

Risk-averse stochastic programming approach for microgrid planning under uncertainty



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ABSTRACT

In the planning of isolated microgrids aiming for a small carbon footprint, the penetration of renewable energy resources is expected to be high. Energy supply from renewable sources are highly variable and renewable energy sources have relatively a large capital investment although with a positive impact on the environment. In planning and designing of renewable energy based microgrids, we introduce the approach of two-stage stochastic programming to incorporate the various possible scenarios for renewable energy generation and cost in the planning of microgrids to tackle uncertainty. Most planning problems are similar to portfolio optimization problems. We wish to minimize risk in the investment due to uncertain nature of the resources and also minimize the expected cost of investment. Therefore, we introduced the idea of *Markovitz* (mean-variance) objective function to minimize the effect of uncertainties in the operation of the microgrid. The model is generic and can be used for any location to suit their geographical topography and demand/supply needs. The result shows the economic advantage of using the risk-averse stochastic programming approach over the deterministic approaches while satisfying environmental objectives.

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1. Introduction

Global environmental concerns and the ever increasing need for energy, combined with steady progress in renewable energy technologies have provided immense thrust in industry and academia to explore solutions for energy, which are cheap, environmentally friendly, reliable and self-sustaining. Extensive research has been carried out in the past few decades towards the design of systems which encompass the above-mentioned features. Hybrid power systems (HPS) use solar, wind, bio gas, hydro power and other renewable sources of energy with or without grid connectivity; in this paper we consider HPS without grid connectivity and using only wind and solar renewable energy technologies. These HPS are also referred to as *Microgrids*. A large spectrum of mathematical tools has been employed in an attempt to find an optimal mix of such resources to develop reliable systems. In Ref. [1] Ofry et al. developed a graphical method based on the loss of power supply probability to design a stand-alone solar electrical system. The idea adopted by Ref. [1] was to minimize a linear cost

function comprising the cost of the battery and the solar arrays.

In Ref. [2], the author presented an analytical technique for the design of standalone solar and battery systems. They present an analogy between the battery storage and reservoir, queues and stocks and approached the problem by formulating the energy deficit as *Markov process*. They discretized the probability distribution for the energy deficit and solar power generated and converted them into finite states and formulated a state transition probability matrix. A similar approach was adopted by Chandy et al. [3], where they discretized the battery state and modeled it as a Markov process. They considered the state of energy deficit as an absorbing state and hence, at any instant, evaluated the probability of loss of power supply by finding the probability of the absorbing state. Similar stochastic models are already in use in hydrology for modeling a reservoir, which has a direct analogy to a battery in our case. In Ref. [4], Ponnambalam et al. presented an analysis of a multi-reservoir system based on the development of first and second moment expressions for the stochastic storage state variables. The expressions in Ref. [4] give explicit consideration to the maximum and minimum storage bounds in the reservoir system. Their formulation provided analytical results for various parameters such as the variance of storage, reliability levels and failure probabilities, which are of also of significant importance to a power

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system. The ideas of using indicator functions from Ref. [4] was extended in Ref. [5] to use the F-P (Fletcher-Ponnambalam) method for the capacity design of a battery bank in renewable energy systems with constant demand and uncertain supply.

Optimization especially with integer variables is always a challenging task, so global optimization techniques, such as Genetic Algorithms or Evolutionary Algorithms are employed extensively in the design of HPS. In Ref. [6], the authors use the genetic algorithmic framework for optimal sizing and operation of an HPS. Given the non-linearity in the system model and the system components, it becomes a very difficult and challenging optimization problem. In Ref. [6], the authors divided the algorithm into two parts: one for the optimal sizing and the other for the optimal operation of the HPS. This results in an optimal selection of a HPS configuration and an operating strategy for the given site. Genetic Algorithms have also been used in Ref. [7] for distributed energy resource selection, sizing and effective coordination. The problem was formulated as a mixed integer non-linear problem which minimizes the total capital cost, operational and maintenance cost subject to constraints as energy limits, emission limits, and loss of power supply probability. Simulated Annealing based approach for optimal sizing and siting is used in Ref. [8].

A report by the Consortium for Electric Reliability Technology Solutions (CERTS) [9], further explains the concept of Microgrids. The CERTS Microgrid concept assumes an aggregation of loads and micro sources operating as a single system providing both power and heat. The majority of the micro sources must be power electronic based to provide the required flexibility to ensure operation as a single aggregated system. This control flexibility allows the CERTS Microgrid to present itself to the bulk power system as a single controlled unit that meets local needs for reliability and security. This report is the basis for many research articles published recently in the context of microgrids and RES.

In Ref. [10], a more recent algorithm, DIRECT (Dividing Rectangles), was used to solve the horizon planning optimization problem for sizing of a wind/PV system. DIRECT was developed by Ref. [11] as a global optimization method. It is an effective deterministic algorithm [10]. It finds the minimum of a Lipschitz continuous function without knowing the Lipschitz constant. In DIRECT an assumption is made that the rate-of-change of the objective function and constraints are bounded. In brief, the entire search space is divided into a set of rectangles, and optimal direction is determined by evaluating the objective function at the center points of the subdivided boxes. In this case, they used a few varieties of renewable energy sources types and capacities to choose from, but it made the search space high dimensional.

In another approach to handling uncertainty in renewable sources [12], applied stochastic optimization to identify the size of the storage in isolated grids with a wind-diesel HPS. Energy storage is important in such systems as it is a means for optimizing the energy use and for reducing the consumption of the diesel fuel. An important inference of the work is that the storage size and cost of delivered energy is dependent on wind penetration levels, storage efficiency and diesel operating strategy. They considered various scenarios of wind and demand profiles. They also employed the two-stage stochastic programming technique where the first stage variables being power rating and energy rating of the energy storage along with the initial energy storage, whereas the second stage variables constituted the diesel generator power, dump load, binary variables associated with the diesel generator dispatch and the energy discharged from the storage at any given instant of time.

An integrated approach to solve the problem of PV-Wind-Diesel-Battery HPS planning has been presented in Ref. [13]. They address the problem as a multi-objective optimization problem with two objectives: minimizing the total cost and minimizing the

total CO₂ emissions, while capping the expected unserved energy. Direct and indirect assessments of emissions of all the components are obtained using life cycle assessment (LCA) techniques. They apply the approach to a city with 50,000 thousand residents. The results obtained from the linear programming model were used to construct the Pareto front, which represented the best trade-off between cost and emissions under different reliability conditions.

There have been numerous attempts for the design of micro-power systems using various open source applications; HOMER [13] developed by National Renewable Energy Laboratory is commonly used. It performs techno-economic analysis and prioritizes solutions based on cost. One of the successful attempts towards microgrid design using HOMER is [14]. Unfortunately, the software has many approximations and assumptions which need to be addressed using a detailed mathematical formulation to handle the uncertainty in the renewable energy sources and demand.

Microgrids are not much different from distribution systems [15]; the planning problem including the renewable energy sources (RES) in the distribution system is modeled as a mixed integer non-linear programming problem with an objective of minimizing the systems' annual energy loss. The constraints in the optimization model include the voltage limits, the feeders' capacity, the maximum penetration limit and the discrete size of the available DG units. The technique has been employed in various rural scenarios and results have shown a considerable reduction in costs.

The approach developed by Ref. [16] claimed to have a two-stage stochastic programming model for planning and operation of the distributed energy systems, which is proposed here in this paper. Their title however, is misleading as they do not use stochastic programming to solve their model, but utilized a two-stage decomposition and a genetic algorithm to solve the planning problem. They employed the standard approach of Monte Carlo simulations to deal with the uncertainty in the second stage. A detailed comparison of the results was done to compare the benefits of the model as compared to the deterministic one. In Ref. [17], they developed a comprehensive microgrid planning model considering uncertainties. They used a robust optimization approach by decomposing the problem into two parts: the investor problem and the operations problem. Their results were promising, but risk was not considered explicitly in their formulation. Lastly, in Ref. [18], a comprehensive review of computational techniques used in the planning of microgrids is discussed. Our research fills in the gap in this area by considering the uncertainty and risk in the planning of microgrids with renewable energy technologies.

In section 2 we describe the mathematical model for planning of microgrid, present our computational results in Section 3 and conclude with directions for future research in Section 4.

2. Mathematical system model

2.1. Stochastic two-stage model for the optimal design of microgrid

Deterministic models lack the ability to handle uncertainty in demand and supply of renewable energy in the planning of microgrids. Therefore, we use a two-stage stochastic programming with recourse for the planning of microgrid under uncertainty. The stochastic nature is captured in the model in the form of scenarios of the random variables. Our first stage design variables are for determining the capacity of solar power, wind power, diesel generation and storage. The recourse or the second stage decisions are the operating variables to decide the amount of power that needs to be generated from the diesel generators and/or supplied from the batteries. It is important to note that there is an optimal solution for each uncertain energy scenario and this is captured using these second stage variables. Therefore, minimizing the capital

investment cost along with the operating costs seems to be a feasible approach.

Fig. 1, shows an example of a microgrid with renewable generation, traditional diesel generation, battery storage and load.

The two-stage stochastic programming model for planning of microgrids is presented below. Equations (2-1)–(2-17) constitute the mathematical formulation of the two-stage stochastic optimization problem for the planning and operation of the microgrid.

In the model below N_{DG} is the total number of non-renewable generators, N_{WT} is the number of wind turbines, N_{PV} is the number of PV panels, IC_{DG} (diesel generation), IC_{PV} (PV panels), IC_{WT} (wind turbine), IC_{BAT} (batteries) is the annual installation cost of generation sources and storage (\$) considering the entire service of each component. Annual operation and management cost for generation sources is given by OM_{DG} , OM_{PV} , OM_{WT} , OM_{BAT} in (\$/year) and FC_{DG} is the fuel cost in \$/kWh. The charging power Chb and power supplied by all the batteries $DChb$ is given in kW respectively. The maximum allowable charge of a battery is C_{MAX} (kWh). The power output from generation sources are O_{DG} , O_{PV} and O_{WT} (kW). The demand D is in kW while D_{US} is the unserved demand in kW, C_{US} is the cost of unserved power in \$/kW. C_{TAX} and C_{INT} are the carbon tax in (\$/kg) and CO_2 intensity in kg/kWh. The rated capacity of diesel generator and battery are given by RC_{DG} and RC_B (kW). The energy stored in the battery is Ce (kWh). The self-discharge and discharge efficiencies of a battery are γ_{sd} , γ_d in %. The maximum allowable depth of discharge of a battery in percentage is DOD_{MAX} . T_b is the life in terms of charging-discharging cycles of a single battery. It is important to note here that the aging of technologies such as wind turbines, solar panels, batteries, and diesel generation is considered in their annual installation cost with a discount rate of 8%.

The Stochastic two-stage model for planning is presented in Equations (2-1)–(2-18).

Objective Function

$$\begin{aligned} \min & \bar{N}_{PV}(IC_{PV} + OM_{PV}(N_{PV})) + \bar{N}_{WT}(IC_{WT} + OM_{WT}(N_{WT})) \\ & + N_{DG}(IC_{DG} + OM_{DG}(O_{DG})) + \bar{N}_{BAT}(IC_{BAT} + OM_{BAT}) + \frac{1}{ns} \left(\sum_t \right. \\ & \times \sum_s (C_{us} D_{us_t}^s) + FC_{DG} \left(\sum_s \sum_t O_{DG_t}^s \right) + \left(\sum_s \right. \\ & \times \sum_t O_{DG_t}^s \cdot C_{Tax} \cdot C_{Int} \left. \right) \left. \right) \end{aligned} \quad (2-1)$$

1) Supply-Demand balance at any time interval is given as: power demand is equal to the sum of power supply from RES (O_{PV} and O_{WT} (kW)), diesel (O_{DG}) and battery ($DChb$). A certain amount of energy may go unserved (D_{US}) or extra energy beyond the storage capacity may need to be dumped.

$$\begin{aligned} D(t, s) = & O_{DG}(t, s) + N_{PV} \cdot O_{PV}(t, s) + N_{WT} \cdot O_{WT}(t, s) \\ & + DChb(t, s) + D_{us}(t, s) - Chb(t, s) \end{aligned} \quad (2-2)$$

$$\forall t \in 1..time, s \in 1..ns$$

2) Diesel Generator output limit: Power output of the diesel generators O_{DG} cannot exceed their gross rated capacity (RC_{DG}) where $SR(t, s)$ is the spinning reserve. Here spinning reserve is also a function of the s as we need to make sure the desired quantity of power is available in each scenario.

$$O_{DG}(t, s) + SR(t, s) \leq N_{DG} \cdot RC_{DG} \quad \forall t \in 1..time, s \in 1..ns \quad (2-3)$$

3) Battery State of Charge (SoC): An energy conservation constraint which links the energy stored in the battery at any time with charging and discharging processes. Equation (2-16) prevents the simultaneous occurrence of charging and discharging.

$$\begin{aligned} Ce(t, s) = & Ce(t-1, s) \cdot (1 - \gamma_{sd}) + Chb(t, s) \\ & - \frac{DChb(t, s)}{\gamma_d} \quad \forall t \in 1..time, s \in 1..ns \end{aligned} \quad (2-4)$$

4) Battery capacity limits: The battery has an upper limit with respect to its SoC, there is a maximum depth of discharge (DOD) that can be reached in a cycle

$$(1 - DOD_{MAX}) \cdot C_{MAX} \cdot N_B \leq Ce(t, s) \leq C_{MAX} \cdot N_B, \quad \forall s \in 1..ns \quad (2-5)$$

5) The electric power charged to or discharged from the battery cannot exceed its rated capacity

$$Chb(t, s) \leq N_B \cdot RC_B, \quad \forall t, s \quad (2-6)$$

$$DChb(t, s) \leq N_B \cdot RC_B, \quad \forall t, s \quad (2-7)$$

6) Reliability Constraint: It is to ensure that a desired loss of load doesn't exceed the maximum allowed expected unserved energy limit (EUE_{MAX})

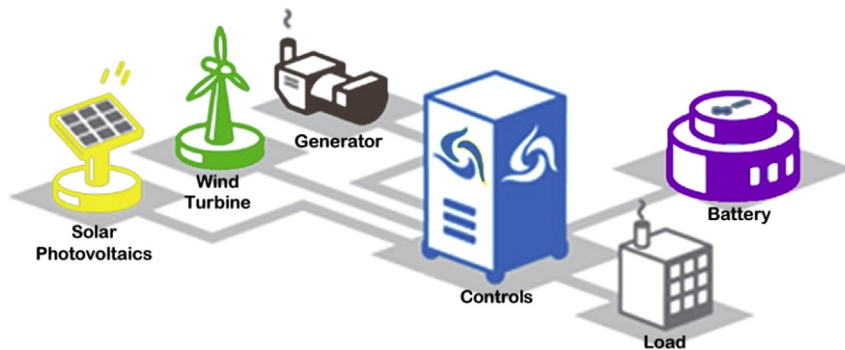


Fig. 1. A pictorial representation of a microgrid.

$$\frac{\sum_{t=1}^{8760} D_{us,t}^s}{\sum_{t=1}^{8760} D_t^s} \leq EUE_{MAX}, \quad \forall s \in 1..ns \quad (2-8)$$

7) Budget/Resource Constraints: To make sure the number of RES and storage units used in the system do not go beyond budget and the solution is practical. This constraint may also be modeled using the monetary value of the components.

$$N_{(DG,min)} \leq N_{DG} \leq N_{(DG,max)} \quad (2-9)$$

$$N_{(PV,min)} \leq N_{PV} \leq N_{(PV,max)} \quad (2-10)$$

$$N_{(WT,min)} \leq N_{WT} \leq N_{(WT,max)} \quad (2-11)$$

$$N_{(BAT,min)} \leq N_{BAT} \leq N_{(BAT,max)} \quad (2-12)$$

8) Initial State of Battery in each scenario

$$Ce_t^s = \frac{C_{MAX} \times N_{BAT}}{2} \forall s \in 1..ns, t = 1 \quad (2-13)$$

Another major component considered as a part of the modeling of such a system was the renewable energy penetration. As the capital cost of diesel generation is quite low as compared to the renewable resources of energy the optimizer would tend to provide most of the energy from it. Hence we explicitly need to specify and optimize the percentage contribution of the renewable source along maintaining the specified reliability measure as presented next.

9) Minimum penetration levels of RES

$$\sum_{t=1}^{time} N_{wt} * O_{wt}(t, s) + \sum_{t=1}^{time} N_{pv} * O_{pv}(t, s) \geq RP * \sum_{t=1}^{time} (D(t, s) - D_{us}(t, s)) \quad \forall s \in 1..ns \quad (2-14)$$

where RP is the percentage penetration level of renewable energy.

These above constraints force the optimizer to ensure a required penetration level of renewable generation in the microgrid.

10) Spinning Reserve

$$SR(t, s) \geq SRmin * D(t, s) \quad \forall t \in 1..8760, s \in 1..ns \quad (2-15)$$

11) Preventing simultaneous charging and discharging of batteries: This constraint eliminates the possibility of the simultaneous charging and discharging of the batteries

$$Chb(t, s) \times DChb(t, s) = 0 \quad \forall t, s \quad (2-16)$$

12) Energy to Power Ratio (E/P): The energy capacity of any battery for a certain power rating is determined based on its E/P ratio which is a relationship between the power and the energy size for a certain energy storage technology. The energy size represents the maximum amount of energy that can be stored for a certain time. Whereas the power size is the rate at which the energy storage is capable of discharging/charging power continually.

$$EPR \text{ or } \frac{E}{P} = \frac{\text{Energy Capacity, kWh}}{\text{Power Rating, kW}} \quad (2-17)$$

$$\underline{EPR} \cdot N_B \cdot RC_B \leq Ce(t, s) \leq \overline{EPR} \cdot N_B \cdot RC_B \quad \forall t, m \quad (2-18)$$

2.2. Quantification of the uncertainties

In this study, three types of parameters, with significant volatility are selected as uncertain parameters i.e. energy demands, solar energy, and wind energy availability. The probability distributions of these parameters are quantified. We utilized the Kumaraswamy distribution, which is equivalent to the beta distribution but with a simple analytical form allowing for fast simulation of random variables from the distribution. Any well-known mathematical distribution can be easily modeled using the either Beta or Kumaraswamy distribution with variation in their parameters. We modeled all the parameters considering the hourly and seasonal variations (autumn, winter, and spring). The data used in our study is for the Greater Toronto Area (GTA) in Ontario, Canada. The demand data is obtained for a hypothetical rural community with a peak demand of 17 kW and an average daily energy demand of 220 kWh/day. For an accurate modeling of wind resources, we utilized copula-based dependence modeling with Kumaraswamy distribution as the marginal distribution [19].

There is no standard distribution for energy demand, solar energy and wind energy. Advantages of using Kumaraswamy distribution over the beta distribution are [20]:

- a simple explicit formula for its distribution function not involving any special functions;
- a simple and explicit formula for the quantile function;
- as a consequence of the simplicity of the quantile function, a simple formula for random variate generation;
- explicit formulae for L-moments;
- simpler formulae for moments of order statistics.

The Kumaraswamy distribution is given by the Equations (2-19) and (2-20), where $f(x)$ is the PDF, and $F(x)$ is the CDF.

$$f(x) = abx^{a-1}(1-x^a)^{b-1} \quad (2-19)$$

where $a \geq 0$, $b > 0$ and $x \in [0,1]$

$$F(x) = \left[1 - (1 - x^a)^b \right] \quad (2-20)$$

2.3. A risk-averse model for microgrid planning using the Markovitz approach

Risk affects the planning of power transmission systems, distribution systems, and microgrids; most important are the economic and the environmental risks. The Markovitz approach [23] considers variance of costs or benefits as indicative of risk; the higher this value, the higher the risk. The recent literature in planning and design of micro power system do mention uncertainties, but most of them do not consider risk as a part of the design. Even though [12,16,21,22] state the importance of considering risk in system design, they do not consider risk explicitly in the design process when using either global optimization techniques or Monte Carlo simulations.

We extend our stochastic two-stage model to consider risk by

incorporating the ideas from portfolio optimization. We modify the objective function of Equation (2-1) to a new objective function as per Markovitz objective function which considers risk explicitly [23]. Use of risk averse modeling is not found in any power system planning studies of microgrids. We utilized the theory of [23] in our objective function as below:

As the variance of the costs is reflected by the second stage variables we evaluate the variance of the second stage variables using the scenarios. We assume the second stage objective function is denoted by A then the new second stage objective function would become Equation (2-24) explained through Equations (2-21)–(2-23)

$$\text{Minimize } A + \theta \sqrt{\text{Var}(A)} \quad (2-21)$$

where $\text{Var}(A)$ is given by the following Equations

$$\text{Var}(A) = E[A^2] - (E[A])^2 \quad (2-22)$$

where the second term is a simple square of the earlier objective function while the first term is evaluated as following:

$$E[A^2]_s = \sum_{s=1}^n A_s^2 \times \text{Pr}(s) \quad (2-23)$$

where $\text{Pr}(s)$ is the probability of occurrence of scenario s , and A_s^2 is the cost function squared for each scenario.

Based on Equations (2.21)–(2-23), our new objective function with risk transforms into a non-linear objective function with risk as shown in Equation (2-24)

$$\begin{aligned} A &= \bar{N}_{PV}(IC_{PV} + OM_{PV}(O_{PV})) + \bar{N}_{WT}(IC_{WT} + OM_{WT}(O_{WT})) \\ &\quad + N_{DG}(IC_{DG} + OM_{DG}(O_{DG})) + \bar{N}_{BAT}(IC_{BAT} + OM_{BAT}) \\ B &= \frac{1}{n} \left(\sum_t \sum_s (C_{us} D_{us}^s) + FC_{DG} \left(\sum_s \sum_t O_{DG}^{s,t} \right) \right. \\ &\quad \left. + \left(\sum_s \sum_t O_{DG}^{s,t} * CTax * CInt \right) \right) \\ \min E[A + B] + \theta \text{Var}(B) \end{aligned} \quad (2-24)$$

All the constraints remain the same. The new objective function considers the variance of the uncertainties explicitly in the objective function.

3. Results and analysis

To evaluate the microgrid configuration which satisfies our technical, economical and environmental constraints a detailed sensitivity analysis needs to be performed. We evaluate the importance of using a two-stage stochastic programming approach using two main indices used in the literature on stochastic programming namely, Value of Stochastic Solution (VSS) and Expected Value of Perfect Information (EVPI). These two indices are widely used to evaluate the significance of using the stochastic programming approach.

We present here a case study for planning of microgrids using the two-stage stochastic programming framework presented in Section 2.1. We assume a hypothetical rural community with scaled peak power and average daily energy consumption as provided in Ref. [14], and is presented in Fig. 2. To use accurate and reliable data for solar and wind we used the data available in Ref. [24] for the Greater Toronto Area (GTA), in Ontario, Canada. Various parameters

such as rated capacities, min and max values, efficiency of storage etc., used in the optimization are given in Tables 1–4.

3.1. Comparing deterministic and stochastic objective values

It is difficult the gauge the benefit of stochastic model over the deterministic model by simply observing the objective function values. Therefore, even if these two models come out with about the same optimal objective value, one does not know much about whether or not it is wise to work with a stochastic model. We evaluate the benefit of using the stochastic model over the deterministic model based on two common metrics, the expected value of perfect information (EVPI) and the value of stochastic solution (VSS) [25].

Evaluation of EVPI and VSS requires solutions from both the stochastic and the deterministic models. The solution from the deterministic model with random parameters replaced by their expected values is called the expected value (EV) solution. The results from the stochastic model are referred to as recourse problem (RP) solution. Lastly, using the RP model and by fixing the first stage variables to the solution obtained from the EV solution and calculating the objective function value similar to the stochastic model is the EEV solution. Evaluation of the average deterministic solution over all possible scenarios using the Monte-Carlo simulations is called the wait-and-see solution (WSS).

3.2. Expected value of perfect information

The difference between the two objective values, RP and WSS, is called the expected value of perfect information (EVPI). EVPI measures the maximum amount a decision maker would be ready to pay in return for complete and accurate information about the future. In other words, EVPI can be considered as the cost of using prediction techniques, so a decision maker gains more information about the future outcome of the stochastic parameters. EVPI can be calculated using Equation (3-1).

$$\text{EVPI} = \text{RP} - \text{WSS} \quad (3-1)$$

A larger value of EVPI indicates that randomness plays an important role in the problem.

3.3. Value of stochastic solution

The value of Stochastic Solution or VSS is computed by subtracting the solution of the recourse problem (RP) from the solution obtained from EEV problem. It justifies the application of the stochastic model especially when it becomes fairly high [25]. The value of VSS is an indicator of the impact of uncertain variables on the solution. Mathematically, VSS can be computed according to Equation (3-2).

$$\text{VSS} = \text{EEV} - \text{RP} \quad (3-2)$$

3.4. Scenario generation for stochastic programming

Two-stage stochastic programming problem needs to deal with the issue of scenario generation. In our problem, the first stage variables are the capital investment decisions for installation capacity of wind, solar and diesel generation with storage; whereas the second stage variables are the operating variables. The uncertain parameters are the demand of electricity and the renewable energy available at any location. We modeled the uncertain parameters using Kumaraswamy distribution. We use the same

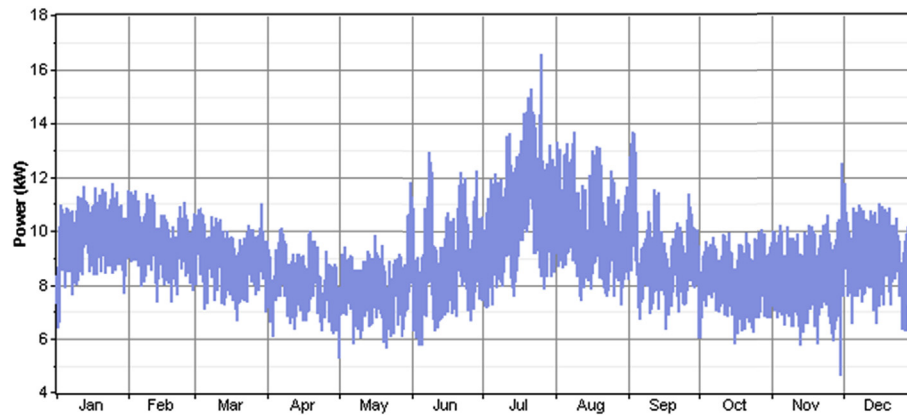


Fig. 2. Mean annual demand profile for the hypothetical rural community.

Table 1

Input data for the cost of options available.

Options	Capital cost	Replacement cost	O&M cost	Life
Wind	\$7800/turbine	\$9000/turbine	\$30/year	15 years
Solar	\$7.5/W	\$7.5/W	0	20 years
Battery	\$75/battery	\$75/battery	\$2/battery/year	845 kWh
Grid extension	\$2000/km	\$2000/km	\$100/km/year	
Diesel	For a 4.25 kW–12.5 kW \$2550 - \$7000	\$2550	\$0.15/h	5000 h

Table 2

Equivalent annual cost, life, and sizes of various components.

Options	Capital cost	Annualized cost	Life	Size
Wind	\$7800/turbine	\$678.5/turbine or \$226.16/kW	20 years	3kW/Turbine
Solar	\$1350/panel	\$117.31/panel or \$7.5/W	25 years	180 Wp
Battery	\$75/battery	\$6.51/battery or \$0.09/kWh	845 kWh	225 Ah, 12 V
Diesel	For 8 kW, \$5500	\$477.95	5000 h	8 kW

Table 3

Battery data for optimization.

Battery type	Li-ion
Charging efficiency	95%
Discharging efficiency	95%
Maximum DOD	100%
E/P ratio range	1–6
Battery rating	225 Ah, 12 V

Kumaraswamy distribution for the generation of the scenarios of supply for the microgrid under consideration. We used copula-based dependence model for the generation of wind energy from different locations with Kumaraswamy distribution modeling the marginal distribution [19]; this is a significant work that is presented in an independent work of ours [26]. We modeled the hourly demand of solar energy using the Kumaraswamy distribution and used the same for generating scenarios for the stochastic programming formulation.

Table 4

Values of parameters used in the optimization problem.

Sl.No.	Parameter name	Description	Value
1	FC_{DG}	Fuel Cost in \$/kWh	\$0.264
2	C_{MAX}	Max. allowed charging of battery in kWh	3 kWh
3	C_{TAX}	Carbon Tax in \$/kg-CO ₂	0.04 \$/kg-CO ₂
4	C_{INT}	Carbon emission intensity kg-CO ₂ /kWh	0.758 kg-CO ₂ /kWh
5	RC_{DG}	Rated capacity of each diesel generator in kVA	5 kVA
6	RC_B	Rated capacity of each battery in kW	2 kW
7	γ_{SD}	Self discharge efficiency of the battery in %	0.5%
8	γ_D	Discharge efficiency of the battery in %	95%
9	DOD_{MAX}	Max. depth of discharge of the battery in %	100%
10	SR_{MIN}	Min. spinning reserve required in % of demand	20%
11	$N_{(DG,min)}, N_{(PV,min)}, N_{(WT,min)}, N_{(BAT,min)}$	Min. values of all components	0
12	$N_{(DG,max)}, N_{(PV,max)}, N_{(WT,max)}, N_{(BAT,max)}$	Max. values of all components	NA

The annual mean demand profile for the hypothetical rural community under consideration is shown in Fig. 2. As stated earlier the peak demand is 17 kW and the average annual energy demand is 220 kWh/day.

Fig. 3 and Fig. 4 show the monthly averages for the wind speed and solar radiation. We took this data for the region of Greater Toronto Area (GTA) [24].

The risk is an important factor to consider when investing in projects involving large capital investment. Given the information about the uncertainties in demand and supply, we can generate Mean-Variance frontier using the Markovitz objective function. The data in Tables 1–4 were obtained from recent literature [14,27].

Table 1 lists costs and life of various components used in the optimization model for microgrid planning. Life of a battery is given in kWh which is the lifetime throughput of energy from one battery. It is used as a standard in battery modeling using HOMER. We evaluate the equal annual cost of each component as shown in Table 2.

We used the data for battery from Ref. [28] as shown below in Table 3. It is known that the life of a battery is affected by the depth of discharge and the number of cycles, we consider the depth of discharge as 100% and represent the life of a battery in kWh as described in Ref. [13].

Various parameters (min and max values, rated values) used in the constraints of the optimization problem are listed out in Table 4.

Fuel cost per kWh is considered as \$0.264 as per [21] in the optimization problem. We evaluated N_{PV} , N_{WT} , N_{BAT} , N_{DG} and the total installation cost and operational cost including the fuel cost/kWh for the diesel generators. We introduced the constraints and a penalty on the unserved demand. Budget constraints were introduced concerning the maximum number of solar panels, wind turbines, and batteries. We considered four distinct cases based on [14], to determine the most favorable option for microgrid planning as shown in Table 5.

3.5. Results for the two-stage stochastic programming model

The optimal microgrid designs for the various cases considered above are obtained using the model as described in Section 2-1 and the results are shown in Table 6 for the parameters of sizes are presented in Tables 1–4. The deterministic solutions are the one where just one scenario (corresponding to the mean values) is used in the optimization problem.

In this section, we present the results of our primary two-stage stochastic model vs the standard deterministic model. The best approach to compare these results is by using the metrics defined in Section 3. We shall find the solution of the RP, EEV, and WSS to find the values of EVPI and VSS, which shall be useful for analyzing the advantages of the stochastic solution.

Table 6 presents the objective values of the deterministic and the two-stage stochastic solution for 200 scenarios of random variables. Each scenario comprises of hourly annual data representing 8760 h. The data is generated carefully to keep into

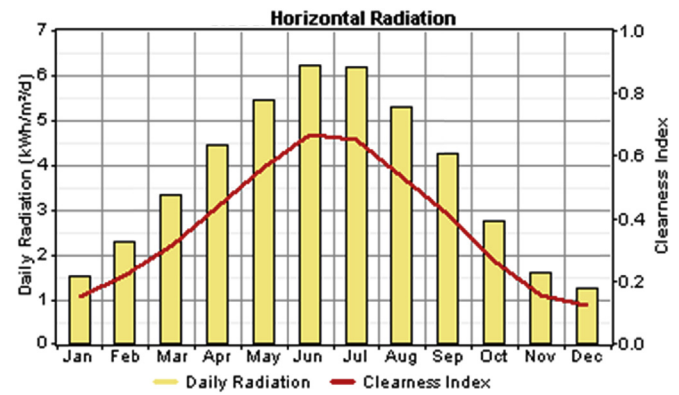


Fig. 4. Average annual profile of solar radiation.

consideration the day/night variations and the seasonal variation in both the supply and the demand [26]. The second stage decisions enable us to compare the impacts of storage and diesel generation regarding costs as well as environmental impacts indirectly through the amount of diesel usage (see also Fig. 5).

It is important to note here that problem size is very large given the number of scenarios, the problem size increases exponentially. In our case for 200 scenarios with hourly data, the total number of variables (battery and diesel generation) spanned to a minimum of $200 \times 8760 = 1,752,000$. The problem is computationally expensive and takes approximately 54 min on a computer with 16 GB RAM, Intel Core i7 processor run using AMPL programming environment.

Based on the solution in Table 6, we can calculate $VSS = EEV - RP = \$3573.28$. It is important to see that VSS indicates a 5% benefit in solving the stochastic problem rather than the deterministic equivalent. The value of EVPI for the above model is $EVPI = RP - WSS$, where WSS was obtained using the Monte-Carlo simulations to be \$34607.20. Hence $EVPI = \$78567.89 - \$34607.20 = \$43961$, thus the uncertainty is significantly costly.

The above results indicate the advantage of using a stochastic approach over a deterministic or simulation based approach. It indicates the advantage of using one approach over the other but also indicates one can save approximately 5% in costs. This is just an example; for larger variances and with larger costs, this saving can be even higher explained further later with results.

We will analyze the important results of Table 7 next, where we do not constraint the renewable energy penetration, but let the optimizer evaluate the penetration with specified loss of load tolerance. In the results presented in Table 6, the renewable energy penetration was set to be at least 80%.

Table 7 presents the results of a comparison between the results of the deterministic model and the stochastic model. Overall, stochastic solutions perform better than the deterministic solution in the cases of high reliability or low percentage of EUEmax (Expected Energy Unserved), where the allowance for demand going unserved is limited to 0% and 10% whereas deterministic solution performs better at low reliability levels or high percentage of EUEmax. This indicates the importance of using the stochastic solution for a higher reliability when there is uncertainty, as indicated by the values of the overall objective function.

The output of the second stage variables, mainly the output from batteries and diesel generators for a 24 h period are presented in Fig. 5. The results from the second stage variable can be summarized where the overall attempt is to provide the demand with 100% reliability and minimize the non-renewable sources of energy's penetration. We see that diesel generation is needed still but its

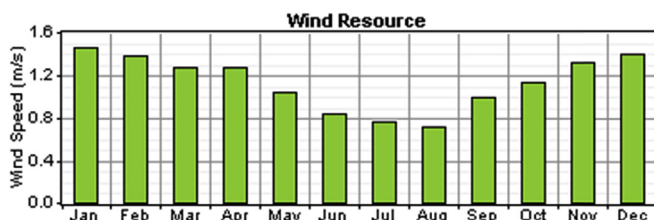


Fig. 3. Average annual profile of wind speed.

Table 5
Summary of cases studied.

Case	Description of the case
1	Diesel dependent microgrid (base case)
2	Renewable based microgrid (wind, solar PV, battery, converter)
3	Diesel-renewable mixed microgrid (diesel, wind, solar PV, battery, converter)
4	Microgrid-connected to external grid

Table 6
Optimization results for main decision variables in varying problem formulations.

	EV	RP	EEV
Demand (kWh/year)	79,999.4	79,999.4	79,999.4
Objective function cost (\$/year)	61,135.94	78,567.89	82,141.17
Diesel, (kWh/year)	2815.73	5830.88	8729.96
Solar, (kWh/year)	12,519.2	18,664.8	12,520.1
Wind, (kWh/year)	125,314	197,320	125,314

random variables as that gives us a better indication of the impact of stochastic programming based solutions over deterministic solutions.

We can see in Fig. 6 and Fig. 7, as we increase the coefficient of variation (CV; defined in Equation (3-3)) for the random parameters (in our case solar and wind energy) we see that the VSS and EVPI both increase demonstrating the importance of using a stochastic programming approach. Here in Equation (3-3), σ is stan-

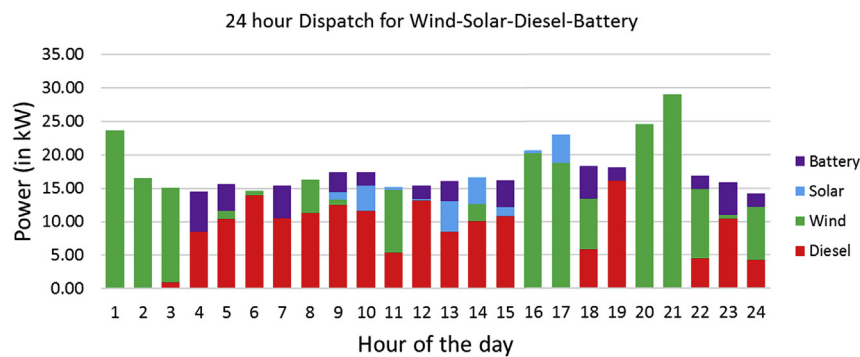


Fig. 5. Power dispatch for a 24 h period using wind, solar, battery, and diesel.

Table 7
Comparison on penetration levels using deterministic and stochastic programming approach.

Component	Method	EUEmax = 0%	EUEmax = 10%	EUEmax = 20%	EUEmax = 30%	EUEmax = 40%
Demand (kWh/year)	Stochastic	79,999.40	79,999.40	79,999.40	79,999.40	79,999.40
	Deterministic	79,999.40	79,999.40	79,999.40	79,999.40	79,999.40
Objective function cost (\$/year)	Stochastic	104,100.00	78,568.00	58,382.00	49,503.00	42,566.14
	Deterministic	114,832.10	83,322.45	53,993.76	41,173.91	37,196.06
Diesel (kWh/year)	Stochastic	14,137.38	5830.88	1523.98	677.67	0
	Deterministic	17,890.09	7804.72	0	0	0
Solar (kWh/year)	Stochastic	18,520.12	18,664.78	16,297.70	15,174.03	13,031.59
	Deterministic	14,344.70	14,601.00	11,444.68	0	0
Wind (kWh/year)	Stochastic	122,019.90	123,948.50	104,361.20	83,800.92	68,287.73
	Deterministic	160,039.40	160,322.90	143,441.60	103,867.70	68,744.95
Unserved power (kWh/year)	Stochastic	0	7871.14	13,789.35	18,802.02	24,310.21
	Deterministic	0	7999.90	15,999.80	23,999.70	31,999.60
Renewable energy contribution	Stochastic	140,540.00	142,613.30	120,658.90	98,974.95	81,319.32
	Deterministic	174,384.10	174,923.90	154,886.30	103,867.70	68,744.95

usage has considerably reduced for the high reliability cases, given the penetration of renewable sources of energy and storage. The importance of second stage variables is also illustrated later.

It is also interesting to note as we move towards low reliability levels (or higher percentage of EUEmax) the diesel generation is reduced to zero and the entire demand is supplied by the renewable resource (wind/solar) with support from energy storage. For higher percentages of EUEmax, we see that stochastic solution provides some energy using diesel generation while reducing the capital investment in renewable technologies.

In addition to the above analysis, one important thing to observe is the variation of VSS and EVPI with increasing variability in the

dard deviation and μ is mean of wind or solar energy, respectively in different figures.

$$CV = \frac{\sigma}{\mu} \quad (3-3)$$

In Figs. 2 and 3 we mention mean of hourly CVs because the CV varies within the hour of the day. Both VSS and EVPI are indicators of the importance of stochastic methods in solving problems with a higher degree of uncertainty in the random variables. An increase in the value indicates the benefit one can obtain from using the stochastic method.

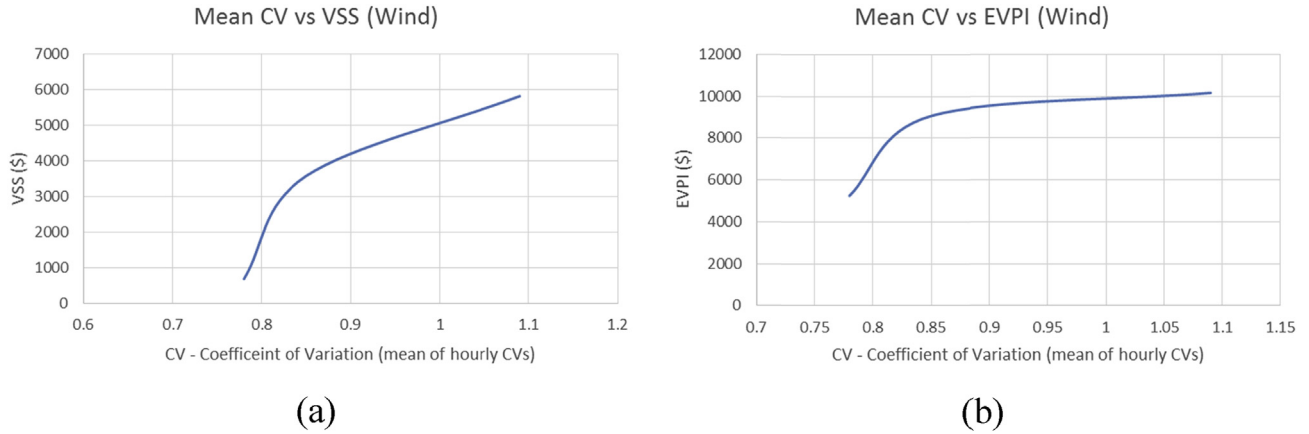


Fig. 6. Variation of (a) VSS and (b) EVPI vs varying coefficient of variation (CV) for Wind.

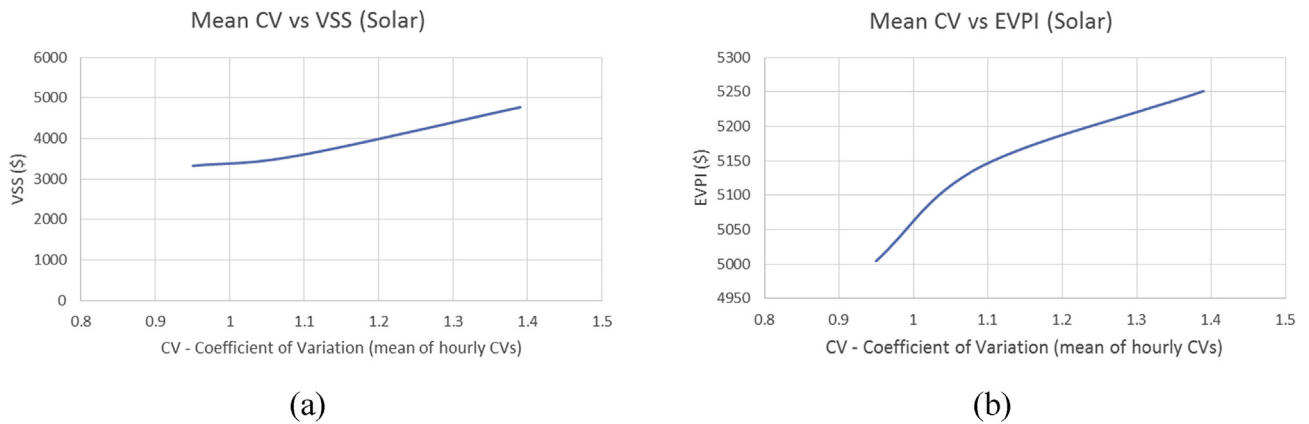


Fig. 7. Variation of (a) VSS and (b) EVPI vs varying coefficient of variation (CV) for Solar.

3.6. Results for the Markovitz model

In this section, we shall see the results of the model using our approach for risk-averse two-stage stochastic programming technique because risk is an important factor to consider when investing in projects involving large capital investment. Given the information about the uncertainties in demand and supply, we can generate mean-variance frontier using the Markovitz objective function. The data used in our problem was obtained from recent literature [14,27].

We shall vary the parameter θ in Equations 2–21, between 0.01 and 0.9 and obtain a mean-variance frontier and unspecified penetration levels for solar and wind energy systems. The parameter θ does not have any physical significance; it is used for analyzing the tradeoff between the expected cost and the variance/risk. This enables one to decide an appropriate configuration for a system based on the risk one is willing to take. In general, we shall consider a complete system with wind, solar, diesel, and batteries. We assumed the expected percentage of unserved energy can be left at 10% annually as infrequent brownout spread across the year are not troublesome in remote places where electricity is not even available, and no critical tasks are dependent on it. In Fig. 8, we see the variation of the standard deviation vs the risk parameter θ and observe that larger the risk weighting parameter, the lesser is the standard deviation indicating that with if one wants to minimize the effect of uncertainty compensation comes with diesel generation or storage.

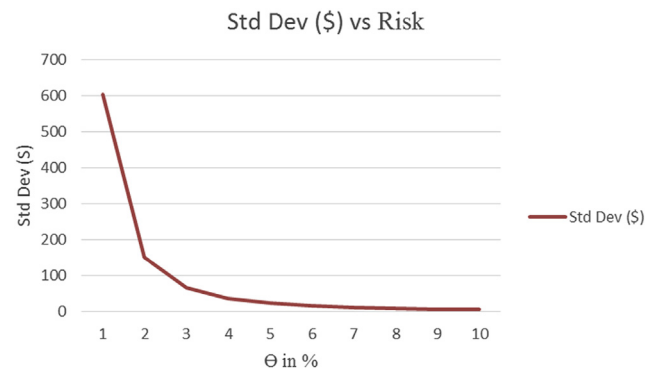


Fig. 8. Standard Deviation vs Risk parameter.

It may also be observed that by varying the risk parameter we can obtain efficient frontiers which are an indication of the optimal strategy for storage utilization, diesel generation or electricity from the grid if available. Efficient frontiers are plots (Fig. 9) between standard deviation and expected cost for the second stage variables i.e. the diesel generation. The efficient frontier is shown for a specific case with 10% expected unserved energy in Fig. 9, where the risk decreases along the x-axis towards the right. It is seen that the expected cost increases as we try to minimize the risk or the standard deviation because in an attempt to minimize the variation one needs to compensate the variations by either diesel generation

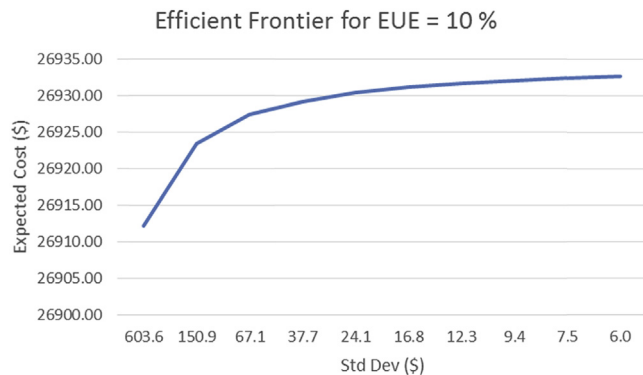


Fig. 9. Efficient frontier for 10% load unserved.

or storage, which adds to the cost.

If we analyze Fig. 9, it is clear that by investing more one can obtain a larger reduction in standard deviation which also demonstrates the utility of the risk-averse model.

4. Conclusions and future work

In this paper, we solve a two-stage stochastic programming problem with various objective functions as mentioned, using the MINOS solver [29]. We utilized the AMPL [30] programming environment to model the mathematical model. Since the programming problem was quite large given the 8760 h in a year and 200 scenarios, the problem was computationally intensive. The problem size increased on modifying the objective function from the two-stage stochastic optimization framework to mean-variance formulation.

It has been mentioned in the literature that uncertainty in the parameters leads to infeasible or economically expensive designs for microgrid. In order to mitigate such possibility, we used the approach of stochastic programming using the two stage stochastic programming paradigm for microgrid planning which provided economically and environmentally acceptable designs with specified reliabilities.

We extended the two-stage stochastic programming model to a risk-averse model using the Markovitz objective function. We considered the microgrid planning problem as an investment problem where the risk is due to uncertainty in the resources and one can determine solutions of desirable risk with corresponding optimal expected values.

In future works, improvements can be done in the optimization model for microgrid planning by considering variety of renewable resources and storage with varying capacities which would give the planners a more diverse range of possibilities in the system design but may lead to further computational challenges in solving the problem.

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