Understanding

Options

What is The

OPTIONS

An option is a contract that represents the right to buy or sell a financial product at an agreed-upon price for a specific period of time. You can typically buy and sell an options contract at any time before expiration.

#1 European option

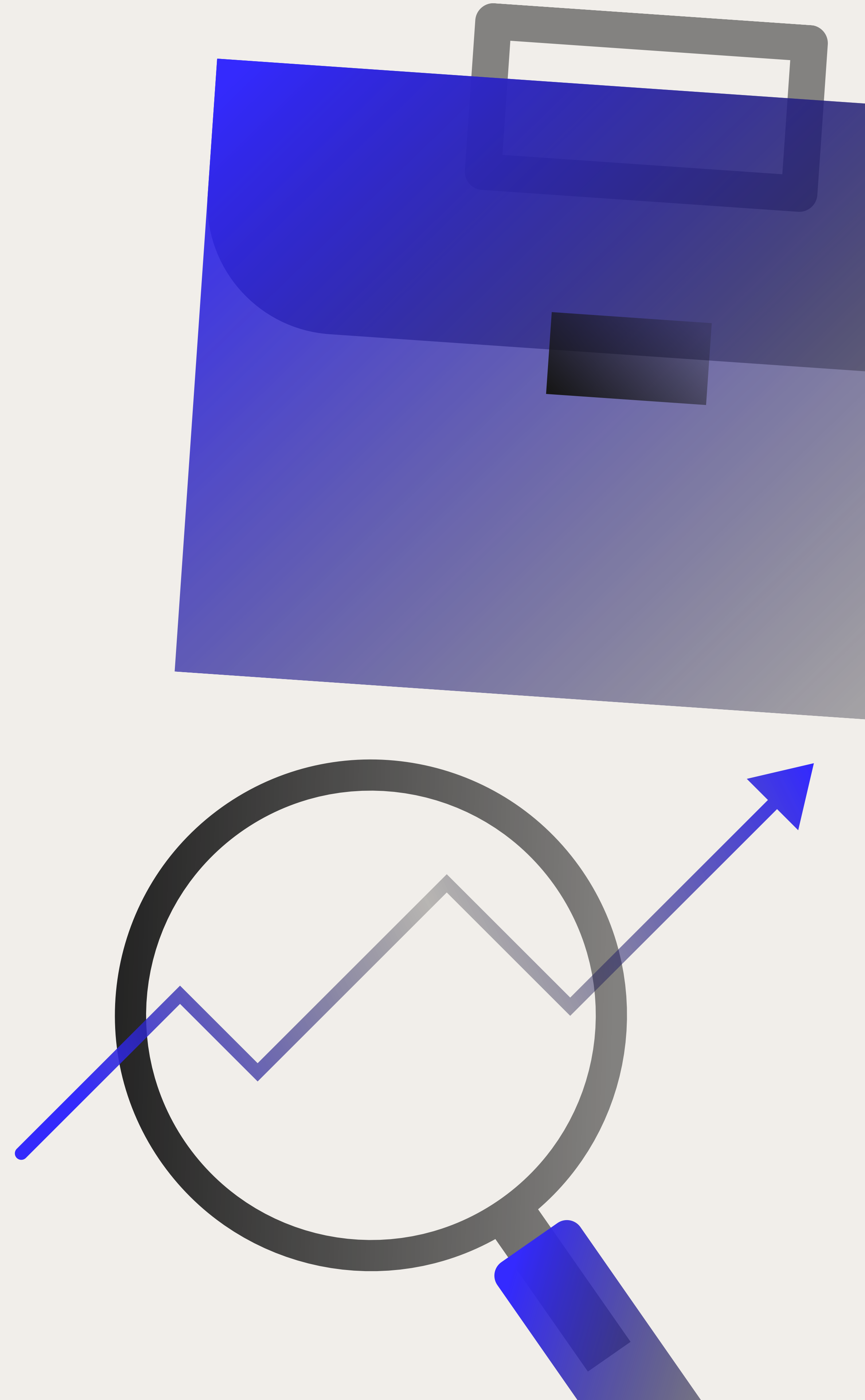
Exercised only on expiry

#2 Bermuda option

Exercised only on specified days

#3 American option

can be Exercised on any day before expiry



TYPES OF OPTIONS

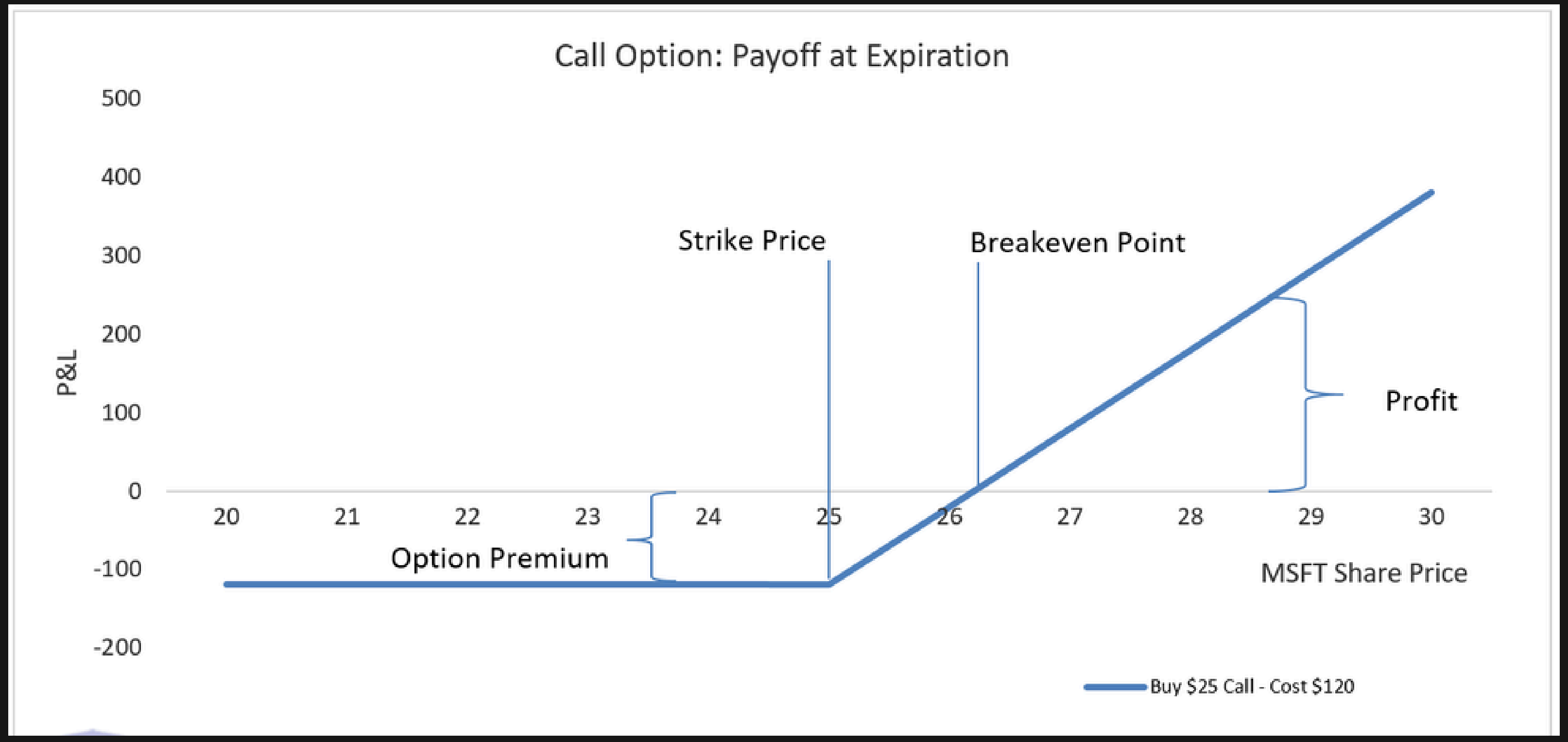
Based acquisition or disposal rights

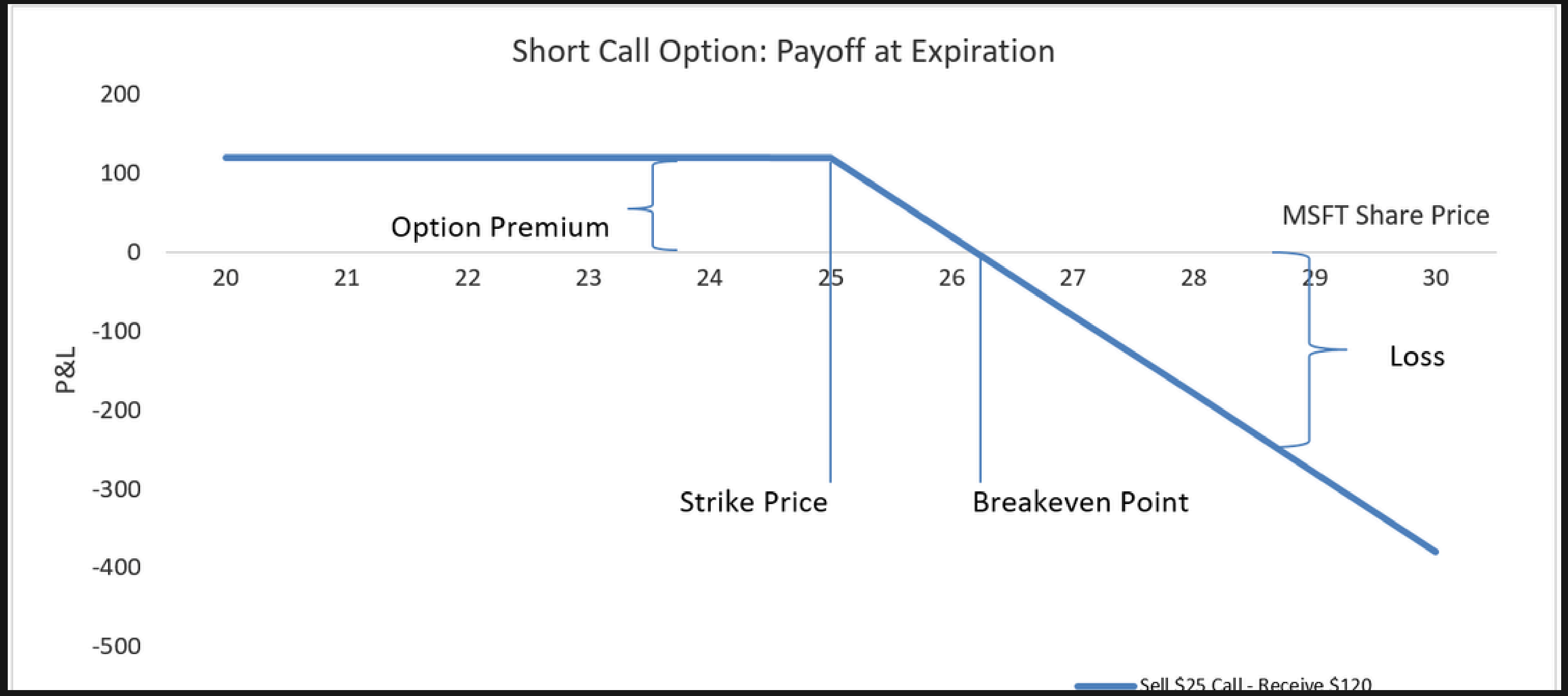
#1 Call option

It is the right to **buy** a particular asset for an agreed amount at a specified time in the future

#2 Put option

It is the right to **sell** a particular asset for an agreed amount at a specified time in the future





Monte Carlo Simulation

Monte Carlo simulation is a powerful and versatile tool used to model complex systems that incorporate uncertainty and randomness. Here we use the Geometric Brownian motion, which assumes that stock prices follow a stochastic process with continuous time and log-normal distribution. We will be looking mainly for the European call options.

Geometric Brownian motion

The stock price evolution is modelled using the following stochastic differential equation (SDE) under the GBM framework:

$$dS=\mu*S*dt+\sigma*S*(dW_t)$$

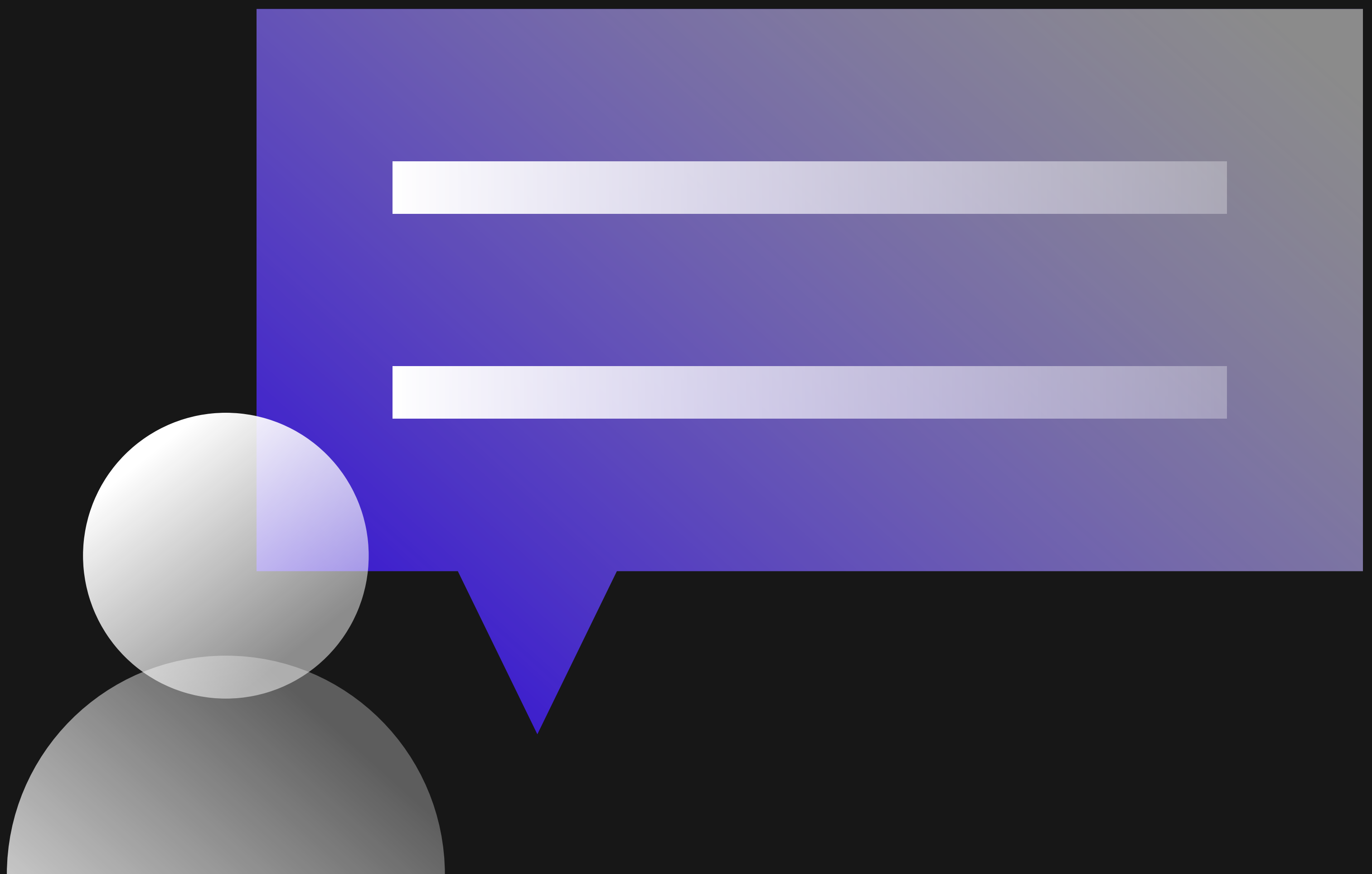
$$S_T=S_0 \exp[(\mu-\sigma^2/2)T+\sigma*W_t]$$

Wiener Coefficient (Wt)

Wt (Wiener process) is the source of randomness in the GBM model. The Wiener process captures the random fluctuations in the stock market. It goes after the fact that stock markets experience unpredictable movements in small intervals of time. These movements have no consistent upward or downward bias.

$$Wt2 - Wt1 \qquad N\,(0,\ t2-t1)$$

Here N is the Normal distribution with mean 0 and variance Δ T. The random shocks of the stock market are captured by the σ*Wt, where Wt are random values generated from normal distribution.



Monte-Carlo approaches Black Scholes

As the number of stock markets simulated reach infinity it approaches the value of Black-Scholes models call option price due to the Law of Large Numbers and Central Limit theorem. The Central Limit Theorem states that under appropriate conditions, the distribution of a normalized version of sample mean converges to a standard normal distribution.

The sudden spikes are not considered in the Black-Scholes Model. These spikes are considered in Monte-Carlo simulation through random normed distribution. If the number of stimulated graphs does indeed reach infinity all the sudden spikes are cancelled making it a risk-neutral measure.

The Monte-Carlo estimate for the call option

$$C_{\mathrm{MC}} = e^{-rT} \frac{1}{N} \sum_{i=1}^{N} \mathrm{Payoff}(S_T^{(i)})$$

The Black-Schole estimate for call option

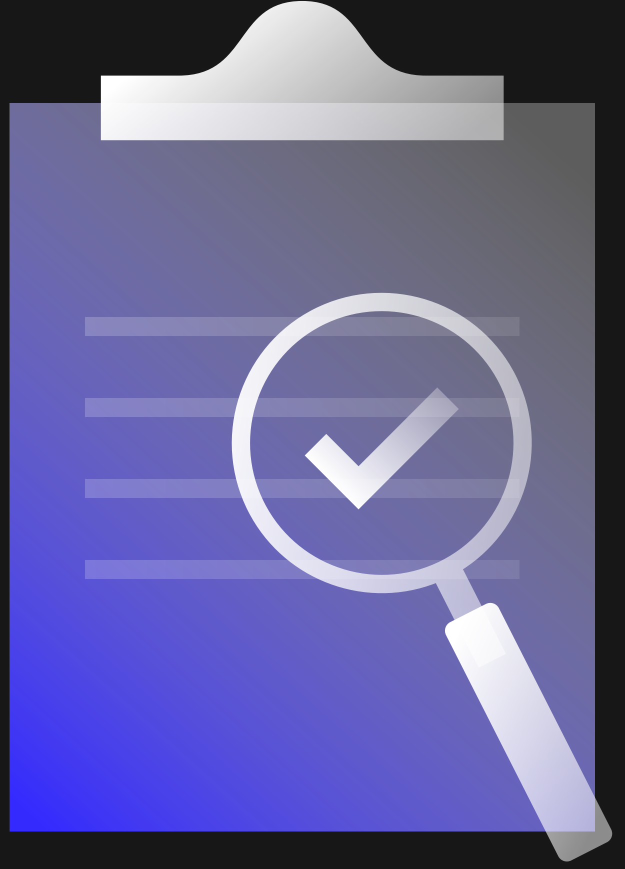
$$C_{\mathrm{BS}} = e^{-rT} \mathbb{E}[\mathrm{Payoff}(S_T)]$$

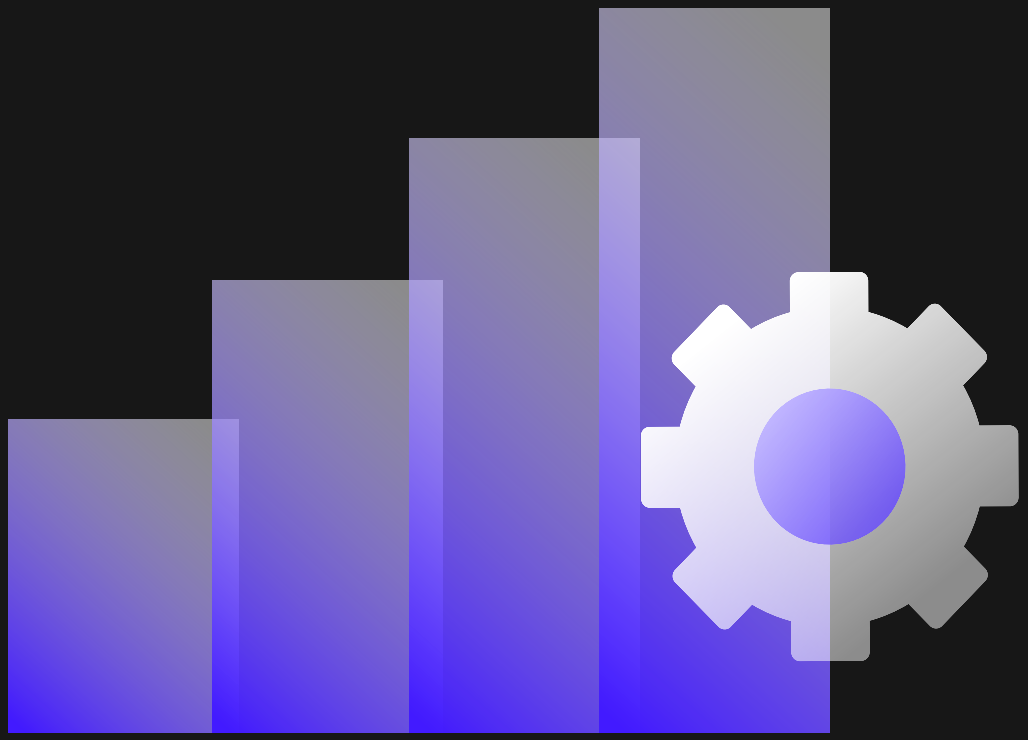
By the Law of Large Numbers, as N tends to infinity, the Monte Carlo Estimate C_{MC} converges almost surely to the expected value of payoff:-

$$C_{\mathrm{MC}} \xrightarrow{a.s.} C_{\mathrm{BS}}$$

By the Central Limit Theorem, the distribution of Monte Carlo estimates approaches normal distribution centered at C_{BS} as the mean and variance decreases as N increases.

$$\sqrt{N}(C_{\mathrm{MC}} - C_{\mathrm{BS}}) \xrightarrow{d} \mathcal{N}(0, \sigma^2)$$





Black-Scholes Model

Black-Scholes-Merton model is a mathematical equation used to determine the fair prices of options given a set of parameters.

$$C(S, t) = S*N(d_1) - Ke^{-rt}N(d_2)$$

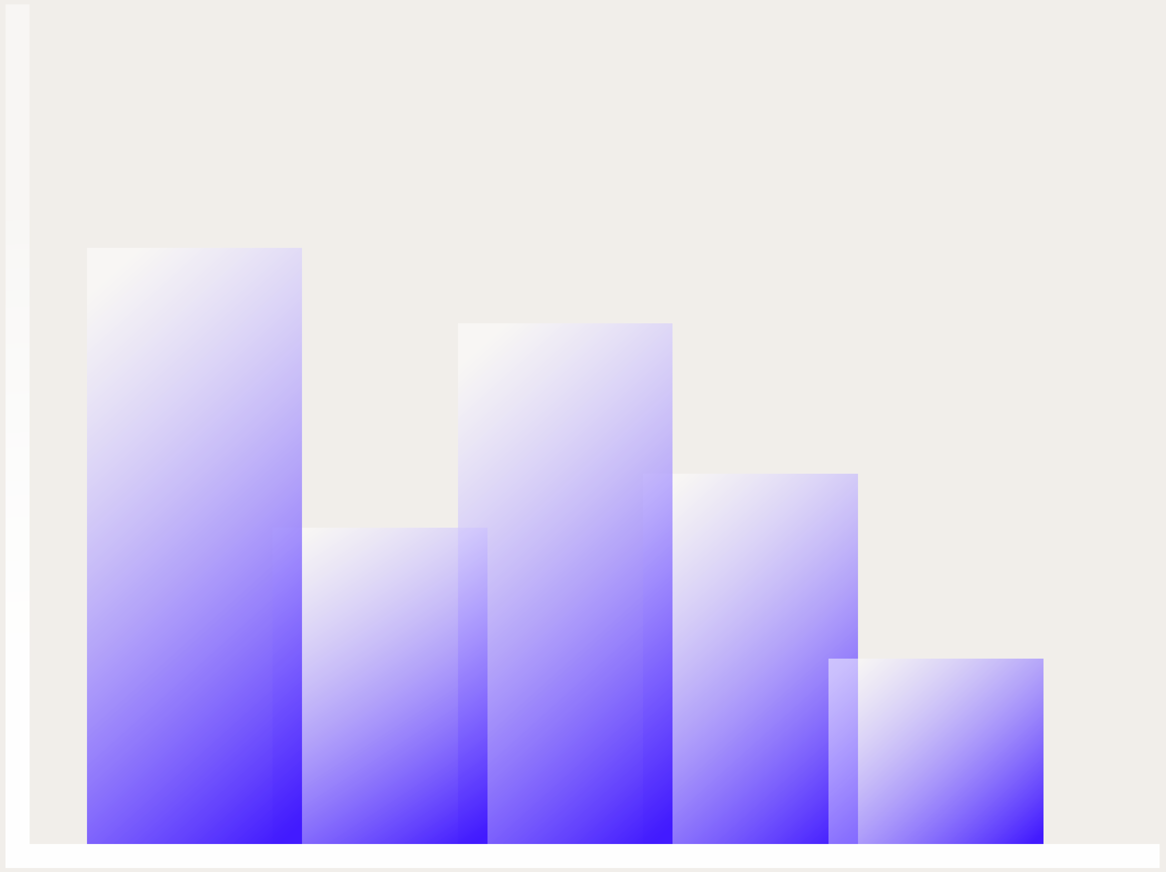
$$P(S, t) = Ke^{-rt}N(-d_2) - S*N(-d_1)$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

- S: asset price (market price)
- t: time to maturity
- K: strike price (price at which the option can be exercised)
- σ: volatility
- r: risk free interest rate (%rate/100)

Black-Scholes Model

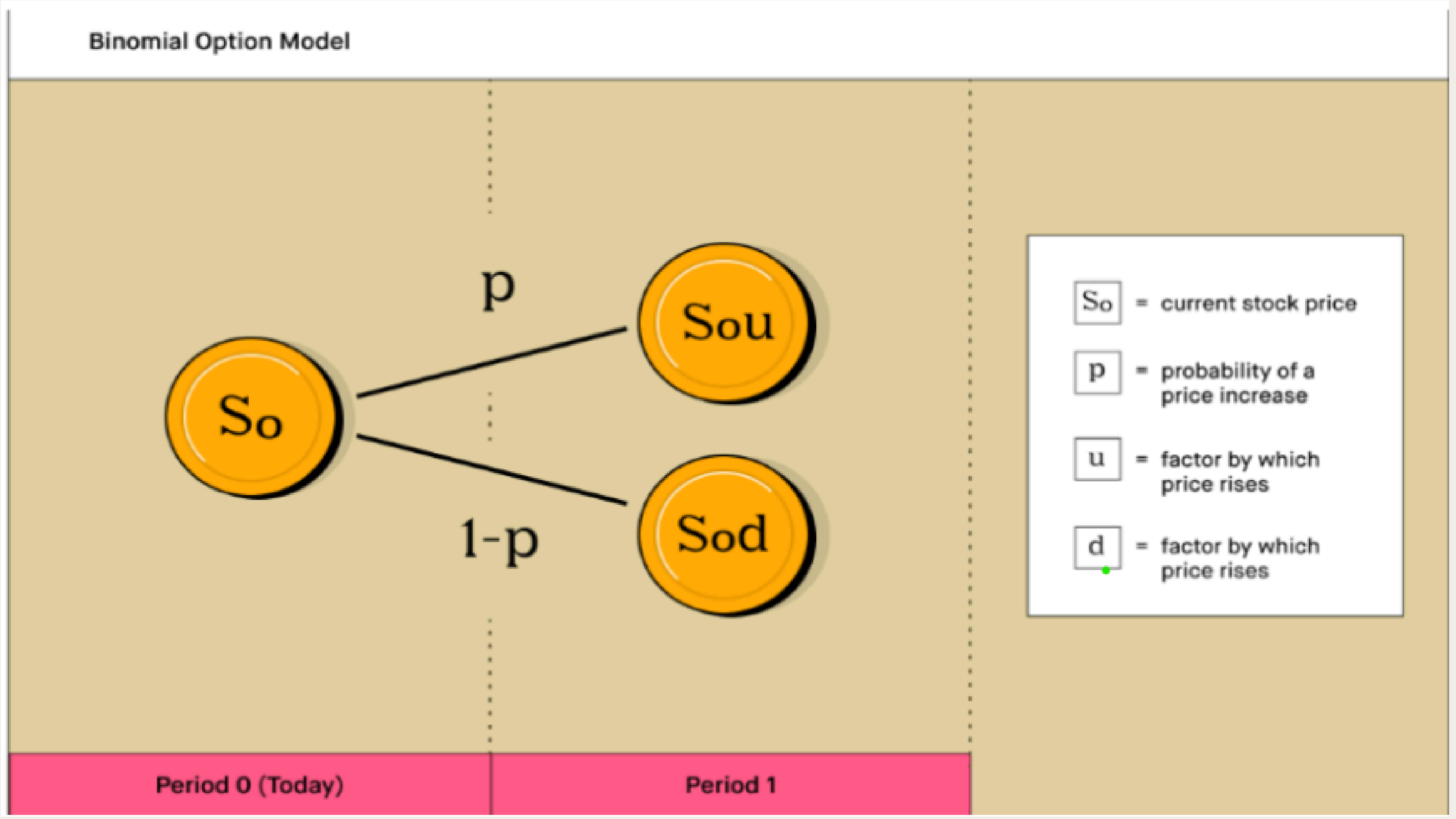


Assumptions of the model:

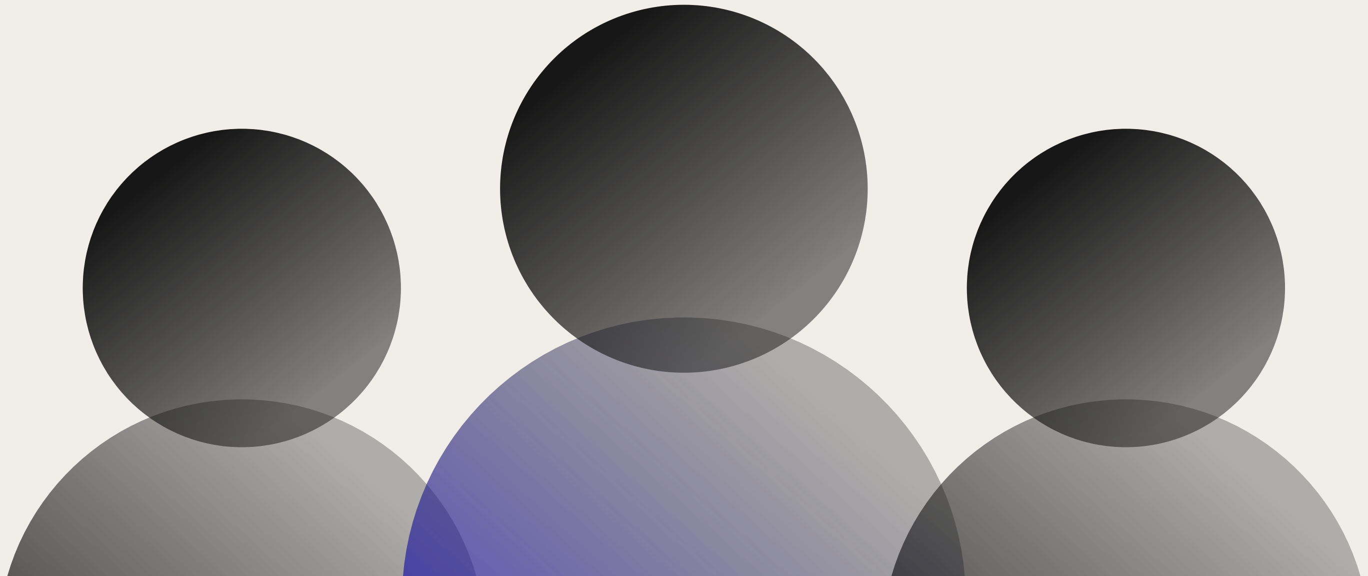
- Constant risk free interest rates and volatality (standard deviation of the stock's logarithmic returns)
- No dividends
- Frictionless market (no processing costs in transactions)
- Normally distributed returns (logarithm of returns is normally distributed)
- European style options

Binomial Pricing

Binomial pricing is a relatively simple and more intuitive approach as compared to Black-Scholes model. In each step, it assumes that the option price can do up(by an up-factor) or down(by a down factor), the down factor is in-effect the reciprocal of the up-factor.



Now, there are also probabilities associated with the price going up or down and both add to 1. The image attached to the right gives the perfect description of how this works.
Further, it breaks the life of a stock into multiple periods and in each period, the above mentioned procedure is applied.



Binomial Pricing

As the number of time steps increase, the Binomial model becomes accurate and also converges to the Black-Scholes model.

The option prices at each previous node (going back in time) are calculated using a discounted average of the values at the next time step. The formula used here is,
(here, r=Risk free interest rate and dt=T/N.)

$$optionValue = e^{-r.dt} \times (p \times valueUp + (1 - p) \times valueDown).$$

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Key Stock Market Terms

Conclusion

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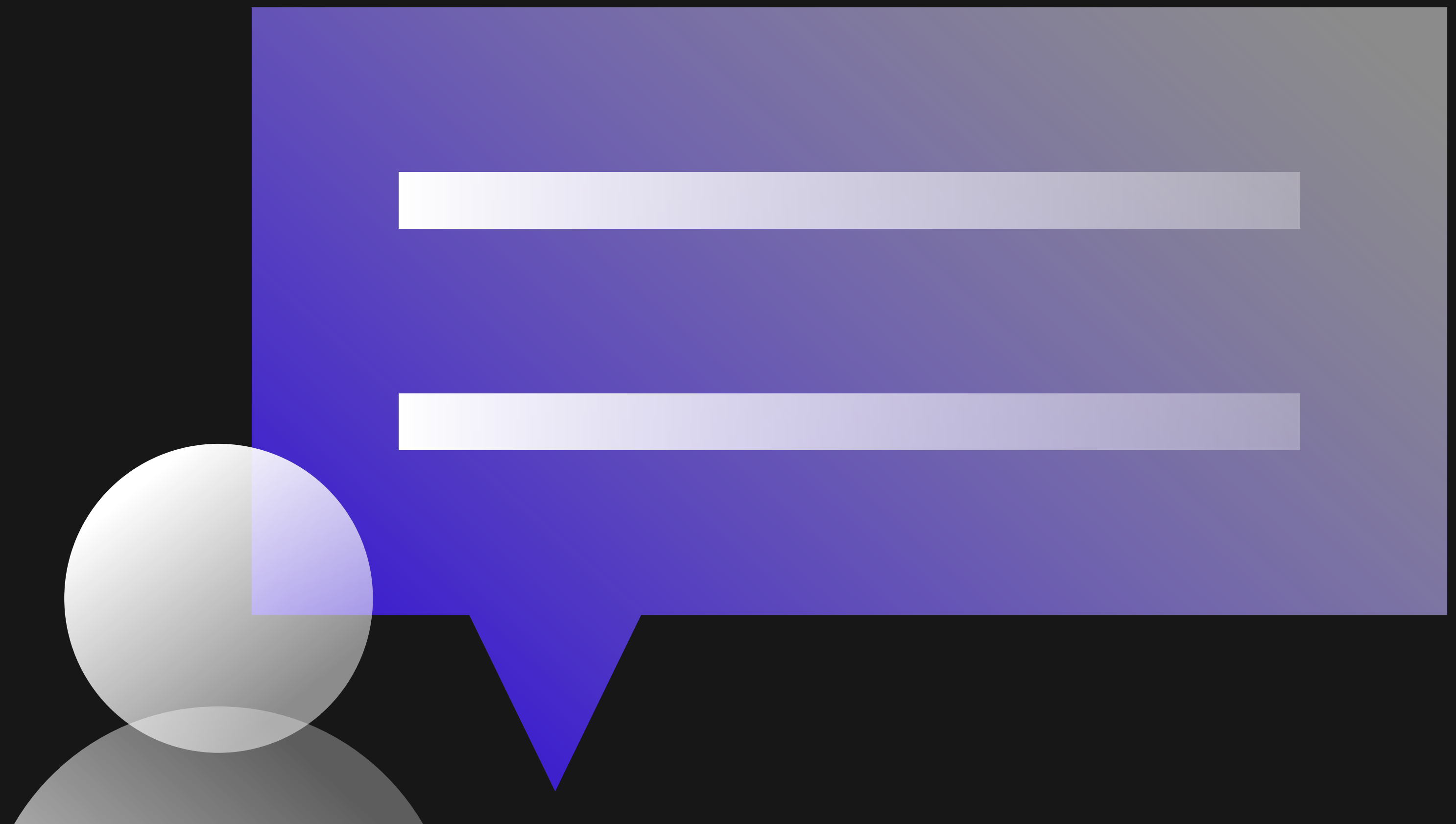

Thank You