Experimentation

- Tried implementing a basic KAN for MNIST dataset

  Note: Since the proposed pykan library is a Python wrapper for working with knowledge graphs and structured data, it is not useful for our purpose, forcing us to look for alternatives.

- Implemented Quanvolutional Neural Networks for comparison with our original approach.

  Note: Since the MNIST dataset consists of basic visual images, the accuracy of all methods is expected to be quite similar.

- Implementation of SineKANS, Multi-Layer Perceptrons for comparision. SineKANs are expected to produce the best result according to the paper [4].
- Lastly, the goal of the project is to explore the extension of KANs to QKANs and further investigate potential applications in LHC-related research. I have explained this aspect in detail in my proposal.

1. **KAN for MNIST dataset**

A. Implentation of splines by directly using BSpline functionality from SciPy.

```
import torch
import os
import torch.nn as nn
import torch.optim as optim
import torch.nn.functional as F
from torch.autograd import Function
from scipy.interpolate import BSpline #One of the few libraries for implementing B-Splines
import numpy as np
from torchvision import datasets, transforms
from torch.utils.data import DataLoader
import matplotlib.pyplot as plt


class BSplineActivation(Function):
    @staticmethod #Static throughout training and testing
    def forward(ctx, input, knots, coeffs, degree):
        ctx.save_for_backward(input, knots, coeffs) #Save input tensors
        ctx.degree = degree

        input_np = input.detach().cpu().numpy()
        knots_np = knots.detach().cpu().numpy()
        coeffs_np = coeffs.detach().cpu().numpy()

        #Define B-Spline
        spline = BSpline(knots_np, coeffs_np, degree)
        output = torch.tensor(spline(input_np), dtype=torch.float32).to(input.device)

        return output

    @staticmethod
    def backward(ctx, grad_output):
        input, knots, coeffs = ctx.saved_tensors
        degree = ctx.degree

        #Computing B-Spline derivative
        knots_np = knots.detach().cpu().numpy()
        coeffs_np = coeffs.detach().cpu().numpy()
        spline = BSpline(knots_np, coeffs_np, degree)
        deriv_spline = spline.derivative()

        input_np = input.detach().cpu().numpy()
        grad_input = torch.tensor(deriv_spline(input_np), dtype=torch.float32).to(input.device) * grad_output

        return grad_input, None, None, None #No grad calc for inputs, knots, coeffs

class KAN(nn.Module):
    def __init__(self, input_dim, hidden_dim, output_dim, spline_degree=3, num_knots=10):
        super(KAN, self).__init__()
```

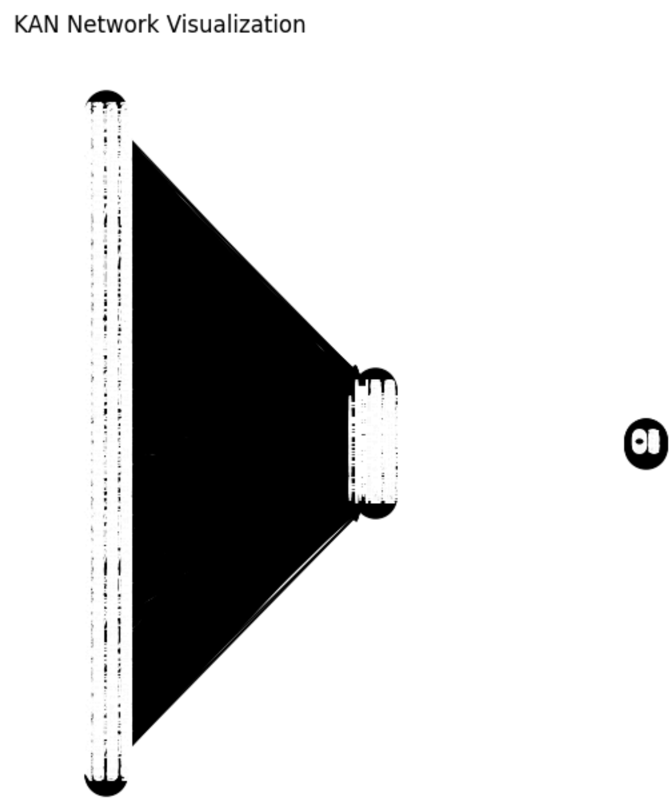
```
        self.fc1 = nn.Linear(input_dim, hidden_dim)
        self.fc2 = nn.Linear(hidden_dim, output_dim)

        # Learnable B-Spline parameters
        self.knots = nn.Parameter(torch.linspace(-1, 1, num_knots))
        self.coeffs = nn.Parameter(torch.randn(num_knots))
        self.spline_degree = spline_degree

    def forward(self, x):
        x = self.fc1(x)
        x = BSplineActivation.apply(x, self.knots, self.coeffs, self.spline_degree)  # KAN activations are placed on edges and not nodes, ur
        x = self.fc2(x)
        return x

# Loading MNIST dataset
transform = transforms.Compose([
    transforms.ToTensor(),
    transforms.Normalize((0.1307,), (0.3081,))  # MNIST mean/std
])

train_dataset = datasets.MNIST(root='./data', train=True, transform=transform, download=True)
test_dataset = datasets.MNIST(root='./data', train=False, transform=transform, download=True)

train_loader = DataLoader(train_dataset, batch_size=64, shuffle=True)
test_loader = DataLoader(test_dataset, batch_size=1000, shuffle=False)

#Instantiate the model, device, loss type and optimizer
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
model = KAN(input_dim=28*28, hidden_dim=128, output_dim=10).to(device)
criterion = nn.CrossEntropyLoss()
optimizer = optim.Adam(model.parameters(), lr=0.0005)

save_dir = "training_loss_images"
os.makedirs(save_dir, exist_ok=True)

# Training Loop
epochs = 60
epoch_train_losses=[]
for epoch in range(epochs):
    model.train()
    train_losses=0.0
    for images, labels in train_loader:
        images = images.view(images.size(0), -1).to(device)  # Flatten images
        labels = labels.to(device)

        optimizer.zero_grad()
        output = model(images)
        loss = criterion(output, labels)
        loss.backward()
        optimizer.step()
        train_losses += loss.item()

    avg_loss = train_losses / len(train_loader)
    epoch_train_losses.append(avg_loss)  # Store the loss for plotting

    plt.figure()
    plt.plot(range(epoch + 1), [train_losses / len(train_loader) for epoch in range(epoch + 1)], label="Train Loss")
    plt.xlabel('Epoch')
    plt.ylabel('Loss')
    plt.title(f'Epoch {epoch + 1}/{epochs}')
    plt.legend()
    plt.savefig(f"{save_dir}/epoch_{epoch + 1}.png")
    plt.close()
    print(f"Epoch {epoch+1}/{epochs}, Loss: {train_losses / len(train_loader):.4f}")

# Evaluate Model
model.eval()
correct = 0
total = 0
epoch_test_losses = []
test_losses=0.0
with torch.no_grad():
    for images, labels in test_loader:
        images = images.view(images.size(0), -1).to(device)
        labels = labels.to(device)

        outputs = model(images)
```

```
        loss = criterion(outputs, labels)  # Compute loss
        test_losses += loss.item()  # Accumulate loss
        _, predicted = torch.max(outputs, 1)
        total += labels.size(0)
        correct += (predicted == labels).sum().item()

    avg_loss_t=test_losses/len(test_loader)
    epoch_test_losses.append(avg_loss_t)

print(f"Test Accuracy: {100 * correct / total:.2f}%")
```

```
Epoch 4/60, Loss: 0.3755
Epoch 5/60, Loss: 0.5134
Epoch 6/60, Loss: 0.3888
Epoch 7/60, Loss: 0.2496
Epoch 8/60, Loss: 0.7703
Epoch 9/60, Loss: 0.2163
Epoch 10/60, Loss: 0.4059
Epoch 11/60, Loss: 0.4355
Epoch 12/60, Loss: 0.5213
Epoch 13/60, Loss: 0.2938
Epoch 14/60, Loss: 0.5935
Epoch 15/60, Loss: 0.3874
Epoch 16/60, Loss: 0.3595
Epoch 17/60, Loss: 0.5784
Epoch 18/60, Loss: 0.3475
Epoch 19/60, Loss: 0.5121
Epoch 20/60, Loss: 0.4963
Epoch 21/60, Loss: 0.3350
Epoch 22/60, Loss: 0.6306
Epoch 23/60, Loss: 0.3411
Epoch 24/60, Loss: 0.7612
Epoch 25/60, Loss: 0.5438
Epoch 26/60, Loss: 0.4499
Epoch 27/60, Loss: 0.5460
Epoch 28/60, Loss: 0.4262
Epoch 29/60, Loss: 0.6974
Epoch 30/60, Loss: 0.5206
Epoch 31/60, Loss: 0.6578
Epoch 32/60, Loss: 0.5642
Epoch 33/60, Loss: 0.4833
Epoch 34/60, Loss: 0.7785
Epoch 35/60, Loss: 0.5020
Epoch 36/60, Loss: 0.6313
Epoch 37/60, Loss: 0.7022
Epoch 38/60, Loss: 0.6897
Epoch 39/60, Loss: 0.8176
Epoch 40/60, Loss: 0.6815
Epoch 41/60, Loss: 0.6452
Epoch 42/60, Loss: 0.7640
Epoch 43/60, Loss: 0.7229
Epoch 44/60, Loss: 0.7098
Epoch 45/60, Loss: 1.0493
Epoch 46/60, Loss: 0.8276
Epoch 47/60, Loss: 0.8740
Epoch 48/60, Loss: 0.8977
Epoch 49/60, Loss: 0.8853
Epoch 50/60, Loss: 0.8466
Epoch 51/60, Loss: 0.9949
Epoch 52/60, Loss: 1.0410
Epoch 53/60, Loss: 1.0361
Epoch 54/60, Loss: 1.0284
Epoch 55/60, Loss: 1.1861
Epoch 56/60, Loss: 1.1984
Epoch 57/60, Loss: 0.9220
Epoch 58/60, Loss: 1.4596
Epoch 59/60, Loss: 1.1105
Epoch 60/60, Loss: 1.0231
Test Accuracy: 95.02%
```

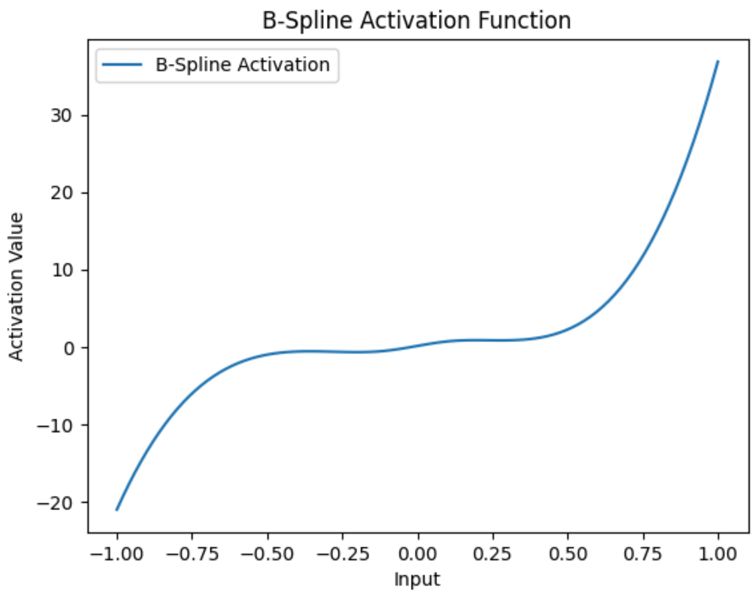
```
import torch
import networkx as nx
import matplotlib.pyplot as plt
import numpy as np

# Extract model weights
fc1_weights = model.fc1.weight.detach().cpu().numpy()
fc2_weights = model.fc2.weight.detach().cpu().numpy()

# Define nodes
input_nodes = [f"X{i}" for i in range(fc1_weights.shape[1])]  # Input layer
```

```
hidden_nodes = [f"H{i}" for i in range(fc1_weights.shape[0])]  # Hidden layer
output_nodes = [f"O{i}" for i in range(fc2_weights.shape[0])]  # Output layer

# Create graph
G = nx.DiGraph()

# Add nodes
G.add_nodes_from(input_nodes, layer=0)
G.add_nodes_from(hidden_nodes, layer=1)
G.add_nodes_from(output_nodes, layer=2)

# Add edges with weights
for i, h in enumerate(hidden_nodes):
    for j, x in enumerate(input_nodes):
        if abs(fc1_weights[i, j]) > 0.1:  # Only draw significant weights
            G.add_edge(x, h, weight=fc1_weights[i, j])

for i, o in enumerate(output_nodes):
    for j, h in enumerate(hidden_nodes):
        if abs(fc2_weights[i, j]) > 0.1:
            G.add_edge(h, o, weight=fc2_weights[i, j])

# Draw the network
pos = nx.multipartite_layout(G, subset_key="layer")
plt.figure(figsize=(8, 6))
nx.draw(G, pos, with_labels=True, node_color="black", font_color="white", node_size=500, edge_color="black")
plt.title("KAN Network Visualization")
plt.show()
```

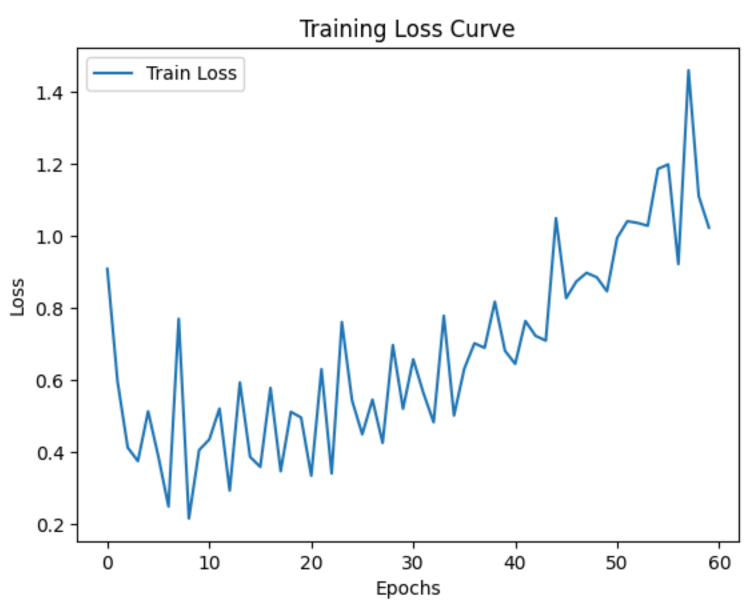



```
import numpy as np
import matplotlib.pyplot as plt
from scipy.interpolate import BSpline

# Get current knots & coefficients after training
knots_np = model.knots.detach().cpu().numpy()
coeffs_np = model.coeffs.detach().cpu().numpy()
spline_degree = model.spline_degree

# Create a B-Spline curve
x_vals = np.linspace(-1, 1, 100)
spline = BSpline(knots_np, coeffs_np, spline_degree) #k,c,t
```

```
y_vals = spline(x_vals)

# Plot
plt.plot(x_vals, y_vals, label="B-Spline Activation")
plt.xlabel("Input")
plt.ylabel("Activation Value")
plt.title("B-Spline Activation Function")
plt.legend()
plt.show()
```




```
import matplotlib.pyplot as plt

# Example loss tracking (assuming losses are stored in lists during training)
plt.plot(epoch_train_losses, label='Train Loss')
plt.xlabel("Epochs")
plt.ylabel("Loss")
plt.legend()
plt.title("Training Loss Curve")
plt.show()
```



We ended up overshooting! This was because of setting the epochs = 60. Setting the number of epochs between [15,30] works well.

B. Alternate Implementation of splines without the manual SciPy use (Ref [5]).

```
import torch
import torch.nn as nn
import torch.nn.functional as F
import torchvision
import torchvision.transforms as transforms
from torch.utils.data import DataLoader
import numpy as np
import time


# Function to precompute B-spline basis functions
def precompute_bspline_basis(num_splines, degree, num_points=100):
    knots = np.linspace(0, 1, num_splines + degree + 1)
    x = np.linspace(0, 1, num_points)
    basis = np.zeros((num_points, num_splines))

    def cox_de_boor(x, k, d, knots):
        if d == 0:
            return np.where((knots[k] <= x) & (x < knots[k+1]), 1.0, 0.0)
        else:
            a = (x - knots[k]) / (knots[k+d] - knots[k] + 1e-8)
            b = (knots[k+d+1] - x) / (knots[k+d+1] - knots[k+1] + 1e-8)
            return a * cox_de_boor(x, k, d-1, knots) + b * cox_de_boor(x, k+1, d-1, knots)

    for i in range(num_splines):
        basis[:, i] = cox_de_boor(x, i, degree, knots)

    return torch.tensor(basis, dtype=torch.float32)

# Precompute basis functions
num_splines = 10
degree = 3
basis = precompute_bspline_basis(num_splines, degree).to('cuda')  # Move the precomputed basis to GPU

# Define the PrecomputedB_Spline class
class PrecomputedB_Spline(nn.Module):
    def __init__(self, precomputed_basis):
        super(PrecomputedB_Spline, self).__init__()
        self.precomputed_basis = precomputed_basis
        self.coefficients = nn.Parameter(torch.randn(precomputed_basis.size(1)) * 0.1).to('cuda')  # Initialize coefficients
        self.w = nn.Parameter(torch.ones(1).to('cuda'))

    def forward(self, x):
        idx = (x * (self.precomputed_basis.size(0) - 1)).long()
        idx = torch.clamp(idx, 0, self.precomputed_basis.size(0) - 1)
        basis = self.precomputed_basis[idx]
        spline = torch.matmul(basis, self.coefficients)
        b = x / (1 + torch.exp(-x))  # Silu function
        return self.w * (b + spline)


class KANLayer(nn.Module):
    def __init__(self, in_features, out_features, precomputed_basis):
        super(KANLayer, self).__init__()
        self.in_features = in_features
        self.out_features = out_features
        self.b_splines = nn.ModuleList(
            [PrecomputedB_Spline(precomputed_basis) for _ in range(out_features)]
        )
        self.weights = nn.Parameter(torch.randn(out_features, in_features) * 0.1).to('cuda')
        self.bias = nn.Parameter(torch.zeros(out_features)).to('cuda')
        self.batch_norm = nn.BatchNorm1d(out_features)

    def forward(self, x):
        batch_size = x.size(0)
        activation_output = []
        for i in range(self.out_features):
            activation = self.b_splines[i]
            linear_combination = torch.matmul(x, self.weights[i]) + self.bias[i]
            activation_output.append(activation(linear_combination).unsqueeze(1))
        output = torch.cat(activation_output, dim=1)
        return self.batch_norm(output)

# Define the KANModel class
```

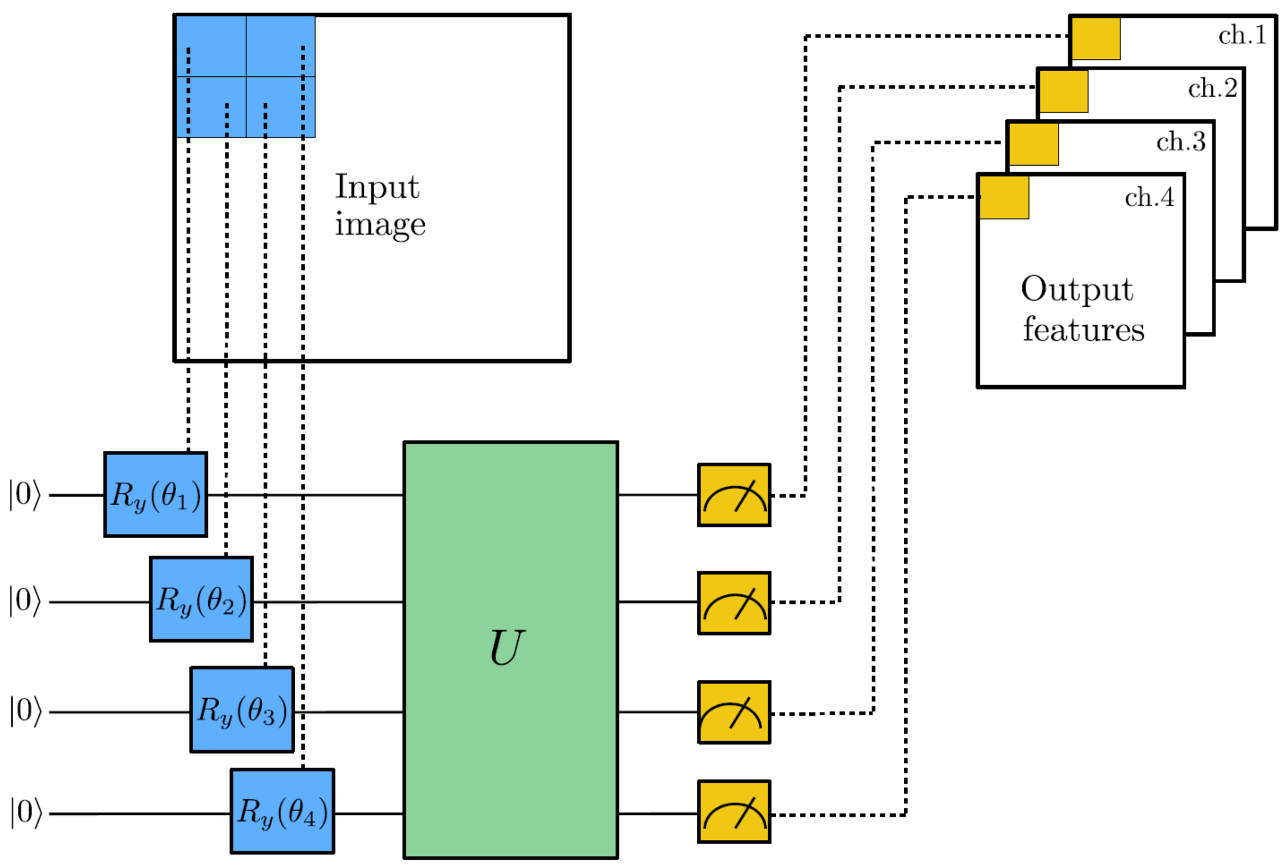
```
class KANModel(nn.Module):
    def __init__(self, precomputed_basis):
        super(KANModel, self).__init__()
        self.layer1 = KANLayer(784, 1024, precomputed_basis)
        self.layer2 = KANLayer(1024, 512, precomputed_basis)
        self.layer3 = KANLayer(512, 256, precomputed_basis)
        self.layer4 = KANLayer(256, 10, precomputed_basis)

    def forward(self, x):
        x = x.view(-1, 784)
        x = self.layer1(x)
        x = self.layer2(x)
        x = self.layer3(x)
        x = self.layer4(x)
        return x


# Load the MNIST dataset
print("Loading MNIST dataset...")
transform = transforms.Compose([transforms.ToTensor(), transforms.Normalize((0.5,), (0.5,))])

trainset = torchvision.datasets.MNIST(root='./data', train=True, download=True, transform=transform)
trainloader = DataLoader(trainset, batch_size=256, shuffle=True)

testset = torchvision.datasets.MNIST(root='./data', train=False, download=True, transform=transform)
testloader = DataLoader(testset, batch_size=256, shuffle=False)
print("MNIST dataset loaded.")

# Define the model, loss function, and optimizer
print("Initializing model...")
model = KANModel(basis).to('cuda')
criterion = nn.CrossEntropyLoss()
optimizer = torch.optim.Adam(model.parameters(), lr=0.0167, betas=(0.577, .839))
scheduler = torch.optim.lr_scheduler.StepLR(optimizer, step_size=5, gamma=0.1)
print("Model initialized.")

# Train the model
print("Starting training...")
start_time = time.time()
for epoch in range(10):  # Increased number of epochs for example
    running_loss = 0.0
    for i, data in enumerate(trainloader, 0):
        inputs, labels = data
        inputs, labels = inputs.to('cuda'), labels.to('cuda')

        optimizer.zero_grad()
        outputs = model(inputs)
        loss = criterion(outputs, labels)
        loss.backward()
        optimizer.step()

        running_loss += loss.item()
        if i % 100 == 99:    # Print every 100 mini-batches
            print(f'[Epoch {epoch + 1}, Batch {i + 1}] loss: {running_loss / 100:.3f}')
            running_loss = 0.0

    scheduler.step()
    print(f'Finished epoch {epoch + 1} loss: {running_loss / 100:.3f}')
end_time = time.time()
training_time = end_time - start_time
print(f'Total training time: {training_time:.2f} seconds')

print('Finished Training')

# Evaluate the model
print("Evaluating model...")
start_eval_time = time.time()
correct = 0
total = 0
with torch.no_grad():
    for data in testloader:
        images, labels = data
        images, labels = images.to('cuda'), labels.to('cuda')
        outputs = model(images)
        _, predicted = torch.max(outputs.data, 1)
        total += labels.size(0)
        correct += (predicted == labels).sum().item()
```

```
end_eval_time = time.time()
evaluation_time = end_eval_time - start_eval_time
print(f'Accuracy of the network on the 10000 test images: {100 * correct / total}%')
print(f'Total evaluation time: {evaluation_time:.2f} seconds')
```

```
Loading MNIST dataset...
100%|██████████| 9.91M/9.91M [00:00<00:00, 16.6MB/s]
100%|██████████| 28.9k/28.9k [00:00<00:00, 494kB/s]
100%|██████████| 1.65M/1.65M [00:00<00:00, 4.61MB/s]
100%|██████████| 4.54k/4.54k [00:00<00:00, 7.77MB/s]
MNIST dataset loaded.
Initializing model...
Model initialized.
Starting training...
[Epoch 1, Batch 100] loss: 1.149
[Epoch 1, Batch 200] loss: 0.441
Finished epoch 1 loss: 0.131
Total training time: 352.83 seconds
Finished Training
Evaluating model...
Accuracy of the network on the 10000 test images: 89.47%
Total evaluation time: 20.81 seconds
```

2. **Quanvolutional Neural Networks**

Let us try comparing the results of ths classical KAN with quanvolutional neural networsk by benchmarking accuracy similarly on the MNIST dataset.




```
!pip install pennylane
import pennylane as qml
from pennylane import numpy as np
from pennylane.templates import RandomLayers
import tensorflow as tf
from tensorflow import keras
import matplotlib.pyplot as plt
```

```
Collecting pennylane
  Downloading PennyLane-0.40.0-py3-none-any.whl.metadata (10 kB)
Requirement already satisfied: numpy<2.1 in /usr/local/lib/python3.11/dist-packages (from pennylane) (2.0.2)
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Collecting rustworkx>=0.14.0 (from pennylane)
  Downloading rustworkx-0.16.0-cp39-abi3-manylinux_2_17_x86_64.manylinux2014_x86_64.whl.metadata (10 kB)
Requirement already satisfied: autograd in /usr/local/lib/python3.11/dist-packages (from pennylane) (1.7.0)
Collecting tomlkit (from pennylane)
  Downloading tomlkit-0.13.2-py3-none-any.whl.metadata (2.7 kB)
Collecting appdirs (from pennylane)
  Downloading appdirs-1.4.4-py2.py3-none-any.whl.metadata (9.0 kB)
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  Downloading autoray-0.7.1-py3-none-any.whl.metadata (5.8 kB)
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Collecting pennylane-lightning>=0.40 (from pennylane)
  Downloading PennyLane_Lightning-0.40.0-cp311-cp311-manylinux_2_28_x86_64.whl.metadata (27 kB)
Requirement already satisfied: requests in /usr/local/lib/python3.11/dist-packages (from pennylane) (2.32.3)
Requirement already satisfied: typing-extensions in /usr/local/lib/python3.11/dist-packages (from pennylane) (4.12.2)
Requirement already satisfied: packaging in /usr/local/lib/python3.11/dist-packages (from pennylane) (24.2)
Collecting diastatic-malt (from pennylane)
  Downloading diastatic_malt-2.15.2-py3-none-any.whl.metadata (2.6 kB)
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Downloading autoray-0.7.1-py3-none-any.whl (930 kB)
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Downloading PennyLane_Lightning-0.40.0-cp311-cp311-manylinux_2_28_x86_64.whl (2.4 MB)
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Downloading rustworkx-0.16.0-cp39-abi3-manylinux_2_17_x86_64.manylinux2014_x86_64.whl (2.1 MB)
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Downloading appdirs-1.4.4-py2.py3-none-any.whl (9.6 kB)
Downloading diastatic_malt-2.15.2-py3-none-any.whl (167 kB)
   ━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 167.9/167.9 kB 6.6 MB/s eta 0:00:00
Downloading tomlkit-0.13.2-py3-none-any.whl (37 kB)
Downloading scipy_openblas32-0.3.29.0.0-py3-none-manylinux_2_17_x86_64.manylinux2014_x86_64.whl (8.6 MB)
   ━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 8.6/8.6 MB 27.3 MB/s eta 0:00:00
Installing collected packages: appdirs, tomlkit, scipy-openblas32, rustworkx, autoray, diastatic-malt, pennylane-lightning, pennylane
Successfully installed appdirs-1.4.4 autoray-0.7.1 diastatic-malt-2.15.2 pennylane-0.40.0 pennylane-lightning-0.40.0 rustworkx-0.16.0 sc
```

```python
n_epochs = 60   # Number of optimization epochs
n_layers = 1    # Number of random layers
n_train = 50    # Size of the train dataset
n_test = 30     # Size of the test dataset

SAVE_PATH = "../_static/demonstration_assets/quanvolution/"  # Data saving folder
PREPROCESS = True
np.random.seed(0)
tf.random.set_seed(0)


mnist_dataset = keras.datasets.mnist
(train_images, train_labels), (test_images, test_labels) = mnist_dataset.load_data()

# Reduce dataset size
train_images = train_images[:n_train]
train_labels = train_labels[:n_train]
test_images = test_images[:n_test]
test_labels = test_labels[:n_test]

# Normalize pixel values within 0 and 1
train_images = train_images / 255
test_images = test_images / 255

# Add extra dimension for convolution channels
train_images = np.array(train_images[..., tf.newaxis], requires_grad=False)
test_images = np.array(test_images[..., tf.newaxis], requires_grad=False)
```

```
Downloading data from https://storage.googleapis.com/tensorflow/tf-keras-datasets/mnist.npz
11490434/11490434 ━━━━━━━━━━━━━━━━━━━━ 1s 0us/step
```

```
dev = qml.device("default.qubit", wires=4)

rand_params = np.random.uniform(high=2 * np.pi, size=(n_layers, 4))

@qml.qnode(dev)
def circuit(phi):
    # Encoding of 4 classical input values
    for j in range(4):
        qml.RY(np.pi * phi[j], wires=j)

    # Random quantum circuit
    RandomLayers(rand_params, wires=list(range(4)))

    # Measurement producing 4 classical output values
    return [qml.expval(qml.PauliZ(j)) for j in range(4)]
```

```
np.shape(train_images[0])
```

```
(28, 28, 1)
```

```
def quanv(image):
    """Convolves the input image with many applications of the same quantum circuit."""
    out = np.zeros((14, 14, 4))

    for j in range(0, 28, 2):
        for k in range(0, 28, 2):
            q_results = circuit(
                [
                    image[j, k, 0],
                    image[j, k + 1, 0],
                    image[j + 1, k, 0],
                    image[j + 1, k + 1, 0]
                ]
            )
            # Assign expectation values to different channels of the output pixel (j/2, k/2)
            for c in range(4):
                out[j // 2, k // 2, c] = q_results[c]
    return out
```

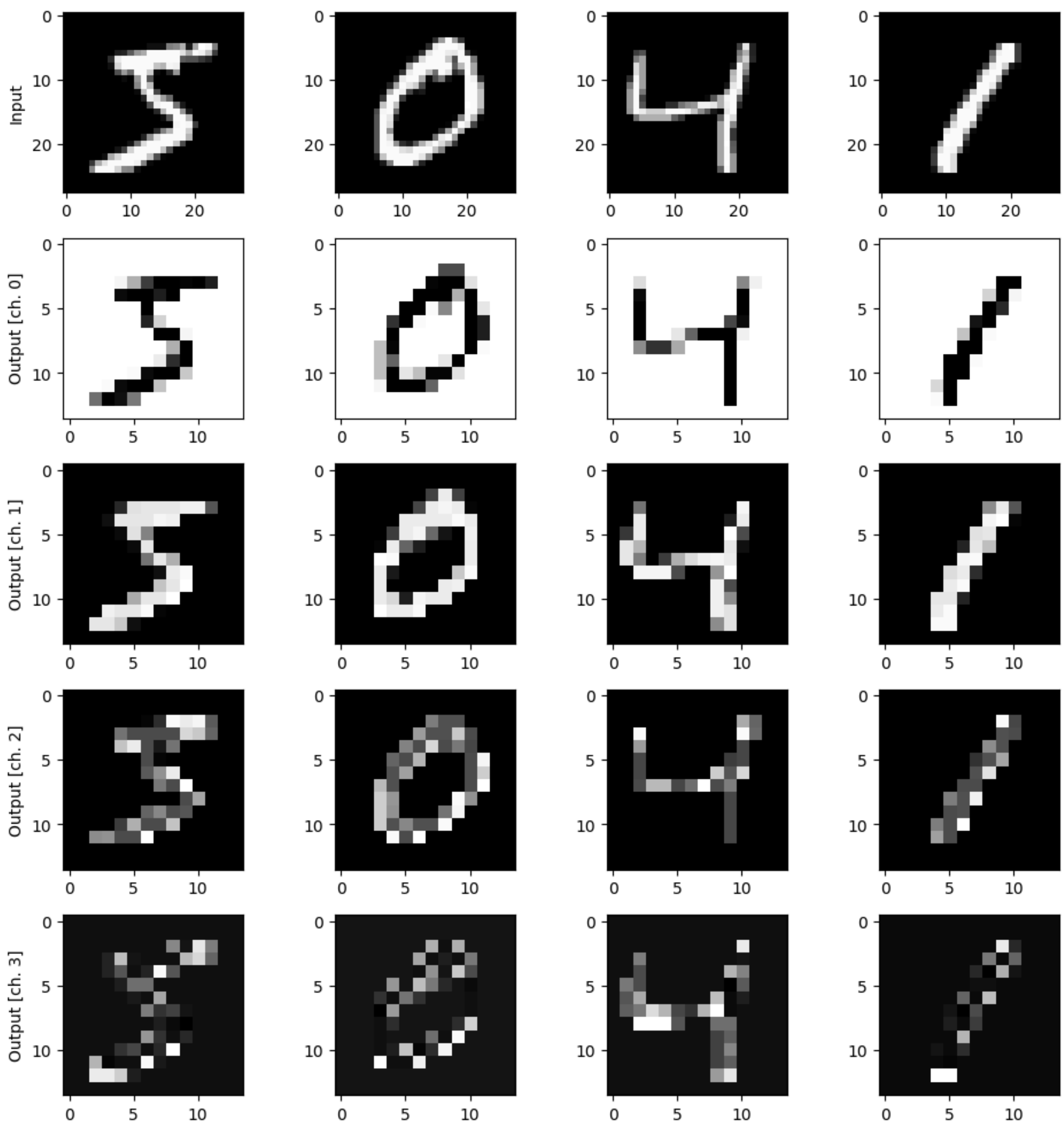
```
PREPROCESS=True
```

```
if PREPROCESS == True:
    q_train_images = []
    print("Quantum pre-processing of train images:")
    for idx, img in enumerate(train_images):
        print("{}/{}        ".format(idx + 1, n_train), end="\r")
        q_train_images.append(quanv(img))
    q_train_images = np.asarray(q_train_images)

    q_test_images = []
    print("\nQuantum pre-processing of test images:")
    for idx, img in enumerate(test_images):
        print("{}/{}        ".format(idx + 1, n_test), end="\r")
        q_test_images.append(quanv(img))
    q_test_images = np.asarray(q_test_images)

    # Create the directory if it doesn't exist
    import os
    os.makedirs(SAVE_PATH, exist_ok=True)

    # Save pre-processed images
    np.save(os.path.join(SAVE_PATH, "q_train_images.npy"), q_train_images)
    np.save(os.path.join(SAVE_PATH, "q_test_images.npy"), q_test_images)


# Load pre-processed images
q_train_images = np.load(os.path.join(SAVE_PATH, "q_train_images.npy"))
q_test_images = np.load(os.path.join(SAVE_PATH, "q_test_images.npy"))
```

```
Quantum pre-processing of train images:
```

Quantum pre-processing of test images:

```
n_samples = 4
n_channels = 4
fig, axes = plt.subplots(1 + n_channels, n_samples, figsize=(10, 10))
for k in range(n_samples):
    axes[0, 0].set_ylabel("Input")
    axes[0, k].imshow(train_images[k, :, :, 0], cmap="gray")

    # Plot all output channels
    for c in range(n_channels):
        axes[c + 1, 0].set_ylabel("Output [ch. {}]".format(c))
        axes[c + 1, k].imshow(q_train_images[k, :, :, c], cmap="gray")

plt.tight_layout()
plt.show()
```




```
def model():
  model=keras.models.Sequential([
      keras.layers.Flatten(),
      keras.layers.Dense(20, activation="softmax")
  ])
```

```
    model.compile(
        optimizer='adam',
        loss="sparse_categorical_crossentropy",
        metrics=["accuracy"],
    )

    return model


q_model = model()

q_history = q_model.fit(
    q_train_images,
    train_labels,
    validation_data=(q_test_images, test_labels),
    batch_size=4,
    epochs=n_epochs,
    verbose=2,
)
```

```
Epoch 32/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0764 - val_accuracy: 0.7333 - val_loss: 1.0436
Epoch 33/60
13/13 - 0s - 10ms/step - accuracy: 1.0000 - loss: 0.0725 - val_accuracy: 0.7333 - val_loss: 1.0397
Epoch 34/60
13/13 - 0s - 13ms/step - accuracy: 1.0000 - loss: 0.0690 - val_accuracy: 0.7000 - val_loss: 1.0359
Epoch 35/60
13/13 - 0s - 20ms/step - accuracy: 1.0000 - loss: 0.0657 - val_accuracy: 0.7000 - val_loss: 1.0324
Epoch 36/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0627 - val_accuracy: 0.7000 - val_loss: 1.0291
Epoch 37/60
13/13 - 0s - 23ms/step - accuracy: 1.0000 - loss: 0.0599 - val_accuracy: 0.7000 - val_loss: 1.0260
Epoch 38/60
13/13 - 0s - 9ms/step - accuracy: 1.0000 - loss: 0.0572 - val_accuracy: 0.7000 - val_loss: 1.0231
Epoch 39/60
13/13 - 0s - 9ms/step - accuracy: 1.0000 - loss: 0.0548 - val_accuracy: 0.7000 - val_loss: 1.0204
Epoch 40/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0525 - val_accuracy: 0.7000 - val_loss: 1.0177
Epoch 41/60
13/13 - 0s - 10ms/step - accuracy: 1.0000 - loss: 0.0504 - val_accuracy: 0.7000 - val_loss: 1.0153
Epoch 42/60
13/13 - 0s - 9ms/step - accuracy: 1.0000 - loss: 0.0484 - val_accuracy: 0.7000 - val_loss: 1.0129
Epoch 43/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0465 - val_accuracy: 0.7000 - val_loss: 1.0107
Epoch 44/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0448 - val_accuracy: 0.7000 - val_loss: 1.0086
Epoch 45/60
13/13 - 0s - 10ms/step - accuracy: 1.0000 - loss: 0.0431 - val_accuracy: 0.7000 - val_loss: 1.0066
Epoch 46/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0416 - val_accuracy: 0.7000 - val_loss: 1.0047
Epoch 47/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0401 - val_accuracy: 0.7000 - val_loss: 1.0029
Epoch 48/60
13/13 - 0s - 12ms/step - accuracy: 1.0000 - loss: 0.0387 - val_accuracy: 0.7000 - val_loss: 1.0011
Epoch 49/60
13/13 - 0s - 20ms/step - accuracy: 1.0000 - loss: 0.0374 - val_accuracy: 0.7000 - val_loss: 0.9995
Epoch 50/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0362 - val_accuracy: 0.7000 - val_loss: 0.9979
Epoch 51/60
13/13 - 0s - 10ms/step - accuracy: 1.0000 - loss: 0.0350 - val_accuracy: 0.7000 - val_loss: 0.9964
Epoch 52/60
13/13 - 0s - 9ms/step - accuracy: 1.0000 - loss: 0.0339 - val_accuracy: 0.7000 - val_loss: 0.9950
Epoch 53/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0328 - val_accuracy: 0.7000 - val_loss: 0.9936
Epoch 54/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0318 - val_accuracy: 0.7000 - val_loss: 0.9923
Epoch 55/60
13/13 - 0s - 20ms/step - accuracy: 1.0000 - loss: 0.0308 - val_accuracy: 0.7000 - val_loss: 0.9911
Epoch 56/60
13/13 - 0s - 9ms/step - accuracy: 1.0000 - loss: 0.0299 - val_accuracy: 0.7000 - val_loss: 0.9898
Epoch 57/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0290 - val_accuracy: 0.7000 - val_loss: 0.9887
Epoch 58/60
13/13 - 0s - 12ms/step - accuracy: 1.0000 - loss: 0.0282 - val_accuracy: 0.7000 - val_loss: 0.9876
Epoch 59/60
13/13 - 0s - 12ms/step - accuracy: 1.0000 - loss: 0.0274 - val_accuracy: 0.7000 - val_loss: 0.9865
Epoch 60/60
13/13 - 0s - 13ms/step - accuracy: 1.0000 - loss: 0.0266 - val_accuracy: 0.7000 - val_loss: 0.9855
```

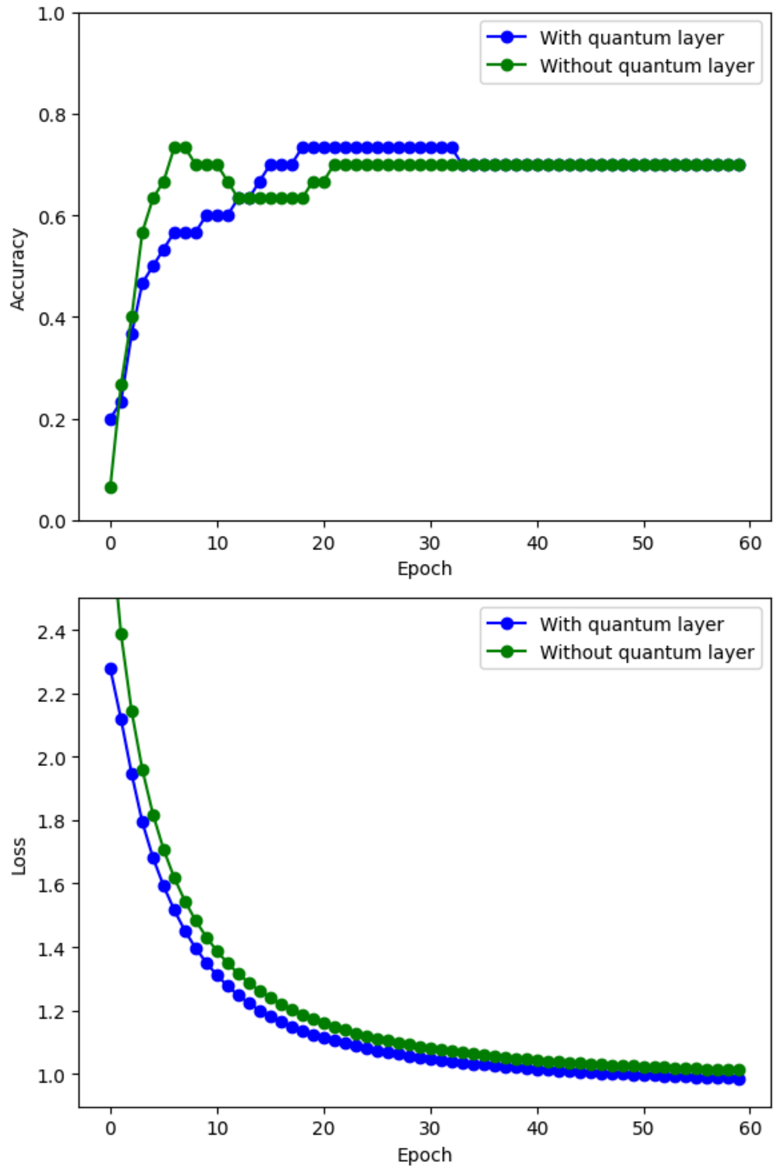
```
c_model = model()
```

```
c_history = c_model.fit(
    train_images,
    train_labels,
    validation_data=(test_images, test_labels),
    batch_size=4,
    epochs=n_epochs,
    verbose=2,
)
```

```
Epoch 32/60
13/13 - 0s - 10ms/step - accuracy: 1.0000 - loss: 0.1134 - val_accuracy: 0.7000 - val_loss: 1.0756
Epoch 33/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.1078 - val_accuracy: 0.7000 - val_loss: 1.0710
Epoch 34/60
13/13 - 0s - 23ms/step - accuracy: 1.0000 - loss: 0.1026 - val_accuracy: 0.7000 - val_loss: 1.0667
Epoch 35/60
13/13 - 0s - 12ms/step - accuracy: 1.0000 - loss: 0.0978 - val_accuracy: 0.7000 - val_loss: 1.0627
Epoch 36/60
13/13 - 0s - 12ms/step - accuracy: 1.0000 - loss: 0.0934 - val_accuracy: 0.7000 - val_loss: 1.0590
Epoch 37/60
13/13 - 0s - 13ms/step - accuracy: 1.0000 - loss: 0.0893 - val_accuracy: 0.7000 - val_loss: 1.0555
Epoch 38/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0854 - val_accuracy: 0.7000 - val_loss: 1.0522
Epoch 39/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0818 - val_accuracy: 0.7000 - val_loss: 1.0491
Epoch 40/60
13/13 - 0s - 12ms/step - accuracy: 1.0000 - loss: 0.0784 - val_accuracy: 0.7000 - val_loss: 1.0462
Epoch 41/60
13/13 - 0s - 10ms/step - accuracy: 1.0000 - loss: 0.0753 - val_accuracy: 0.7000 - val_loss: 1.0435
Epoch 42/60
13/13 - 0s - 9ms/step - accuracy: 1.0000 - loss: 0.0723 - val_accuracy: 0.7000 - val_loss: 1.0409
Epoch 43/60
13/13 - 0s - 9ms/step - accuracy: 1.0000 - loss: 0.0696 - val_accuracy: 0.7000 - val_loss: 1.0385
Epoch 44/60
13/13 - 0s - 9ms/step - accuracy: 1.0000 - loss: 0.0670 - val_accuracy: 0.7000 - val_loss: 1.0362
Epoch 45/60
13/13 - 0s - 9ms/step - accuracy: 1.0000 - loss: 0.0645 - val_accuracy: 0.7000 - val_loss: 1.0340
Epoch 46/60
13/13 - 0s - 10ms/step - accuracy: 1.0000 - loss: 0.0622 - val_accuracy: 0.7000 - val_loss: 1.0320
Epoch 47/60
13/13 - 0s - 9ms/step - accuracy: 1.0000 - loss: 0.0600 - val_accuracy: 0.7000 - val_loss: 1.0300
Epoch 48/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0579 - val_accuracy: 0.7000 - val_loss: 1.0282
Epoch 49/60
13/13 - 0s - 10ms/step - accuracy: 1.0000 - loss: 0.0560 - val_accuracy: 0.7000 - val_loss: 1.0265
Epoch 50/60
13/13 - 0s - 9ms/step - accuracy: 1.0000 - loss: 0.0541 - val_accuracy: 0.7000 - val_loss: 1.0248
Epoch 51/60
13/13 - 0s - 9ms/step - accuracy: 1.0000 - loss: 0.0523 - val_accuracy: 0.7000 - val_loss: 1.0233
Epoch 52/60
13/13 - 0s - 10ms/step - accuracy: 1.0000 - loss: 0.0507 - val_accuracy: 0.7000 - val_loss: 1.0218
Epoch 53/60
13/13 - 0s - 9ms/step - accuracy: 1.0000 - loss: 0.0491 - val_accuracy: 0.7000 - val_loss: 1.0204
Epoch 54/60
13/13 - 0s - 9ms/step - accuracy: 1.0000 - loss: 0.0476 - val_accuracy: 0.7000 - val_loss: 1.0190
Epoch 55/60
13/13 - 0s - 9ms/step - accuracy: 1.0000 - loss: 0.0461 - val_accuracy: 0.7000 - val_loss: 1.0178
Epoch 56/60
13/13 - 0s - 10ms/step - accuracy: 1.0000 - loss: 0.0448 - val_accuracy: 0.7000 - val_loss: 1.0165
Epoch 57/60
13/13 - 0s - 10ms/step - accuracy: 1.0000 - loss: 0.0434 - val_accuracy: 0.7000 - val_loss: 1.0154
Epoch 58/60
13/13 - 0s - 9ms/step - accuracy: 1.0000 - loss: 0.0422 - val_accuracy: 0.7000 - val_loss: 1.0143
Epoch 59/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0410 - val_accuracy: 0.7000 - val_loss: 1.0133
Epoch 60/60
13/13 - 0s - 10ms/step - accuracy: 1.0000 - loss: 0.0399 - val_accuracy: 0.7000 - val_loss: 1.0123
```

```
import matplotlib.pyplot as plt

fig, (ax1, ax2) = plt.subplots(2, 1, figsize=(6, 9))

ax1.plot(q_history.history["val_accuracy"], "-ob", label="With quantum layer")
ax1.plot(c_history.history["val_accuracy"], "-og", label="Without quantum layer")
ax1.set_ylabel("Accuracy")
ax1.set_ylim([0, 1])
ax1.set_xlabel("Epoch")
ax1.legend()

ax2.plot(q_history.history["val_loss"], "-ob", label="With quantum layer")
ax2.plot(c_history.history["val_loss"], "-og", label="Without quantum layer")
ax2.set_ylabel("Loss")
```

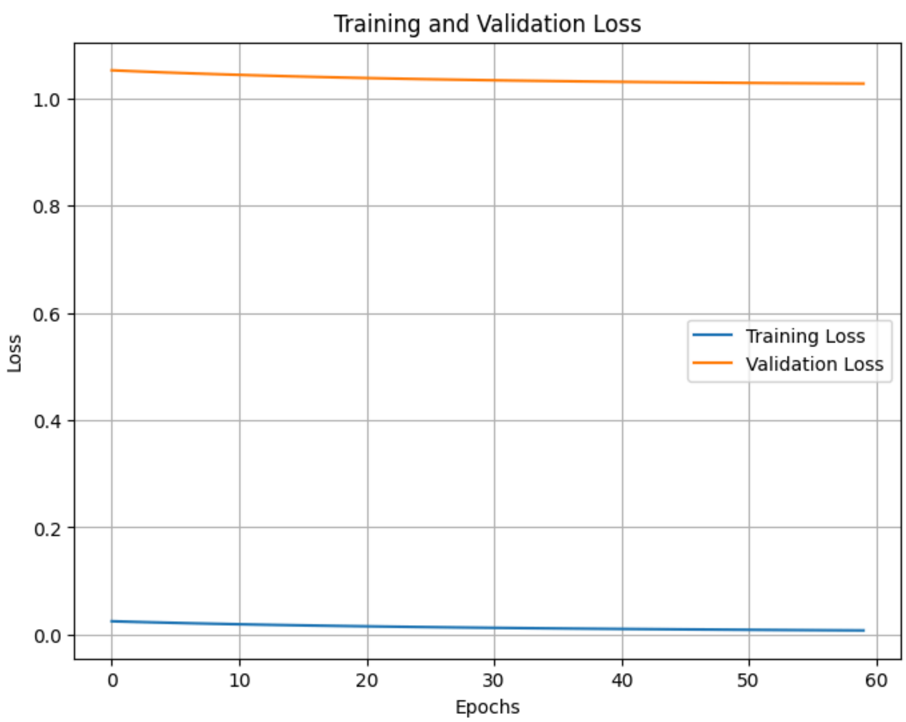
```
ax2.set_ylim(top=2.5)
ax2.set_xlabel("Epoch")
ax2.legend()
plt.tight_layout()
plt.show()
```



```
import matplotlib.pyplot as plt

# Fit the model and store the training history
history = q_model.fit(
    q_train_images,
    train_labels,
    validation_data=(q_test_images, test_labels),
    batch_size=4,
    epochs=n_epochs,
    verbose=2,
)

# Plot the loss
plt.figure(figsize=(8, 6))
plt.plot(history.history['loss'], label='Training Loss')
plt.plot(history.history['val_loss'], label='Validation Loss')
plt.title('Training and Validation Loss')
plt.xlabel('Epochs')
plt.ylabel('Loss')
plt.legend()
plt.grid(True)
plt.show()
```

```
Epoch 1/60
13/13 - 0s - 16ms/step - accuracy: 1.0000 - loss: 0.0249 - val_accuracy: 0.6333 - val_loss: 1.0524
Epoch 2/60
13/13 - 0s - 18ms/step - accuracy: 1.0000 - loss: 0.0243 - val_accuracy: 0.6333 - val_loss: 1.0514
Epoch 3/60
13/13 - 0s - 9ms/step - accuracy: 1.0000 - loss: 0.0236 - val_accuracy: 0.6333 - val_loss: 1.0505
Epoch 4/60
13/13 - 0s - 10ms/step - accuracy: 1.0000 - loss: 0.0230 - val_accuracy: 0.6333 - val_loss: 1.0495
Epoch 5/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0224 - val_accuracy: 0.6333 - val_loss: 1.0486
Epoch 6/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0219 - val_accuracy: 0.6333 - val_loss: 1.0478
Epoch 7/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0213 - val_accuracy: 0.6333 - val_loss: 1.0469
Epoch 8/60
13/13 - 0s - 12ms/step - accuracy: 1.0000 - loss: 0.0208 - val_accuracy: 0.6333 - val_loss: 1.0461
Epoch 9/60
13/13 - 0s - 24ms/step - accuracy: 1.0000 - loss: 0.0203 - val_accuracy: 0.6333 - val_loss: 1.0454
Epoch 10/60
13/13 - 0s - 23ms/step - accuracy: 1.0000 - loss: 0.0198 - val_accuracy: 0.6333 - val_loss: 1.0446
Epoch 11/60
13/13 - 0s - 19ms/step - accuracy: 1.0000 - loss: 0.0193 - val_accuracy: 0.6333 - val_loss: 1.0439
Epoch 12/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0189 - val_accuracy: 0.6333 - val_loss: 1.0432
Epoch 13/60
13/13 - 0s - 9ms/step - accuracy: 1.0000 - loss: 0.0185 - val_accuracy: 0.6333 - val_loss: 1.0425
Epoch 14/60
13/13 - 0s - 9ms/step - accuracy: 1.0000 - loss: 0.0180 - val_accuracy: 0.6333 - val_loss: 1.0419
Epoch 15/60
13/13 - 0s - 13ms/step - accuracy: 1.0000 - loss: 0.0176 - val_accuracy: 0.6333 - val_loss: 1.0413
Epoch 16/60
13/13 - 0s - 23ms/step - accuracy: 1.0000 - loss: 0.0173 - val_accuracy: 0.6333 - val_loss: 1.0407
Epoch 17/60
13/13 - 0s - 24ms/step - accuracy: 1.0000 - loss: 0.0169 - val_accuracy: 0.6333 - val_loss: 1.0401
Epoch 18/60
13/13 - 0s - 13ms/step - accuracy: 1.0000 - loss: 0.0165 - val_accuracy: 0.6333 - val_loss: 1.0396
Epoch 19/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0162 - val_accuracy: 0.6667 - val_loss: 1.0390
Epoch 20/60
13/13 - 0s - 25ms/step - accuracy: 1.0000 - loss: 0.0158 - val_accuracy: 0.6667 - val_loss: 1.0385
Epoch 21/60
13/13 - 0s - 24ms/step - accuracy: 1.0000 - loss: 0.0155 - val_accuracy: 0.6667 - val_loss: 1.0380
Epoch 22/60
13/13 - 0s - 21ms/step - accuracy: 1.0000 - loss: 0.0152 - val_accuracy: 0.6667 - val_loss: 1.0375
Epoch 23/60
13/13 - 0s - 12ms/step - accuracy: 1.0000 - loss: 0.0149 - val_accuracy: 0.6667 - val_loss: 1.0370
Epoch 24/60
13/13 - 0s - 24ms/step - accuracy: 1.0000 - loss: 0.0146 - val_accuracy: 0.6667 - val_loss: 1.0366
Epoch 25/60
13/13 - 0s - 23ms/step - accuracy: 1.0000 - loss: 0.0143 - val_accuracy: 0.6667 - val_loss: 1.0362
Epoch 26/60
13/13 - 0s - 15ms/step - accuracy: 1.0000 - loss: 0.0140 - val_accuracy: 0.6667 - val_loss: 1.0357
Epoch 27/60
13/13 - 0s - 14ms/step - accuracy: 1.0000 - loss: 0.0137 - val_accuracy: 0.6667 - val_loss: 1.0353
Epoch 28/60
13/13 - 0s - 24ms/step - accuracy: 1.0000 - loss: 0.0135 - val_accuracy: 0.6667 - val_loss: 1.0349
Epoch 29/60
13/13 - 0s - 10ms/step - accuracy: 1.0000 - loss: 0.0132 - val_accuracy: 0.6667 - val_loss: 1.0346
Epoch 30/60
13/13 - 0s - 12ms/step - accuracy: 1.0000 - loss: 0.0129 - val_accuracy: 0.6667 - val_loss: 1.0342
Epoch 31/60
13/13 - 0s - 10ms/step - accuracy: 1.0000 - loss: 0.0127 - val_accuracy: 0.6667 - val_loss: 1.0338
Epoch 32/60
13/13 - 0s - 10ms/step - accuracy: 1.0000 - loss: 0.0125 - val_accuracy: 0.6667 - val_loss: 1.0335
Epoch 33/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0122 - val_accuracy: 0.6667 - val_loss: 1.0332
Epoch 34/60
13/13 - 0s - 10ms/step - accuracy: 1.0000 - loss: 0.0120 - val_accuracy: 0.6667 - val_loss: 1.0328
Epoch 35/60
13/13 - 0s - 9ms/step - accuracy: 1.0000 - loss: 0.0118 - val_accuracy: 0.6667 - val_loss: 1.0325
Epoch 36/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0116 - val_accuracy: 0.6667 - val_loss: 1.0322
Epoch 37/60
13/13 - 0s - 12ms/step - accuracy: 1.0000 - loss: 0.0114 - val_accuracy: 0.6667 - val_loss: 1.0320
Epoch 38/60
13/13 - 0s - 22ms/step - accuracy: 1.0000 - loss: 0.0112 - val_accuracy: 0.6667 - val_loss: 1.0317
Epoch 39/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0110 - val_accuracy: 0.6667 - val_loss: 1.0314
Epoch 40/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0108 - val_accuracy: 0.6667 - val_loss: 1.0312
Epoch 41/60
13/13 - 0s - 24ms/step - accuracy: 1.0000 - loss: 0.0106 - val_accuracy: 0.6667 - val_loss: 1.0309
Epoch 42/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0104 - val_accuracy: 0.6667 - val_loss: 1.0307
```

```
Epoch 43/60
13/13 - 0s - 23ms/step - accuracy: 1.0000 - loss: 0.0102 - val_accuracy: 0.6667 - val_loss: 1.0304
Epoch 44/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0101 - val_accuracy: 0.6667 - val_loss: 1.0302
Epoch 45/60
13/13 - 0s - 23ms/step - accuracy: 1.0000 - loss: 0.0099 - val_accuracy: 0.6667 - val_loss: 1.0300
Epoch 46/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0097 - val_accuracy: 0.6667 - val_loss: 1.0298
Epoch 47/60
13/13 - 0s - 13ms/step - accuracy: 1.0000 - loss: 0.0096 - val_accuracy: 0.6667 - val_loss: 1.0296
Epoch 48/60
13/13 - 0s - 21ms/step - accuracy: 1.0000 - loss: 0.0094 - val_accuracy: 0.6667 - val_loss: 1.0294
Epoch 49/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0093 - val_accuracy: 0.6667 - val_loss: 1.0292
Epoch 50/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0091 - val_accuracy: 0.6667 - val_loss: 1.0290
Epoch 51/60
13/13 - 0s - 22ms/step - accuracy: 1.0000 - loss: 0.0090 - val_accuracy: 0.6667 - val_loss: 1.0289
Epoch 52/60
13/13 - 0s - 13ms/step - accuracy: 1.0000 - loss: 0.0088 - val_accuracy: 0.6667 - val_loss: 1.0287
Epoch 53/60
13/13 - 0s - 9ms/step - accuracy: 1.0000 - loss: 0.0087 - val_accuracy: 0.6667 - val_loss: 1.0286
Epoch 54/60
13/13 - 0s - 9ms/step - accuracy: 1.0000 - loss: 0.0086 - val_accuracy: 0.6667 - val_loss: 1.0284
Epoch 55/60
13/13 - 0s - 10ms/step - accuracy: 1.0000 - loss: 0.0084 - val_accuracy: 0.6667 - val_loss: 1.0283
Epoch 56/60
13/13 - 0s - 9ms/step - accuracy: 1.0000 - loss: 0.0083 - val_accuracy: 0.6667 - val_loss: 1.0281
Epoch 57/60
13/13 - 0s - 10ms/step - accuracy: 1.0000 - loss: 0.0082 - val_accuracy: 0.6667 - val_loss: 1.0280
Epoch 58/60
13/13 - 0s - 11ms/step - accuracy: 1.0000 - loss: 0.0081 - val_accuracy: 0.6667 - val_loss: 1.0279
Epoch 59/60
13/13 - 0s - 10ms/step - accuracy: 1.0000 - loss: 0.0079 - val_accuracy: 0.6667 - val_loss: 1.0278
Epoch 60/60
13/13 - 0s - 10ms/step - accuracy: 1.0000 - loss: 0.0078 - val_accuracy: 0.6667 - val_loss: 1.0276
```



3. **SineKAN**

```
import torch
import torch.nn.functional as F
import math
from typing import *
import numpy as np
import torch.nn as nn
import torch.optim as optim
import torchvision
import torchvision.transforms as transforms
from torch.utils.data import DataLoader
from tqdm import tqdm

def forward_step(i_n, grid_size, A, K, C):
    ratio = A * grid_size**(-K) + C
    i_n1 = ratio * i_n
    return i_n1

class SineKANLayer(torch.nn.Module):
    def __init__(self, input_dim, output_dim, device='cuda', grid_size=5, is_first=False, add_bias=True, norm_freq=True):
        super(SineKANLayer,self).__init__()
        self.grid_size = grid_size
        self.device = device
        self.is_first = is_first
        self.add_bias = add_bias
        self.input_dim = input_dim
        self.output_dim = output_dim
        self.A, self.K, self.C = 0.9724108095811765, 0.9884401790754128, 0.999449553483052

        self.grid_norm_factor = (torch.arange(grid_size) + 1)
        self.grid_norm_factor = self.grid_norm_factor.reshape(1, 1, grid_size)

        if is_first:
            self.amplitudes = torch.nn.Parameter(torch.empty(output_dim, input_dim, 1).normal_(0, .4) / output_dim  / self.grid_norm_factor)
        else:
            self.amplitudes = torch.nn.Parameter(torch.empty(output_dim, input_dim, 1).uniform_(-1, 1) / output_dim  / self.grid_norm_factor

        grid_phase = torch.arange(1, grid_size + 1).reshape(1, 1, 1, grid_size) / (grid_size + 1)
        self.input_phase = torch.linspace(0, math.pi, input_dim).reshape(1, 1, input_dim, 1).to(device)
        phase = grid_phase.to(device) + self.input_phase

        if norm_freq:
            self.freq = torch.nn.Parameter(torch.arange(1, grid_size + 1).float().reshape(1, 1, 1, grid_size) / (grid_size + 1)**(1 - is_fir
        else:
            self.freq = torch.nn.Parameter(torch.arange(1, grid_size + 1).float().reshape(1, 1, 1, grid_size))

        for i in range(1, self.grid_size):
            phase = forward_step(phase, i, self.A, self.K, self.C)
        self.register_buffer('phase', phase)

        if self.add_bias:
            self.bias  = torch.nn.Parameter(torch.ones(1, output_dim) / output_dim)

    def forward(self, x):
        x_shape = x.shape
        output_shape = x_shape[0:-1] + (self.output_dim,)
        x = torch.reshape(x, (-1, self.input_dim))
        x_reshaped = torch.reshape(x, (x.shape[0], 1, x.shape[1], 1))
        s = torch.sin(x_reshaped * self.freq + self.phase)
        y = torch.einsum('ijkl,jkl->ij', s, self.amplitudes)
        if self.add_bias:
            y += self.bias
        y = torch.reshape(y, output_shape)
        return y

class SineKAN(torch.nn.Module):
    def __init__(
        self,
        layers_hidden: List[int],
        grid_size: int = 8,
        device: str = 'cuda',
    ) -> None:
        super().__init__()
```

```
        self.layers = torch.nn.ModuleList([
            SineKANLayer(
                in_dim, out_dim, device, grid_size=grid_size, is_first=True
            ) if i == 0 else SineKANLayer(
                in_dim, out_dim, device, grid_size=grid_size,
            ) for i, (in_dim, out_dim) in enumerate(zip(layers_hidden[:-1], layers_hidden[1:]))
        ])

    def forward(self, x):
        for layer in self.layers:
            x = layer(x)
        return x


transform = transforms.Compose(
    [transforms.ToTensor(), transforms.Normalize((0.1307,), (0.3081,))]
)
trainset = torchvision.datasets.MNIST(
    root="./data", train=True, download=True, transform=transform
)
valset = torchvision.datasets.MNIST(
    root="./data", train=False, download=True, transform=transform
)
train_loader = DataLoader(trainset, batch_size=64, num_workers=2, shuffle=True)
val_loader = DataLoader(valset, batch_size=64, num_workers=2, shuffle=False)


epochs = 5
lrs = [2e-4, 3e-3, 4e-2]
gammas = [0.8, 0.9]
hdims = [64, 128, 256]
best_accs = []
for lr in lrs:
    for gamma in gammas:
        for hdim in hdims:
            torch.manual_seed(42)
            best_acc = 0
            # Define model
            model = SineKAN(layers_hidden=[28 * 28, hdim, 10], grid_size=8)
            device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
            model.to(device)
            # Define optimizer
            optimizer = optim.AdamW(model.parameters(), lr=lr)
            # Define learning rate scheduler
            scheduler = optim.lr_scheduler.ExponentialLR(optimizer, gamma=gamma)

            # Define loss
            criterion = nn.CrossEntropyLoss()
            for epoch in range(epochs):    # Train
                model.train()
                with tqdm(train_loader) as pbar:
                    for i, (images, labels) in enumerate(pbar):
                        images = images.view(-1, 28 * 28).to(device)
                        optimizer.zero_grad()
                        output = model(images)
                        loss = criterion(output, labels.to(device))
                        loss.backward()
                        optimizer.step()
                        accuracy = (output.argmax(dim=1) == labels.to(device)).float().mean()
                        pbar.set_postfix(loss=loss.item(), accuracy=accuracy.item(), lr=optimizer.param_groups[0]['lr'])

                # Validation
                model.eval()
                val_loss = 0
                val_accuracy = 0
                with torch.no_grad():
                    for images, labels in val_loader:
                        images = images.view(-1, 28 * 28).to(device)
                        output = model(images)
                        val_loss += criterion(output, labels.to(device)).item()
                        val_accuracy += (
                            (output.argmax(dim=1) == labels.to(device)).float().mean().item()
                        )
                val_loss /= len(val_loader)
                val_accuracy /= len(val_loader)
                if val_accuracy > best_acc:
                    best_acc = val_accuracy
```

```
            # Update learning rate
            scheduler.step()

            print(
                f"Epoch {epoch + 1}, Val Loss: {val_loss}, Val Accuracy: {val_accuracy}"
            )
        best_accs.append(best_acc)
        print(f"LR: {lr} Gamma: {gamma} Hdim: {hdim} Best Accuracy: {best_acc}")
```

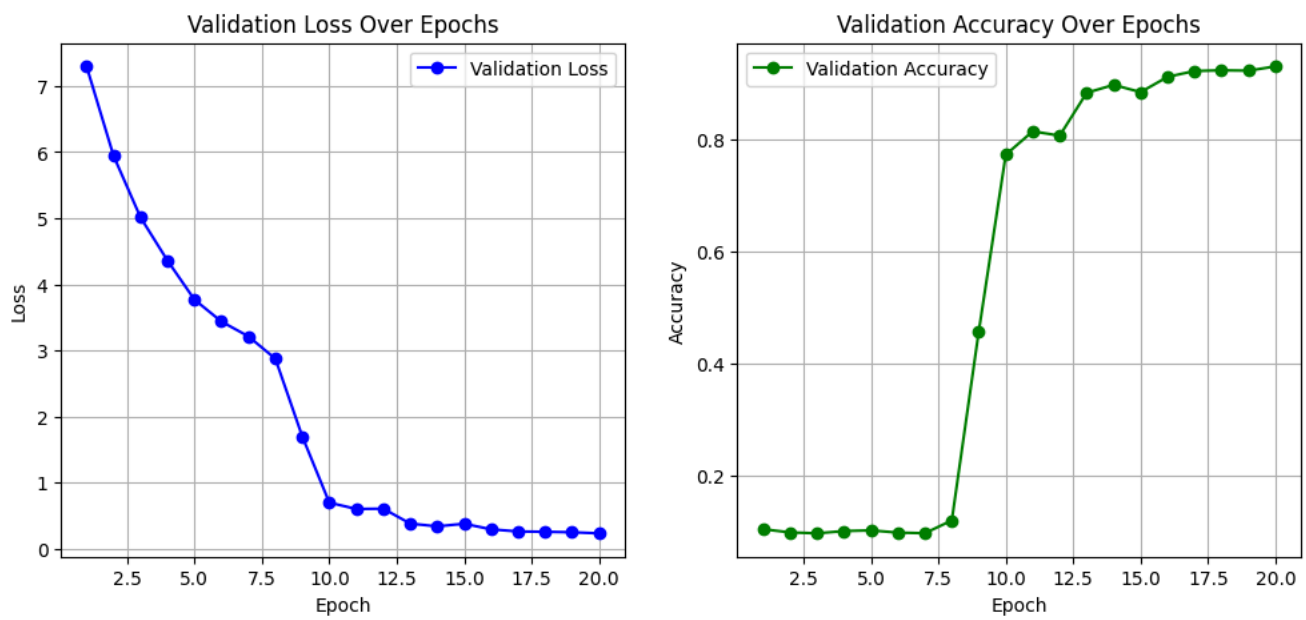
```
100%|██████████| 938/938 [00:20<00:00, 44.68it/s, accuracy=0.938, loss=0.136, lr=0.0002]
Epoch 1, Val Loss: 0.25346964880064793, Val Accuracy: 0.9266520700636943
100%|██████████| 938/938 [00:15<00:00, 62.50it/s, accuracy=0.938, loss=0.297, lr=0.00016]
Epoch 2, Val Loss: 0.19701114199628497, Val Accuracy: 0.942078025477707
100%|██████████| 938/938 [00:15<00:00, 61.28it/s, accuracy=0.938, loss=0.2, lr=0.000128]
Epoch 3, Val Loss: 0.14599578013750397, Val Accuracy: 0.9561106687898089
100%|██████████| 938/938 [00:19<00:00, 46.92it/s, accuracy=1, loss=0.0342, lr=0.000102]
Epoch 4, Val Loss: 0.13135418584748249, Val Accuracy: 0.9593949044585988
100%|██████████| 938/938 [00:15<00:00, 60.96it/s, accuracy=1, loss=0.0731, lr=8.19e-5]
Epoch 5, Val Loss: 0.11268040606200364, Val Accuracy: 0.9650676751592356
LR: 0.0002 Gamma: 0.8 Hdim: 64 Best Accuracy: 0.9650676751592356
100%|██████████| 938/938 [00:15<00:00, 59.54it/s, accuracy=0.875, loss=0.274, lr=0.0002]
Epoch 1, Val Loss: 0.20217574968530683, Val Accuracy: 0.9417794585987261
100%|██████████| 938/938 [00:14<00:00, 63.32it/s, accuracy=1, loss=0.0719, lr=0.00016]
Epoch 2, Val Loss: 0.1455211911148801, Val Accuracy: 0.9586982484076433
100%|██████████| 938/938 [00:16<00:00, 58.51it/s, accuracy=0.938, loss=0.12, lr=0.000128]
Epoch 3, Val Loss: 0.11302672866079364, Val Accuracy: 0.9665605095541401
100%|██████████| 938/938 [00:15<00:00, 60.74it/s, accuracy=1, loss=0.0345, lr=0.000102]
Epoch 4, Val Loss: 0.1042141501315733, Val Accuracy: 0.9677547770700637
100%|██████████| 938/938 [00:14<00:00, 63.16it/s, accuracy=0.969, loss=0.047, lr=8.19e-5]
Epoch 5, Val Loss: 0.09780232257515216, Val Accuracy: 0.9707404458598726
LR: 0.0002 Gamma: 0.8 Hdim: 128 Best Accuracy: 0.9707404458598726
100%|██████████| 938/938 [00:15<00:00, 62.32it/s, accuracy=1, loss=0.0408, lr=0.0002]
Epoch 1, Val Loss: 0.18200375050140224, Val Accuracy: 0.948546974522293
100%|██████████| 938/938 [00:15<00:00, 58.76it/s, accuracy=0.875, loss=0.367, lr=0.00016]
Epoch 2, Val Loss: 0.12366194087733176, Val Accuracy: 0.9625796178343949
100%|██████████| 938/938 [00:15<00:00, 62.17it/s, accuracy=1, loss=0.0248, lr=0.000128]
Epoch 3, Val Loss: 0.09775582525232558, Val Accuracy: 0.9709394904458599
100%|██████████| 938/938 [00:15<00:00, 62.40it/s, accuracy=0.969, loss=0.0793, lr=0.000102]
Epoch 4, Val Loss: 0.08831097605077633, Val Accuracy: 0.971437101910828
100%|██████████| 938/938 [00:16<00:00, 58.47it/s, accuracy=1, loss=0.00568, lr=8.19e-5]
Epoch 5, Val Loss: 0.08067609585642185, Val Accuracy: 0.9739251592356688
LR: 0.0002 Gamma: 0.8 Hdim: 256 Best Accuracy: 0.9739251592356688
100%|██████████| 938/938 [00:14<00:00, 62.57it/s, accuracy=0.938, loss=0.136, lr=0.0002]
Epoch 1, Val Loss: 0.25346964880064793, Val Accuracy: 0.9266520700636943
100%|██████████| 938/938 [00:15<00:00, 62.05it/s, accuracy=0.938, loss=0.289, lr=0.00018]
Epoch 2, Val Loss: 0.19540891957700632, Val Accuracy: 0.9423765923566879
100%|██████████| 938/938 [00:15<00:00, 60.78it/s, accuracy=0.938, loss=0.194, lr=0.000162]
Epoch 3, Val Loss: 0.14697432232676608, Val Accuracy: 0.9563097133757962
100%|██████████| 938/938 [00:15<00:00, 61.94it/s, accuracy=1, loss=0.023, lr=0.000146]
Epoch 4, Val Loss: 0.12716912634368202, Val Accuracy: 0.9598925159235668
100%|██████████| 938/938 [00:15<00:00, 61.06it/s, accuracy=1, loss=0.0529, lr=0.000131]
Epoch 5, Val Loss: 0.1096566090935687, Val Accuracy: 0.9651671974522293
LR: 0.0002 Gamma: 0.9 Hdim: 64 Best Accuracy: 0.9651671974522293
100%|██████████| 938/938 [00:15<00:00, 61.85it/s, accuracy=0.875, loss=0.274, lr=0.0002]
Epoch 1, Val Loss: 0.20217574968530683, Val Accuracy: 0.9417794585987261
100%|██████████| 938/938 [00:15<00:00, 61.10it/s, accuracy=1, loss=0.0781, lr=0.00018]
Epoch 2, Val Loss: 0.14685676535472844, Val Accuracy: 0.9575039808917197
100%|██████████| 938/938 [00:15<00:00, 59.49it/s, accuracy=0.938, loss=0.116, lr=0.000162]
Epoch 3, Val Loss: 0.1151969745498637, Val Accuracy: 0.9655652866242038
100%|██████████| 938/938 [00:15<00:00, 61.70it/s, accuracy=0.969, loss=0.0552, lr=0.000146]
Epoch 4, Val Loss: 0.1159010350772411, Val Accuracy: 0.9637738853503185
100%|██████████| 938/938 [00:15<00:00, 62.39it/s, accuracy=1, loss=0.0434, lr=0.000131]
Epoch 5, Val Loss: 0.10195070593232278, Val Accuracy: 0.9694466560509554
LR: 0.0002 Gamma: 0.9 Hdim: 128 Best Accuracy: 0.9694466560509554
100%|██████████| 938/938 [00:16<00:00, 58.39it/s, accuracy=1, loss=0.0408, lr=0.0002]
Epoch 1, Val Loss: 0.18200375050140224, Val Accuracy: 0.948546974522293
100%|██████████| 938/938 [00:15<00:00, 61.56it/s, accuracy=0.906, loss=0.349, lr=0.00018]
```

```
import matplotlib.pyplot as plt

epochs = 20
lrs = [4e-2]
gammas = [0.8]
hdims = [128]
best_accs = []

for lr in lrs:
    for gamma in gammas:
        for hdim in hdims:
            torch.manual_seed(42)
            best_acc = 0
```

```
    # Define model
    model = SineKAN(layers_hidden=[28 * 28, hdim, 10], grid_size=8)
    device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
    model.to(device)

    # Define optimizer
    optimizer = optim.AdamW(model.parameters(), lr=lr)

    # Define learning rate scheduler
    scheduler = optim.lr_scheduler.ExponentialLR(optimizer, gamma=gamma)

    # Define loss function
    criterion = nn.CrossEntropyLoss()

    # Lists to store validation loss and accuracy
    val_losses = []
    val_accuracies = []

    for epoch in range(epochs):
        model.train()
        with tqdm(train_loader) as pbar:
            for i, (images, labels) in enumerate(pbar):
                images = images.view(-1, 28 * 28).to(device)
                optimizer.zero_grad()
                output = model(images)
                loss = criterion(output, labels.to(device))
                loss.backward()
                optimizer.step()
                accuracy = (output.argmax(dim=1) == labels.to(device)).float().mean()
                pbar.set_postfix(loss=loss.item(), accuracy=accuracy.item(), lr=optimizer.param_groups[0]['lr'])

        # Validation
        model.eval()
        val_loss = 0
        val_accuracy = 0
        with torch.no_grad():
            for images, labels in val_loader:
                images = images.view(-1, 28 * 28).to(device)
                output = model(images)
                val_loss += criterion(output, labels.to(device)).item()
                val_accuracy += (
                    (output.argmax(dim=1) == labels.to(device)).float().mean().item()
                )
        val_loss /= len(val_loader)
        val_accuracy /= len(val_loader)

        # Append values to lists
        val_losses.append(val_loss)
        val_accuracies.append(val_accuracy)

        if val_accuracy > best_acc:
            best_acc = val_accuracy

        # Update learning rate
        scheduler.step()

        print(f"Epoch {epoch + 1}, Val Loss: {val_loss:.4f}, Val Accuracy: {val_accuracy:.4f}")

    best_accs.append(best_acc)
    print(f"LR: {lr} Gamma: {gamma} Hdim: {hdim} Best Accuracy: {best_acc:.4f}")

    # Plot Validation Loss and Accuracy
    plt.figure(figsize=(12, 5))

    # Plot Loss
    plt.subplot(1, 2, 1)
    plt.plot(range(1, epochs + 1), val_losses, marker='o', label='Validation Loss', color='blue')
    plt.xlabel('Epoch')
    plt.ylabel('Loss')
    plt.title('Validation Loss Over Epochs')
    plt.legend()
    plt.grid()

    # Plot Accuracy
    plt.subplot(1, 2, 2)
    plt.plot(range(1, epochs + 1), val_accuracies, marker='o', label='Validation Accuracy', color='green')
```

```
    plt.xlabel('Epoch')
    plt.ylabel('Accuracy')
    plt.title('Validation Accuracy Over Epochs')
    plt.legend()
    plt.grid()

    plt.show()
```

```
100%|██████████| 938/938 [00:15<00:00, 60.41it/s, accuracy=0.156, loss=5.72, lr=0.04]
Epoch 1, Val Loss: 7.2959, Val Accuracy: 0.1052
100%|██████████| 938/938 [00:16<00:00, 55.40it/s, accuracy=0.0938, loss=6.2, lr=0.032]
Epoch 2, Val Loss: 5.9440, Val Accuracy: 0.0995
100%|██████████| 938/938 [00:15<00:00, 59.27it/s, accuracy=0.188, loss=4.77, lr=0.0256]
Epoch 3, Val Loss: 5.0121, Val Accuracy: 0.0978
100%|██████████| 938/938 [00:15<00:00, 59.23it/s, accuracy=0.125, loss=4.89, lr=0.0205]
Epoch 4, Val Loss: 4.3592, Val Accuracy: 0.1019
100%|██████████| 938/938 [00:15<00:00, 59.04it/s, accuracy=0.0625, loss=3.6, lr=0.0164]
Epoch 5, Val Loss: 3.7688, Val Accuracy: 0.1031
100%|██████████| 938/938 [00:16<00:00, 57.79it/s, accuracy=0.0625, loss=4.06, lr=0.0131]
Epoch 6, Val Loss: 3.4369, Val Accuracy: 0.0991
100%|██████████| 938/938 [00:16<00:00, 57.13it/s, accuracy=0.0625, loss=3.15, lr=0.0105]
Epoch 7, Val Loss: 3.2133, Val Accuracy: 0.0980
100%|██████████| 938/938 [00:16<00:00, 58.50it/s, accuracy=0.0938, loss=2.68, lr=0.00839]
Epoch 8, Val Loss: 2.8744, Val Accuracy: 0.1208
100%|██████████| 938/938 [00:16<00:00, 55.47it/s, accuracy=0.562, loss=1.42, lr=0.00671]
Epoch 9, Val Loss: 1.6969, Val Accuracy: 0.4583
100%|██████████| 938/938 [00:15<00:00, 59.72it/s, accuracy=0.719, loss=0.58, lr=0.00537]
Epoch 10, Val Loss: 0.6985, Val Accuracy: 0.7732
100%|██████████| 938/938 [00:16<00:00, 58.37it/s, accuracy=0.875, loss=0.426, lr=0.00429]
Epoch 11, Val Loss: 0.6025, Val Accuracy: 0.8149
100%|██████████| 938/938 [00:16<00:00, 58.08it/s, accuracy=0.812, loss=0.774, lr=0.00344]
Epoch 12, Val Loss: 0.6071, Val Accuracy: 0.8071
100%|██████████| 938/938 [00:16<00:00, 58.58it/s, accuracy=0.875, loss=0.301, lr=0.00275]
Epoch 13, Val Loss: 0.3812, Val Accuracy: 0.8836
100%|██████████| 938/938 [00:16<00:00, 58.14it/s, accuracy=0.844, loss=0.497, lr=0.0022]
Epoch 14, Val Loss: 0.3400, Val Accuracy: 0.8977
100%|██████████| 938/938 [00:15<00:00, 59.13it/s, accuracy=0.906, loss=0.185, lr=0.00176]
Epoch 15, Val Loss: 0.3791, Val Accuracy: 0.8848
100%|██████████| 938/938 [00:16<00:00, 56.28it/s, accuracy=1, loss=0.0994, lr=0.00141]
Epoch 16, Val Loss: 0.2947, Val Accuracy: 0.9120
100%|██████████| 938/938 [00:15<00:00, 59.52it/s, accuracy=0.938, loss=0.114, lr=0.00113]
Epoch 17, Val Loss: 0.2624, Val Accuracy: 0.9225
100%|██████████| 938/938 [00:16<00:00, 55.89it/s, accuracy=0.875, loss=0.334, lr=0.000901]
Epoch 18, Val Loss: 0.2577, Val Accuracy: 0.9237
100%|██████████| 938/938 [00:16<00:00, 58.60it/s, accuracy=0.969, loss=0.107, lr=0.000721]
Epoch 19, Val Loss: 0.2503, Val Accuracy: 0.9231
100%|██████████| 938/938 [00:15<00:00, 59.19it/s, accuracy=0.875, loss=0.321, lr=0.000576]
Epoch 20, Val Loss: 0.2342, Val Accuracy: 0.9306
LR: 0.04 Gamma: 0.8 Hdim: 128 Best Accuracy: 0.9306
```




```
#Weight Decay Regularization

epochs = 5
```

```
epochs = 5
wds = [0.01, 0.1, 0.25, 0.5, 0.75, 1.]
wd_accs = np.empty((6, epochs))
for h, wd in enumerate(wds):
    torch.manual_seed(42)
    best_acc = 0
    # Define model
    model = SineKAN(layers_hidden=[28 * 28, hdim, 10], grid_size=8)
    device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
    model.to(device)
    # Define optimizer
    optimizer = optim.AdamW(model.parameters(), lr=4e-4, weight_decay=wd)
    # Define learning rate scheduler
    scheduler = optim.lr_scheduler.ExponentialLR(optimizer, gamma=gamma)

    # Define loss
    criterion = nn.CrossEntropyLoss()
    for epoch in range(epochs):
        # Train
        model.train()
        with tqdm(train_loader) as pbar:
            for i, (images, labels) in enumerate(pbar):
                images = images.view(-1, 28 * 28).to(device)
                optimizer.zero_grad()
                output = model(images)
                loss = criterion(output, labels.to(device))
                loss.backward()
                optimizer.step()
                accuracy = (output.argmax(dim=1) == labels.to(device)).float().mean()
                pbar.set_postfix(loss=loss.item(), accuracy=accuracy.item(), lr=optimizer.param_groups[0]['lr'])

        # Validation
        model.eval()
        val_loss = 0
        val_accuracy = 0
        with torch.no_grad():
            for images, labels in val_loader:
                images = images.view(-1, 28 * 28).to(device)
                output = model(images)
                val_loss += criterion(output, labels.to(device)).item()
                val_accuracy += (
                    (output.argmax(dim=1) == labels.to(device)).float().mean().item()
                )

        val_loss /= len(val_loader)
        val_accuracy /= len(val_loader)
        wd_accs[h, epoch] = val_accuracy

        # Update learning rate
        scheduler.step()

        print(
            f"Epoch {epoch + 1}, Val Loss: {val_loss}, Val Accuracy: {val_accuracy}"
        )


# Checking best dims

epochs = 5
hdims = [16, 32, 64, 128, 256, 512]
hdim_accs = np.empty((6, epochs))
for h, hdim in enumerate(hdims):
    torch.manual_seed(42)
    # Define model
    model = SineKAN(layers_hidden=[28 * 28, hdim, 10], grid_size=8)
    device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
    model.to(device)
    # Define optimizer
    optimizer = optim.AdamW(model.parameters(), lr=4e-4, weight_decay=.5)
    # Define learning rate scheduler
    scheduler = optim.lr_scheduler.ExponentialLR(optimizer, gamma=0.9)

    # Define loss
    criterion = nn.CrossEntropyLoss()
    for epoch in range(epochs):
        # Train
        model.train()
        with tqdm(train_loader) as pbar:
```

```
            for i, (images, labels) in enumerate(pbar):
                images = images.view(-1, 28 * 28).to(device)
                optimizer.zero_grad()
                output = model(images)
                loss = criterion(output, labels.to(device))
                loss.backward()
                optimizer.step()
                accuracy = (output.argmax(dim=1) == labels.to(device)).float().mean()
                pbar.set_postfix(loss=loss.item(), accuracy=accuracy.item(), lr=optimizer.param_groups[0]['lr'])

        # Validation
        model.eval()
        val_loss = 0
        val_accuracy = 0
        with torch.no_grad():
            for images, labels in val_loader:
                images = images.view(-1, 28 * 28).to(device)
                output = model(images)
                val_loss += criterion(output, labels.to(device)).item()
                val_accuracy += (
                    (output.argmax(dim=1) == labels.to(device)).float().mean().item()
                )
        val_loss /= len(val_loader)
        val_accuracy /= len(val_loader)
        hdim_accs[h, epoch] = val_accuracy

        # Update learning rate
        scheduler.step()

        print(
            f"Epoch {epoch + 1}, Val Loss: {val_loss}, Val Accuracy: {val_accuracy}"
        )



# Checking best hidden layers

epochs = 5
n_layers = [1, 2, 3, 4]
layer_accs = np.empty((4, epochs))
for h, n_layer in enumerate(n_layers):
    torch.manual_seed(42)
    # Define model
    model = SineKAN(layers_hidden=[28 * 28] + [128]*n_layer + [10], grid_size=8)
    device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
    model.to(device)
    # Define optimizer
    optimizer = optim.AdamW(model.parameters(), lr=4e-4, weight_decay=.5)
    # Define learning rate scheduler
    scheduler = optim.lr_scheduler.ExponentialLR(optimizer, gamma=0.9)

    # Define loss
    criterion = nn.CrossEntropyLoss()
    for epoch in range(epochs):
        # Train
        model.train()
        with tqdm(train_loader) as pbar:
            for i, (images, labels) in enumerate(pbar):
                images = images.view(-1, 28 * 28).to(device)
                optimizer.zero_grad()
                output = model(images)
                loss = criterion(output, labels.to(device))
                loss.backward()
                optimizer.step()
                accuracy = (output.argmax(dim=1) == labels.to(device)).float().mean()
                pbar.set_postfix(loss=loss.item(), accuracy=accuracy.item(), lr=optimizer.param_groups[0]['lr'])

        # Validation
        model.eval()
        val_loss = 0
        val_accuracy = 0
        with torch.no_grad():
            for images, labels in val_loader:
                images = images.view(-1, 28 * 28).to(device)
                output = model(images)
                val_loss += criterion(output, labels.to(device)).item()
                val_accuracy += (
```

```
                    (output.argmax(dim=1) == labels.to(device)).float().mean().item()
                )
        val_loss /= len(val_loader)
        val_accuracy /= len(val_loader)
        layer_accs[h, epoch] = val_accuracy

        # Update learning rate
        scheduler.step()

        print(
            f"Epoch {epoch + 1}, Val Loss: {val_loss}, Val Accuracy: {val_accuracy}"
        )
```

4. **Convolutional Neural Network**

```
import torch
import torch.nn as nn
import torch.optim as optim
import torchvision
import torchvision.transforms as transforms
from torch.utils.data import DataLoader

# Define device
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")

# Transformations
transform = transforms.Compose([
    transforms.ToTensor(),
    transforms.Normalize((0.1307,), (0.3081,))  # Normalization for MNIST
])

# Datasets
train_dataset = torchvision.datasets.MNIST(root="./data", train=True, download=True, transform=transform)
test_dataset = torchvision.datasets.MNIST(root="./data", train=False, download=True, transform=transform)

# Dataloaders
train_loader= DataLoader(dataset= train_dataset, batch_size=64, shuffle=True)
test_loader= DataLoader(dataset= test_dataset, batch_size=64, shuffle=False)

# CNN Module
class CNN(nn.Module):
  def __init__(self):
    super(CNN, self).__init__()
    self.conv1= nn.Conv2d(in_channels=1, out_channels=32, kernel_size=2, stride=1, padding=1)
    self.conv2= nn.Conv2d(in_channels=32, out_channels=64, kernel_size=2, stride=1, padding=1)
    self.pool= nn.MaxPool2d(kernel_size=2, stride=2)
    self.fc1= nn.Linear(in_features=64*7*7, out_features=128)
    self.fc2 = nn.Linear(in_features=128, out_features=10)
    self.relu = nn.ReLU()
    self.dropout = nn.Dropout(p=0.4)


  def forward(self, x):
      x= self.pool(self.relu(self.conv1(x)))
      x= self.pool(self.relu(self.conv2(x)))
      x= x.view(-1, 64*7*7)
      x=self.relu(self.fc1(x))
      x= self.dropout(x)
      x= self.fc2(x)
      return x

model= CNN().to(device)
criterion = nn.CrossEntropyLoss()
optimizer = optim.Adam(model.parameters(), lr=0.001)
```

```
100%|██████████| 9.91M/9.91M [00:00<00:00, 16.3MB/s]
100%|██████████| 28.9k/28.9k [00:00<00:00, 482kB/s]
100%|██████████| 1.65M/1.65M [00:00<00:00, 4.43MB/s]
100%|██████████| 4.54k/4.54k [00:00<00:00, 8.64MB/s]
```

```
#Training Loop & Testing Loop
epochs= 10
train_losses= []
```