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Parameter Evaluation

Assignment 1

#1 $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-y)^2}{2\sigma^2}}$

$x_1, x_2, x_3 \ldots x_n$ - Sample of Size n

$L(x_1, x_2 \ldots x_n) = f(x_1) \cdot f(x_2) \ldots f(x_n)$

$= \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-y)^2}{2\sigma^2}}\right) \cdot \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-y)^2}{2\sigma^2}}\right) \ldots$

Taking log on bs

$\ln(L) = -\frac{n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^{n} \left(\frac{(x_i-y)^2}{2\sigma^2}\right)$ — (1)

$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1}^{n} -\left(\frac{2(x_i-\mu)}{2\sigma^2}\right) = 0$

$= \sum_{i=1}^{n} (x_i - \mu) = 0$

$n\bar{x} - n\mu = 0$

$\bar{x} = \mu$

Hence $\theta_1 = \bar{x}$ is therefore Sample mean

$\frac{\partial \ln(L)}{\partial \sigma^2} = \frac{-n}{2\sigma^2} + \sum_{i=1}^{n} -\left(\frac{x_i-\mu}{2(\sigma^2)}\right) = 0$

$n = \sum_{i=1}^{n} + \left(\frac{x_i-\mu}{\sigma^2}\right)$

$$\sigma^2 = \frac{1}{n}\left(\sum_{i=1}^{n}(x_i-\mu)^2\right)$$

$$\boxed{\text{hence } \theta_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2}$$ Ans

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#2 Binomial distribution $\rightarrow {}^{n}C_{x_i}\,\theta^{x_i}(1-\theta)$

$$L = \prod_{i=1}^{n} {}^{n}C_{x_i}\,\theta^{x_i}(1-\theta)^{n-x_i}$$

log on bs

$$\log L = \sum_{i=1}^{n} \log\left({}^{n}C_{x_i}\right) + \log\theta^{x_i} + \log(1-\theta)^{n-x_i}$$

differential w.r.t θ

$$\frac{d\log(L)}{d\theta} = 0$$

$$\frac{1}{\theta}\sum x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta}\sum x_i = 0$$

$$\frac{1}{\theta(1-\theta)}\sum x_i = \frac{n}{(1-\theta)}$$

$$\boxed{\theta = \frac{\sum x_i}{n^2}}$$ Ans