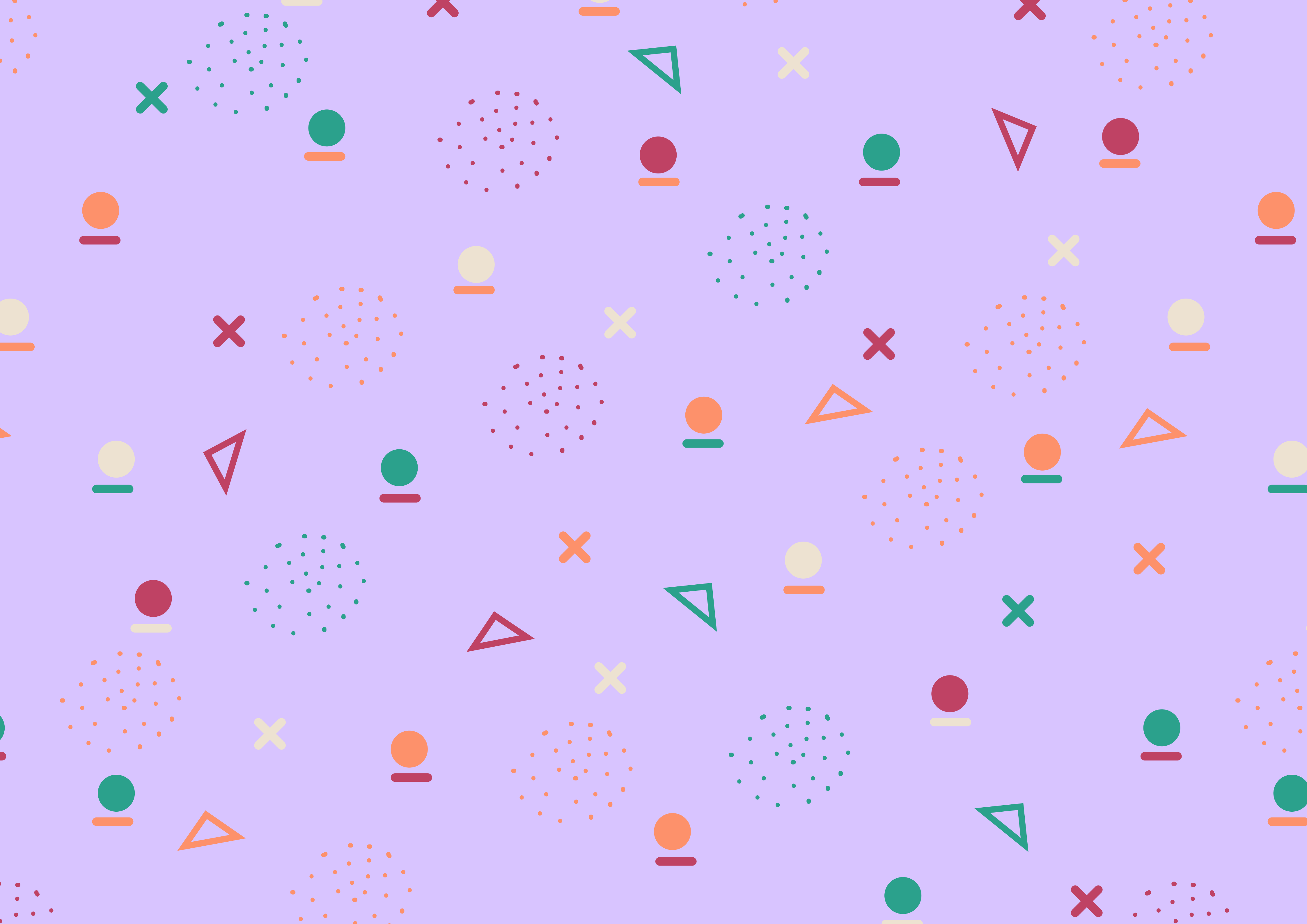


① $P(1,1) = 1/9$ $P(2,1) = 1/3$ $P(3,1) = 1/9$

$P(1,2) = 1/9$ $P(2,2) = 0$ $P(2,3) = 1/18$

$P(1,3) = 0$ $P(2,3) = 1/6$ $P(3,3) = 1/9$

$E\left[X/Y=y\right]$?

i) $E\left[X/_{Y=1}\right] = \sum_x x \cdot P_{X/Y}\left(x/_{y=1}\right)$

$$= (1) \cdot \frac{P(X=1, Y=1)}{P(Y=1)} + (2) \cdot \frac{P(X=2, Y=1)}{P(Y=1)} + (3) \frac{P(X=3, Y=1)}{P(Y=1)}$$

$$P(Y=1) = \sum_x P(X=x, Y=1) = P(X=1, Y=1) + P(X=2, Y=1) + P(X=3, Y=1)$$

$$= 1/9 + 1/3 + 1/9 = 5/9$$

$$= \frac{9}{5}\left\{ 1/9 + 2 \cdot 1/3 + 3 \cdot 1/9 \right]$$

$$\frac{9}{5}\left\{ \frac{1}{9} + \frac{2}{3} + 1/3 \right\} = \cancel{\frac{9}{5}}\left\{ \frac{\cancel{10}^{2}}{\cancel{9}} \right\} = \underline{\underline{2}}$$

ii) $E\{X/Y=2\} = \sum x \cdot P_{X/Y}\left(x/y=2 \right)$

$$P_Y(2) = \sum_x P(X=x, Y=2)$$

$$= 1/9 + 0 + 1/18 = \frac{3}{18} = 1/6$$

$$E\{X/Y=2\} = 6\left\{ 1 \cdot 1/9 + 2(0) + 3 \cdot (1/18) \right]$$

$$= 6\left\{ 1/9 + 1/6 \right\} = \cancel{6}\left\{ \frac{2+3}{\cancel{18}_{3}} \right\} = 5/3$$

$$P_Y(3) = \sum_x P(X=x, Y=3)$$

$$= 0 + 1/6 + 1/9 = \frac{9+6}{54} = 15/54$$

iii) $E\{X/Y=3\} = \sum_x x \cdot P_{X/Y}(x/y)$

$$= \frac{54}{15} \cdot \left[1 \cdot 0 + 2 \cdot 1/6 + 3 \cdot 1/9\right]$$

$$= \frac{54}{15}\left[1/3 + 1/3\right] = \frac{\overset{18}{\cancel{54}}}{15}\left[2/\cancel{3}\right] = 36/15$$

(2)

$1 \rightarrow p$

$0 \rightarrow 1-p$

Outcome sequence can be

11 0 - - - - (or) 001 - -

111 0 - - - - . . 0001 . - -

1111 0 - - - - 00001 . - -

11111 0 - - - - 000001 . - -

. - - - . - -

Let X be the random variable which associates the length of run

$$E[X] = \sum_{x} x \, P_X(x)$$

$$= \sum_{x=1}^{\infty} x \cdot \left[p^x (1-p) + p \cdot (1-p)^x \right]$$

$$= \underline{(1-p)} \sum_{x=1}^{\infty} x \cdot p^{x} + p \sum_{x=1}^{\infty} (1-p)^{x} \cdot x$$

$$S = \sum_{i} i \cdot r^{i}$$

$$S = 1 \cdot r + 2 \cdot r^{2} + 3 \cdot r^{3} + \cdots$$

$$rS = \quad 1 \cdot r^{2} + 2r^{3} + \cdots$$

$$S(1-r) = r + r^{2} + r^{3} + \cdots$$

$$= r\left(\frac{1}{1-r}\right)$$

$$S = \frac{r}{(1-r)^{2}}$$

hence

$$E\{x\} = \frac{p}{(1-p)} + \frac{1-p}{p}$$

ii) Expectation of 2nd run

$Y =$ 2nd run length $\underline{\underline{l-1}}$ after 2nd run ending

① 0 $\rightarrow$

110

1110

11110

111110

$$\sum_{x=1} \left(p^x (1-p) + p(1-p)^x \right)$$

③ $P(X=i, Y=j) = {}^jC_i \frac{e^{-2\lambda} \lambda^j}{j!} \quad 0 \le i \le j$

$$P_Y(j) = \sum_i P(X=i, Y=j)$$

$$= \sum_{i} {}^{j}C_i \frac{e^{-2\lambda} \cdot \lambda^{j}}{j!}$$

$$\frac{e^{-2\lambda} \cdot \lambda^{j}}{j!} \sum_{i=0}^{j} {}^{j}C_i \quad — (1)$$

$$\sum_{i=0}^{n} {}^{n}C_i = {}^{n}C_0 + {}^{n}C_1 + {}^{n}C_2 + \cdots + {}^{n}C_n$$

$$(a+b)^n = \sum_{r=0}^{n} {}^{n}C_r \, a^{r} \, b^{(n-r)}$$

$a=1 \quad b=1$

$$(1+1)^n = \sum_{r=0}^{n} {}^{n}C_r$$

$$\boxed{2^n = \sum_{r=0}^{n} {}^{n}C_r} \quad — (2)$$

(2) in (1)

$$\boxed{P_Y(j) = \frac{e^{-2\lambda}\lambda^j}{j!} \cdot 2^j}$$

ii) $P_X(i) = \sum_j P(X=i, Y=j)$

$$= \sum_j {}^jC_i \frac{e^{-2\lambda}\lambda^j}{j!} = e^{-2\lambda} \sum_j \frac{{}^jC_i \lambda^j}{j!}$$

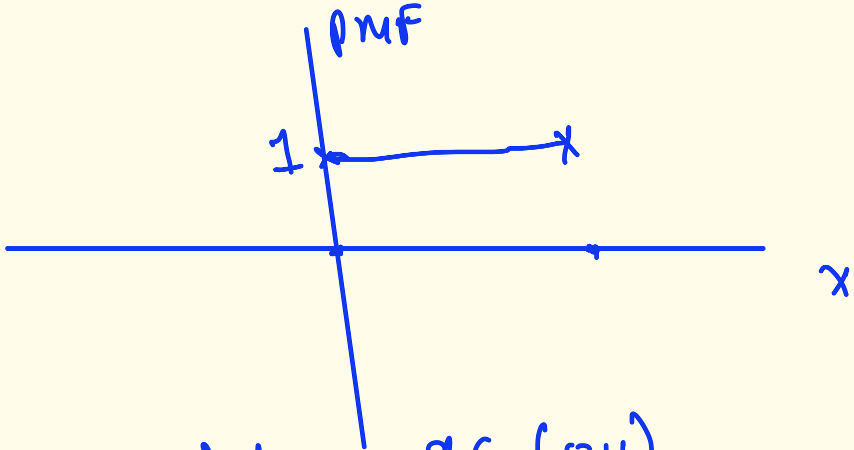
iii) PMF of $Y-X$

$P_{Y-X}(Y-X)$

$Y-X = Z \qquad Y = Z+X$

$$P_Z(z) = \sum_{x=0}^{\infty} P(X=x, Y=z+x)$$

(4) X is uniform over $(0,1)$



$$\int_0^1 c\cdot dx = 1$$

$$c(1) = 1 \qquad c = 1$$

$$f_X(x) = \begin{cases} 1 & x \in (0,1) \\ 0 & \end{cases}$$

i) $$E[X^n] = \int_x x^n \cdot f_X(x)\,dx$$

$$= \int_0^1 x^n \cdot 1\,dx = \left[\frac{x^{n+1}}{n+1}\right]_0^1 = \frac{1}{n+1} - 0 = \frac{1}{n+1}$$

ii) $\mathrm{Var}[X^n] = E[(X^n - E[X^n])^2]$ $\quad$ $\mathrm{Var}[X] = E[(X-\mu)^2]$

$$= E[(X^n - 1/(n+1))^2]$$

$$= E\left[X^{2n} + (1/(n+1))^2 - 2\frac{1}{n+1}X^n\right]$$

$$= E[X^{2n}] - \frac{2}{n+1}E[X^n] + \frac{1}{(n+1)^2}$$

$$= \int_x x^{2n} f(x)\,dx - \frac{2}{n+1}\int_x x^n\,dx + \frac{1}{(n+1)^2}$$

$$= \left[\frac{x^{2n+1}}{2n+1}\right]_0^1 - \frac{2}{n+1}\cdot\left[\frac{x^{n+1}}{n+1}\right]_0^1 + \frac{1}{(n+1)^2}$$

$$= \frac{1}{2n+1} - \frac{2}{(n+1)^2} + \frac{1}{(n+1)^2}$$

$$= \frac{1}{2n+1} - \frac{1}{(n+1)^2}$$

20 $P_{XY}(x_i, y_j) = \begin{cases} K(2x_i + y_j) & x_i = 1,2 \; ; \; y_j = 1,2 \\ 0 \end{cases}$

(a) $\sum P_{XY}(x_i, y_j) = 1$

$$K(2+1) + K(4+1) + K(2+2) + K(4+2) = 1$$

$$K(3+5+4+6) = 1$$

$$K(18) = 1 \qquad \boxed{K = 1/18}$$

(b) Marginal pmf of X and Y

$$P_X(x) = \sum_y P_{X,Y}(X=x, Y=y)$$

$$= \sum_y (1/18)(2x+y)$$

$$= (1/18)(2x+1) + (1/18)(2x+2)$$

$$\boxed{P_X(x) = (1/18)(4x+3)}$$

$$\boxed{P_X(x) = \begin{cases} 7/18 & x=1 \\ 11/18 & x=2 \end{cases}}$$

$$P_Y(y) = \sum_x P(X=x, Y=y)$$

$$= \sum_x \frac{1}{18}(2x+y)$$

$$= \frac{1}{18}(2+y) + \frac{1}{18}(4+y) = \frac{1}{18}(6+2y)$$

$$\boxed{P_Y(y) = \frac{1}{18}(6+2y) = \begin{cases} 1/3 & y=1 \\ 2/3 & y=2 \end{cases}}$$

(C) $\boxed{P_{XY}(X=x, Y=y) = P_X(x) \cdot P_Y(y)}$ → Then X and Y are independent

$$\frac{1}{18}(4x+3) \cdot \frac{1}{18}(6+2y)$$

$$\frac{1}{324}\left(24x + 8xy + 18 + 6y\right)$$

$$\frac{\left(24x + 6y + 8xy + 18\right)}{324}$$

$$\frac{1}{18}(2x+y) \neq \frac{24x + 6y + 8xy + 18}{324}$$

hence they are not indep —

18 $X \rightarrow$ successes

$X \in \{0, n\} \cap \mathbb{Z}$

$Y \rightarrow$ Failures

$Y \in \{0, n\} \cap \mathbb{Z}$

$$P_Z(z) = P_{X,Y}(X = x,\ Y = x - z)$$

$\boxed{X - Y = z,}$ $\boxed{X + Y = n}$ $\boxed{X - (n - X) = z}$

$\boxed{2X - n = z}$

$$P_Z(z) = {}^nC_x \cdot p^x (1-p)^{x-z}$$

$$P_Z(z) = {}^nC_{\frac{n+z}{2}} (p)^{\frac{n+z}{2}} (1-p)^{\frac{n-z}{2}}$$

$$E[Z] = \sum_z z \cdot {}^nC_{\frac{n+z}{2}} (p)^{\frac{n+z}{2}} (1-p)^{\frac{n-z}{2}}$$

$$z = 2x - n$$

$$\sum_x (2x-n) \, {}^nC_x (p)^x (1-p)^{n-x}$$

$$= 2 \sum_{x=0}^{n} x \, {}^nC_x \, p^x (1-p)^{n-x} - n \sum {}^nC_x (p)^x (1-p)^{n-x}$$

$$= 2np - n$$

$= 2np - n$

$Var(z) = E\{(z - \mu_z)^2\} = E\{z^2\} - \mu_z^2$

$= \sum_x (2x-n)^2 \cdot {}^nC_x\, p^x (1-p)^{n-x} - (2np-n)^2$

$= \sum_x (4x^2 + n^2 - 4xn) \cdot {}^nC_x\, p^x (1-p)^{n-x} - (2np-n)^2$

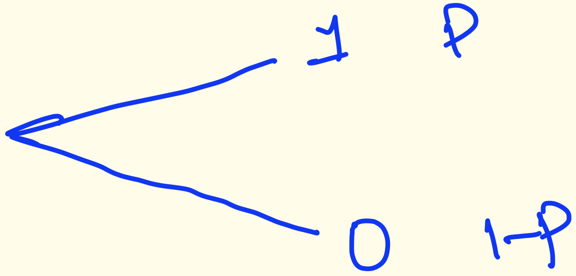
$= 4(np(1-p)) + n^2 - 4n^2(p) - (2np-n)^2$

$= 4np - 4np^2 + n^2 - 4n^2p - (4n^2p^2 + n^2 - 4n^2p)$

$= 4np - 4np^2 + n^2 - \cancel{4n^2p} - 4n^2p^2 - n^2 \cancel{+4n^2p}$

$= 4np - 4np^2 + n^2 - 4n^2p^2 - n^2$

17)



$X \rightarrow$ no of transmissions in time period

$Y =$ no of transmission of 1's in time period

$$P_X(k) = \frac{e^{-\lambda}\lambda^{k}}{k!}$$

$$P_Y(y) = \sum_{x=y}^{\infty} \frac{e^{-\lambda}\lambda^{x}}{x!}\cdot (p)^{y}(1-p)^{x-y}\,{}^{x}C_{y}$$

$$p^{y} \sum_{x=y}^{\infty} \frac{e^{-\lambda}\lambda^{x}}{x!}(1-p)^{x-y}\,{}^{x}C_{y}$$

$$e^{-\lambda}\cdot p^{y} \sum_{x=y}^{\infty} \frac{\lambda^{x}}{x!}(1-p)^{x-y}\,{}^{x}C_{y}$$

$$e^{-\lambda} p^{y} \sum_{x=y}^{\infty} \frac{\lambda^{x}}{\cancel{x!}} (1-p)^{x-y} \frac{\cancel{x!}}{(x-y)!\, y!}$$

$$\frac{e^{-\lambda} p^{y}}{y!} \sum_{x=y}^{\infty} \frac{\lambda^{x} (1-p)^{x-y}}{(x-y)!}$$

$$\frac{e^{-\lambda} p^{y} \cdot \lambda^{y}}{y!} \sum_{x=y}^{\infty} \frac{\left(\lambda(1-p)\right)^{x-y}}{(x-y)!} = \frac{e^{-\lambda} \cdot p^{y} \cdot \lambda^{y}}{y!} \times e^{\lambda(1-p)}$$

$$= \frac{e^{-\lambda p} \cdot p^{y} \cdot \lambda^{y}}{y!}$$

$$= \frac{e^{-\lambda p} \cdot (\lambda p)^{y}}{y!}$$

hence proved//

(1b) $P_{M,N}(m,n) = \begin{cases} C & m \geq 0, n \geq 0, m+n < L \\ 0 & \text{otherwise} \end{cases}$

(a) $\Rightarrow \sum_{m,n} P_{M,N}(m,n) = 1$

$\Rightarrow$ $m=0 \Rightarrow n = 0, 1, 2 \ldots L-1 \Rightarrow L \rightarrow 1$

$m=1 \Rightarrow n = 0, 1, 2 \ldots L-2 \Rightarrow L-1 \rightarrow 1$

$\vdots$

$m = L-1 \Rightarrow n = 0$

$\Rightarrow 1$

$$\Rightarrow \quad \frac{(L)(L+1)}{2} c = 1$$

$$\Rightarrow \quad C = \frac{2}{(L)(L+1)}$$

(b) $P_M(m)$, $P_N(n)$

$m+n < L$

$\boxed{n < L-m}$

m

$$P_M(m) = \sum_n P_{M,N}(m,n)$$

$$= \sum_{n=0}^{L-m-1} C \quad \Rightarrow \quad C[L-m]$$

$$P_n(n) = \sum_{m=0}^{L-n-1} \Rightarrow \boxed{c\{L-n\}}$$

(c) $Pr(M+N< L/2)$

case i) $L/2$ is an integer

Let $L/2 = K$

$$Pr(M+N < K) = \boxed{\frac{(K)(K+1)}{2} \times C}$$

case ii) $L/2$ is not an integer

$[L/2] = K$ where $[.]$ is GIF.

$Pr(M+N \leq K)$

because $M+N < L/2$ is same as $M+N \leq K$

$= Pr(M+N < K \cup M+N = K) = Pr(M+N < K) + Pr(M+N = K)$

$$= \left((K+1) + \frac{K(K+1)}{2}\right) \times C$$

$$= \boxed{\frac{(K+1)(K+2)}{2} \times C}$$

(15) Let the number of Paper we hand him be represented by R.V "X"

$X = [6, \infty) \cap \mathbb{Z}$

$$E\{X\} = \sum_{x} x\, P_X(x)$$

$$P_X(x) = {}^{x}C_1 \cdot \left(\cancel{1/6}\right) \times {}^{6}C_1 \times (1/x) \cdot \frac{(x-1)}{x} \cdot \frac{(x-2)}{x} \cdots \frac{(x-5)}{6}$$

$$P_X(x) = \frac{{}^{x}C_1}{x} \cdot \frac{{}^{x-1}C_1}{x} \cdot \frac{{}^{x-2}C_1}{x} \frac{{}^{x-3}C_1}{x} \frac{{}^{x-4}C_1}{x} \frac{{}^{x-5}C_1}{x}$$

$$P_X(x) = \frac{(\cancel{x})(x-1)(x-2)(x-3)(x-4)(x-5)}{x^{\cancel{6}\,5}} = \frac{(x-1)(x-2)\cdots(x-5)}{x^5}$$

$$E\{X\} = \sum_{x=6}^{\infty} \frac{(x-1)(x-2)(x-3)(x-4)(x-5)}{x^4}$$

(13) $z = x + y$

$$P_x(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!} \qquad P_{x,y}(x,y) = P_x(x) \cdot P_y(y)$$

$$P_z(z) = \sum_x P_{x,y}(x, z-x)$$

$$x \geq 0$$
$$y \geq 0$$

$$= \sum_{x=0}^{z} \frac{e^{-\lambda_1} \cdot \lambda_1^x}{x!} \cdot \frac{e^{-\lambda_2} \cdot \lambda_2^{z-x}}{(z-x)!}$$

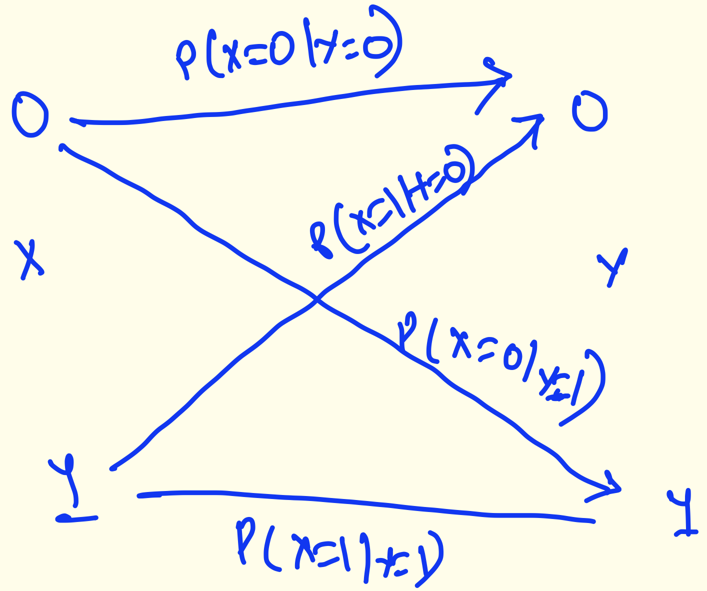
$$= e^{-(\lambda_1+\lambda_2)} \sum_{x=0}^{z} \frac{\lambda_1^x \cdot \lambda_2^{z-x}}{x!(z-x)!} \cdot \frac{z!}{z!}$$

$$= \frac{e^{-(\lambda_1+\lambda_2)}}{z!} \cdot \sum_{x=0}^{z} {}^zC_x \, \lambda_1^x \cdot \lambda_2^{z-x} = \frac{e^{-(\lambda_1+\lambda_2)}}{z!} (\lambda_1+\lambda_2)^z$$

hence proved that $P_Z(z) = \dfrac{e^{-(\lambda_1+\lambda_2)} \cdot (\lambda_1+\lambda_2)^z}{z!}$

hence $\lambda_1+\lambda_2$ is the Parameter

(12) $P(X=0) = 0.5$ $\quad P(Y=1/X=0) = 0.1$ $\quad P(Y=0|X=1) = 0.2$



$$\frac{P(X=0, Y=1)}{P(X=0)} = 0.1 \qquad \frac{P(Y=0, X=1)}{P(X=1)} = 0.2$$

$$\frac{P(x=0, y=1)}{P(x=0)} + \frac{P(x=0, y=0)}{P(x=0)} = 1$$

$$0.1 + \frac{P(x=0, y=0)}{0.5} = 1$$

$$P(x=0, y=0) = (0.9).1/2 = 0.45$$

$$P(x=0, y=1) = 0.05$$

$$\frac{P(y=0, x=1)}{P(x=1)} = 0.2$$

$$P(x=0) + P(x=1) = 1$$

$$P(x=1) = 0.5$$

$$\boxed{P(y=0, x=1) = 0.1}$$

$$(0.2) + \frac{P(x=1, y=1)}{P(x=1)} = 1$$

$$\frac{P(x=1, y=1)}{P(x=1)} = 0.8$$

$$P(x=1, y=1) = 0.5 \times 0.8$$
$$= 0.4$$

$$P(x=1, y=1) = 0.4$$

joint pmf's

a) $P_{X,Y}(0,0) = 0.45$, $P_{X,Y}(0,1) = 0.05$

$P_{X,Y}(1,0) = 0.1$, $P_{X,Y}(1,1) = 0.4$

ⓑ Marginal pmf of X, Y

i)

$P_X(x=0) = 0.5$

$P_X(x=1) = 0.5$

ii) $P_Y(y=1) = \sum_{x} P_{XY}(X=x, Y=1)$

$= P(X=0, Y=1) + P(X=1, Y=1)$

$= 0.05 + 0.4 = 0.45$

$$P_Y(y=0) = \sum_{x} P_{X,Y}(X=x, Y=0)$$

$$= P(X=0, Y=0) + P(X=1, Y=0)$$

$$= 0.45 + 0.1 = 0.55.$$

(c) $P_{X,Y}(x, y) = P_X(x) \cdot P_Y(y)$

Then X and Y are independent

$P_{X,Y}(0,0) = 0.45$ $\quad P_X(0) = 0.5$ $\quad P_Y(0) = 0.55$

$$0.45 \neq (0.5)(0.55)$$

$$0.45 \neq 0.275$$

hence they are not independent

(10) $X = r\cos\frac{2\pi\theta}{8}$ $Y = r\sin\frac{2\pi\theta}{8}$ $\theta \in \{0,1,2,3,4,5,6,7\}$

$\theta = 0 \Rightarrow (0,0)$

$\theta = 1 \Rightarrow (r/\sqrt{2}, r/\sqrt{2})$

$\theta = 2 \Rightarrow (0, r)$

$\theta = 3 \Rightarrow (-r/\sqrt{2}, r/\sqrt{2})$

$\theta = 4 \Rightarrow (-r, 0)$

$\theta = 5 \Rightarrow (-r/\sqrt{2}, -r/\sqrt{2})$

$\theta = 6 \Rightarrow (0, -r)$

$\theta = 7 \Rightarrow (r/\sqrt{2}, -r/\sqrt{2})$

joint PMF of X, Y is $= 1/8$

$$P_{X,Y}(X=x, Y=y) = 1/8$$

(11) marginal PMF of X and Y

$$P_X(x) = \sum_y P_{XY}(X=x, Y=y)$$

$$P_X(0) = \sum_y P_{XY}(X=0, Y=y)$$

$$= 3/8$$

$$P_X(r) = 0$$

$$P_X(r/\sqrt{2}) = 1/4$$

$$P_X(r/\sqrt{2}) = 1/4$$

$$P_Y(0) = 1/4$$

$$P_Y(r/\sqrt{2}) = 1/4$$

$$P_Y(-r/\sqrt{2}) = 1/4$$

$$P_Y(r) = 1/8$$

$$P_Y(-r) = 1/8$$

(b) $P(X=0) = 3/8$

$$P(Y \le r/\sqrt{2}) = 1/4 + 1/4 + 1/4 + 1/8 = 3/4 + 1/8 = \frac{7}{8}$$

$$P(X \geq r/\sqrt{2}, Y \geq r/\sqrt{2}) = \underset{X, Y \sqrt{2}}{P(r, r)} + P_{XY}(r, r/\sqrt{2}) + P(r/\sqrt{2}, r/\sqrt{2})$$

$$= 1/8 + 0 + 1/8 = 1/4$$

(9) There are m bits

X represent the number of patterns tested until correct password

$\Rightarrow$ 2^m patterns are

$X \in [1, 2^m] \cap \mathbb{Z}$

Y represent the password has not been found after k tries

$P_{X/Y}(X = x; Y = k)$

$$\boxed{P_X(x) = \frac{1}{2^m}}$$

if Given until k trails The pattern has not found it means $p_{X/x}(x|k) = \frac{1}{2^m - k}$

as the probability of those trails is given '0'

$$E[x] = \sum_{k+1}^{2^m} x \cdot \frac{1}{2^m - k}$$

$$= \frac{1}{2^m - k} \left\{ (k+1) + (k+2) + (k+3) + \cdots 2^m \right\}$$

$$= \frac{1}{2^m - k} \times \left[\frac{2^m (2^m + 1)}{2} - \frac{(k)(k+1)}{2} \right]$$

⑧ error Prob = 10^{-6}

transmission occur in block $\rightarrow$ 10000 bits

$N \rightarrow$ number ~~of~~ errors

$$P\{N=0\} = (1-10^{-6})^{10000}$$

$$(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + - -$$

$b \approx 0$

hence $\quad {}^nC_0 a^n + n a^{n-1} b \quad = \quad a^n + n a^{n-1} b$

$$P\{N=0\} = 1 + (10000)(1)(-10^{-6})$$

$$= 1 + (1)(-10^{-2})$$

$$= 1 - 10^{-2} = 0.99$$

$P\{N=0\} \approx 0.99$

$$P\{N\leq 3\} = (1-10^{-6})^{10000} + 10000(1-10^{-6})^{9999}(10^{-6})$$
$$+ {}^{10000}C_2(1-10^{-6})^{9998}(10^{-6})^2 + {}^{10000}C_3(1-10^{-6})^{9997}(10^{-6})^3$$

(b) $P\{N=0\} = (1-P)^{10000} = 0.01$

$$= (1-P) = (0.01)^{1/10000}$$

$$\Rightarrow \boxed{P = 1-(0.01)^{\frac{1}{10000}}}$$

(7) n letter $\longrightarrow$ n envelopes

Places in a random order

mean = variance 1

$$\boxed{P = 1/n}$$

$$\boxed{\text{no. of Dearrangement} \Rightarrow D_n}$$

Given there are $\underline{x}$ number of matched pairs

$$P_x(x) = {}^nC_x \cdot P^x \cdot (D_{n-x}) (1-P)^{n-x}$$

$$= {}^nC_x \cdot p^x \cdot (1-p)^{n-x} \cdot (D_{n-x})$$

$$= \frac{n!}{\cancel{(n-x)!}\, x!} \cdot p^x \cdot (1-p)^{n-x} \cdot \cancel{(n-x)!} \left\{ 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots \frac{(-1)^{(n-x)}}{(n-x)!} \right.$$

$$\lim_{n\to\infty} P_X(x) = \frac{e^{-1}}{(x!)} \lim_{n\to\infty} \underline{n!\, p^x} \cdot (1-p)^{n-x}$$

$$= \frac{e^{-1}}{(x!)} \lim_{n\to\infty} n! \left(1/n\right)^x \left(1 - 1/n\right)^{n-x}$$

$$= \frac{e^{-1}}{x!} \lim_{n\to\infty} n! \left(1/n\right)^{x} \cdot \left(1-1/n\right)^{n-x}$$

$$= \frac{e^{-1}}{x!} \lim_{n\to\infty} n! \left(1/n\right)^{x} \left(1-1/n\right)^{n} \qquad e^{\cancel{n} \cdot 1/\cancel{n}}$$

$$= \frac{\cancel{e^{-1}}}{x!} \cdot \cancel{e} \lim_{n\to\infty} n! \left(1/n\right)^{x}$$

⑥

$$P(X=n+k \mid X>n) = P(X=k) \quad \text{if } X = \text{Geometric}$$

$$\Rightarrow \frac{(1-p)^{n+k}}{\sum_{x=n+1}^{\infty} (1-p)^x} = \frac{\frac{(1-p)^{n+k}}{(1-p)^{n+1}}}{(1-p)} = (1-p)^k$$

$$= P(X=k)$$

⑤

$$P_X(k) = {}^nC_k \cdot p^k \cdot (1-p)^k$$

$$r = \frac{P(X=k+1)}{P(X=k)} = \frac{{}^nC_{(k+1)}}{{}^nC_k} \cdot \frac{p}{1-p} = \frac{n-k}{k+1} \cdot \frac{(p)}{(1-p)}$$

if $r>1$ the PMF is ↑ in nature

$r<1$ The PMF is $\downarrow$ in nature

hence $\frac{n-k}{k+1} \frac{(p)}{(1-p)} < 1$ That k is the value at which the func^n decreases

$$(n-k)p < (k+1)(1-p)$$

a) $(n+1)p = l$

$\Rightarrow k(l-p) < lp - m$

$k = l-1$ or $k = l$

$k = (n+1)p - 1$ or $k = (n+1)p$

(19) $\alpha = 10 \,/\text{min}$

$$P_X(k) = \frac{(\lambda)^k \cdot e^{-\lambda}}{k!}$$

a) $$P_X(11) = \frac{(10)^{11} e^{-11}}{11!}$$

b) N booths to a toll Plaza

$\lambda = 30 \rightarrow$ at plaza

$\alpha = \frac{30}{N}$ at each booths

min number of booths need to be employed so that probability is atleast 0.05 that not more than 5 cars approach the Booth

$$P_y(K) = \frac{e^{-30/N} \cdot \left(-30/N\right)^{K}}{K!}$$

$$\boxed{30/N = \alpha}$$

Now

$$P(Y \leq 5) = \frac{e^{-\alpha} \cdot \alpha^5}{5!} + \frac{e^{-\alpha} \cdot \alpha^4}{4!} + \frac{e^{-\alpha} \alpha^3}{3!} + \frac{e^{-\alpha} \cdot \alpha^2}{2!} + \frac{e^{-\alpha} \cdot \alpha^1}{1!} + \frac{e^{-\alpha} \cdot \alpha^0}{0!}$$

$$\leq 0.05$$

$$e^{-\alpha}\left\{1+\frac{\alpha}{1!}+\frac{\alpha^2}{2!}+\frac{\alpha^3}{3!}+\frac{\alpha^4}{4!}+\frac{\alpha^5}{5!}\right] \leq 0.05$$

now

$$\sum_{x=0}^{5}\frac{K^x}{x!} \leq 0.05$$

$$\frac{1}{(K-1)}\sum_{x=0}^{5}\frac{K^x-K^{x-1}}{x!} \leq 0.05$$

find $\boxed{K \Rightarrow 30/N}$. find N

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$P = 1/3$

2 hours $\rightarrow$ duration of trip

$N \rightarrow RV$ $\rightarrow$ Time total

$N = 2\text{hrs} \times T$

$T =$ no. of. trips

average time for escape

$E\{N\} = 2E\{T\}$

$P_X(k) = P(1-P)^{k-1}$

$P_T(k) = P(1-P)^{k-1}$

$E\{T\} = 1/p = 3$ $E\{N\} = 6$