Chapter-5

RANDOM VARIABLE AND MATHEMATICAL EXPECTATION

Random Variable → There are two types of random variable

a) **Discrete Random variable** → A Random variable which takes the integer values for e.g. If we toss a coin then head or tail appear and X be the discrete random variable which represent the number of heads as shown below:

Outcomes	H	T
$X = x$:	1	0

ii) ~~Cont~~ **Probability mass function** → Let 'X' be a discrete random variable takes the values $x_1, x_2, \ldots x_n$ with their probabilities $p_1, p_2, \ldots p_n$ respectively. Then, $p_i = P(X = x_i)$ is a probability mass function if it satisfy the following condition;

i) $p_i = p(X = x_p) \geq 0$ probability negative हुँदैन

ii) $\sum_{i=1}^{n} p_i = 1$ i.e. total probability is 1

For eg.

X	1	2	3	4
p_i	k	2k	3k	4k

→ Sum of this row gives total probability 1.

find the value of k?

Here,

$$\text{Since } \sum p_i = 1.$$

$$\text{or, } k + 2k + 3k + 4k = 1$$

$$\text{or, } 10k = 1$$

$$\text{or, } k = 0.1$$

Expection of Discrete random variables:

Expection of random variable X is the sum of the product of values taken by random variables and their corresponding probabilities. Let random variable 'x' takes the values $x_1, x_2, \ldots x_n$ with corresponding probabilities $p_1, p_2, \ldots, p_n$ then,

$$E(X) = x_1 p_1 + x_2 p_2 + \ldots + x_n p_n.$$
$$= \sum_{i=1}^{n} x_i p_i$$

Similarly

$$E(X^2) = \sum_{i=1}^{n} x_i^2 . p_i$$

Properties of Expection

(i) $E(\text{constant}) = \text{constant}$

for e.g. $E(4) = 4$

(ii) $E[ax+b) = a.E(x) + b.$

(iii) $E(x+y) = E(x) + E(y).$

(iv) $E(xy) = E(x).E(y)$ (If x and y are independent random variables)

Variance (of Random variable)

Here, Variance of $X = E(X^2) - \{E(X)\}^2$

$$= \sum_{i=1}^{n} x_i^2 p_i - \left\{\sum_{i=1}^{n} x_i p_i\right\}^2$$

Properties:

i) Variance (constant) = 0

ii) Variance $(\underset{\text{constant}}{c}x) = c^2 . V(x)$ ← variance

iii) $V(ax+b) = a^2 . V(x)$

Scanned with CamScanner

Date. ______
Page No. ______

Numerical Questions:

Q1 A random variable x has the following probability distribution.

X	0	1	2	3	4	5	6	7	8
P(X)	a	3a	5a	7a	9a	11a	13a	15a	17a

i) Determine value of a
ii) Find $P(X<3)$, $P(X \geq 3)$, $P(0<X<5)$.

Soln:

X	P(X)
0	$a = 1/81$
1	$3a = 3/81$
2	$5a = 5/81$
3	$7a = 7/81$
4	$9a = 9/81$
5	$11a = 11/81$
6	$13a = 13/81$
7	$15a = 15/81$
8	$17a = 17/81$
	$\Sigma P(X) = 81a$

Since,

(i) $\Sigma P(x) = 1$

or, $81a = 1$

or, $a = \frac{1}{81}$

(ii) Now, $P(x<3)$

$= P(x=0) + P(x=1) + P(x=2)$

$= 1/81 + 3/81 + 5/81$

$= 9/81$

$= 1/9$

Date.
Page No.

(iii) $P(X \geq 3)$

$= 1 - P(x < 3)$

$= 1 - 1/9$

$= \frac{8}{9}$

(iv) $P(0 < x < 5)$

$= P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= 3/81 + 5/81 + 7/81 + 9/81$

$= \frac{24}{81}$

$= \frac{8}{27}$ //

Q3. Two unbaised dice are thrown, Find the expected values of the sum of numbers of points on them.

Soln.

When we throw two dice then, the possible outcomes are as follows:

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

For the calculation of expected and variance of the sum of the number of points on the dice.

2 sum हुने [illegible] se total 36 Probability 1/36.

[illegible] sum [illegible] 2 [illegible]

X	X^2	$P(X=x)$ or $P(X)$	$XP(x)$
2	4	1/36	$2\times 1/36 = 2/36$
3	9	2/36	6/36
4	16	3/36	12/36
5	25	4/36	20/36
6	36	5/36	30/36
7	49	6/36	42/36
8	64	5/36	40/36
9	81	4/36	36/36
10	100	3/36	30/36
11	121	2/36	22/36
12	144	1/36	12/36
		$\sum P(X=x)=1$	$\sum X.P(X) = 252/36 = 7$

Now,

$$E(X) = \sum X.P(X)$$
$$= 7$$

~~and $E(X^2) = \sum X^2 P(X)$~~

~~$= 4\times 4/36 + 9\times 6/36 + 16\times 10/36 + 25\times 20/36 + 36\times 30/36 + 49\times 42/36 + 64\times 40/30 + 81\times 36/36 + 100\times 30/36 + 121\times 22/36 + 144\times 12/36$~~

$=$

and $E(X^2) = \sum X^2 . P(X)$.

$$= 4\times 1/36 + 9\times 2/36 + 16\times 3/36 + 25\times 4/36 + 36\times 5/36 + 49\times 6/36$$
$$+ 64\times 5/36 + 81\times 4/36 + 100\times 3/26 + 121\times 2/36 + 144\times 1/36$$

$$= 54.83.$$

Date.
Page No.

Now, $V(X) = E(X^2) - \{E(X)\}^2$
$= 54.83 - (7)^2$
$= 54.83 - 49$
$= 5.83$ //

Imp Q.2 A coin is tossed until a tail appears. What is the expected number of toss required?

Soln.

Outcomes	No. of toss (X)	P(X=x)
T	1	1/2
HT	2	1/4
HHT	3	1/8
HHHT	4	1/16
HHHHT	5	1/32
⋮	⋮	⋮

Now,

Expection of no. of tosses i.e. $E(X) = 1\times\frac{1}{2} + 2\times\frac{1}{4} + 3\times\frac{1}{8} + 4\times\frac{1}{16} + 5\times\frac{1}{32} + \ldots$

$S = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \ldots$

and $\frac{S}{2} = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \ldots$

then, $S - \frac{S}{2} = \frac{1}{2} + \left(\frac{2}{4} - \frac{1}{4}\right) + \left(\frac{3}{8} - \frac{2}{8}\right) + \left(\frac{4}{16} - \frac{3}{16}\right) + \ldots$

$\frac{S}{2} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$

$\frac{S}{2} = \frac{\frac{1}{2}}{1 - \frac{1}{2}} = \frac{1}{2} \times \frac{2}{1} = 1$ $\left[\because S = \frac{a}{1-a} \text{ in G.S}\right]$

or $S = 2$

So, Expected No. of tosses is 2.

Date.
Page No.

Q.No.7 An impoter is offered a shipment of machine tools for ~~Rs 700, 900, 1100, 1300, 1500 and 1700 with probabilities 1/12, 1/12;~~ Rs 140,000 and the probability that he will be able to sell them for Rs. 180,000 Rs 170,000 or Rs 150,000 are 0.32, 0.55 and 0.13 respectively. What is the importers expected gross profit?

Soln

Sell $(X=x)$	Probability (P_i)
180,000	0.32
170,000	0.55
150000	0.13

Now,

$$E(X) = \sum x_i P_i$$
$$= 18000 \times 0.32 + 170000 \times 0.55 + 150,000 \times 0.13$$
$$= 170600$$

$\therefore$ Gross profit $= 170,000 - 140,000$
$= 30600$

Q9 Three bags contain 3 vido cards and 2 network cards, 5 video cards and 6 network cards, 2 video cards and 4 network cards respectively. One card is drawn from each urn. Find the expected number of video cards.

Soln

I	II	III
3V	5V	2V
2N	6N	4N
5	11	6

For selecting one card from each urn ~~i.e. network card~~ and for the calculation of expected number of network card we have the following possible outcomes.

Now, for Expected no. of network card

$X=x$	$P(X=x)$	$X.P(x)$
0	$30/330$	0
1	$116/330$	$116/330$
2	$136/330$	$272/330$
3	$48/330$	$144/330$

$\Sigma X\, P(x) = 532/330 = 1.6$

Calculation rough

$P(V_1 V_2 V_3)$

$= \frac{3}{5} \times \frac{5}{11} \times \frac{2}{6}$

$= 330$

$P(N_1 V_2 V_3$ or $V_1 N_2 V_3$ or $V_1 V_2 N_3)$

$= \frac{2}{5} \times \frac{5}{11} \times \frac{2}{6} + \frac{3}{5} \times \frac{6}{11} \times \frac{3}{6} + \frac{3}{5} \times \frac{5}{11} \times \frac{4}{6}$

$= 116/330$

$P(N_1 \cdot N_2 V_3 \text{ or } N_1 V_2 N_3 \text{ or } V_1 N_2 N_3)$

$= P(N_1 N_2 V_3) + P(N_1 V_2 N_3) + P(V_1 N_2 N_3)$

$= 2/5 \times 6/11 \times 2/6 + 2/5 \times 5/11 \times 4/6 + 3/5 \times 6/11 \times 4/6$

$= 136/330$

$P(N_1 N_2 N_3)$

$= P(N_1) \cdot P(N_2) \cdot P(N_3)$

$= 2/5 \times 6/11 \times 4/6$

$= 48/330$

$\int x \cdot dx = \frac{x^2}{2}$

$\int x^n \cdot dx = \frac{x^{n+1}}{n+1}$

$\int_0^\infty x^n e^{-x} \cdot dx = \Gamma n = (n-1)!$

Date.

Page No.

Continious Random Variable:

Let x be a continious random variable and $f(x)$ is the continious probability function of x and it is also known as probability density function.

The probability of x lies in the interval ~~P(a~~ $[a,b]$ is given by $P(a \le x \le b)$

$$= \int_a^b f(x) \cdot dx$$

The function $f(x)$ is called probability density function if it satisfies the following conditions

i) $f(x) \geq 0$ ← i.e, probability can not be -ve

ii) $\int_{-\infty}^{\infty} f(x) \cdot dx = 1$ ← i.e, total probability is 1

Q11 Suppose a continious random variable x has the density function $f(x) = \begin{cases} k(1-x)^2 & \text{for } 0 < x < 1 \\ 0; & \text{elsewhere} \end{cases}$

Find the value of (i) k (ii) $P(0 < x < 0.5)$ and (iii) $E(x)$.

Soln Given, $f(x) = \begin{cases} k(1-x)^2 & \text{for } 0 < x < 1 \\ 0; & \text{elsewhere} \end{cases}$

(i) Since, $\int_{-\infty}^{\infty} f(x) \cdot dx = 1$

So, $\int_0^1 f(x) \cdot dx = 1$

or, $\int_0^1 k(1-x)^2 dx$

$$\text{or, } k\left[\frac{(1-x)^3}{3\times(-1)}\right]_0^1 = 1.$$

$$\text{or, } k\left[0+\frac{1}{3}\right] = 1$$

$$\text{or, } \frac{k}{3} = 1$$

$$\text{or, } k = 3$$

Now,

(ii) $f(x) = 3(1-x)^2$

So, $P(0 < x < 0.5)$

$$= \int_0^{0.5} f(x)\,dx$$

$$= \int_0^{0.5} 3(1-x)^2\,dx$$

$$= 3\left[\frac{(1-x)^3}{3(-1)}\right]_0^{0.5}$$

$$= 3\left[\frac{-0.125}{3}+\frac{1}{3}\right]$$

$$= 0.875.$$

(iii) $E(x)$

$$= \int_0^1 x.f(x)\,dx$$

$$= \int_0^1 3x(1-x)^2\,dx.$$

Date.
Page No.

$$= \int_0^1 3x(1-2x+x^2)\,dx$$

$$= \int_0^1 3x - 6x^2 + 3x^3 \; dx$$

$$= \left[\frac{3x^2}{2} - \frac{6x^2}{3} + \frac{3x^4}{4} \right]_0^1$$

$$= \frac{3}{2} \times 1 - 6/3 + 3/4$$

$$= \frac{3}{2} - 6/3 + 3/4$$

$$= \frac{18 - 24 + 9}{12}$$

$$= \frac{27 - 24}{12}$$

$$= \frac{3}{12}$$

$$= \frac{1}{4}$$

Q.14 Find the mean of the random variable having probability density function $f(x) = xe^{-x}$, $x \geq 0$.

Sol^n

Given, $f(x) = xe^{-x}$, $x \geq 0$

Here,

$$\text{Mean } E(x) = \int_0^\infty x \cdot f(x) \, dx$$

$$= \int_0^\infty x \cdot xe^{-x} \, dx$$

Scanned with CamScanner

$$= \int_0^\infty x^2 \cdot e^{-x} \cdot dx$$

$$= \int_0^\infty e^{-x} \cdot x^{3-1} \cdot dx$$

$$= (3-1)!$$
$$= 2!$$
$$= 2 \text{ Ans.}$$

By gama distribution

$$\int_0^\infty e^{-x} x^{n-1} dx$$
$$\sqrt{n}$$
$$= (n-1)!$$

Q.15. The pdf of a continious random variable is given by

$$f(x) = \begin{cases} 12x(1-x)^2, & 0 \le x \le 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Find $E(x)$, $E(x^2)$.

Sol^n

Given, $f(x) = \begin{cases} 12x(1-x)^2, & 0 \le x \le 1 \\ 0, & \text{elsewhere.} \end{cases}$

Now,

$$E(x) = \int_0^1 x \cdot f(x) \cdot dx.$$

$$= \int_0^1 x \cdot 12x(1-x)^2 dx$$

$$= \int_0^1 12x^2(1+x^2-2x) dx.$$

$$= 12\left[\int_0^1 x^2 \cdot dx + \int_0^1 x^4 dx - \int_0^1 2x^3 \cdot dx\right]$$

$$= 12\left\{\left[\frac{x^3}{3}\right]_0^1 + \left[\frac{x^5}{5}\right]_0^1 - 2\left[\frac{x^4}{4}\right]_0^1\right\}$$

Date.
Page No.

$$= 12\left[\frac{1}{3}+\frac{1}{5}-\frac{2}{5}\right]$$

$$= 12\left[\frac{20+12-30}{6}\right]$$

$$= \frac{12\times 2}{60}$$

$= 2/5$ Ans.

& $E(x^2) = \int_0^1 x^2 f(x).dx$

$$= \int_0^1 x^2 . 12x(1-x)^2 dx.$$

$$= \int_0^1 12x^3(1+x^2-2x).dx$$

$$= 12\left\{\int_0^1 x^3 dx + \int_0^1 x^5 . dx - 2\int_0^1 x^4 dx\right\}$$

$$= 12\left\{\left[\frac{x^4}{4}\right]_0^1 + \left[\frac{x^6}{6}\right]_0^1 - 2\left[\frac{x^5}{5}\right]_0^1\right\}$$

$$= 12\left\{\frac{1}{4}+\frac{1}{6}-\frac{2}{5}\right\}$$

$$= 12\left(\frac{15+10-24}{60}\right)$$

$$= \frac{12\times 1}{60}$$

$= \frac{1}{5}$ Ans.

Date.
Page No.

Q.17. A continious random variable X has a pdf $f(x) = 3x^2,\ 0 \le x \le 1$. Find a and b such that (i) $P(X \le a) = P(X > a)$ (ii) $P(X > b) = 0.05$.

Soln

Given, Pdf $f(x) = 3x^2,\ 0 \le x \le 1$ according to question

(i). $P(x \le a) = P(x > a)$

or, $\int_0^a f(x).dx = \int_a^1 f(x).dx.$

or, $\int_0^a 3x^2 dx = \int_a^1 3x^2 dx.$

or, $3\left[\frac{x^3}{3}\right]_0^a = 3\left[\frac{x^3}{3}\right]_a^1$

or, $\frac{a^3 - 0}{3} = \frac{1 - a^3}{3}$

or, $a^3 = 1 - a^3$

or, $2a^3 = 1$

or, $a^3 = 1/2$

or, $a = \sqrt[3]{1/2}$

or, $a = 0.79$ Ans

(ii). $P(x > b) = 0.05$

Soln

$\int_b^1 f(x).dx = 0.05$

or, $\int_b^1 3x^2 dx = 0.05$

or, $3\left[\frac{x^3}{3}\right]_b^1 = 0.05$

or, $1-b^3=0.05$

or, $b^3=1-0.05$

or, $b=\sqrt[3]{0.95}$

or, $b=0.98$ <u>Ans</u>

<u>Q.18</u> <u>Find mean and S.D. of a distribution having pdf</u> <u>$f(x)=ke^{-x/\sigma}, 0<x<\infty$ and $\sigma>0$.</u>

<u>Soln</u>

Given, pdf is $f(x)=k.e^{-x/\sigma}, 0<x<\infty, \sigma>0$

we have,

$$\int_{-\infty}^{\infty} f(x).dx=1.$$

or, $$\int_{-\infty}^{\infty} k.e^{-x/\sigma}.dx=1.$$

Let, $\frac{x}{\sigma}=y.$

or, $\frac{1}{\sigma}=\frac{dy}{dx}$

or, $dx=\sigma.dy.$

When, $x=0, y=0$

& $x=\infty, y=0$

$\therefore \int_0^{\infty} ke^{-x/\sigma}dx = \int_0^{\infty} k.e^{-y/\sigma}.dy=1$

$\Rightarrow k.\sigma\int_0^{\infty} e^{-y}.dy=1.$

$\Rightarrow k.\sigma\left[\frac{e^{-y}}{-1}\right]_0^{\infty}=1$

$\Rightarrow k.\sigma=1$

$\Rightarrow k=\frac{1}{\sigma}$

Now,

$$\text{Mean } E(x) = \int_0^\infty x \cdot f(x) \cdot dx.$$

$$= \int_0^\infty x \cdot \frac{1}{\sigma} \cdot e^{-x/\sigma} \cdot dx$$

$$= \frac{1}{\sigma} \int_0^\infty x \cdot e^{-x/\sigma} \cdot dx.$$

Let, $\frac{x}{\sigma} = y \Rightarrow x = \sigma y.$

$$\frac{1}{\sigma} = \frac{dy}{dx}$$

or, $dx = \sigma \cdot dy.$

Mean or $E(x) = \frac{1}{\cancel{\sigma}} \int_0^\infty \sigma \cdot y \cdot e^{-y} \cdot \cancel{\sigma} \cdot dy$

$$= \sigma \int_0^\infty y^{2-1} \cdot e^{-y} \cdot dy$$

$$= \sigma (2-1)!$$

$$= \sigma \cdot 1!$$

$= \sigma$ Ans

$$E(x)^2 = \int_0^\infty x^2 f(x) \cdot dx.$$

$$= \int_0^\infty x^2 \cdot \frac{1}{\sigma} \cdot x^{-x/\sigma} \cdot dx$$

$$= \int_0^\infty \sigma^2 y^2 \cdot \frac{1}{\sigma} \cdot e^{-y} \cdot \sigma \cdot dy$$

$$= \sigma^{-2} \int_0^\infty y^2 \cdot e^{-y} \cdot dy.$$

$$= \sigma^2 \int_0^\infty y^{3-1} \cdot e^{-y} \, dy \quad = \sigma^2 (3-1)! = \sigma^2 (2)!$$

$$= \sigma^2 \cdot 2$$

Date.
Page No.

$$\text{Variance} = E(x^2) - \{E(x)\}^2$$
$$= 2\sigma^2 - \sigma$$
$$= \sigma^2$$
$$\therefore S.D = \sqrt{\text{varience}}$$
$$= \sqrt{\sigma^2}$$
$$= \sigma \quad \underline{\text{Ans}}.$$

Q.19. X is a random variable taking values $-3, -2, -1, 0, 1, 2, 3$ with following probability:

x	-3	-2	-1	0	1	2	3
$P(x)$	$1/9$	$1/9$	$1/9$	$1/3$	$1/9$	$1/9$	$1/9$

Find (i) First four moments about origin (ii) first four moments about mean (iii) Measure of central tendency, dispersion, skewness and kurtosis.

Sol^n:

For the calculation of first four moments about origin.

$$\mu_1' = E(x) = \Sigma x.\,P(X=x)$$
$$\mu_2' = E(x^2) = \Sigma x^2.\,P(X=x)$$
$$\mu_3' = E(x^3) = \Sigma x^3.\,P(X=x)$$
$$\mu_4' = E(x^4) = \Sigma x^4.\,P(X=x)$$

$X = x = x_i$	$P(X=x) = P_i$	x^2	x^3	x^4	$x.P(X=x)$	$x^2.P(X=x)$	$x^3P(X=x)$	$x^4P(X=x)$
-3	$1/9$	9	27	81	$-3/9$	$9/9$	$27/9$	$81/9$
-2	$1/9$	4	8	16	$-2/9$	$4/9$	$8/9$	$16/9$
-1	$1/9$	1	1	1	$-1/9$	$1/9$	$1/9$	$1/9$
0	$1/3$	0	0	0	0	0	0	0
1	$1/9$	1	1	1	$1/9$	$1/9$	$1/9$	$1/9$
2	$1/9$	4	8	16	$2/9$	$4/9$	$8/9$	$16/9$
3	$1/9$	9	27	81	$3/9$	$9/9$	$27/9$	$81/9$
	$\Sigma P_i = 1$				$\Sigma X.P(X=x) = 0$	$\Sigma X.P(x^2) = 3.11$	$\Sigma X.P(x^3) = 0$	$\Sigma X.P(x^4) = 21.77$

Date.
Page No.

$\therefore \mu_1' = E(x) = \Sigma x. P(X=x) = 0.$

$\mu_2' = E(x^2) = \Sigma x^2. P(X=x) = 3.11$

$\mu_3' = E(x^3) = \Sigma x^3. P(X=x) = 0.$

$\mu_4' = E(x^4) = \Sigma. x^4. P(X=x) = 21.77$

Measure of Central tendency $(\mu_1') = 0$

dispersion $= \mu_2 =$ variance
$= \sqrt{3.11}$
$= 1.76$

skewness $(\mu_3') = 0.$

~~Kurtosis.~~ For Kurtosis

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

Date.
Page No.

Q2. Let $P(x,y) = \frac{2x+y}{28}$, $x = 1,3$ and $y = 2,4$; $P(y) = \frac{y+4}{14}$, find $E(X/Y=2)$ and $V(X/Y=2)$.

Soln
Given, $P(x,y) = \frac{2x+y}{28}$ and $x = 1,3$

and $P(y) = \frac{y+4}{14}$, $y = 2,4$.

Now,

$$E(X/Y=2) = \sum_{x=1,3} X \cdot P(X/Y=2)$$

$$= \sum_{x=1,3} X \cdot \frac{P(X, Y=2)}{P(Y=2)}$$

$$= \sum_{x=1,3} X \cdot \frac{\frac{2x+2}{28}}{6/14}$$

$$= \sum_{x=1,3} X \cdot \frac{(2x+2)}{28} \times \frac{14}{6}$$

$$= \sum_{x=1,3} X \cdot \frac{(x+1)}{6}$$

$$= \frac{2}{6} + \frac{12}{6}$$

$$= \frac{2+12}{6}$$

$$= 14/6$$

$$= 7/3$$

$$E(X^2/Y=2) = \sum_{x=1,3} X^2 \cdot P(X/Y=2)$$

$$= \sum_{x=1,3} X^2 \cdot \frac{P(X, Y=2)}{P(Y=2)}$$

$$= \sum_{x=1,3} X^2 \frac{(X+1)}{6} = \frac{2}{6} + 6 = \frac{38}{6} = 6.33$$

Scanned with CamScanner

Date. ______
Page No. ______

Now,

$$V(X/Y=2) = E(X^2/Y=2)\ \{E(X/Y=2)\}^2$$

$$= 6.33 - \left(7/3\right)^2$$

$$= 8/9.$$