① ⓐ $H(z) = \dfrac{2-2z^{-1}}{1+\frac{4}{3}z^{-1}+\frac{1}{3}z^{-2}} = \dfrac{2-2z^{-1}}{(1+\frac{1}{3}z^{-1})(1+z^{-1})}$

$= \dfrac{k_1}{1+\frac{1}{3}z^{-1}} + \dfrac{k_2}{1+z^{-1}}$

$2-2z^{-1} = k_1 + k_1 z^{-1} + k_2 + \frac{1}{3}k_2 z^{-1}$

$k_1 + k_2 = 2$ $\qquad$ $k_1 + k_2 = 2$

$k_1 z^{-1} + \frac{1}{3}z^{-1}k_2 = -2z^{-1} \Rightarrow - \quad k_1 + \frac{1}{3}k_2 = -2$

$\frac{2}{3}k_2 = 4$

$k_2 = 6$

$H(z) = \dfrac{-4}{1+\frac{1}{3}z^{-1}} + \dfrac{6}{1+z^{-1}}$ $\qquad$ $k_1 = -4$

$h[n] = -4\left(\frac{1}{3}\right)^n u[n] + 6(-1)^n u[n]$

ⓑ step response $= h[n] * u[n] = H(z) \cdot \dfrac{1}{1-z^{-1}}$

$= \dfrac{-4}{(1+\frac{1}{3}z^{-1})(1-z^{-1})} + \dfrac{6}{(1+z^{-1})(1-z^{-1})}$

$\dfrac{A}{1+\frac{1}{3}z^{-1}} + \dfrac{B}{1-z^{-1}} + \dfrac{C}{1+z^{-1}} + \dfrac{D}{1-z^{-1}} = \dfrac{-1}{1+\frac{1}{3}z^{-1}} + \dfrac{-3}{1-z^{-1}} + \dfrac{3}{1+z^{-1}} + \dfrac{3}{1-z^{-1}}$

$-4 = A - Az^{-1} + B + B\frac{1}{3}z^{-1}$ $\qquad$ $6 = C - Cz^{-1} + D + Dz^{-1}$

$A + B = -4$ $\qquad$ $A + B = -4$ $\qquad$ $C + D = 6$

$-Az^{-1} + \frac{1}{3}Bz^{-1} = 0$ $\qquad$ $-A + \frac{1}{3}B = 0$ $\qquad$ $-C + D = 0$

$-A + \frac{1}{3}B = 0$ $\qquad$ $\frac{4}{3}B = -4$ $\qquad$ $C = 3 \quad D = 3$

$B = -3$

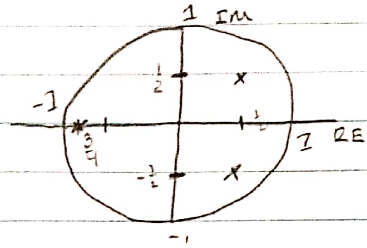
$A = -1$

$s[n] = \left(\frac{1}{3}\right)^n u[n] - 3u[n] + 3(-1^n)u[n] + 3u[n]$

$= \frac{1}{3}^n u[n] + 3(-1)^n u[n]$

ⓒ $1 + \frac{4}{3}y[n] + \frac{1}{3}y[n-1] = 2 - 2x[n]$

(2) x[n]

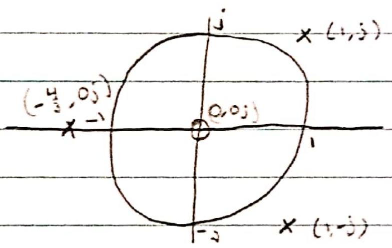


$x[n+3] = x[-(n-3)] \rightarrow z^{-3}X(z^{-1})$

inverse poles: $(.5 \pm .5j)^{-1} = \frac{1}{.5 \pm .5j} = \frac{2}{1 \pm j} = 1 \pm j$

$(-\frac{3}{4})^{-1} = -\frac{4}{3}$

Zero at $0 + 0j$



ROC: $|z| < \frac{4}{3}$ and $|z| \neq 0$

(3) (a) $y[n] - y[n-1] + c\,y[n-2] = x[n]$

$H(z) = \frac{1}{1 - z^{-1} + cz^{-2}}$

$|\text{Poles}| = \sqrt{\frac{1}{4}(1+4c-1)} = \sqrt{c}$

Poles: $\frac{1 \pm \sqrt{1-4c}}{2} \Rightarrow$ $c <= \frac{1}{4}$ poles are real

$c > \frac{1}{4}$ poles are complex

causal when: if $c <= \frac{1}{4}$ $\frac{1+(1-4c)^{\frac{1}{2}}}{2} < |z|$

if $c > \frac{1}{4}$ $\sqrt{c} < |z|$

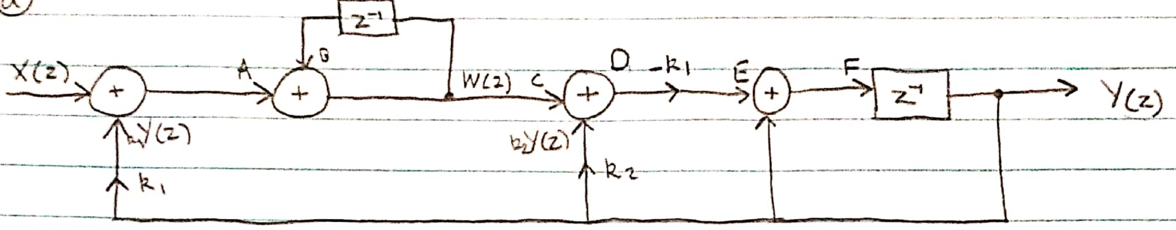
(b) Anti-causal when: if $c <= \frac{1}{4}$ $\frac{1}{2}(1-(1-4c)^{\frac{1}{2}}) > |z|$

if $c > \frac{1}{4}$ $\sqrt{c} > |z|$

(c) non causal if $c <= \frac{1}{4}$ $\frac{1}{2}(1-(1-4c)^{\frac{1}{2}}) < |z| < \frac{1}{2}(1-(1-4c)^{\frac{1}{2}})$

if $c > \frac{1}{4}$ $\sqrt{c} = |z|$

④ ⓐ



$A = X(z) - \frac{1}{2}Y(z)$

$C = W(z) = X(z) - \frac{1}{2}Y(z) + z^{-1}W(z)$

$B = z^{-1}W(z)$

$D = W(z) + 2Y(z)$

$E = \frac{1}{2}W(z) + Y(z)$

$F = \frac{1}{2}W(z) + Y(z) + Y(z) = \frac{1}{2}W(z) + 2Y(z)$

$Y(z) = \frac{1}{2}z^{-1}W(z) + 2z^{-1}Y(z)$

$W(z)(1-z^{-1}) = X(z) - \frac{1}{2}Y(z)$

$W(z) = (1-z^{-1})^{-1}X(z) - \frac{1}{2}(1-z^{-1})^{-1}Y(z)$

$Y(z) = \frac{1}{2}z^{-1}(1-z^{-1})^{-1}X(z) - \frac{1}{4}(1-z)^{-1}z^{-1}Y(z) + 2z^{-1}Y(z)$

$Y(z)(1-z^{-1}) = \frac{1}{2}z^{-1}X(z) - \frac{1}{4}z^{-1}Y(z) + 2z^{-1}Y(z) - 2z^{-2}Y(z)$

$Y(z)(1-z^{-1}) = \frac{1}{2}z^{-1}X(z) + \frac{7}{4}z^{-1}Y(z) - 2z^{-2}Y(z)$

$Y(z)\left(1-z^{-1} - \frac{7}{4}z^{-1} + 2z^{-2}\right) = \frac{1}{2}z^{-1}X(z)$

$Y(z)\left(1 - \frac{11}{4}z^{-1} + 2z^{-2}\right) = \frac{1}{2}z^{-1}X(z)$

$$H(z) = \frac{\frac{1}{2}z^{-1}}{1-\frac{11}{4}z^{-1}+2z^{-2}} \qquad \text{ROC: } |z| < \sqrt{2}$$

④ ⓑ $H(z) = \frac{\frac{1}{2}z^{-1}}{1 - \frac{11}{4}z^{-1} + 2z^{-2}}$ $\qquad p_{1,2} = \frac{11}{8} \pm \frac{\sqrt{7}}{8}i$

$= \frac{A}{(1-p_1 z^{-1})} + \frac{B}{(1-p_2 z^{-1})}$

$\frac{1}{2}z^{-1} = A - p_2 A z^{-1} + B - p_1 B z^{-1}$

$A + B = 0 \qquad \frac{1}{2} = -p_2 A - p_1 B$

$0 = p_2 A + p_2 B$

$\frac{1}{2} = (-p_1 + p_2) B$

$B = \frac{1}{2(p_2 - p_1)}$

$A = \frac{-1}{2(p_2 - p_1)}$

$B = \frac{1}{2\left(\frac{11}{8} - \frac{\sqrt{7}}{8}i - \frac{11}{8} - \frac{\sqrt{7}}{8}i\right)}$

$B = \frac{-1}{2\frac{\sqrt{7}}{4}j} = \frac{-j}{-\frac{\sqrt{7}}{2}} = \frac{2j}{\sqrt{7}}$

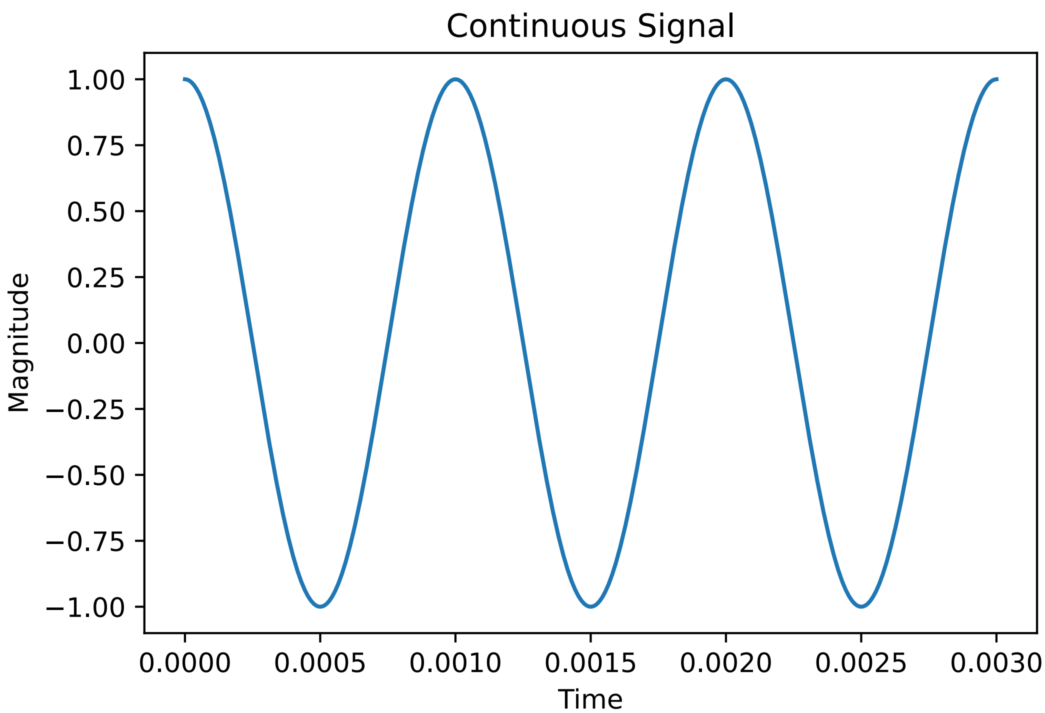
$A = -\frac{2j}{\sqrt{7}}$

$H(z) = \frac{-\frac{2j}{\sqrt{7}}}{1 - p_1 z^{-1}} + \frac{\frac{2j}{\sqrt{7}}}{1 - p_2 z^{-1}}$

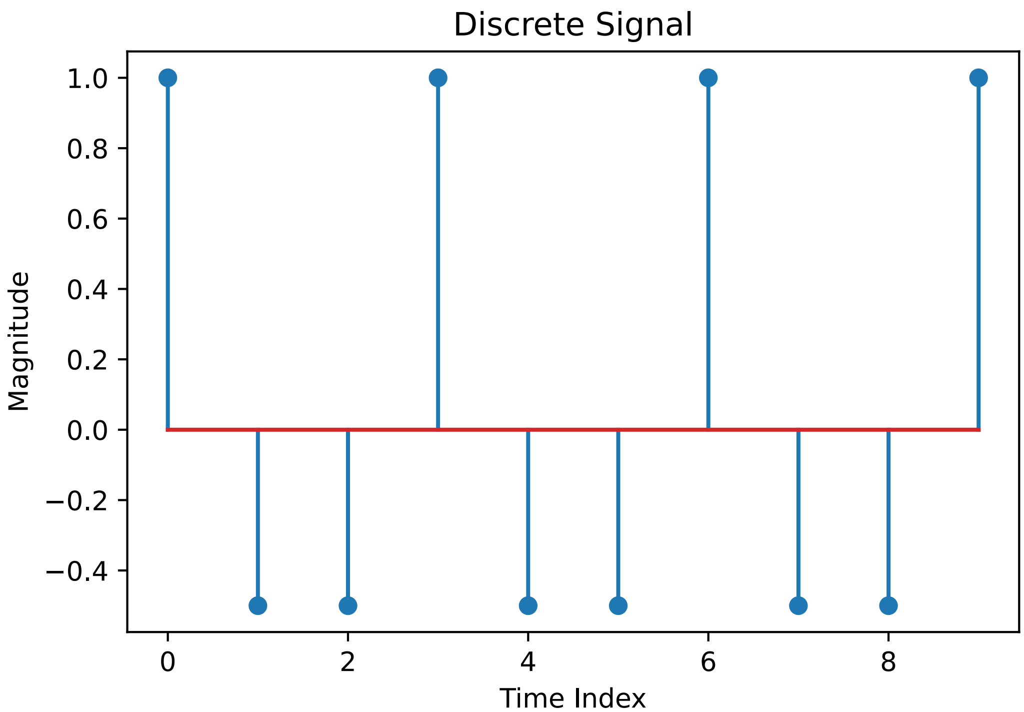
$h[n] = -\frac{2j}{\sqrt{7}}\left(\frac{11}{8} + \frac{\sqrt{7}}{8}j\right)^n u[n] + \frac{2j}{\sqrt{7}}\left(\frac{11}{8} - \frac{\sqrt{7}}{8}\right)^n u[n]$

In [2]:
```
import numpy as np
import matplotlib.pyplot as plt
```

In [17]:
```
t=np.linspace(0,.003,3000)
a=np.cos(2000*np.pi*t)
plt.plot(t,a)
plt.ylabel('Magnitude')
plt.xlabel('Time')
plt.title('Continuous Signal')
plt.show()
```

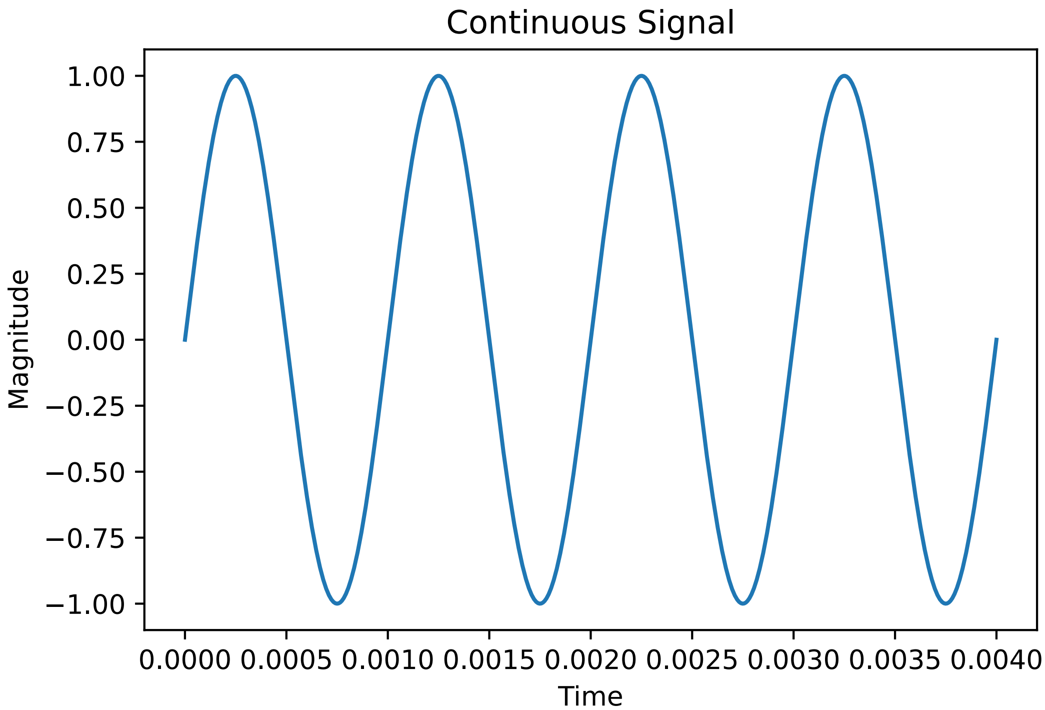


In [24]:
```
n=np.linspace(0,9,10)
an=np.cos(2*np.pi*n/3)
plt.stem(an)
plt.ylabel('Magnitude')
plt.xlabel('Time Index')
plt.title('Discrete Signal')
plt.show()
```

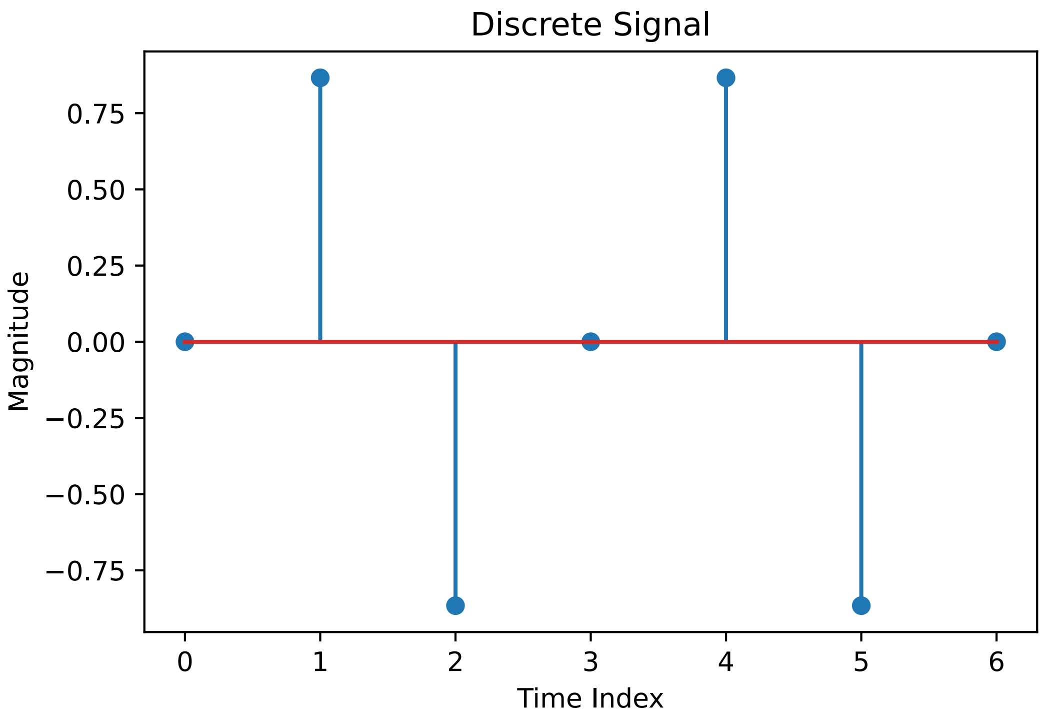


In [32]:

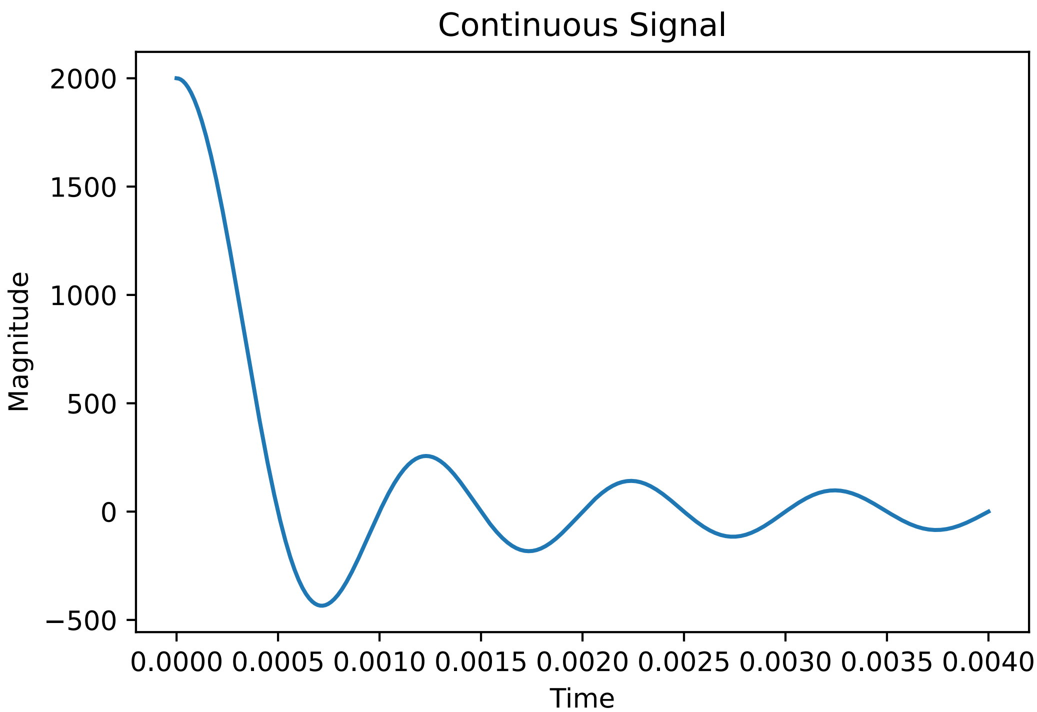
```
t=np.linspace(0,.004,3000)
b=np.sin(2000*np.pi*t)
plt.plot(t,b)
plt.ylabel('Magnitude')
plt.xlabel('Time')
plt.title('Continuous Signal')
plt.show()
```



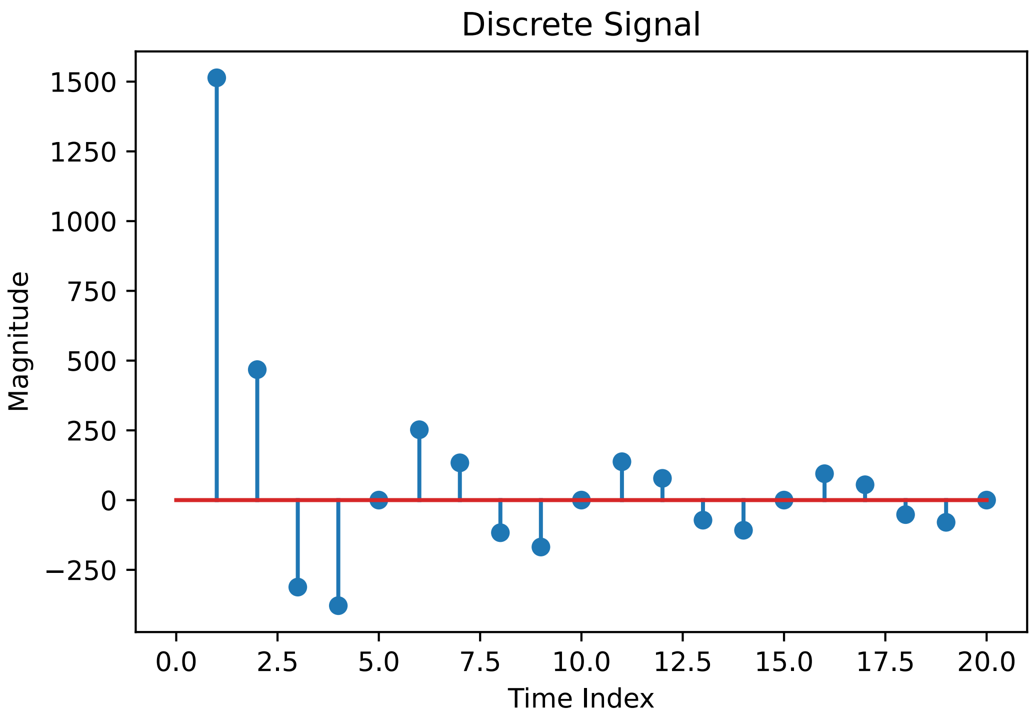
In [33]:
```
n=np.linspace(0,6,7)
bn=np.sin(2*np.pi*n/3)
plt.stem(bn)
plt.ylabel('Magnitude')
plt.xlabel('Time Index')
plt.title('Discrete Signal')
plt.show()
```



In [34]:
```
t=np.linspace(0,.004,3000)
c=np.sin(2000*np.pi*t)/(np.pi*t)
c[0]=2000
plt.plot(t,c)
plt.ylabel('Magnitude')
plt.xlabel('Time')
plt.title('Continuous Signal')
plt.show()
```



In [35]:
```
n=np.linspace(0,20,21)
cn=np.sin(2*np.pi*n/5)/(np.pi*n/5000)
plt.stem(cn)
plt.ylabel('Magnitude')
plt.xlabel('Time Index')
plt.title('Discrete Signal')
plt.show()
```



In [ ]: