Assignment - 1

1. $P(A) = 0.3 \quad P(B) = 0.4 \quad P(A \cap B) = 0.2$

a) exactly one of the events A or B : $P(A \cup B) - P(A \cap B)$

$= P(A) + P(B) - 2P(A \cap B)$

$= 0.3$

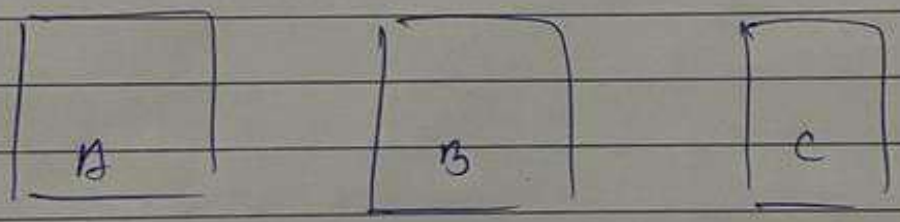
b) atleast one of the events A or B : $P(A \cup B) =$

$= P(A) + P(B) - P(A \cap B)$

$= 0.5$

c) none of A and B : $P(\overline{(A \cup B)}) = 0.5$

2.



Let A be the chosen door by the contestant

$P(A) = \frac{1}{3}$

If Gate A has car, then

$P(\text{car in A and goat in B/C}) = \frac{1}{2}$

Similarly

$P(\text{car in B/C and goat in A}) = \frac{1}{3}$

Now,

$P(\text{Car in A} \mid \text{Goat in C/B}) = \frac{1}{6}$ $P(\text{Goat in B}) = \frac{1}{3}$

$P(\text{Car in B/C} \mid \text{Goat in C/B}) = \frac{1}{3}$ $P(\text{Goat in B/C}) = \frac{2}{3}$

Thus, It is good to switch gates.

3. A : 3 balls drawn are red
B : All balls are red

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(B)\,P(A|B)}{P(A)}$$

$$P(A) = \frac{1}{6} \times \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} + \frac{1}{6} \times \frac{4}{6} \times \frac{3}{5} \times \frac{2}{4}$$

$$\frac{1}{6} \times \frac{5 \times 4 \times 3}{6 \times 5 \times 4} + \frac{1}{6} \times \frac{6 \times 5 \times 4}{6 \times 5 \times 4}$$

$$= \frac{4}{7}$$

4.

a) $P(X < 0.5) = 0.3$

b) $P(0.25 < X < 0.75) = P(X = 0.4) + P(X = 0.5) = 0.4$

c) $P(X = 0.2 \mid X = 0.6) = \frac{P(X = 0.2 \cap X = 0.6)}{P(X = 0.6)}$

$= \frac{P(X = 0.2)}{P(X = 0.2) + P(X = 0.4) + P(X = 0.5)} = \frac{0.1}{0.5} = \frac{1}{5}$

5. i) Since, F is right continuous.

$$\frac{4c^2 - 9c + 6}{4} = 1$$

$$4c^2 - 9c + 2 = 0$$

$c = 2, \frac{1}{4}$

$$\boxed{c = \frac{1}{4}}$$

2) $P(1 < X < 2) = F(2^-) - F(1) = 0$

$P(2 \leq X < 3) = F(3^-) - F(2^-) = 1 - \frac{11}{12} = \frac{1}{12}$

$P(0 < X \leq 1) = F(1) - F(0) = \frac{11}{12} - \frac{2}{3} = \frac{1}{4}$

$P(1 \leq X \leq 2) = F(2) - F(1^-) = \frac{11}{12} - \frac{2}{3} = \frac{1}{4}$

$P(X \geq 3) = 1 - F(3^-) = 0$

6.

a) $E(x) = \int_0^1 x f(x) = \int_0^1 x\,dx = \frac{1}{2}$

b) $E(x^2) = \int_0^1 x^2 f(x) = \int_0^1 x^2\,dx = \frac{1}{3}$

$V(x) = E(x^2) - (E(x))^2 = 1/3 - 1/4 = 1/12$

c) $E(x^2) + E(Y^2) = 1$

$E(Y^2) = 1 - E(x^2) = \frac{2}{3}$

$Var(Y) = E(Y^2) - (E(Y))^2$

$\frac{5}{9} = \frac{2}{3} - (E(Y))^2$

$E(Y) = \frac{1}{3}$

d) $E(x+Y) = E(x) + E(Y) = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

1 a) 0.3

b) 0.5

c) 0.5

2. It is good to switch gates

3. $\frac{4}{7}$

4. a) 0.3

b) 0.4

c) 0.2

5. i) $\frac{1}{4}$

2) 0, $\frac{1}{12}$, $\frac{1}{4}$, $\frac{1}{4}$, 0

6. a) $\frac{1}{2}$

b) $\frac{1}{12}$

c) $\frac{1}{3}$

d) $\frac{5}{6}$