. For base 2, the radix point is also known as the BINARY point. And so on.

^ The DECIMAL Number System

For base 10, the a_j coefficients are one of 0, 1, 2, 3, 4, 5, 6, 7, 8 & 9, and the subscript j gives the power of 10 by which the coefficient must be multiplied.

Decimal to Decimal Conversion –

1. $(7392)_{10} = (7 \times 10^3) + (3 \times 10^2) + (9 \times 10^1) + (2 \times 10^0)$
2. $(12.36)_{10} = (1 \times 10^1) + (2 \times 10^0) + (3 \times 10^{-1}) + (6 \times 10^{-2})$

^ The BINARY Number System

For base 2, the a_j coefficients are one of 0 & 1, and the subscript j gives the power of 2 by which the coefficient must be multiplied.

Binary to Decimal Conversion –

1. $(1011)_2 = (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$

$= (11)_{10}$

2. $(11010.11)_2 = (1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) + (1 \times 2^{-1}) + (1 \times 2^{-2})$

$= (26.75)_{10}$

^ The OCTAL Number System

For base 8, the a_j coefficients are one of 0, 1, 2, 3, 4, 5, 6 & 7, and the subscript j gives the power of 8 by which the coefficient must be multiplied.

Octal to Decimal Conversion –

1. $(6471)_8 = (6 \times 8^3) + (4 \times 8^2) + (7 \times 8^1) + (1 \times 8^0)$

$= (3385)_{10}$

2. $(231.4)_8 = (2 \times 8^2) + (3 \times 8^1) + (1 \times 8^0) + (4 \times 8^{-1})$

$= (153.5)_{10}$

^ The HEXADECIMAL Number System

For base 16, the a_j coefficients are one of 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A (10), B (11), C (12), D (13), E (14) & F (15), and the subscript j gives the power of 16 by which the coefficient must be multiplied.

Hexadecimal to Decimal Conversion –

1. $(B65F)_{16} = (11 \times 16^3) + (6 \times 16^2) + (5 \times 16^1) + (15 \times 16^0)$

$= (46687)_{10}$

2. $(3F9.C)_{16} = (3 \times 16^2) + (15 \times 16^1) + (9 \times 16^0) + (12 \times 16^{-1})$

$= (1017.75)_{10}$

^ Arithmetic operations with unsigned numbers in base r follow the SAME rules as for unsigned decimal numbers. For eg.,

1. $(23.5)_{10} + (1.67)_{10} = (25.17)_{10}$

```
      0              1 0            01 0            01 0
    23.50           23.50           23.50           23.50
           --->            --->            --->
  + 01.67         + 01.67         + 01.67         + 01.67
  -------         -------         -------         -------
      . 7             .17            5.17           25.17
```

2. $(11.1)_2 - (1.01)_2 = (10.01)_2$

```
           0 10                 0 10                 0 10                 0 10
    1 1 . 1 0            1 1 . 1 0            1 1 . 1 0            1 1 . 1 0
               --->                 --->                 --->
  - 0 1 . 0 1          - 0 1 . 0 1          - 0 1 . 0 1          - 0 1 . 0 1
  ------------         ------------         ------------         ------------
        .   1                . 0 1            0 . 0 1            1 0 . 0 1
```

3. $(3.24)_8 \times (7.1)_8 = (27.464)_8$

```
                                                                 3
    324            324            324            324            324

  x  71          x  71          x  71          x  71          x  71
  -----   --->   -----   --->   -----   --->   -----   --->   -----   --->
      4             24            324            324            324

                                                    0             40
                                                 -----          -----
```

```
   23             23             23
   324            324            324            3.24

  x  71          x  71          x  71          x  7.1
  -----   --->   -----   --->   -----   --->   ------
    324            324            324          27.464

    140        + 27140        + 27140
  -----          -----          -----
                                27464
```

> Note that in octal,
>
> 7 x 2 = 16
>
> 7 x 3 = 25
>
> 7 x 4 = 34
>
> and so on

4. $(2B.E)_{16}$ / $(3.4)_{16}$ = $(D.8)_{16}$

```
      D                      D                      D.                     D.8
    ┌------                ┌------                ┌------                ┌------
3.4 │ 2B.E             3.4 │ 2B.E             3.4 │ 2B.E             3.4 │ 2B.E
----┘                  ----┘                  ----┘                  ----┘
   - 2A.4                 - 2A.4                 - 2A.4                 - 2A.4
    ------   --->          ------   --->          ------   --->          ------
                             1.A                     1A                     1A

                                                                          - 1A
                                                                         ------
                                                                             0
```

In this example, the quotient TERMINATED and the remainder became ZERO. But, in other cases, just like with unsigned decimal numbers, division with unsigned numbers in base r may also result in a NON-TERMINATING quotient.

^ Decimal to Binary Conversion

1. $(41)_{10}$ = $(101001)_2$

```
  2 | 41 | 1              2 | 41 | 1              2 | 41 | 1
---|----|---    --->    ---|----|---    --->    ---|----|---    --->
   | 20 |                 2 | 20 | 0              2 | 20 | 0
                        ---|----|---            ---|----|---
                           | 10 |                 2 | 10 | 0
                                                ---|----|---
                                                   |  5 |
```

41 = (2 x 20) + 1

```
  2 | 41 | 1              2 | 41 | 1              2 | 41 | 1   ↑
---|----|---    --->    ---|----|---    --->    ---|----|---   |
  2 | 20 | 0              2 | 20 | 0              2 | 20 | 0   |
---|----|---            ---|----|---            ---|----|---   |
  2 | 10 | 0              2 | 10 | 0              2 | 10 | 0   |
---|----|---            ---|----|---            ---|----|---   |
  2 |  5 | 1              2 |  5 | 1              2 |  5 | 1   |
---|----|---            ---|----|---            ---|----|---   |
   |  2 |                 2 |  2 | 0              2 |  2 | 0   |
                        ---|----|---            ---|----|---   |
                           |  1 |                 2 |  1 | 1   |
                                                ---|----|---
                                                   |  0 |
```

Read the digits from BOTTOM to TOP

2. $(0.6875)_{10} = (0.1011)_2$

$0.6875 \times 2 = 1.375$ 1

$0.375 \times 2 = 0.75$ 0

$0.75 \times 2 = 1.5$ 1

$0.5 \times 2 = 1$ 1

Read the digits from TOP to BOTTOM

In this example, the binary number TERMINATED and the fraction part became ZERO.

But, in other cases, the result may also be NON-TERMINATING.

3. $(41.6875)_{10} = (101001.1011)_2$

Since $(41)_{10} = (101001)_2$ and $(0.6875)_{10} = (0.1011)_2$,

therefore $(41.6875)_{10} = (101001.1011)_2$.

^ Decimal to Octal Conversion

Similar to 'Decimal to Binary Conversion', except the division & multiplication need to be done by 8.

^ Decimal to Hexadecimal Conversion

Similar to 'Decimal to Binary Conversion', except the division & multiplication need to be done by 16.

^ Binary to Octal and Octal to Binary Conversions

Binary	000	001	010	011	100	101	110	111
Octal	0	1	2	3	4	5	6	7

The PARTITIONING of the binary number into groups of 3 digits begins from the BINARY POINT and proceeds to the LEFT and to the RIGHT.

1. $(10110001101011.11110000011)_2 = (26153.7406)_8$

$$(\ \underline{10}\ \underline{110}\ \underline{001}\ \underline{101}\ \underline{011}\ .\ \underline{111}\ \underline{100}\ \underline{000}\ \underline{11}\ )_2$$

$$= (\ \underline{010}\ \underline{110}\ \underline{001}\ \underline{101}\ \underline{011}\ .\ \underline{111}\ \underline{100}\ \underline{000}\ \underline{110}\ )_2 \quad \text{(by adding LEADING \& TRAILING 0s)}$$

$$= (\ 2\ \ 6\ \ 1\ \ 5\ \ 3\ .\ 7\ \ 4\ \ 0\ \ 6\ )_8$$

2. $(673.124)_8 = (110111011.0010101)_2$

$$(\ 6\ \ 7\ \ 3\ .\ 1\ \ 2\ \ 4\ )_8$$

$$= (\ 110\ 111\ 011\ .\ 001\ 010\ 100\ )_2$$

$$= (\ 110\ 111\ 011\ .\ 001\ 010\ 1\ )_2 \quad \text{(by removing the TRAILING 0s)}$$

^ Binary to Hexadecimal and Hexadecimal to Binary Conversions

Binary	0000	0001	0010	0011	0100	0101	0110	0111
Hexadecimal	0	1	2	3	4	5	6	7

Binary	1000	1001	1010	1011	1100	1101	1110	1111
Hexadecimal	8	9	A	B	C	D	E	F

The PARTITIONING of the binary number into groups of 4 digits begins from the BINARY POINT and proceeds to the LEFT and to the RIGHT.

1. $(10110001101011.1111001)_2 = (2C6B.F2)_{16}$

$(\ \underline{10}\ \underline{1100}\ \underline{0110}\ \underline{1011}\ .\ \underline{1111}\ \underline{001}\ )_2$

$= (\ \underline{0010}\ \underline{1100}\ \underline{0110}\ \underline{1011}\ .\ \underline{1111}\ \underline{0010}\ )_2$ (by adding LEADING & TRAILING 0s)

$= (\ 2\quad C\quad 6\quad B\ .\ F\quad 2\ )_{16}$

2. $(306.D)_{16} = (1100000110.1101)_2$

$(\ 3\quad 0\quad 6\ .\ D\ )_{16}$

$= (\ 0011\ 0000\ 0110\ .\ 1101\ )_2$

$= (\ 11\ 0000\ 0110\ .\ 1101\ )_2$ (by removing the LEADING 0s)

^ Octal to Hexadecimal and Hexadecimal to Octal Conversions

First convert to Binary, and then convert to Octal/Hexadecimal.

^ Binary number are difficult to work with ...