CPhO 2018 - Solutions

The following are unofficial solutions to CPhO 2018. Many are incomplete and some may be incorrect, so use at your own caution. The respective authors for each solution will be mentioned. If you would like to submit a solution or expand on anything, please shoot an email to `hello@physoly.tech`

Problem 1

The user chem123 submitted the following answers:

1) $\frac{GM}{R+h} \leq v^2 \leq \frac{2GM}{R+h}$

2) $v = \sqrt{V_0^2 + \frac{2GMh}{R(R+h)}}$ at an angle of $\theta = \sin^{-1}[\frac{V_0(R+h)}{r\sqrt{V_0^2+\frac{2GMh}{R(R+h)}}}]$ with the normal at the point where it strikes earth

Problem 2

The user chem123 submitted the following answers:

1. $A_0 = \frac{2*\mu mg}{k}$

2. $A_0 = \frac{4*\mu mg}{k}$

Problem 3

The user PhysicsMonster_01 (and one of the curators for this site!) posted the following solution:

Before starting with my solution, I want to list some important things that will be used in the solution. First, the distance of centre of mass from the point O:

$$x_{\text{COM}} = \frac{mx + M\frac{L}{2}}{M+m}$$

Secondly, the moment of inertia about point O (we don't find it with respect to the new COM just yet, it'll not be of much use until later.):

$$I_O = mx^2 + \frac{ML^2}{3}$$

Now let us begin.

The horizontal component of linear momentum as well as the angular momentum about point O remains conserved. This gives us the two equations

$$mv_o = (M+m)\omega x_{\text{COM}}$$

which on solving gives $\omega = \frac{mv_o}{mx+M\frac{L}{2}}$. The other equation of conservation of angular momentum about a fixed point O is

$$-mv_o x\ \hat{k} = -(mx^2 + \frac{ML^2}{3})\omega\hat{k}$$

On solving these together, we get

$$\boxed{x = \frac{2L}{3}}$$

b)

$$F = mg + \frac{M}{4}g + \cos^2\theta(3mg + \frac{9}{4}Mg) - \cos\theta(\frac{2Mg}{2} + 2mg) + \frac{m^2{v_0}^2}{L}\frac{\sin\theta}{\frac{M}{2} + \frac{2m}{3}}$$

d)

$$\Delta x = \frac{\pi^2 g}{2(\frac{mv_0}{(\frac{2m}{3}+\frac{M}{2})L})^2 + \frac{12g}{L}} - \frac{mL}{3m+3M}$$

Problem 4

The user chem123 submitted the following answers:

Using Biot-Savart law, we get the magnetic field at any (x, y) as :

$$\vec{B} = \vec{B_1} + \vec{B_2} + \vec{B_3} + \vec{B_4}$$

$$\vec{B} = \frac{\mu_0 I * [(y-a)\vec{i} - (x+a)\vec{j}]}{2\pi[(x+a)^2 + (y-a)^2]} + \frac{\mu_0 I * [(x-a)\vec{j} - (y-a)\vec{i}]}{2\pi[(x-a)^2 + (y-a)^2]} + \frac{\mu_0 I * [(y+a)\vec{i} - (x-a)\vec{j}]}{2\pi[(x-a)^2 + (y+a)^2]} + \frac{\mu_0 I * [-(y+a)\vec{i} + (x+a)\vec{j}]}{2\pi[(x+a)^2 + (y+a)^2]}$$

$$\vec{B} = \frac{\mu_0 * I}{2\pi}\left(\frac{[(y-a)\vec{i} - (x+a)\vec{j}]}{[(x+a)^2 + (y-a)^2]} + \frac{[(x-a)\vec{j} - (y-a)\vec{i}]}{[(x-a)^2 + (y-a)^2]} + \frac{[(y+a)\vec{i} - (x-a)\vec{j}]}{[(x-a)^2 + (y+a)^2]} + \frac{[-(y+a)\vec{i} + (x+a)\vec{j}]}{[(x+a)^2 + (y+a)^2]}\right)$$

2) $\vec{B}_{approx.} = \frac{\mu_0 * I}{\pi a^2} * (x\vec{i} - y\vec{j})$

Problem 5

No solution was posted for this problem.

Problem 6

This solution was posted by sturdyoak2012, the user who originally translated the problem.

Consider the outer surface of the shell. It absorbs heat from the cosmic microwave background, conducts heat from the inner surface of the shell, and radiates heat. Since the net heat flow is zero, and the outer surface of the shell is a black body, we can write

$$44 + A\sigma T^4 = A\sigma T_2^4.$$

where T is the temperature of the CMBR. Solving,

$$T_2 = \left(\frac{44}{4\pi R_2^2\sigma} + T^4\right)^{0.25} \approx 88.6\text{ K}.$$

Consider the shell with thickness dR inside the spherical shell. The thermal power conducted through it is 44.0 W. Hence, we can write

$$44 = -\kappa(4\pi R^2)\frac{dT}{dR}.$$

Hence,

$$\int_{0.9}^{1.0} \frac{1}{R^2} dR = -\frac{\pi\kappa}{11}\int_{T_1}^{T_2} dT.$$

Solving, $T_1 = 128$ K.

Consider the inner surface of the spherical shell. 44 watts leave it through the shell. It radiates inwards, some of which is reflected back and absorbed, the rest is re-reflected, etc. It also absorbs some radiation from the ball, some of which is reflected and reflected back, etc...

Let $P = 4\pi R_1^2 E\sigma T_1^4$ be the energy radiated by the inner surface of the shell. Then $P(1-e)$ watts are reflected back. $P(1-e)E$ of this is absorbed, and $P(1-e)(1-E)$ watts are reflected. When this reflected energy is reflected back, it is reduced to $P(1-e)^2(1-E)$ watts, $P(1-e)^2(1-E)E$ of which is absorbed. In total,

$$E\left(P(1-e) + P(1-e)^2(1-E) + P(1-e)^3(1-E)^2 + \cdots\right) = \frac{PE(1-e)}{1-(1-e)(1-E)}$$

watts are absorbed.

We also need to consider radiation from the ball. Let $Q = 4\pi r^2 e\sigma T_0^4$ be the watts radiated from the ball. A similar calculation gives

$$E\left(Q + Q(1-E)(1-e) + Q(1-E)^2(1-e)^2 + \cdots\right) = \frac{QE}{1-(1-E)(1-e)}$$

watts absorbed from the radiation from the ball.

Since the power going out of the inner surface is equal to the power going in, we can write

$$44 + P = \frac{QE + PE(1-e)}{1-(1-e)(1-E)}.$$

This is

$$44 + 4\pi R_1^2 E\sigma T_1^4 = \frac{4\pi R_1^2 E\sigma T_1^4 E(1-e) + 4\pi r^2 e\sigma T_0^4 E}{1-(1-e)(1-E)}.$$

Solving, $T_0 = 296$ K.