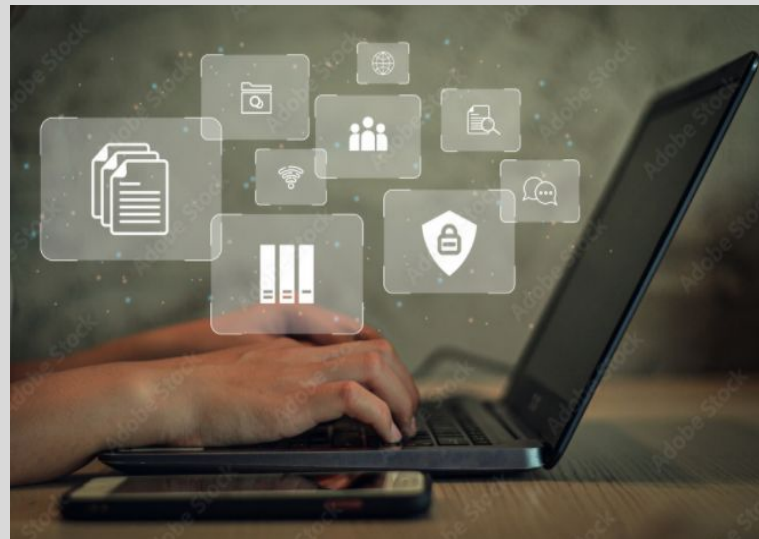




Fundamentals of Computer Science (MCAC-0017)

Topic: **Boolean Algebra**



INTRODUCTION

- ✓ Developed by English Mathematician George Boole in between 1815 - 1864.
- ✓ It is described as an algebra of logic or an algebra of two values i.e True or False.
- ✓ The term logic means a statement having binary decisions i.e True/Yes or False/No.

APPLICATION OF BOOLEAN ALGEBRA



- It is used to perform the logical operations in digital computer.
- In digital computer True represent by '1' (high volt) and False represent by '0' (low volt)
- Logical operations are performed by logical operators. The fundamental logical operators are:
 1. AND (conjunction)
 2. OR (disjunction)
 3. NOT (negation/complement)

AND operator

It performs logical multiplication and denoted by (.) dot.

X	Y	X.Y
0	0	0
0	1	0
1	0	0
1	1	1

OR operator

It performs logical addition and denoted by (+) plus.

X	Y	X+Y
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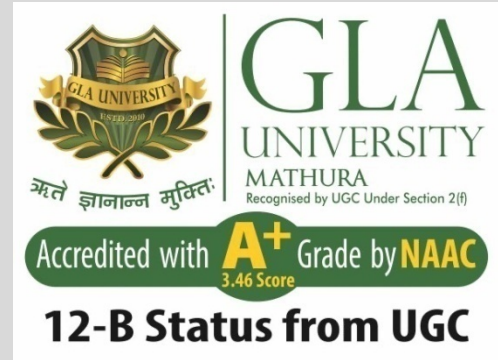
0	0	0
---	---	---

0	1	1
---	---	---

1	0	1
---	---	---

1	1	1
---	---	---

NOT operator



It performs logical negation and denoted by (-) bar. It operates on single variable.

\bar{x} (means complement of x)

0 1

1 0

Truth Table

- Truth table is a table that contains all possible values of logical variables/statements in a Boolean expression.

No. of possible combination = 2^n , where n =number of variables used in a Boolean expression.

Truth Table

The truth table for $XY + Z$ is as follows:

Dec	X	Y	Z	XY	XY+Z
0	0	0	0	0	0
1	0	0	1	0	1
2	0	1	0	0	0
3	0	1	1	0	1
4	1	0	0	0	0
5	1	0	1	0	1
6	1	1	0	1	1
7	1	1	1	1	1

Tautology & Fallacy

If the output of Boolean expression is always True or 1 is called Tautology.

If the output of Boolean expression is always False or 0 is called Fallacy.

Exercise

1. Evaluate the following Boolean expression using Truth Table.

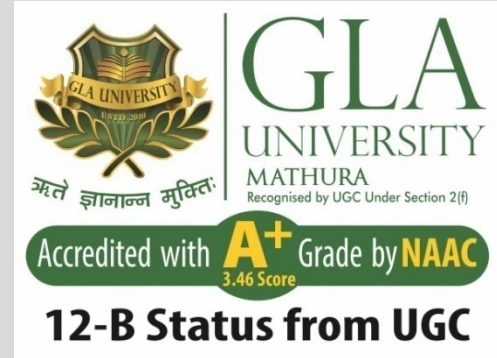
(a) $X'Y' + X'Y$ (b) $X'YZ' + XY'$

(c) $XY'(Z + YZ') + Z'$

2. Verify that $P + (PQ)'$ is a Tautology.

3. Verify that $(X + Y)' = X'Y'$

Implementation



Boolean Algebra applied in computers electronic circuits. These circuits perform Boolean operations and these are called logic circuits or logic gates.

Basic Theorem of Boolean Algebra

T1 : Properties of 0

$$(a) 0 + A = A$$

$$(b) 0 A = 0$$

T2 : Properties of 1

$$(a) 1 + A = 1$$

$$(b) 1 A = A$$

Basic Theorem of Boolean Algebra

T3 : Commutative Law

$$(a) A + B = B + A$$

$$(b) A B = B A$$

T4 : Associate Law

$$(a) (A + B) + C = A + (B + C)$$

$$(b) (A B) C = A (B C)$$

T5 : Distributive Law

$$(a) A (B + C) = A B + A C$$

$$(b) A + (B C) = (A + B) (A + C)$$

$$(c) A + A'B = A + B$$

Basic Theorem of Boolean Algebra

T6 : Indempotence (Identity) Law

$$(a) A + A = A$$

$$(b) A A = A$$

T7 : Absorption (Redundance) Law —

$$(a) A + A B = A$$

$$(b) A (A + B) = A$$

Basic Theorem of Boolean Algebra

T8 : Complementary Law

(a) $X + X' = 1$

(b) $X \cdot X' = 0$

T9 : Involution

(a) $x'' = x$

T10 : De Morgan's Theorem

(a) $(X + Y)' = X' \cdot Y'$

(b) $(X \cdot Y)' = X' + Y'$

De Morgan's Theorem 1

Theorem 1 $\overline{A \cdot B} = \overline{A} + \overline{B}$

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

NAND = Bubbled OR



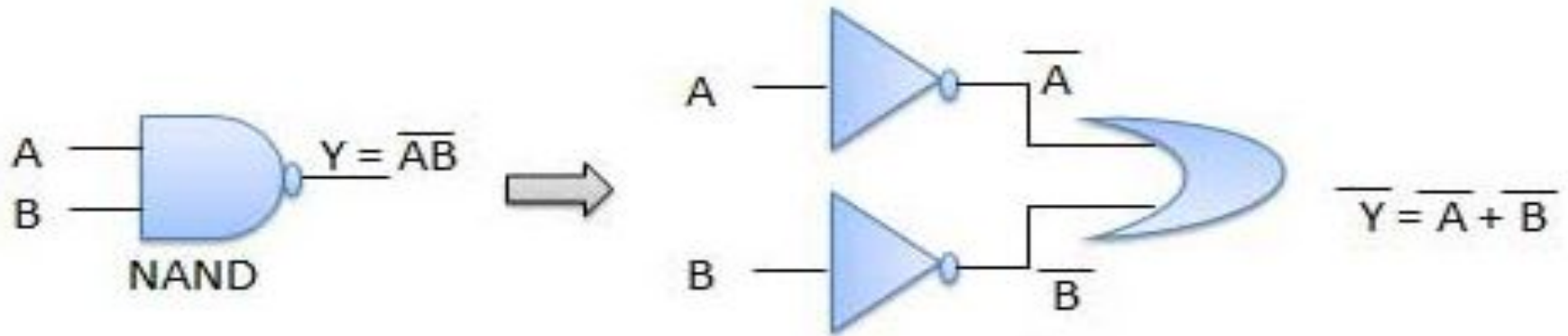
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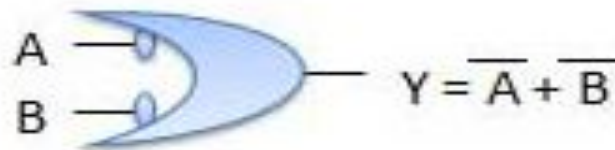
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De Morgan's Theorem 1

Theorem 1 $\overline{A \cdot B} = \overline{A} + \overline{B}$



NAND \equiv Bubbled OR



Bubbled OR

De Morgan's Theorem 1



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Theorem 1 $\overline{A \cdot B} = \overline{A} + \overline{B}$

A	B	$\overline{A \cdot B}$	\overline{A}	\overline{B}	$\overline{A} + \overline{B}$
0	0	1	1	1	1
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	0	0	0

De Morgan's Theorem 2



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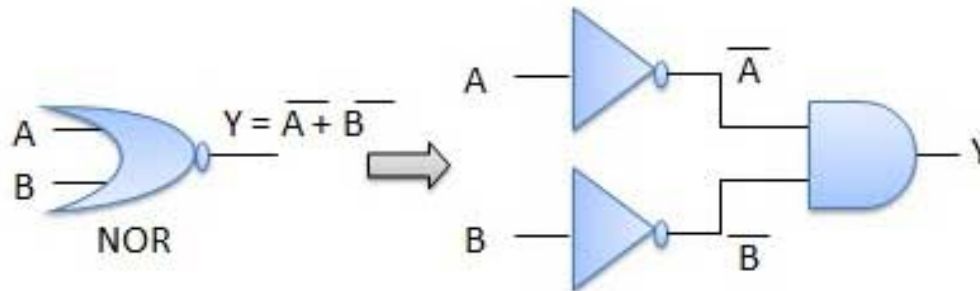
Theorem 2 $\overline{A + B} = \overline{A} . \overline{B}$

$$\overline{A + B} = \overline{A} . \overline{B}$$

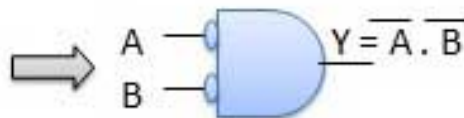
NOR = Bubbled AND

De Morgan's Theorem 2

Theorem 2 $\overline{A + B} = \overline{A} . \overline{B}$



NOR \equiv Bubbled AND



Bubbled AND

De Morgan's Theorem 2

Theorem 2 $\overline{A + B} = \overline{A} \cdot \overline{B}$

A	B	$\overline{A + B}$	\overline{A}	\overline{B}	$\overline{A} \cdot \overline{B}$
0	0	1	1	1	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	0

Thank You