_/_/_

~~Bsab/∴BSV~~

Assignment #3
(AMS 102)

1. $P[X = x \mid Y=1] = \begin{cases} \frac{1/9}{5/9}, & x=1 \\ \frac{1/3}{5/9}, & x=2 \\ \frac{1/9}{5/9}, & x=3 \end{cases}$

$= \begin{cases} 1/5, & x=1 \\ 3/5, & x=2 \\ 1/5, & x=3 \end{cases}$

$\therefore E[X, Y=1] = \frac{1}{5} + \frac{6}{5} + \frac{3}{5} = \underline{\underline{2}}$

$P[X \mid Y=2] = \begin{cases} (1/9)/1/6 = 2/3, & X=1 \\ (0)/1/6 = 0, & X=2 \\ (1/18)/1/6 = 1/3, & X=3 \end{cases}$

$E[X, Y=2] = \frac{2}{3} + 1 = \underline{\underline{\frac{5}{3}}}$

$P[X, Y=3] = \begin{cases} 0/(5/18) = 0, & X=1 \\ (1/6)/(5/18) = 3/5, & X=2 \\ (1/9)/(5/18) = 2/5, & X=3 \end{cases}$

$E[X, Y=3] = 6/5 + 6/5 = \underline{\underline{12/5}}$

$\therefore E[X \mid Y=i] = \begin{cases} 2, & i=1 \\ 5/3, & i=2 \\ 12/5, & i=3 \end{cases}$

2. ~~1-5~~ (a) $X \to$ length of 1st run

$$p_X(x) = \begin{cases} p(1-p) + (1-p)p & , x=1 \\ p^2(1-p) + (1-p)^2 p & , x=2 \\ \vdots & \\ p^x(1-p) + (1-p)^x p & , x=x \end{cases}$$

$$E[X] = \sum_{1}^{\infty} x\left(p^x(1-p) + (1-p)^x p\right)$$

$$\cancel{= \sum_{1}^{\infty} x p^x - x p^{x+1}} = (1-p)\sum x p^x + p \sum x(1-p)^x$$

Note :- $\boxed{\sum x p^x = \frac{p}{(1-p)^2}, \ p<1}$

$$\therefore E[X] = (1-p)\frac{p}{(1-p)^2} + p\frac{(1-p)}{(p)^2}$$

$= \frac{p}{1-p} + \frac{1-p}{p}$

(b) $X \to$ dean of 2nd run

~~$p_X(y) = \{ p(1-p) + p(1-p$~~

$$p_X(y) = \begin{cases} p(1-p) + p(1-p), & y = 1 \\ p^2(1-p) + (1-p)^2 p, & y = 2 \\ \vdots \\ p^n(1-p) + (1-p)^n p, & y = n \end{cases}$$

∴ similarly, $E[X] = \frac{p}{1-p} + \frac{1-p}{p}$

3. $p(X=i, Y=j) = {}^jC_i \frac{e^{-2\lambda} \lambda^j}{j!} \quad 0 \le i \le j$

(a) $p_Y(Y=j) = \sum_{\text{All } x=i} {}^jC_i \frac{e^{-2\lambda} \lambda^j}{j!} = \frac{e^{-2\lambda} \lambda^j}{j!} \sum {}^jC_i$

$= \frac{e^{-2\lambda} \lambda^j}{j!} 2^j$

(b) $p_X(X=i) = \sum_{y=j} {}^jC_i \frac{e^{-2\lambda} \lambda^j}{j!}$

$= e^{-2\lambda} \sum {}^jC_i \frac{\lambda^j}{j!} = e^{-2\lambda} \sum \frac{j!}{(j-i)!\, i!} \frac{\lambda^j}{j!}$

$= \lambda^i \frac{e^{-2\lambda}}{i!} \underbrace{\sum \frac{\lambda^{j-i}}{(j-i)!}}_{\to e^{\lambda}} = \frac{e^{-\lambda} \lambda^i}{i!}$

__/__/____

(c) $p(X=i, Y=j) = {}^{j}C_{i}\, e^{-2\lambda} \frac{\lambda^{j}}{j!}$ — (1)

$p(Y-X=k)$, let $X=t$, $Y=(k+t)$

$= p(t, k+t) = {}^{k+t}C_{t}\, e^{-2\lambda} \frac{\lambda^{k+t}}{(k+t)!}$

$= \frac{e^{-2\lambda}\lambda^{k}\lambda^{t}}{(k!)(t)!} = \frac{e^{-\lambda}\lambda^{k}}{k!} \cdot \frac{e^{-\lambda}\lambda^{t}}{t!}$

(3?/17 → easy)

~~X ~ f(x) x → length of first~~

4. $f_X(x) = \begin{cases} 1 & , x \in (0,1) \\ 0 & , \text{o/w} \end{cases}$

$$E[x^n] = \int x^n f(x)\,dx = \int_0^1 x^n\,dx = \frac{x^{n+1}}{n+1}$$

$$\text{Var}(x^n) = E[x^{2n}] - (E[x^n])^2 = \frac{x^{2n+1}}{2n+1} - \frac{x^{2n+2}}{(n+1)^2}$$

5. ~~when success,~~

~~$P(X=k) = {}^nC_k (p)^k (1-p)^{n-k}$~~

~~As ${}^nC_k$ first increases till value reaches n/2 or (n+1)/2 whenever and then decreases, P(X=k) first increases monotonically, then decreases after reaching largest.~~

~~(a)~~

~~3. P(X = x, Y = y) = ...~~

~~6. To prove $P(X \geq n+k \mid X \geq n) = P(X = k)$~~

5. When increasing, $P(k) < P(k+1)$

$$\Rightarrow {}^nC_k (p)^k (1-p)^{n-k} < {}^nC_{k+1} (p)^{k+1} (1-p)^{n-(k+1)}$$

$$\Rightarrow \frac{n!}{k!\,(n-k)!}(1-p) < \frac{n!}{(n-(k+1))!\,(k+1)!}\,p$$

~~$\Rightarrow \frac{p(k+1)}{(n-k)(1-p)} > 1$~~

_/_/_

$\Rightarrow \frac{p(n-k)}{(k+1)(1-p)} > 1 \Rightarrow pn - \cancel{pk} > k+1-p-\cancel{pk}$

$\Rightarrow p(n+1)-1 > k$ — ①

Also when decreasing,

$\cancel{p(k) > p(k+1)}$

$p(n+1)-1 < k$ — ②

(a) as from ① & ②,
when $k = (n+1)p - 1$,
func^n is max. (When $(n+1)p$ is an integer)

(b) When $(n+1)p$ is not an integer (As k has to be integer), $k \in ((n+1)p-1, (n+1)p)$

8. (a) $P[N=0] = (1-10^{-6})^{10^5}$

$P[N \leq 3] = P[0] + P[1] + P[2] + P[3]$
$= (1-10^{-6})^{10^5} + {}^{10^5}C_1 (1-10^{-6})^{9999}(10^{-6})$
$+ {}^{10^5}C_2 (1-10^{-6})^{9998}(10^{-6})^2 + {}^{10^5}C_3 (1-10^{-6})^{9997}(10^{-6})^3$

(b) $P[N \geq 1] = 99\%$
$\Rightarrow P[0] = 1\% = 0.01 = e^{-\lambda}$

6. $P(X=k) = (1-p)^{k-1} p$ — ②

$$P(X=m+k \mid X>m) = \frac{P(X=m+k \wedge X>m)}{P(X>m)}$$

(as $k \geq 1 \Rightarrow X>m$) $\Rightarrow P(X=m+k \wedge X>m)$

$= P(X=m+k)$

$\Rightarrow$ ~~$= P(X=m+k)$~~

~~$\therefore P(X=m+k \mid X>m) = P(X=m+k)$~~

$\therefore$ Hence,

$$P(X=m+k \mid X>m) = \frac{P(X=m+k)}{P(X>m)} \quad - ①$$

$$P(X>m) = 1 - P(X \leq m)$$

$$= 1 - \left((1-p)^{m-1} p + \dots p\right)$$

$$= 1 - \frac{p(1-(1-p)^m)}{1-(1-p)} = \underline{\underline{(1-p)^m}}$$

$$\therefore ① = \frac{P(X=m+k)}{(1-p)^m} = \frac{(1-p)^{m+k-1} \cdot p}{(1-p)^m}$$

$$= (1-p)^{k-1} p = ②$$

H.P.

_/_/_

7. $X \Rightarrow$ correctly matched pair, $X \in [0, n]$

~~P(1st letter going into correct envelope) = 1/n~~
~~P(2nd " " " " ") = 1/n~~

Indicator func^n

$I_1, I_2, \dots I_n$

$I_i = 1$ if correct match / $I_i = 0$ if incorrect

$X = \sum_{i=1}^{n} I_i$ $\qquad E[I_i] = 1/n$

$$E[X] = \sum_1^n E[I_i] = \sum_1^n \left(\frac{1}{n}\right) = 1$$

$$Var(X) = Var\left(\sum_1^n I_i\right)$$

They are independent

$$Var(X+Y) = Var(X) + Var(Y) + 2\,Cov(X, Y)$$

$$\therefore Var(X) = \sum_1^n Var(I_i) + 2\sum_{i \neq j} Cov(I_i\, I_j)$$

$$Var(I_i) = E[I_i^2] - (E[I_i])^2 \qquad (\text{as } E[I_i^2] = E[I_i] \text{ as } I_i^2 = I_i)$$
$$= E[I_i](1 - E[I_i])$$
$$= \frac{1}{n}\left(1 - \frac{1}{n}\right) = \frac{n-1}{n^2}$$

$$Covar[I_i, I_j] = E[I_i, I_j] - \frac{1}{n} \cdot \frac{1}{n}$$

$$= \frac{n-(n-1)}{n^2(n-1)} = \frac{1}{n^2(n-1)}$$

_/_/_

$\therefore Var(X) = \sum_{1}^{n} \frac{n-1}{n^2} + 2 \sum_{i \neq j} \frac{1}{n^2(n-1)}$

$= \frac{n-1}{n} + 2\,{}^{n}C_2 \left(\frac{1}{n^2(n-1)} \right)$

$= \frac{n-1}{n} + \frac{1}{n} = \frac{n}{n} = \underline{\underline{1}}$

As $n \to \infty$ the prob. of success becomes minimal so we can ~~write it assume~~ say it converges to poisson

for ~~poisson~~ poison $E[X] = \lambda = 1$

$Var(X) = \lambda = 1$

$p_X(\lambda) = \begin{cases} \left(\frac{e^{-\lambda} \lambda^k}{k!} \right) & \lambda > 0 \\ 0 & o/w \end{cases}$

$= \begin{cases} \frac{e^{-1} 1^k}{k!} & \lambda > 0 \\ 0 \end{cases} = \begin{cases} \frac{1}{k!\,e} & \lambda > 0 \\ 0 & o/w \end{cases}$

8. ~~$p = 10^{-6}$~~ $p(e) = 10^{-6}$; 10,000. bits.

(a) $n = 10^4$, $p = 10^{-6}$

$\therefore np = 10^{-2} \Rightarrow$ rate $= \lambda = E[X]$

$p(N) = \frac{e^{-\lambda} \lambda^N}{N!}$ $\quad p(0) = e^{-\lambda} = e^{-10^{-2}}$

$= \frac{1}{e^{1/100}} \approx 0.99$

$p(N \leq 3) = p(1) + p(2) + p(3) + p(0)$

~~= 0.99~~ +

_/_/_

$$= 0.99 + \frac{e^{-10^{-2}} \times (10^{-2})}{1!} + \frac{e^{-10^{-2}} \times (10^{-2})^2}{2!} + \frac{e^{-10^{-2}} \times (10^{-2})^3}{3!}$$

$= \ldots\ldots\ldots$

(b) $p(N \geq 1) = 0.99$

$\therefore 1 - p(N=0) = 0.99$

$0.01 = p(N=0) = e^{-\lambda}$

$\Rightarrow e^{\lambda} = 100 \Rightarrow \underline{\lambda = \ln(100)}$

$\therefore$ at $p = \frac{\lambda}{n} = \frac{\ln(100)}{10^4}$

9. $X \in [1, 2^m]$

(a) $p(X=x) = \frac{1}{2^m} \quad \forall \quad 1 \leq x \leq 2^m$

$$P(X=x \mid X>k) = \frac{P(X=x \cap X>k)}{P(X>k)}$$

$P(X>k) = 1 - P(X \leq k) = 1 - \frac{k}{2^m}$

$$= \frac{1/2^m}{\frac{2^m - k}{2^m}} = \frac{1}{2^m - k}$$

(b) $$E[X \mid X>k] = \frac{1}{2^m - k} \sum_{x=k+1}^{2^m} x$$

$$= \frac{1}{2^m - k}\left(\frac{(2^m)(2^m+1)}{2} - \frac{(k)(k+1)}{2}\right)$$

$$= \frac{1}{\cancel{2^m - k}}\left(\frac{\cancel{(2^m - k)}(2^m + k + 1)}{2}\right) = \frac{2^m + k + 1}{2}$$

$$x = r\cos\left(\frac{\pi\theta}{4}\right)$$

$$y = r\sin\left(\frac{\pi\theta}{4}\right)$$

10. (a)
$$X = \begin{cases} r, & \theta = 0 \\ r/\sqrt{2}, & \theta = 1, 7 \\ 0, & \theta = 2, 6 \\ -r/\sqrt{2}, & \theta = 3, 5 \\ -r, & \theta = 4 \end{cases}$$

$$Y = \begin{cases} 0, & \theta = 0, 4 \\ r/\sqrt{2}, & \theta = 1, 3 \\ r, & \theta = 2 \\ -r/\sqrt{2}, & \theta = 5, 7 \\ -r, & \theta = 6 \end{cases}$$

$$\therefore (X, Y) = \begin{cases} (r, 0), & \theta = 0 \\ (r/\sqrt{2}, r/\sqrt{2}), & \theta = 1 \\ (0, r), & \theta = 2 \\ (-r/\sqrt{2}, r/\sqrt{2}), & \theta = 3 \\ (-r, 0), & \theta = 4 \\ (-r/\sqrt{2}, -r/\sqrt{2}), & \theta = 5 \\ (0, -r), & \theta = 6 \\ (r/\sqrt{2}, -r/\sqrt{2}), & \theta = 7 \end{cases}$$

(b) $p_{(X,Y)}(x, y) = 1/8$

11. (a)
$$p_X(x) = \begin{cases} 1/8, & x = r, -r \\ 1/4, & x = r/\sqrt{2}, 0, -r/\sqrt{2} \end{cases}$$

$$p_Y(y) = \begin{cases} 1/8, & y = r, -r \\ 1/4, & y = r/\sqrt{2}, 0, -r/\sqrt{2} \end{cases}$$

__/__/____

(b) $p(A) = 1/4$ $\cancel{p(B) =}$

$p(B) = p_Y\{ y \le r/\sqrt{2}) = 1 - p_Y(y = r)$

$= 7/8$

$p(C) = \cancel{p_{(X,Y)}((x,y) : (x,y) \in \{(r/\sqrt{2}, r/\sqrt{2}), (r, r), (r/\sqrt{2}, r), (r, r/\sqrt{2})\}}$

$= p_{(X,Y)}(r/\sqrt{2}, r/\sqrt{2}) = 1/8$

$p(D) = p_X(x = -r) = 1/8$

12. (a) $p_X(x=0) = p_X(x=1) = 0.5$

$p(Y=1 \mid X=0) = 0.1$, $p(Y=0 \mid X=0) = 0.9$

$p(Y=0 \mid X=1) = 0.2$, $p(Y=1 \mid X=1) = 0.8$

$p_{(X,Y)}(x,y) = \cancel{p_X(x) \times} \; p_X(X=x) \cdot p(Y=y \mid X=x)$

$\cancel{p_{(X,Y)}(1,0) = p_X(x=1) \times p_Y(Y=0 \mid X=1)}$

$\cancel{(X=0, Y=0) = p_X(x=0) \times p(Y=0 \mid X=0)}$

$$p_{(X,Y)}(x,y) = \begin{cases} 0.1, & (x,y) = (1,0) \\ 0.05 & (x,y) = (0,1) \\ 0.45, & (x,y) = (0,0) \\ 0.4, & (x,y) = (1,1) \end{cases}$$

(b) $p_X(X=x) = \{ 0.5, \; x \in \{0,1\}$

$p_Y(Y=y) = \sum_x p_{(X,Y)}(X=x, Y=y)$

$$p_Y(Y=y) = \begin{cases} 0.55, & Y=0 \\ 0.45, & Y=1 \end{cases}$$

_/_/_

(c) For independent X & Y, $p_{(X,Y)}(x,y) = p_X(x) \times p_Y(y)$

$\Rightarrow$ ~~LHS = $\frac{1}{2} \times 0.55$~~ $p_X(0)\, p_Y(0) = \frac{1}{2} \times 0.55 = 0.275$

$\neq p_{(X,Y)}(0,0)$

13. $p_X(x) = \dfrac{e^{-\lambda_1}(\lambda_1)^{x}}{x!}$, $p_Y(y) = \dfrac{e^{-\lambda_2}(\lambda_2)^{y}}{y!}$

$Z = X + Y$ $\quad z = x + y$

~~$p_Z(z) = p_Z(x+y)$~~

$p_Z(Z = m) = p_Z(x + y = m)$

$= \sum p_{X,Y}(X = k, Y = m-k)$

$= \sum_{k=0}^{m} p(X=k)\, p(Y=m-k)$

$= \sum_{k=0}^{m} \dfrac{e^{-\lambda_1}\lambda_1^{k}}{k!} \dfrac{e^{-\lambda_2}\lambda_2^{m-k}}{(m-k)!} = \sum_{0}^{m} \dfrac{e^{-(\lambda_1+\lambda_2)}\lambda_1^{k}\lambda_2^{m-k}}{k!\,(m-k)!}$

$= \dfrac{e^{-(\lambda_1+\lambda_2)}}{m!} \sum_{0}^{m} \dfrac{\lambda_1^{k}\lambda_2^{m-k}\, m!}{k!\,(m-k)!}$

$= \dfrac{e^{-(\lambda_1+\lambda_2)}}{m!} \sum_{0}^{m} {}^{m}C_{k}\, \lambda_1^{k} \lambda_2^{m-k}$

$= \dfrac{e^{-(\lambda_1+\lambda_2)}}{m!} (\lambda_1+\lambda_2)^{m}$

_/_/_

14. ~~$p_X(x) =$~~ $X \to$ no. of hrs doctor to escape.

$$p_X(x) = \begin{cases} 1/3 & , x = 2 \\ \left(\frac{2}{3}\right)\frac{1}{3} & , x = 4 \\ \left(\frac{2}{3}\right)^2 \frac{1}{3} & , x = 6 \\ \vdots & \\ \left(\frac{2}{3}\right)^{\left(\frac{x}{2}-1\right)} (1/3) & , x = x \end{cases}$$

$$E[x] = \sum x\, p_X(x) = \sum_{2}^{\infty} x\, (2/3)^{\left(\frac{x}{2}-1\right)} \frac{1}{3}$$

$$= \frac{1}{3} \sum_{1}^{\infty} 2x \left(\frac{2}{3}\right)^{x-1} = \sum_{1}^{\infty} x \left(\frac{2}{3}\right)^{x} = \frac{2/3}{(1-2/3)^2}$$

$$= \underline{6 \text{ hrs}}$$

17. ~~$p_x(N) = \frac{e^{-\lambda}(\lambda)^N}{N!}$~~

~~Let ...~~

15. 1st paper

$\downarrow$ new grade $p = 1$

$\frac{1}{5}$ of old grade $\quad$ $\frac{5}{6}$ prob of new

$\frac{2}{6}$ of old $\quad$ $\frac{4}{6}$ of new

$\frac{3}{6}$ of old $\quad$ $\frac{3}{6}$ of new

$\frac{4}{6}$ of old $\quad$ $\frac{2}{6}$ of new

$\frac{5}{6}$ of old $\quad$ $\frac{1}{6}$ of old

If prob. of finding a subcase is $\frac{x}{y}$

then min no. req. is $\frac{y}{x}$

prob: $\rightarrow \frac{x}{y}$ $\quad$ no. req. $= \frac{y}{x}$

$$1 + \frac{6}{5} + \frac{6}{4} + \frac{6}{3} + \frac{6}{2} + \frac{6}{1}$$

$$= 14.7$$

15)

16. (a) $\sum_{m+n \leq L} P_{M,N}(m,n) = 1$

$$\therefore \quad C\left(\frac{(L+1)^2-(L+1)}{2}\right) = 1$$

$$\Rightarrow C = \frac{2}{(L+1)(L+1-1)} = \frac{2}{(L+1)L}$$

(b) $P_{M,N}(m,n) = \frac{2}{(m+n+1)(m+n)}$

$$P_M(m) = \sum_{n=0}^{L-m-1} \frac{2}{(L+1)L} = \frac{(L-m)2}{(L+1)L}$$

$$P_M(m) = \frac{2(L-m)}{L(L+1)}$$

(c) $P(M+N<L/2)$

$$= \frac{C}{2}\left(\left(\frac{L}{2}+1\right)^2 - \left(\frac{L}{2}+1\right)\right)$$

$$= \frac{C}{2}\left(\left(\frac{L}{2}+1\right)\left(\frac{L}{2}\right)\right)$$

$$= \frac{1}{2} \cdot \frac{2}{(L+1)L}\left(\frac{L+2}{2}\right)\frac{L}{2} = \frac{L+2}{4(L+1)}$$

0 1 2 3 ... L/2
1
2
3
⋮
L/2

_/_/_

17. $X \to$ no. of transmissions

$$p_X(x) = \begin{cases} \dfrac{\lambda^x e^{-\lambda}}{x!} & \forall \lambda > 0 \\ 0 & o/w \end{cases}$$

No. of 1's $\to$ binomial

$$p_X(x=k) = \sum_{n=k} (x=k \mid N=n) \cdot p(N=n)$$

$$= \sum_{n=k}^{\infty} {}^{n}C_{k}\, p^{k} (1-p)^{n-k} \frac{\lambda^n e^{\lambda}}{n!}$$

$$= \frac{p^k}{k!} \sum_{k}^{\infty} \frac{(1-p)^{n-k} \lambda^n e^{-\lambda}}{(n-k)!}$$

$$A = n-k$$

$$p(x=k) = \frac{p^k}{k!} \sum_{A=0}^{\infty} \frac{(1-p)^A \lambda^{A+k} e^{-\lambda}}{A!}$$

$$p_X(X=k) = \frac{p^k \lambda^k e^{-\lambda}}{k!} \sum_{A=0}^{\infty} \frac{\{(1-p)\lambda\}^A}{A!}$$

$$= e^{(1-p)\lambda} = \frac{(p\lambda)^k e^{-\lambda p}}{k!}$$

$\therefore$ poison pmf $= p\lambda$

18. $X =$ no. of success, $Y =$ no. of failure, $P(\text{success}) = p$

$$p_X(x) = {}^{n}C_x\, p^x (1-p)^{n-x} \qquad p_Y(y) = {}^{n}C_y (1-p)^y (p)^{n-y}$$

$$Z = X - Y = X - (n - X) = 2X - n$$

$\therefore$ ~~$p_Z(z) = p_X\left(\frac{z+n}{2}\right) = {}^{n}C_{\frac{z+n}{2}}\, p$~~

$$Z = \begin{cases} -n & 0 \text{ success} \\ -n+2 & 1 \text{ "} \\ -n+4 & 2 \text{ "} \\ \vdots & \vdots \\ n & 0 \text{ failure} \end{cases}$$

__/__/__

Let $\frac{z-m}{2} = k$

$$P(x=k) = {}^{m}C_{\frac{z-m}{2}}\, p^{\frac{z-m}{2}} (1-p)^{\frac{3m-z}{2}}$$

$$E[z] = E[2x-m]$$
$$= 2E[x] - m$$
$$= 2mp - m$$
$$E[z] = m(2p-1)$$
$$Var[z] = Var[2x-m]$$
$$= 4\,Var[x]$$
$$= 4mp(1-p)$$

19. (a) $P(11) = \frac{e^{-10}\lambda^{11}}{11!}$

$\lambda = 10$, $\therefore P(11) = \frac{e^{-10}\times 10^{11}}{11!}$

(b) not more than 5 cars $\therefore \leq 5$ cars

$\therefore P(x \leq 5) = \sum_{0}^{5} \frac{e^{-30/N} \times (30/N)^k}{k!}$

solving this, we can find N.

20. $p_{XY}(x_i, y_j) = \begin{cases} k(2x_i + y_j), & x_i = 1,2,\ y_j = 1,2 \\ 0 & o/w \end{cases}$

$p(1,1) = 3k$ $\quad p(2,1) = 5k$

$p(1,2) = 4k$ $\quad p(2,2) = 6k$

$18k = 1$

$k = 1/18$

(a) $p(X=i) = \sum_{\forall j} p_{X,Y}(i,j)$

(b) $p(X=1) = 7/18$ / $p(X=2) = 11/18$

$\therefore\ p(Y=1) = 8/18 \quad , \quad p(Y=2) = 10/18$