$$p(1,1)=\frac{1}{9} \qquad p(1,3)=0$$

$$p(2,1)=1/3 \qquad p(2,3)=1/6$$

$$p(3,1)=1/9 \qquad p(3,3)=1/9$$

$$p(1,2)=1/9$$

$$p(2,2)=0$$

$$p(3,2)\ 1/18$$

$$P_Y(1)=\frac{1}{9}+\frac{1}{9}+\frac{3}{9}=\frac{5}{9}$$

$$P_Y(2)=\frac{2}{18}+\frac{1}{18}=\frac{3}{18}=\frac{1}{6}$$

$$P_Y(3)=\frac{1}{6}+\frac{1}{9}=\frac{15}{9\cdot 6}=\frac{5}{18}$$

$E[X/Y=i] \quad i=1,2,3$

for $i=1$ $E[X/Y=1]$,

$$E[X/Y=1]=\sum x\cdot p(X/Y=1)$$

$$=1\cdot p(X=1/Y=1)+2\cdot p(X=2/Y=1)+3\cdot p(X=3/Y=1)$$

$$E[X/Y=1]=1\cdot\left(\frac{\frac{1}{9}}{\frac{5}{9}}\right)+2\cdot\left(\frac{\frac{1}{3}}{\frac{5}{9}}\right)+3\cdot\frac{\left(\frac{1}{9}\right)}{\left(\frac{5}{9}\right)}$$

$$\frac{1}{5}+\frac{6}{5}+\frac{3}{5}=\frac{10}{5}=2$$

for $i=2$

$$E[X/Y=2]=\sum x\cdot p(X/Y=2)$$

$$=1\cdot p(X=1/Y=2)+2\cdot p(X=2/Y=2)+3\cdot p(X=3/Y=2)$$

$$=\left(1\cdot\frac{2}{18}+2\cdot 0+3\cdot\frac{1}{18}\right)\times\frac{1}{\left(\frac{1}{6}\right)}$$

$$=\frac{5}{18}\times 6=5/3$$

for $i=3$,

$$E[X/Y=3]=\sum x\cdot p(X/Y=3)$$

$$=\frac{\left(1\cdot 0+2\cdot\frac{1}{6}+3\cdot\frac{1}{9}\right)}{\left(\frac{5}{18}\right)}$$

$$=\frac{\left(\frac{1}{3}\right)}{\left(\frac{5}{18}\right)}=\frac{6}{5}$$

for $i=2$

$$E[X/y=2] = \sum x\, p(X/y=2)$$

$$= 1\cdot\frac{(\frac{1}{18})}{\frac{1}{12}} + 2\cdot\frac{(0)}{(\frac{1}{6})} + 3\cdot\frac{(\frac{1}{18})}{(\frac{1}{6})}$$

$$= 1\cdot\left(\frac{2}{3}\right) + 0 + 3\cdot\left(\frac{1}{3}\right)$$

$$= \frac{2}{3}+1$$

$$= 5/3$$

---

2) 1 → Pr(each element being 1 in seq of Binary data) = p

Pr(each eleemnt in seq of Binary data being 0) = 1−p

maximal seq of consecutive values having identical outcomes is called a run

i.e if outcome seq is 1, 1, 0, 1, 1, 1, 0

first run is of length − 2

Sec — ——— 1

Third — ——— 3.

1) → Expected length of first run

x: length of first run → $\{1, 2, 3, \ldots n, \infty\}$

i.e it can have some k no of 0's / 1's

$$\therefore \quad p(x=k) = p^k + (1-p)^k$$

$$\therefore \quad E[X] = \sum_{x=1}^{\infty} x\cdot p_X(x) = \sum_{x=1}^{\infty} x\cdot p^x + \sum_{x=1}^{\infty} x(1-p)^x$$

$$= (1\cdot p + 2\cdot p^2 + 3p^3 + \cdots) + (1(1-p) + 2(1-p)^2 + 3(1-p)^3 + \cdots$$

$$= \frac{p}{(1-p)^2} + \frac{(1-p)}{p^2}$$

$$= 3p^2 - 3p + 1$$

And also for second run it remains same $= 3p^2 - 3p + 1$

3) $P(X=i, Y=j) = {}^{j}C_{i}\, e^{-2\lambda} \cdot \frac{\lambda^{j}}{j!}$ $\qquad 0 \le i \le j$

a) for PMF of y i.e Marginal PMF

$$p_y(y) = \sum_{\forall x} P(X=i, Y=j)$$

as $X = i$

$X \to i$ from 0 to j

$$= \sum_{0}^{j} {}^{j}C_{i}\, e^{-2\lambda} \frac{\lambda^{j}}{j!}$$

$$= \frac{e^{-2\lambda}\lambda^{j}}{j!} \times \sum_{i=0}^{j} {}^{j}C_{i} = \frac{e^{-2\lambda} \lambda^{j} \times 2^{j}}{j!}$$

$$= \frac{e^{-2\lambda}(2\lambda)^{j}}{j!}$$

b) for marginal PMF of X

only condition for j is $j \ge i$

$\therefore$ j runs from i to ∞

So $j = i + z$

$z \to$ from 0 to ∞

$$p_x(x) = \sum_{\forall y} P(X=i, Y=j)$$

$$= \sum_{j=i}^{\infty} \frac{{}^{j}C_{i}\, e^{-2\lambda}\lambda^{j}}{j!}$$

$$= \sum_{z=0}^{\infty} \frac{{}^{i+z}C_{i}\, e^{-2\lambda}\, \lambda^{(i+z)}}{(i+z)!}$$

$$= e^{-2\lambda} \sum_{z=0}^{\infty} \frac{{}^{i+z}C_{i}\, \lambda^{i+z}}{(i+z)!} = e^{-2\lambda} \sum_{z=0}^{\infty} \frac{\cancel{(i+z)!}}{i! \times z!} \frac{\lambda^{i+z}}{\cancel{(i+z)!}}$$

$$\frac{e^{-2\lambda}}{i!}\sum_{z=0}^{\infty}\frac{\lambda^{i+z}}{z!}$$

$$^{i+z}C_i \cdot \frac{k^{i+z}}{(i+z)!}$$

$$= \frac{e^{-2\lambda}}{i!}\lambda^i\sum_{z=0}^{\infty}\frac{\lambda^z}{z!}$$

$$\frac{\cancel{(i+z)!}}{i!\,(z)!}\times\frac{k^{i+z}}{\cancel{(i+z)!}}$$

$$= \frac{e^{-2\lambda}}{i!}\lambda^i\cdot e^{\lambda}$$

$$\boxed{P_X(x) = \frac{\lambda^i}{i!}\cdot e^{-\lambda}}$$

c) Prob mass function of $Y-X$

Let $Z = Y-X$

$$\boxed{X=i}$$

$$Y = z + X$$

$$\boxed{Y = z+i}$$

$$P(Z=z) = P(Y-X=z)$$

$$= \sum_{i=0}^{\infty}P(X=i,\ Y=z+i)$$

$$= \sum_{i=0}^{\infty}\ {}^{i+z}C_i\, e^{-2\lambda}\frac{\lambda^{i+z}}{(i+z)!}$$

$$= e^{-2\lambda}\sum_{i=0}^{\infty}\frac{\cancel{(i+z)!}}{i!\,z!}\times\frac{\lambda^i\cdot\lambda^z}{\cancel{(i+z)!}}$$

$$= \frac{e^{-2\lambda}\lambda^z}{z!}\sum_{i=0}^{\infty}\frac{\lambda^i}{i!}$$

$$= e^{-2\lambda}\left(\frac{\lambda^z}{z!}\right)\cdot e^{\lambda} = e^{-\lambda}\times\frac{\lambda^{j-i}}{j-i}$$

4) $X \to$ uniform over $(0,1)$

$E[X^n] = ?$ $Var[X^n] = ?$

$\Rightarrow$ $f_X(x) = \begin{cases} 1 & 0<x<1 \\ 0 & o/w \end{cases}$

$f_X(x) = \begin{cases} k & 0<x<1 \\ 0 & o/w \end{cases}$

$E[X^n] = \int_{-\infty}^{\infty} f_X(x) \cdot x^n \, dx$

$\Rightarrow \int_{-\infty}^{\infty} f_X(x) = 1$

$= 0 + \int_0^1 x^n (1) dx + 0$

$\Rightarrow \int_0^1 k \, dx = 1$

$\boxed{E[X^n] = \left[\frac{x^{n+1}}{n+1}\right]_0^1} = \frac{1}{n+1}$

$k[1-0] = 1$

$\boxed{k=1}$

$Var[X^n] = E[X^{2n}] - (E[X^n])^2$

$Var[X] = E[X^2] - (E[X])^2$

$= \cancel{\left[\frac{x^{2n+1}}{2n+1}\right]_0^1 - \left(\left[\frac{x^{n+1}}{n+1}\right]\right)^2}$

$= \frac{1}{2n+1} - \left(\frac{1}{n+1}\right)^2$

5) X: Binomial distribution with parameters n, p.

~~$f_X(x)$~~ $\Rightarrow$ $f_X(x=k) = {}^nC_k \, p^k (1-p)^{n-k}$

let $f_X(x=k)$ increases till k

then $f_X(x=k) > f_X(x=k+1)$

${}^nC_k \, \cancel{p^k} (1-p)^{n-k} > {}^nC_{k+1} \, p^{k+1} (1-p)^{n-k-1}$

${}^nC_k (1-p) > {}^nC_{k+1} \, p.$

$\frac{{}^nC_k}{{}^nC_{k+1}} > \frac{p}{1-p}$

$$\frac{\frac{n!}{k!(n-k)!}}{\frac{n!}{(k+1)!(n-k-1)!}} > \frac{p}{1-p}$$

$$\cancel{\Rightarrow} \quad \frac{k+1}{n-k} > \frac{p}{1-p}$$

$$\boxed{(n+1)p - 1 > k_i}$$

$$k < np + p - 1$$

$\Rightarrow$ $p(x=k)$ increases till $k = np+p-1$ then decreases.

a) $(n+1)p \to$ integer then $k = (n+1)p - 1$
$= np + p - 1$

b) If $(n+1)p$ is not integer then ~~max~~ k is at

$$(n+1)p - 1 < k < (n+1)p$$

---

6) $X \to$ Geometric

i.e $\quad p_X(k) = (1-p)^{k-1} p \qquad k \geq 1$

$$p(X = n+k) = (1-p)^{n+k-1} p$$

$$p(X \geq n) = (1-p)^n p + (1-p)^{n+1} p + \dots$$

$$= p(1-p)^n [1 + (1-p) + (1-p)^2 + \dots]$$

$$= p(1-p)^n \cdot \frac{1}{p}$$

$$= (1-p)^n$$

Now $\quad p(X = n+k / X > n) = \frac{p(X = n+k \cap X > n)}{P(X > n)}$

$$= \frac{p(X = n+k)}{p(X > n)}$$

$$\frac{(1-p)^{n+k-1}p}{(1-p)^n}$$

$$= (1-p)^{k-1} \cdot p$$

$$= p(x=k)$$

x has exceeded n, the prob of x being $n+k$ is same as prob of x being k initially

It forgets that x has already exceeded n.

other distributions with this property is exponential distribution

7)

n envelops $\quad$ k^{th} letter in k^{th} envelop

n letters

$$I_k = \begin{cases} 1 & \text{at } k \\ 0 & o/w \end{cases}$$

$Pr(I_k = 1) = \frac{1}{n}$ $\qquad$ $Pr(I_k = 0) = 1 - \frac{1}{n}$

$E[I_k] = 1 \cdot \frac{1}{n} + 0(\quad)$
$= \frac{1}{n}$

let $X =$ # of correctly matched envelops

$X = f(I_k)$

$= \sum_{k=1}^{n} I_k$

$E[X] = \sum_{k=1}^{n} E[I_k] = \sum_{k=1}^{n} \frac{1}{n}$

$= n \cdot \frac{1}{n}$

$= 1$

$Var[X] = E[X^2] - (E[X])^2$

$= E\left[\left(\sum_{k=1}^{n} I_k\right)^2\right] - 1^2$

E[Sum]
$\downarrow$
Sum[E]

$= E\left[\sum_{k=1}^{n} I_k^2 + 2({}^nC_2) \cdot \sum_{j \neq k}\sum I_j I_k\right] - 1$

$\therefore \Rightarrow \sum_{1}^{n} E[I_k^2] + 2\ {}^nC_2 \cdot \underbrace{E[I_j I_k]}_{\frac{1}{n(n-1)}} - 1$

$= n + 2\,{}^nC_2 \cdot \frac{1}{n(n-1)} - 1$

$= 1$

(8) $P(\text{Bit error}) = 10^{-6}$ tran occurs in Blocks of 10^4 Bits $= n$

$N \rightarrow$ no of Errors introduced in transmission Block.

a) $P[N=0] = (1-10^{-6})^{n}\, {}^{n}C_0 = \underbrace{(1-10^{-6})^{10^4}}_{(1-p)^n \rightarrow} \times {}^{10^4}C_0$ for small p

$= e^{-np}$

$\therefore e^{-10^4 \times 10^{-6}}$

$$\boxed{P[N=0] = e^{-10^{-2}}}$$

$P[N \leq 3] =$

$P[N=0] + P[N=1] + P[N=2] + P(N=3]$

$e^{-10^{-2}} \times {}^{10^4}C_0 + {}^{10^4}C_1 \times 10^{-6} (1-10^{-6})^{10^4-1}$

$+ {}^{10^4}C_2 \times (10^{-6})^2 (1-10^{-6})^{10^4-2} + {}^{10^4}C_3 \times (10^{-6})^3 (1-10^{-6})^{10^4-3}$

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b) $P[N \geq 1] = 0.99$

$\Rightarrow P[N = 0] = 0.01$

$$\underbrace{(1-p)^{10^4}}_{\downarrow \; e^{-np}} (1) = 0.01$$

$\Rightarrow e^{-10^4 p} = 0.01$

$$-10^4 p = \ln(0.01)$$

$$p = -\frac{\ln(0.01)}{10^4}$$

9) m-"Bit" password

0's 1s $\rightarrow$ _ _ _ _ - - - _ $\rightarrow 2^m$ possibilities

X : No of patterns tested untill, correct password is found

$\Rightarrow X = \{1, 2, 3, -, k, - 2^m\}$ $\quad p_X(x) = \frac{1}{2^m}$ for any value of X_{2k}

a) $p_{X/\text{Not found After k tries}} \equiv p_{X/A}(x) = \frac{P_r(X = x \cap A)}{P_r(A)}$

A : Password Not found After k tries. So there are still $2^m - k$ tries

$$\therefore p_{X/A}(x) = \frac{1}{2^m - k}$$

b) Conditional Expected value of X given $X > k$.

i-e $\underbrace{E[X/X>k]}_{C}$ $\quad X = \underbrace{\{k+1, k+2, \text{-----} 2^m\}}_{(2^m - k) \text{ Values}}$

$\Rightarrow$ ~~let X = C~~

Where $X/(X>k) \{k+1, k+2, ---, 2^m\}$

$\Rightarrow$ $$E[C] = \sum_{\forall c} c \cdot p_X(c)$$

$$= (k+1)\frac{1}{2^m - k} + (k+2)\frac{1}{2^m - k} + \text{----} + \frac{2^m}{2^m - k}$$

$$\boxed{E[C] = \frac{1}{2^m - k} \sum_{c=k+1}^{2^m} c}$$

$$E[X/X>k] = \frac{1}{2^m - k} \sum_{x=k+1}^{2^m} x$$

$$\downarrow$$

$$\sum_{x=1}^{2^m} x - \sum_{x=1}^{k} x$$

$$\Rightarrow \frac{2^m(2^m+1)}{2} - \frac{k(k+1)}{2}$$

$$\therefore E[X/X>k] = \frac{1}{2^m - k}\left[\frac{2^m(2^m+1)}{2} - \frac{k(k+1)}{2}\right]$$

$$= \frac{2^m(2^m+1) - k(k+1)}{(2^m-k)\ 2}$$

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10)

2 dimensional Signal (X, Y)

$$X = r\cos\left(\frac{2\pi\theta}{8}\right) = r\cos\left(\frac{\pi\theta}{4}\right)$$

$$Y = r\sin\left(\frac{2\pi\theta}{4}\right) = r\sin\left(\frac{\pi\theta}{4}\right)$$

$\theta = \{0, 1, 2, 3, \dots 7\}$ → discree uniform RV

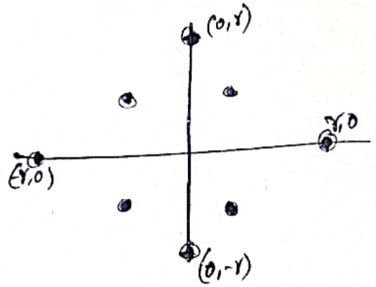
$\Rightarrow p_\theta(\theta) = \frac{1}{8} \quad \forall \theta$

$$\therefore \; X : \left\{ r, \frac{r}{\sqrt{2}}, 0, -\frac{r}{\sqrt{2}}, -r, -\frac{r}{\sqrt{2}}, 0, \frac{r}{\sqrt{2}} \right\}$$

$$Y : \left\{ 0, \frac{r}{\sqrt{2}}, r, \frac{r}{\sqrt{2}}, 0, -\frac{r}{\sqrt{2}}, -r, -\frac{r}{\sqrt{2}} \right\}$$

$\therefore$ (X, Y) can be

$$\underbrace{\begin{matrix} (r, 0) & \left(\pm\frac{r}{\sqrt{2}}, \pm\frac{r}{\sqrt{2}}\right) \\ (0, r) & \\ (-r, 0) & \\ (0, -r) & \end{matrix}}_{A \subseteq R^2}$$



$$p_{X,Y}(x,y) = p_{X,Y}(r,0)$$

$$= p_{X,Y}(x=r \cap y=0)$$

$$= \frac{1}{8}$$

And $p_{X,Y}(x,y) = \frac{1}{8}$

$\forall \; (x,y) \in A$

11)

Marginal PMF of X

$p_X(r) = \frac{1}{8}$ $\qquad p_X(0) = Pr(\theta = 2 \text{ or } \theta = 6)$

$= \frac{1}{4}$

Illy $p_X(-r) = \frac{1}{8}$

$p_X\left(\frac{r}{\sqrt{2}}\right) = \frac{1}{4}$ $\qquad p_X\left(-\frac{r}{\sqrt{2}}\right) = \frac{1}{4}$

Marginal PMF of Y

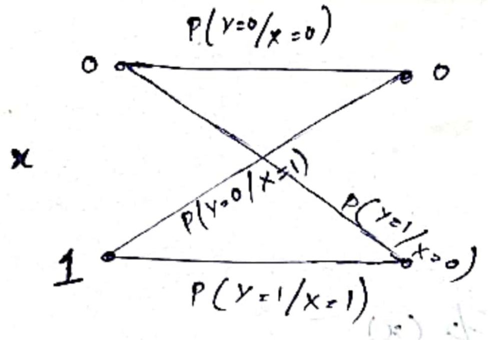
$p_Y(r) = Pr(\theta = 2) = \frac{1}{8}$

$p_Y(0) = Pr(\theta = 0 \text{ or } 4) = \frac{1}{4}$

$p_Y\left(\frac{r}{\sqrt{2}}\right) = \frac{1}{4}$ $\qquad p_Y(-r) = \frac{1}{8}$

$p_Y\left(-\frac{r}{\sqrt{2}}\right) = \frac{1}{4}$

12)



$(X, Y) \to$ bivariate r.v.

$X \to$ input

$Y \to$ output

$p(X=1) = 0.5$

$p(X=0) = 0.5$

$p(Y=1/X=0) = 0.1$

$p(Y=0/X=1) = 0.2$

$$\frac{p(y=1 \cap x=0)}{p(x=0)} = 0.1$$

a) $p(y=1 \cap x=0) = 0.1 \times p(x=0) = \frac{1}{10} \times \frac{1}{2} = \frac{1}{20}$

$P_{X,Y}(0,1) = \frac{1}{20}$

$p(y=0/x=0) = 0.9$

$p(y=1/x=1) = 0.8$

Hy $P_{X,Y}(0,0) = 0.9 \times \frac{1}{2} = \frac{9}{10} \times \frac{1}{2} = \frac{9}{20}$

$P_{X,Y}(1,0) = 0.2 \times \frac{1}{2} = \frac{2}{20} = \frac{1}{10}$

$P_{X,Y}(1,1) = \frac{8}{20} = \frac{4}{10}$

b) $P_X(x) = \sum_{\forall y} P_{X,Y}(x,y)$

$\therefore P_X(1) = \sum_{\forall y} P_{X,Y}(1,y) = P_{X,Y}(1,0) + P_{X,Y}(1,1)$

$= \frac{1}{10} + \frac{4}{10} = \frac{1}{2}$

$P_X(0) = \frac{1}{2}$

$P_Y(1) = \sum_{\forall x} P_{X,Y}(x,1) = P_{X,Y}(0,1) + P_{X,Y}(1,1)$ & $P_Y(0) = \frac{11}{20}$

$= \frac{1}{20} + \frac{8}{20} = \frac{9}{20}$

5hrs.

c)

for checking independence

$$p_{X,Y}(x,y) = p_X(x) \cdot p_Y(y) \quad \forall x,y$$

$$\therefore \quad p_{X,Y}(0,1) = \frac{1}{20} \neq p_X(0) \cdot p_Y(1) = \frac{1}{2} \cdot \frac{9}{20}$$

$$p_{X,Y}(0,0) = \frac{9}{20} \qquad p_X(0) \cdot p_Y(0) = \frac{1}{2} \cdot$$

$$p_{X,Y}(1,0) = \frac{2}{20} \qquad p_X(1)\ p_Y(0) = \frac{1}{2}$$

$$p_{X,Y}(1,1) = \frac{8}{20} \qquad p_X(1)\ p_Y(1) = \frac{1}{2}$$

$\Rightarrow$ X, Y are dependent

13) $X, Y \rightarrow$ independent Poisson random variables with Parameters λ_1 and λ_2 respectively.

Let $Z = X + Y$

Now for PMF of Z

$$p_X(X=x) = \frac{\lambda_1^x e^{-\lambda}}{x!}$$

$$p_Y(Y=y) = \frac{\lambda_2^y e^{-\lambda}}{y!}$$

Now

$$Z = X + Y$$

$$E[Z] = E[X] + E[Y]$$

$$E[Z] = \lambda_1 + \lambda_2$$

Mean of poission RV $= \lambda$

let $z = n$

$$Z = X + Y$$

$$n = k + n-k$$

$$P(Z=n) = \sum_{k=0}^{n} P(X=k) \cdot P(Y=n-k)$$ as X, Y are independent.

$$= \sum_{k=0}^{n} \frac{\lambda_1^k e^{-\lambda_1} \cdot \lambda_2^{n-k} \cdot e^{-\lambda_2}}{k! \quad (n-k)!}$$

$$= e^{-(\lambda_1+\lambda_2)} \underbrace{\sum_{k=0}^{n} \frac{\lambda_1^k}{k!} \cdot \frac{\lambda_2^{n-k}}{(n-k)!}}_{\downarrow}$$

$$\frac{(\lambda_1+\lambda_2)^n}{n!}$$

$\hookrightarrow$ expansion of

$\therefore$ PMF of Z

$$p_Z(Z=z) = \frac{(\lambda_1+\lambda_2)^z \cdot e^{-(\lambda_1+\lambda_2)}}{z!}$$

14) P(Choosing right door) = $1/3$ → Time for travel = 2hr

P(choosing wrong door) = $2/3$ → Time for travel = 2hr

Let T be expected time to escape

$$T = \frac{1}{3}(2) + \frac{2}{3}(2+T)$$

So on simplification

$$T = \frac{2}{3} + \frac{4}{3} + \frac{2T}{3} \Rightarrow \boxed{T = 6}$$

Thus Avg time = 6hrs.

16) integer L

const $- C$

$$p_{M,N}(m,n) \begin{cases} C & m \geq 0, n \geq 0, \; m+n < L \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{\forall m} \sum_{n} mn \cdot p_{M,N}(m,n) = 1$$

$m+n < L$

$n < L - m$

$\Rightarrow$ n can take $L-m-1$ values

$n \to 0$ to $L-m-1 \to L-m$

$\Rightarrow$

$m \to 0 \to L-1$

for $m=0$ $\quad C\left[\frac{L(L+1)}{2}\right] = 1$

$$\boxed{C = \frac{2}{L(L+1)}}$$

$L=1$

(m,n) $\quad (0,0)$

$L=2$

$(0,0)$

$(1,0)$

$(0,1)$

$L=3$

$(0,0)$

$(1,0)$

$(0,1)$

$(1,1)$

$(2,0)$

$(0,2)$

$\Rightarrow$

$$p_M(m) = \sum_{\forall n} p_{M,m}(m,n)$$

for $m=0$

$$\therefore p_M(0) = C \times \sum_n (1)$$

$$= C \cdot L$$

$$= \frac{2L}{L(L+1)}$$

Therefore from m

Similar to n

$$p_N(n) = \begin{cases} \frac{2L}{L(L+1)} & n=0 \\ \frac{2(L-1)}{L(L+1)} & n=1 \\ \vdots & \\ \frac{2}{L(L+1)} & n=L-1 \end{cases}$$

lly for $m=1$ $\quad \boxed{n = L-1}$ $\quad p_M(1) = \frac{2(L-1)}{L(L+1)}$

$$p_M(m) = \begin{cases} \frac{2L}{L(L+1)} & m=0 \\ \frac{2(L-1)}{L(L+1)} & m=1 \\ \vdots & \\ \frac{2}{L(L+1)} & m=L-1 \end{cases}$$

$Pr(M+N<L/2)$ $\quad \sum_m \sum_{n / m+n<L/2} Pr(M+n<L/2)$ for $m=0$, $n=0,1,2,\dots \frac{L}{2}-1$ ($\frac{L}{2}$) | for $m=1$, $N \to$ till $L/2-2$ ($\frac{L}{2}-1$) ... $m=L/2-1$, $N=0$ (1)

So $1+2+\dots \frac{L}{2} = \frac{(\frac{L}{2}+1)\frac{L}{2}}{2} = \frac{L(L+2)}{8}$

$$\Rightarrow Pr(M+N<L/2) = \frac{L(L+2)}{8}(c) = \frac{\cancel{2}}{\cancel{L}(L+1)} \times \frac{\cancel{L}(L+2)}{\cancel{8}4}$$

$$= \frac{L+2}{4(L+1)}$$

19) $x \to$ success of 'n' independent Bernouli trials
$y \to$ failure

$P \to Pr(\text{success})$

$\Rightarrow \quad n = x+y \qquad 1-P \quad Pr(\text{failure})$
$y = n-x$

Given $\quad z = x-y$

$Z = x-(n-x) \qquad n \to$ some const

$Z = 2x-n \qquad Var[Z] = Var[x-y]$

$= Var[2x-n]$

$E[Z] = 2E[x]-n \qquad = 4\,Var[x]$

$\boxed{E[Z] = 2np-n} \qquad = 4npq.$

$$\boxed{Var(Z) = 4np(1-p)}$$

20)

$$p_{X,Y}(x_i, y_j) = \begin{cases} k(2x_i+y_j) & x_i = 1,2 \quad y_j = 1,2 \\ 0 & o/w \end{cases}$$

a) $\Rightarrow p_{X,Y}(1,1) = k(2+1) = 3k$

$p_{X,Y}(1,2) = k(2+2) = 4k$

$p_{X,Y}(2,1) = k(4+1) = 5k$

$p_{X,Y}(2,2) = k(4+2) = 6k$

$\sum p_{X,Y}(x_i, y_j) = 1$

$3k+4k+5k+6k = 1$

$18k = 1$

$\boxed{k = \frac{1}{18}}$

c) $p_{X,Y}(1,1) = 3/18$

$p_X(1) \cdot p_Y(1)$

$= \frac{7}{18} \cdot \frac{8}{18} \neq 3/18$

$\Rightarrow$ dependent

b) $p_X(1) = p_{X,Y}(1,2) + p_{X,Y}(1,1) = 7k = 7/18$

$p_X(2) = 5k+6k = \frac{11}{18}$

$p_Y(1) = 8k = \frac{8}{18}$

$p_Y(2) = 10k = \frac{10}{18}$

(19) a) for exactly 11 Cars

poission distribution with $\alpha = 10$

$$P(x=k) = \frac{e^{-\alpha}\alpha^{k}}{k!}$$

Here $\boxed{k=11}$ $$P(X=11) = \frac{e^{-10}\,10^{11}}{11!}$$

b) Min no of Booths req suchthat prob not more than 5 cars approach each booth is atleast 0.05

$\Rightarrow$ $P(Y \leq 5) \geq 0.05$ Y follows poission $\left(\frac{30}{N}\right)$

$$\Rightarrow \sum_{k=0}^{5} P(Y=k) = \sum_{k=0}^{5} \frac{e^{-\frac{30}{N}}\left(\frac{30}{N}\right)^{k}}{k!}$$

$$\Rightarrow \sum_{k=0}^{5} \frac{e^{-\frac{30}{N}}\left(\frac{30}{N}\right)^{k}}{k!} \geq 0.05$$

$\Rightarrow N \geq 15$

(b)

17) $X \to$ No of 1's transmitted in given interval

$N \to$ No of transmissions " - - - -

$$P(X=x) = \sum_{k=x}^{\infty} P\left(X=x / N=k\right) P(N=k)$$

$$P(X=k) = \frac{e^{-\lambda}\lambda^k}{k!}, \quad k=0,1,2,3,---$$

$$P\left(X=x / N=k\right) = {}^kC_x \, p^x (1-p)^{k-x} \frac{e^{-\lambda}\lambda^k}{k!}$$

So $$P(X=x) = \sum_{k=x}^{\infty} {}^kC_x \, p^x (1-p)^{k-x} \frac{e^{-\lambda}\lambda^k}{k!}$$

$$= e^{-\lambda}\frac{p^x}{x!} \sum_{k=x}^{\infty} \frac{(1-p)^{k-x}\lambda^k}{(k-x)!} \cdot \quad \text{let } (j = k-x)$$

$$= \frac{p^x\lambda^x e^{-\lambda}}{x!} \cdot \sum_{j=0}^{\infty} \frac{(1-p)^j\lambda^j}{j!}$$

$$= \frac{(p\lambda)^x e^{-p\lambda}}{x!}$$

So $$P(X=x) = \frac{(p\lambda)^x e^{-p\lambda}}{x!}$$

So parameter $= p\lambda$

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