1a. $T(x[n]) = e^{x[n]} + 1$

① Assume: $|x[n]| \le B_x < \infty$

Then: $|y[n]| \le |e^{x[n]} + 1| \le e^{B_x} + 1 = B_y < \infty$

Is Stable

② Is causal bc it only depends on current values

③ Define: $x_3[n] = a_1 x_1[n] + a_2 x_2[n]$

$y_3[n] = T\{x_3[n]\} = e^{a_1 x_1[n] + a_2 x_2[n]} + 1 \neq a_1(e^{x_1[n]} + 1) + a_2(e^{x_2[n]} + 1)$

Not Linear

④ $x[n] \rightarrow y[n]$

$x_1[n] = x[n-n_d]$

$y_1[n] = e^{x_1[n]} + 1 = e^{x[n-n_d]} + 1 = y[n-n_d]$

Is time invariant

⑤ Its memoryless since it only depends on current values

1b. $T(x[n]) = b + n^2 x[n]$, $b \neq 0$

① Assume: $|x[n]| \le B_x < \infty$

Then: $|y[n]| \le |b + n^2 x[n]| \le b + n^2 B_x = B_y < \infty$

Is Stable

② Is causal bc it only depends on current values

③ $x_3[n] = a_1 x_1[n] + a_2 x_2[n]$

$y_3[n] = b + n^2(a_1 x_1[n] + a_2 x_2[n]) \neq a_1(b + n^2 x_1[n]) + a_2(b + n^2 x_2[n])$

Not Linear

④ $x[n] \rightarrow y[n]$

$x_1[n] = x[n-n_d]$

$y[n] = b + n^2 x_1[n] = b + n^2 x_1[n-n_d] \neq y[n-n_d]$

Not time invariant

⑤ Its memoryless bc it only depends on current values

2a. $x[n] = \sin\left(\frac{\pi n}{8} + .2\pi\right) + \cos\left(\frac{\pi n}{4} + .3\pi\right)$

$w_0 = \frac{\pi}{8}$ $\boxed{\text{Periodic} \quad N = 16}$

$\Rightarrow \frac{\pi}{8} n = 2\pi k$

2b. $x[n] = e^{jn}$

$w_0 = 1$ $w_0 N = 2\pi k$ $\Rightarrow$ $N = 2\pi \rightarrow$ rational

Not periodic

2c. $x[n] = \underbrace{2\cos\left(\frac{\pi n}{6} + \frac{\pi}{4}\right)}_{w_0 = \frac{\pi}{6}} + \underbrace{\pi e^{j\frac{\pi n}{8}}}_{w_1 = \frac{\pi}{8}} + 5$

$\Downarrow$ $\Downarrow$

$N_0 = 12$ $N_0 = 16$

Periodic $N = \operatorname{lcm}(12, 16) = 60$

3a. $y(n) = x[n] e^{-jw_0 N} * h[n]$

$= \sum_{-\infty}^{\infty} e^{-jw_0 k} x[k] h[n-k]$

Let $x_3[n] = a_1 x_1[n] + a_2 x[n]$

$y_3[n] = \sum_{-\infty}^{\infty} e^{-jw_0 k} (a_1 x_1[k] + a_2 x_2[k]) h[n-k]$

$= a_1 \sum e^{-jw_0 k} x_1[n] h[n-k] + a_2 \sum e^{-jw_0 k} x_2[n] h[n-k]$

$= a_1 y_1[n] + a_2 y_2[n]$

Its linear

3b. Let $x_1[n] = x[n - n_0]$

$y_1[n] = \sum e^{-jw_0(k-n)} x_1[n] h[n-k]$

$= \sum e^{-jw_0(k-n)} x_1[n-k] h[k]$

$= \sum e^{-jw_0(k-n)} x[n - n_0 - k] h[k]$

$\neq y[n - n_0]$

Not time invariant

3c. Assume $x[n] \leq B_x < \infty$

Then $y[n] \leq e^{-jw_0 n} x[n] * h[n] \leq x[n] * h[n] \leq B_x * h[n] \leq B_x$

Stable bc $h[n]$ is stable and $e^{-jw_0 n}$ is bounded by 1

3d. S: $y[n] = \sum_{-\infty}^{\infty} e^{-jw_0 k} x[k] h[n-k]$

$x[n] * (h[n] e^{jw_0 n}) \rightarrow \boxed{C} \rightarrow \sum_{-\infty}^{\infty} e^{-jw_0 k} x[k] h[n-k]$

$= \sum_{-\infty}^{\infty} x[k] h[n-k] e^{jw_0 (n-k)}$

$= \sum_{-\infty}^{\infty} e^{jw_0 n} e^{-jw_0 k} x[k] h[n-k]$

$= e^{jw_0 n} \sum_{-\infty}^{\infty} e^{-jw_0 k} x[k] h[n-k]$

C should be a multiplication by $e^{-jw_0 n}$

4a. e^{j5wn} can be an eigenfunction because it is an LTI system

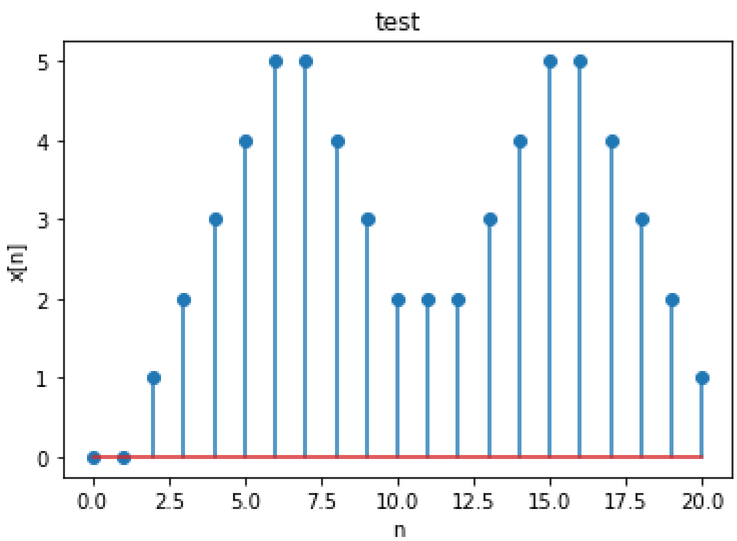
4b. $e^{2jwn} + e^{j3wn}$ cannot be an eigenfunction because the function is not linear

4c. $2^n u[n]$ cannot be an eigenfunction because it is not stable.

In [5]:
```python
import numpy as np
import matplotlib.pyplot as plt
from scipy import signal

arr=np.convolve([1,1,1,1,1], [0,0,1,1,1,1,1,1,0,0,0,1,1,1,1,1,1])
plt.xlabel('n')
plt.ylabel('x[n]')
plt.title('test')
n=np.arange(21)
plt.stem(n, arr)
plt.show()
```

```
<ipython-input-5-c955eab66fa0>:10: UserWarning: In Matplotlib 3.3 individual
lines on a stem plot will be added as a LineCollection instead of individual
lines. This significantly improves the performance of a stem plot. To remove
this warning and switch to the new behaviour, set the "use_line_collection" k
eyword argument to True.
  plt.stem(n, arr)
```



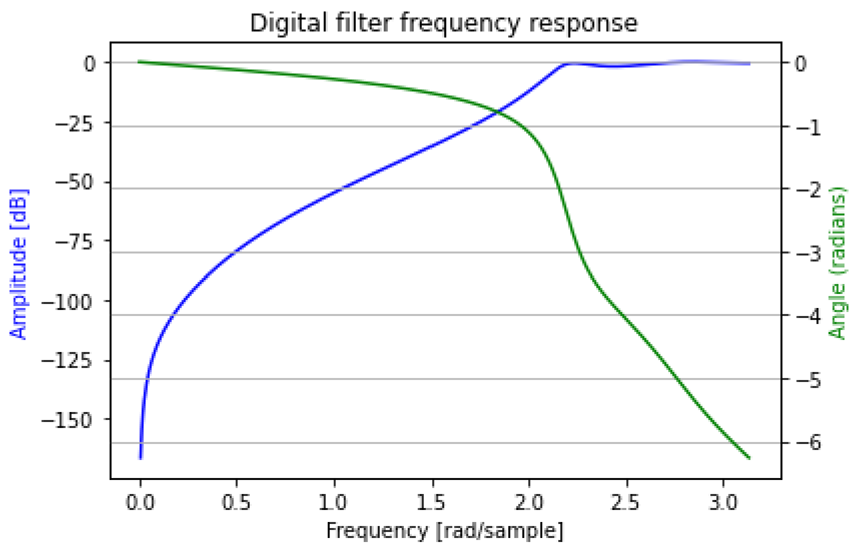
In [6]:

```
numerator=[.008,-.033,.05,-.033,.008]
denominator=[1,2.37,2.7,1.6,.41]
w,h=signal.freqz(numerator,denominator)
fig = plt.figure()
plt.title('Digital filter frequency response')
ax1 = fig.add_subplot(111)

plt.plot(w, 20 * np.log10(abs(h)), 'b')
plt.ylabel('Amplitude [dB]', color='b')
plt.xlabel('Frequency [rad/sample]')

ax2 = ax1.twinx()
angles = np.unwrap(np.angle(h))
plt.plot(w, angles, 'g')
plt.ylabel('Angle (radians)', color='g')
plt.grid()
plt.axis('tight')
plt.show()
```

```
<ipython-input-6-73261741e4e8>:6: MatplotlibDeprecationWarning: Adding an axe
s using the same arguments as a previous axes currently reuses the earlier in
stance.  In a future version, a new instance will always be created and retur
ned.  Meanwhile, this warning can be suppressed, and the future behavior ensu
red, by passing a unique label to each axes instance.
  ax1 = fig.add_subplot(111)
<ipython-input-6-73261741e4e8>:8: RuntimeWarning: divide by zero encountered
in log10
  plt.plot(w, 20 * np.log10(abs(h)), 'b')
```



In [ ]: