1) a) $f_s = \frac{1}{T} = 10000\,Hz$

max frequency of CT = 5000

b) $H(e^{j\omega}) = \dfrac{1 - e^{-j\omega + \frac{\pi}{2}} - e^{-j\omega - j\frac{\pi}{2}} + e^{-2j\omega}}{1 - .9e^{j\omega + \frac{\pi}{2}} - .9e^{-j\omega - j\frac{\pi}{2}} + .81e^{-2j\omega}} = \dfrac{1 + e^{-2j\omega}}{1 + .81e^{-j2\omega}}$

c) $\omega_0 = 60\,Hz$ in rad/sec

$= 2\pi(60) = 120\pi$

$\omega_0 = 0$

2) $X_c(j\Omega) = 0$ for $|\Omega| \geq 2\pi(1000)$

$y[n] = x^2[n] \Rightarrow Y(e^{j\omega}) = \frac{1}{2\pi} X(e^{j\omega}) * X(e^{j\omega})$

$Y(e^{j\omega}): -\pi < \omega < \pi \Rightarrow X(e^{j\omega}): -\frac{\pi}{2} < \omega < \frac{\pi}{2}$

$\omega = \Omega T \rightarrow \frac{\pi}{2} \geq 2\pi(1000)T$

$\frac{1}{4} \geq 1000T$

$\frac{1}{4000} \geq T$

3) a) $x[n] = x_c(nT)$ $\quad y[n] = x[2n]$

$X_c(j\Omega) = 0 \quad |\Omega| > 2\pi(100)$

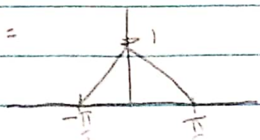
$X(e^{j\omega}) = 0 \quad \underbrace{\frac{\pi}{2} < |\omega| \leq \pi}$ what value of T?

$\Omega_s = 2 \cdot \Omega_N = 2 \cdot 2\pi(100)$

$T = \frac{2\pi}{\Omega_s} = \frac{1}{200}$

b) $T' = 2 \cdot \frac{1}{200} = \frac{1}{100}$

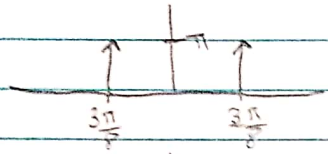
(4) $w[n] = x[n]\cos(3\pi n/8)$ $\quad H(e^{j\omega}) =$



$y[n] = z[2n]$

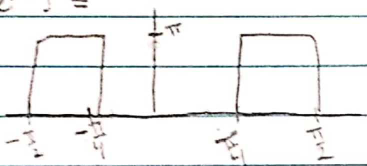
$R[n] = \cos(3\pi n/8)$

$R(e^{j\omega}) = \Sigma\, \pi\,\delta(\omega - \frac{3\pi}{8}) + \pi\delta(\omega + \frac{3\pi}{8})$



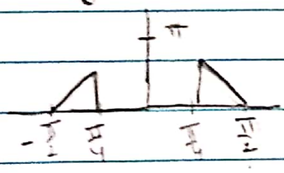
$\Downarrow$

$W(e^{j\omega}) =$



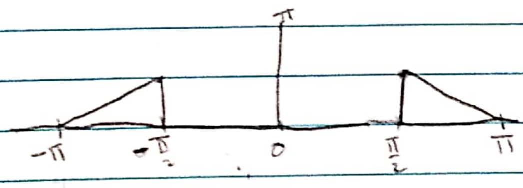
$Z(e^{j\omega}) = W(e^{j\omega}) \cdot H(e^{j\omega})$

$\downarrow$



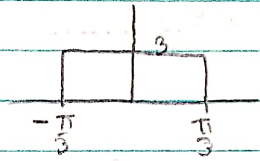
$Z(e^{j\omega}) \rightarrow (\downarrow 2) \rightarrow Y(e^{j\omega})$

$Y(e^{j\omega}) =$



⑤ ⓐ $x[n] = \cos\left(\frac{\pi n}{4}\right)$

$H(e^{j\omega}) =$



$x_d[n] = x[3n]$

$x_c[n] = x_d\left[\frac{n}{3}\right]$

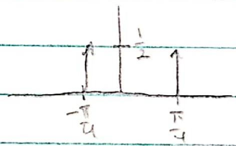
$x_c[n] = x\left[\frac{3n}{3}\right] = x[n]$

$X(e^{j\omega}) = \text{DTFT}\{\cos\frac{\pi n}{4}\}$

$= \text{DTFT}\{\frac{1}{2}(e^{j\frac{\pi n}{4}} + e^{-j\frac{\pi n}{4}})$

$= \frac{1}{2}\delta(\omega - \frac{\pi}{4}) + \frac{1}{2}(\omega + \frac{\pi}{4})$

$\downarrow$

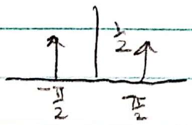


Since $\frac{\pi}{4} < \frac{\pi}{3}$ so $x_r[n] = x[n]$

ⓑ $X(e^{j\omega}) = \text{DTFT}\{\frac{1}{2}(e^{j\frac{\pi 2}{n}} + e^{-j\frac{\pi n}{2}})$

$= \frac{1}{2}\delta(\omega - \frac{\pi}{2}) + \frac{1}{2}\delta(\omega + \frac{\pi}{2})$

$\downarrow$

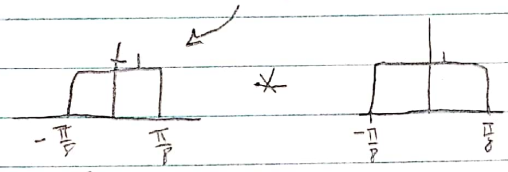


$\Rightarrow \frac{\pi}{2} > \frac{\pi}{3} \Rightarrow X(e^{j\omega})$ doesn't pass through $H(e^{j\omega})$

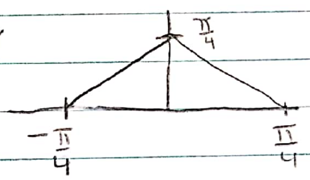
So $x_r[n] \neq x[n]$

(c) $x[n] = \left[\dfrac{\sin(\pi n/8)}{\pi n}\right]^2$

$\text{DTFT}\left\{\dfrac{\sin(\pi n/8)}{\pi n}\right\} = \begin{cases} 1, & |\omega| < \frac{\pi}{8} \\ 0 & \frac{\pi}{8} < |\omega| \leq \pi \end{cases}$



$X(e^{j\omega}) = \text{DTFT}(\text{sinc}(\frac{\pi}{8})) * \text{DTFT}(\text{sinc}(\frac{\pi}{8}))\}$



$\frac{\pi}{4} < \frac{\pi}{3}$ so it passes through $H(e^{j\omega})$

so $X_r[n] = x[n]$

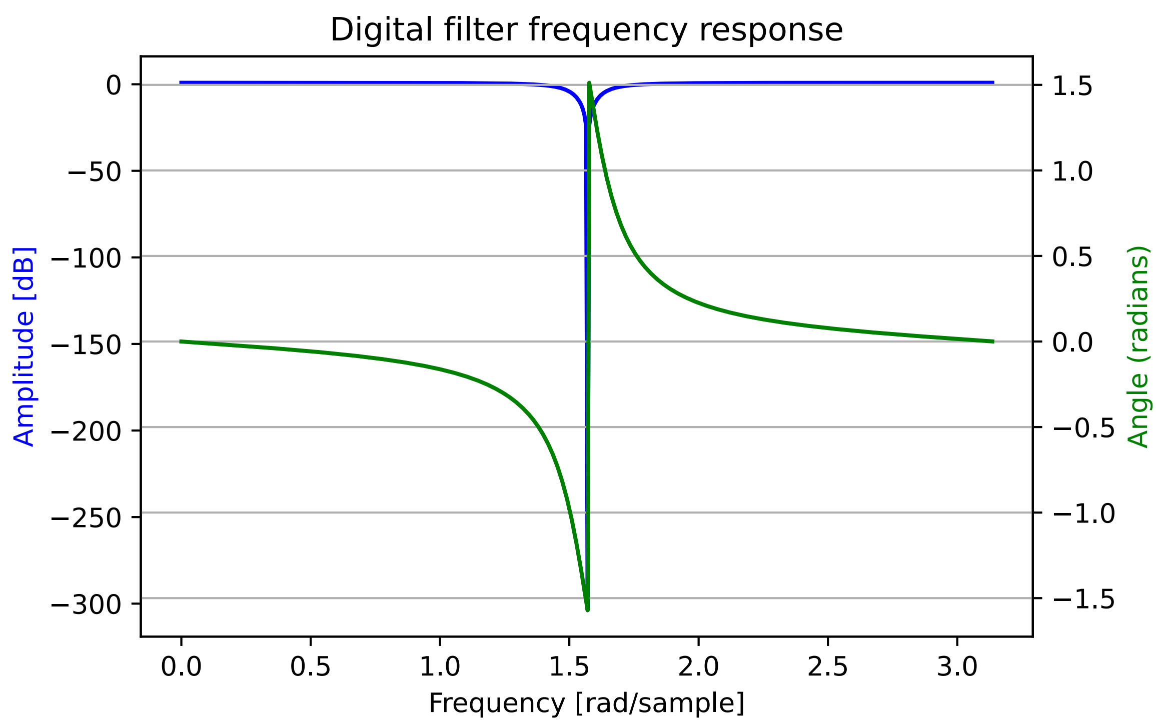
In [6]:

```
import numpy as np
import matplotlib.pyplot as plt
from scipy import signal

numerator=[1,0,1]
denominator=[1,0,.81]
w,h=signal.freqz(numerator,denominator)
fig = plt.figure()
plt.title('Digital filter frequency response')
ax1 = fig.add_subplot(111)

plt.plot(w, 20 * np.log10(abs(h)), 'b')
plt.ylabel('Amplitude [dB]', color='b')
plt.xlabel('Frequency [rad/sample]')

ax2 = ax1.twinx()
angles = np.unwrap(np.angle(h))
plt.plot(w, angles, 'g')
plt.ylabel('Angle (radians)', color='g')
plt.grid()
plt.axis('tight')
plt.show()
```



In [ ]: