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(2)

$$p(1) = p$$
$$p(0) = 1-p$$

Let l be the length of first run

$$p_2(l) = \begin{cases} p(1-p) + (1-p)p, & l=1 \\ p^2(1-p) + (1-p)^2 p, & l=2 \\ p^3(1-p) + (1-p)^3 p, & l=3 \\ p^m(1-p) + (1-p)^m p, & l=m \end{cases}$$

$$E[L] = \sum p_1(l)\cdot l$$
$$= p(1-p) + 2p^2(1-p) + 3p^3(1-p) + \cdots + (1-p)p + 2(1-p)^2 p + 3(1-p)^3 p + \cdots$$

$$E[L] = S_1 + S_2$$

$$S_1 = p(1-p)(1 + 2p + 3p^2 + 4p^3 + \cdots)$$
$$pS_1 = p(1-p)(p + 2p^2 + 3p^3 + \cdots)$$

$$(1-p)S_1 = p(1-p)(1 + p + p^2 + \cdots)$$
$$S_1 = \frac{p(1)}{1-p} = \frac{p}{1-p}$$

$$S_2 = (1-p)p(1 + 2(1-p) + 3(1-p)^2 + \cdots$$

$$S_2 = \frac{1-p}{p}$$

$$E[L] = S_1 + S_2 = \frac{p}{1-p} + \frac{1-p}{p}$$

$$\Rightarrow \frac{p^2 + (1-p)^2}{p(1-p)}$$

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(b) Expected length of second run is independent of expected length of first run.

so expected length of second run will also be same as it does not matter where the second run starts it will be calculated in the same way.

$$E[L] = \frac{p^2+(1-p)^2}{p(1-p)}$$

(3) $P(X=i, Y=j) = \binom{j}{i} \frac{e^{-2\lambda} \lambda^j}{j}$

$0 \leq i \leq j$

(a) $P_Y(y) = p(Y=j) = \sum_{\forall i} P(X=i, Y=j)$

$$= \sum_{\forall i} \binom{j}{i} \frac{e^{-2\lambda} \lambda^j}{j!}$$

$$= {}^jC_i \frac{e^{-2\lambda} \lambda^j}{j!}$$

$$= \frac{e^{-2\lambda} \lambda^j}{j!} \left({}^jC_0 + {}^jC_1 + \dots + {}^jC_{j-1} + {}^jC_j \right)$$

$$\Rightarrow \frac{e^{-2\lambda} \lambda^j 2^j}{j!}$$

(b) $\sum_{j=0}^{\infty} \binom{j}{i} \frac{e^{-2\lambda} \lambda^j}{j!}$

$= e^{-2\lambda} \sum \frac{{}^jC_i \, \lambda^j}{j!}$

$\Rightarrow e^{-2\lambda} \sum \frac{\lambda^j}{i!\,(j-i)!}$

$\frac{e^{-2\lambda}}{i!} \sum \frac{\lambda^j}{(j-i)!}$

$\Rightarrow \frac{e^{-2\lambda} \lambda^i}{i!} \sum \frac{\lambda^{j-i}}{(j-i)!}$

$\Rightarrow \frac{e^{-2\lambda} \lambda^i}{i!} e^{\lambda} = \frac{e^{-\lambda} \lambda^i}{i!}$

(c) PMF of $Y-X$
$(j-i)$

$P(X=i, Y=j) = \binom{j}{i} \frac{e^{-2\lambda} \lambda^j}{j!}$

$(Y-X=k)$ $\quad X=i \rightarrow Y=i+k$

$P(X=i, Y=i+k)$

$P(Y-X=k) = \dfrac{e^{-\lambda}\lambda^{k}}{k!}$

(4) let 'X' be uniform R.V over (0, 1)

$E[X^n] = \int_0^1 x^n dx = 1/n+1$

$E[X^{2n}] = \int_0^1 x^{2n} dx = \dfrac{1}{2n+1}$

$var(X^n) = E[X^{2n}] - (E[X^n])^2$

$= \dfrac{1}{2n+1} - \dfrac{1}{(n+1)^2}$

(5) 'X' be Binomial R.V with parameters 'n' & 'p'

$P(X=k) = {}^nC_k p^k (1-p)^{n-k}$

to show that $P(X=k)$ increases monotonically & then decreases monotonically.

$\dfrac{P(X=k)}{P(X=k+1)} \geq 1 \Rightarrow \dfrac{{}^nC_k (1-p)}{{}^nC_{k+1} (p)} \geq 1$

$\dfrac{(n-k-1)!(k+1)!}{(n-k)!\,k!} = \dfrac{(k+1)(1-p)}{(n-k)(p)} \geq 1$

$k+1-pk-p \geq np-kp$

$k \geq np + p - 1$

$$\frac{p(X=k)}{p(X=k-1)} \geq 1 \Rightarrow \frac{{}^nC_k \, \cancel{p^k} \, (1-p)^{n-k}}{{}^nC_{k-1} \, \cancel{p^{k-1}} \, (1-p)^{n-k+1}}$$

$$\frac{\cancel{(n-k+1)!} \, \cancel{(k-1)!}}{\cancel{(n-k)!} \, \cancel{k!}} \Rightarrow \left(\frac{n-k+1}{k}\right) \frac{p}{1-p}$$

$n-k+1$

k

$np - kp + p > k - kp \qquad k \leq np + p$

$k < (n+1)p - 1$

Similarly for $P(X=k+1) < P(X=k)$

$k > (n+1)p - 1$

therefore $p(x)$ increases monotonically until $x < (n+1)p$ and decreases.

(a) $\frac{P(X=k)}{P(X=k+1)} \geq 1$ & $\frac{P(X=k)}{P(X=k-1)} \leq 1$

& $(n+1)p$ should be an integer

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(ii) If $(n+1)p$ is not an integer, then distribution $P(X=k)$ reaches its max value at only one value of 'k', i.e

$$(n+1)p-1 < k < (n+1)p$$

(5) 'X' is geometric RV

$$P(X=k) = (1-p)^{k-1}(p)$$

$$P\left(X = \frac{m+k}{X>m}\right) = \frac{P(X=m+k)}{P(X>m)}$$

$$\frac{(1-p)^{m+k-1}(p)}{(1-p)^m(p)+(1-p)^{m+1}(p)+\ldots} \Rightarrow \frac{(1-p)^{m+k-1}(p)}{(1-p)^m p[1+(1-p)+\ldots]}$$

$$\frac{1}{1-(1-p)} = \frac{1}{p}$$

$$= (1-p)^{k-1}(p)$$

$$= P(X=k)$$

Hence it is proved that $P\left(X = \frac{m+k}{X>m}\right)$

$= P(X=k)$

It is a lack of memory property because past doesn't affect future prob. of RV.

Geo. distribution is only discrete distribution on +ve integers with this property

(7) $x = \begin{cases} 0 \\ 1 \\ 2 \\ \vdots \\ n \end{cases}$ $E(x) = \sum_{r=1}^{n} r\, p_x(x = r)$

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$$= \sum_{r=1}^{n} r \frac{n!}{(n-r)!\, r!} D_{n-r}$$

$$\Rightarrow \sum_{r=1}^{n} \cancel{r} \frac{\cancel{n!}}{(n-r)!\, \cancel{r!}\, \cancel{n!}} \Rightarrow \left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \ldots + (-1)^{n}\right.$$

$(r-1)!$

$\Rightarrow 1$

$Var(x) = E[x^2] - (E[x])^2$

$\Rightarrow E[x^2] - 1$

(8) Prob of bit error $= 10^{-6} = p$

Block length $= 10{,}000 = 10^4$ bits $= n$

$N \rightarrow$ no. of errors in block

$P(N=0)$, $P(N \leq 3)$ $P(N \geq 1)$

'N' is poisson RV.

In a block $\lambda = np = (10^4)(10^{-6}) = (10^{-2})$

$$p_x(k) = \frac{e^{-\lambda} \lambda^k}{k!} \text{ (PMF of poisson)}$$

(a) $P(N=0) = p(N=0)$

$$= \frac{e^{-\lambda}\lambda^0}{0!} = e^{-\lambda} = e^{-10^{-2}}$$

$$P(N \le 3) = P[N=0] + P[N=1] + P[N=2] + P[N=3]$$

$$= e^{-10^{-2}} + e^{-10^{-2}} 10^{-2} + \frac{e^{-10^{-2}}}{2} 10^{-4} + \frac{e^{-10^{-2}} 10^{-6}}{6}$$

(b) $P(N \ge 1) = 1 - P(N=0)$

$$= 1 - e^{-\lambda} = 1 - e^{-10^{-2}}$$

(9) ~~Fo~~ m-bit password, there are 2^m patterns of passwords

(a) Let R.V 'x' changes from 1 to 2^m

We need to find $P\left(\frac{X=x}{X \ge K}\right)$

password is not found on first 'K' tries so, there are $2^m - K$ tries remaining

$$P\left(\frac{X=x}{X>K}\right) = \frac{1}{2^m - K} \quad (X = K+1, K+2 \ldots 2^m)$$

(b) Conditional expectation of $X > K$

It is a uniform R.V. So, conditional expected value will lie at middle of '$(K+1)$' & '2^m' as prob is uniform over it.

so, $B\left[\frac{X=x}{X=k}\right] = \frac{15+1+2^m}{2}$

(10) $X = r\cos\left(\frac{2\pi\Theta}{8}\right),\ Y = r\sin\left(\frac{2\pi\Theta}{8}\right)$

Θ is discrete R.V $= \{0,1,2\ldots 7\}$

(X, Y) pair B.

X	Y	Θ	
~~r~~	~~0~~	~~0~~	~~$\to 1/8$~~
~~$r/\sqrt{2}$~~	~~$r/\sqrt{2}$~~	~~1~~	~~$\to 1/8$~~
~~0~~	~~r~~	~~2~~	~~$\to 1/8$~~
~~$-r/\sqrt{2}$~~	~~$r/\sqrt{2}$~~	~~3~~	~~$\to$~~

X	Y	Θ	
r	0	0	$\to 1/8$
$r/\sqrt{2}$	$r/\sqrt{2}$	1	$\to 1/8$
0	r	2	$\to 1/8$
$-r/\sqrt{2}$	$r/\sqrt{2}$	3	$\to 1/8$
$-r$	0	4	$\to 1/8$
$-r/\sqrt{2}$	$-r/\sqrt{2}$	5	$\to 1/8$
0	$-r$	6	$\to 1/8$
$r/\sqrt{2}$	$-r/\sqrt{2}$	7	$\to 1/8$

$$p_{X,Y}(x,y) = \begin{cases} 1/8, & (x,y) = (r,0) \\ 1/8, & (x,y) = (r/\sqrt{2}, r/\sqrt{2}) \\ 1/8, & (x,y) = (r/\sqrt{2}, -r/\sqrt{2}) \\ 1/8, & (x,y) = (0,r) \\ 1/8, & (x,y) = (0,-r) \\ 1/8, & (x,y) = (-r,0) \\ 1/8, & (x,y) = \left(\frac{-r}{\sqrt{2}}, \frac{r}{\sqrt{2}}\right) \end{cases}$$

(ii) marginal PMF of 'x'

$p_x(x)$ when $x = r$ $\quad P_x(r) = \sum_y p_{x,y}(r,y)$

$= p_{x,y}(r,r) + p_{x,y}(r,-r) + p_{x,y}(r,0)$

$+ p_{x,y}\left(r, \frac{r}{\sqrt{2}}\right), P_{x,y}\left(r, \frac{-r}{\sqrt{2}}\right)$

$= \frac{1}{8}$

when $x = 0$

$p_x(x=0) = \sum_y p_{x,y}(0,y)$

$\Rightarrow p_{xy}(0,0) + p_{xy}(0,r) + p_{xy}\left(0, \frac{r}{\sqrt{2}}\right)$

$+ p_{xy}(0,-r) + p_{xy}(0, -r/\sqrt{2}) = \frac{1}{4}$

when $\cancel{x} = r/\sqrt{2}$

$p_x\left(\frac{r}{\sqrt{2}}\right) = \sum_y p_{xy}\left(\frac{r}{\sqrt{2}}, y\right)$

$= p_{xy}\left(\frac{r}{\sqrt{2}}, \frac{-r}{\sqrt{2}}\right) + p_{xy}\left(\frac{r}{\sqrt{3}}, \frac{r}{\sqrt{2}}\right) = \frac{1}{4}$

$p_x(-r) = \sum_y p_{xy}(-r,y) = p_{xy}(-r,0)$

$= \frac{1}{8}$

$p_x\left(\frac{-r}{\sqrt{2}}\right) = \sum_y p_{xy}\left(\frac{-r}{\sqrt{2}}, y\right)$

$\Rightarrow p_{xy}\left(\frac{-r}{\sqrt{2}}, \frac{r}{\sqrt{2}}\right) + p_{xy}\left(\frac{-r}{\sqrt{2}}, \frac{-r}{\sqrt{2}}\right) = \frac{1}{4}$

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Marginal pmf of 'y'

when $y = r$

$p_y(r) = \sum_x p_{xy}(x, r)$

$= p_{xy}(0, r) = 1/8$

when $y = r/\sqrt{2}$

$p_y\left(\frac{r}{\sqrt{2}}\right) = \sum_x p_{xy}\left(x, \frac{r}{\sqrt{2}}\right)$

$= 2\, p_{xy}\left(\frac{r}{\sqrt{2}}, \frac{r}{\sqrt{2}}\right) + p_{xy}\left(\frac{-r}{\sqrt{2}}, \frac{r}{\sqrt{2}}\right)$

$= \frac{1}{4}$

when $y = -r$

$p_y(-r) = \sum_x p_{xy}(x, -r)$

$= p_{xy}(0, -r) = 1/8$

(ii) $A = \{x = 0\}$ $\quad p_x(0) = \sum_y p_{x,y}(0, y)$

$= 1/4$

$B = \{y \le \frac{y}{\sqrt{2}}\} = \sum_x p_{xy}\left(x, \frac{-r}{\sqrt{2}}\right)$

$+ \sum_x p_{xy}(x, 0) + \sum_x p_{xy}(x, -y)$

$+ \sum_x p_{xy}(x, -r/\sqrt{2})$

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2) $1/2 + 1/8 + 1/4 = 7/8$

$1 - P\left(X \geq \frac{r}{\sqrt{2}}, Y \geq \frac{r}{\sqrt{2}}\right) = 0$

$PMF\left(X = \frac{r}{\sqrt{2}}, Y = \frac{r}{\sqrt{2}}\right)$

$+ PMF\left(X = r, Y = \frac{r}{\sqrt{2}}\right) +$

$PMF\left(X = \frac{r}{\sqrt{2}}, Y = r\right) + PMF(X = r, Y = r)$

$= \frac{1}{8}$

$D = \{X \leq -\frac{r}{\sqrt{2}}\} = \{x = -r\}$

$= \sum_{y} P_{XY}(-r, y) = 1/8$

3) $P(X = 0) = 0.5$

$P(X = 1) = 1 - P(X = 0) = 1 - 0.5 = 0.5$

$P(Y = 1 \cap X = 0) = 0.1$

$\frac{P(X = 1 \cap X = 0)}{P(X = 0)} = 0.1$

$P(Y = 1 \cap X = 0) = 0.1 \times 0.5 = 0.05$

$P(Y = 0 | X = 0) = 1 - P(Y = 1 | X = 0)$

$= 1 - 0.1 = 0.9$

$P(Y = 0 \cap X = 1) = P(X = 1) \times P(Y = 0 | X = 1)$

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$= 0.5 \times 0.2 = 0.1$

$P(Y=1 \mid X=1) = 1 - P(Y=0 \mid X=1)$

$= 1 - 0.2 = 0.8$

$P(Y=1 \cap X=1) = P(X=1) \times P(Y=1 \mid X=1)$

$= 0.5 \times 0.8 = 0.4$

(a) $P_{X,Y}(x,y) = \begin{cases} 0.45, & (x,y) = (0,0) \\ 0.05, & (x,y) = (0,1) \\ 0.1, & (x,y) = (1,0) \\ 0.4, & (x,y) = (1,1) \end{cases}$

(b) $P_X(x) = \begin{cases} 0.5, & x=0 \\ 0.5, & x=1 \end{cases}$

$P(X=0) = P(Y=0 \cap X=0) + P(Y=0 \cap X=1)$

$= 0.45 + 0.1 = 0.55$

$P(Y=1) = P(Y=1 \cap X=0) + P(X=1 \cap X=1)$

$= 0.4 + 0.05 = 0.45$

$P_Y(y) = \begin{cases} 0.55, & y=0 \\ 0.45, & y=1 \end{cases}$

(c) $P(x=0 \cap y=0) = 0.45$

$P(x=0) \times P(y=0) = 0.55 \times 0.5$

$= 0.275$

$P(x=0 \cap y=0) \neq P(x=0) \times P(y=0)$

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therefore, x and y are dependent.

(17) Let D_1 be selected of door that leads outside and t be the time taken to reach outside.

$$P(D_1) = \frac{1}{3}, \quad P(\bar{D_1}) = 1 - \frac{1}{3} \Rightarrow \frac{2}{3}$$

$$p_T(t) = \begin{cases} 1/3 \; ; & t = 2 \text{ hours} \\ (2/3)(1/3) \; ; & t = 4 \text{ hours} \\ (2/3)^{k-1}(1/3) \; ; & t = 2^k \text{ hours} \end{cases}$$

$$E[t] = \sum t \, p_T(t)$$

$$= (2)\left(\frac{1}{3}\right) + 4\left(\frac{2}{3}\right)\left(\frac{1}{3}\right) + 8\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right) + \ldots$$

$$\Rightarrow \frac{2}{1/3} = 6 \text{ hours.}$$

(16) (a) $P_{M,N}(m, n) = \begin{cases} C & m \geq 0, n \geq 0, m+n < L \\ 0 & \text{otherwise.} \end{cases}$

$$\sum_{n=0}^{L-1} P_{M,N}(m, n) = 1$$

$$c(1+2+3+\ldots L) = 1$$

$$\frac{c(L)(L+1)}{2} = 1$$

$$c = \frac{2}{L^2+L}$$

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(e) $P_M(m) = \sum_{n=0}^{L-m-1} P_{M,N}(m,n)$

$= c[L-m-1-0+1] = c[L-m]$

$P_M(m) = \begin{cases} c[L-m], m \in I \text{ in } [0, L-1] \\ 0, \text{otherwise} \end{cases}$

Similarly,

$P_N(n) = \begin{cases} c[L-n]; n \in I \text{ in } [0, L-1] \\ 0, \text{otherwise} \end{cases}$

(c) Case 1 → L is even

$Pr(M+N < L/2) = \sum_{M+N=0}^{M+N=\frac{L}{2}-1} P_{M,N}(m,n)$

$= c(1+2+3+ \ldots \frac{L}{2})$

$\Rightarrow \cancel{\frac{2}{L^2+L}}\left(\frac{2}{L^2+L}\right) \quad \frac{(L/2 \times (L/2+1))}{2}$

$\Rightarrow \dfrac{\frac{L^2}{4} + \frac{L}{2}}{L^2+L} \Rightarrow \frac{L+2}{4L+4}$

Case 2 :- L is odd

$Pr(M+N < L/2) = \sum_{M+N=0}^{M+N=\frac{L}{2}-\frac{1}{2}} P_{M,N}(m,n)$

$= c(1+2+3+ \ldots \frac{L+1}{2}) \Rightarrow \frac{L+3}{4L}$

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(2) No of transmission $\to N$
No of 15 transmitted $\to X$

$P(X=1 \mid N=1) = p$
$P(X=k \mid N=k) = p^k$

$P(N=m) = \frac{e^{-\lambda}\lambda^m}{m!}$

$P(X=k \mid N=m) = {}^mC_k p^k (1-p)^{m-k}$

$P(Y=k) = \sum_{m=k}^{\infty} P(X=k \mid N=m) \times P(N=m)$

$\Rightarrow \sum_{m=k}^{\infty} {}^mC_k\, p^k (1-p)^{m-k} \times \frac{e^{-\lambda}\lambda^m}{m!}$

$= \sum_{m=k}^{\infty} \frac{m!}{k!(m-k)!} \frac{p^k (1-p)^{m-k} e^{-\lambda}\lambda^m}{m!}$

$= \frac{e^{-\lambda} p^k}{k!} \sum_{m=k}^{\infty} \frac{(1-p)^{m-k}\lambda^m}{(m-k)!}$

$\Rightarrow \frac{e^{-\lambda} p^k}{k!} e^{(1-p)\lambda}$

$P(X=k) = \frac{e^{-p\lambda}(p\lambda)^k}{k!}$, $k = 0, 1, 2 \ldots$

(9) 2^m pattern of password
(a) $P(X = x / x > k) = \frac{1}{2^m - k}$
(b) conditional expectation of $X = Y$

$$E\left[\sum_{x=k} X = x\right] = \frac{k+1+2}{2}$$

(18) $x \to$ Nos. of successes in n trial

$y \to$ No of failures in n trials.

$\boxed{n = x+y}$ $p \to$ prob. of success in 'n' Trials.

$E[x] = np$

$z = X - Y = 2x(-n)$

$E[z] = E[2x-n]$

$= 2E[x] - n = 2np - n$

$\Rightarrow n(2p-1)$

$Var(X) = np(1-p)$

$Var(z) = 4Var(X) = 4np(1-p)$

(20) $P_{X,Y}(x_i, y_i) = \begin{cases} k(2x_i + y_i); \; x_j = 1,2; \\ \quad y_i = 1,2 \\ 0, \; \text{otherwise} \end{cases}$

19) $p_{X,Y}(1,1) + p_{X,Y}(1,2) + p_{X,Y}(2,1) + p_{X,Y}(3,2) = 1$

$k(3) + k(4) + k(5) + k(6) = 1$

$k = \frac{1}{18}$

(b) marginal PMF of X & Y

$p_X(x_i) = \sum_y p_{X,Y}(x_i, y_i)$

$p_X(1) = p_{X,Y}(1,1) + p_{X,Y}(1,1) = 7k$

$= \frac{7}{18}$

$p_X(2) = p_{XY}(2,1) + p_{X\cancel{Y}}(1,1)$

$= 11k = \frac{11}{18}$

$p_X(k) = \begin{cases} 7/18 & ; k=1 \\ 11/18 & ; k=2 \end{cases}$ marginal PMF of X

$P_Y(y) = \sum_x p_{XY}(x_i, y_i)$

$p_Y(1) = p_{XY}(2,1) + p_{X,Y}(1,1) = 8k$

$= \frac{8}{18} = \frac{4}{9}$

$p_Y(2) = p_{XY}(1,2) + p_{XY}(2,2)$

$= 10k = \frac{5}{9}$

$p_Y(k) = \begin{cases} 8/18 & , k=1 \\ 10/18 & , k=2 \end{cases}$ marginal PMF of Y

(c) $p_{XY}(x,y) = P_X(x) P_Y(y)$

$p_{XY}(1,1) = p_X(1)\, p_Y(1)$

$\frac{1}{6} \neq \frac{7}{18} \times \frac{9}{9}$ so, they are not independent

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(15) $P(A) = P(A^-) = P(B^-) = P(B) = P(B^+)$
$= P(C^+) = \frac{1}{6}$

No of paper for getting 1st grade = 1

Prob. getting diff. grade $= 1 - \frac{1}{6} = \frac{5}{6}$

$E(\text{No of paper}) = \frac{1}{5/6} = 1.2$

Prob. for getting diff grade $= 1 - \frac{2}{6}$

$= \frac{2}{3}$

$E[\text{papers}] = \frac{1}{2/3} = 1.5$

Prob. for getting diff grade

$= 1 - \frac{3}{6} = \frac{3}{6}$

total no. of paper req. $= 1 + 2 + 2 + \frac{1}{1/3} + \frac{1}{1/6}$

$= 16$

(18) ~~[x] represent~~

(19) $P(N=0) = p(N=0) = \frac{e^{-\lambda} \lambda^0}{0!} = e^{-\lambda} = e^{-10^{\cdot 2}}$

$$P(N \le 3) = P[N=0] + P[N=1] + P[N=2] + P[N=3]$$

$$= e^{-10^{-2}} + \cancel{10}\, e^{-10^{-2}} 10^{-2} + \frac{e^{-10^{-2}} 10^{-4}}{2} + \frac{e^{-10^{-2}} 10^{-6}}{6}$$

ii) $P(N \ge 1) = 1 - P(N=0)$

$= 1 - e^{-\alpha} = 1 - e^{-10^{-2}}$

(9) (a) 2 toll booth

$\alpha = 10$

$$P_x(k) = \frac{e^{-\alpha} \alpha^k}{k!}$$

$$P(x=11) = \frac{e^{-10} 10^{11}}{11!}$$

ii) $\alpha = 30$

traffic approaching each booth

$\alpha = 30/N$

$X_i \le 5 \rightarrow$ each toll

$i = 1, \cdots N.$

$P(X_1 \le 5, X_2 \le 5 - X_N \le 5)$

$\alpha' = 30/N$

$$= \prod_{i=1}^{N} P(x_i \le 5)$$

$$= \sum_{m=0}^{5} \frac{e^{-\alpha'} (+\alpha')^m}{m!}$$

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Poisson PMF for each booth
N = 1, 2, 3 ...

(a) $P_Y(y) = \sum_x p(x,y)$

(y = 1) $P_Y(1) = p(1,1) + p(2,1) + p(3,1)$

$= \frac{1}{9} + \frac{1}{3} + \frac{1}{9} = \frac{1}{9} + \frac{3}{9} + \frac{1}{9} = \frac{5}{9}$

(y = 2) $P_Y(2) = P(1,2) + P(2,2) + p(3,2)$

$= \frac{1}{9} + 0 + \frac{1}{18} = \frac{1}{6}$

(y = 3) $P_Y(3) = P(1,3) + p(2,3) + p(3,3)$

$= \frac{1}{6} + \frac{2}{18} = \frac{5}{18}$

• $P(X = x / Y = i)$.

$P(X = x | Y = i) = \frac{p(x,i)}{P_Y(i)}$

i = 1

$P(X = 1 | Y = 1) = \frac{P(1,1)}{P_Y(1)} = \frac{1/9}{5/9} = \frac{1}{5}$

$P(X = 2 | Y = 1) = \frac{P(2,1)}{P_Y(1)} = \frac{1/3}{5/9} = \frac{3}{5}$

$$P(X=3|Y=1) = \frac{P(3,1)}{P_Y(1)} = \frac{1/9}{5/9} = \frac{1}{5}$$

for $i=2$

$$P(X=1|Y=2) = \frac{12}{18} = \frac{2}{3}$$

$$P(X=2|Y=2) = 0$$

$$P(X=3|Y=2) = \frac{6}{18} = \frac{1}{3}$$

for $i=3$

$$P(X=1|Y=3) = 0$$

$$P(X=2|Y=3) = 3/5$$

$$P(X=3|Y=3) = 2/5$$

$E[X/Y=i]$

$i=1$

$$E[X|Y=1] = 1 \cdot \frac{1}{5} + 2 \cdot \frac{3}{5} + 3 \cdot \frac{1}{5} = 2$$

$i=2$

$$E[X|Y=2] = 1 \cdot \frac{2}{3} + 0 \cdot 0 + 3 \cdot \frac{1}{3} = \frac{5}{3}$$

$i=3$

$$E[X|Y=3] = 0 \cdot 0 + 2 \cdot \frac{3}{5} + 3 \cdot \frac{2}{5} = \frac{12}{5}$$