Mid Semester Lab Assessment

MA423: Matrix Computations   11 September, 2025   S. Bora

Total Marks: 40

Important instructions:

(i) **Switch to `format long e` for all experiments.**

(ii) **Submit a single livescript file that contains all comments, answers and codes necessary to produce the required outputs. Ensure that the answers are correctly numbered and the file does not include any irrelevant material. The filename of the livescript file should be MA423MSYourrollnumber.mlx**

1. Given an $n \times n$ real invertible matrix A and real column vectors u and v of length n such that $v^t A^{-1} u \neq 1$, the *Sherman-Morrison* formula says that

$$(A - uv^t)^{-1} = A^{-1} + \frac{(A^{-1}u)v^t A^{-1}}{1 - v^t A^{-1} u}.$$

Use this fact to write a MATLAB function program $\texttt{x} = \texttt{Solve}(\texttt{A}, \texttt{u}, \texttt{v}, \texttt{b})$ for solving the system of equations $(A - uv^t)x = b$, in $\frac{2}{3}n^3 + O(n^2)$ flops.

Your program should call exactly three function programs written during regular lab sessions for the main steps and these programs should also be submitted. Codes that do not use the Sherman-Morrison formula will not be considered.

Use the `randn` command to randomly generate A, u, v and x for $n = 8$ and set $b = (A - uv^t)x$. Then solve $(A - uv^t)x = b$ using your code to get x_c the computed x. Display the residual vector and the relative error associated with x_c as well as the condition number associated with the system of equations with respect to the 1-norm. Repeat the process for $n = 10$. What can you say about the backward stability of your code, the sensitivity of the systems of equations to changes and the accuracy of x_c from these numbers? Justify your comments.

[**Marking scheme:** 11 for the 3 function programs written during regular lab sessions + 8 for their efficient use to solve the system + 6 for the sensitivity, stability and accuracy analysis.] **(25)**

2. Write a function program $\texttt{G} = \texttt{mychol}(\texttt{A})$ to find the Cholesky factor of a positive definite matrix A.

Check your output by finding $\|A - G^T G\|_1$ for randomly generated positive definite matrices A of sizes 20×20 and 30×30.

You can use
$\gg \texttt{D} = \texttt{diag}(\texttt{diag}(10 * \texttt{rand}(\texttt{n}))); \texttt{M} = \texttt{randn}(\texttt{n}); [\texttt{Q}, \texttt{R}] = \texttt{qr}(\texttt{M}); \texttt{A} = \texttt{Q}' * \texttt{D} * \texttt{Q};$
to generate a random $n \times n$ positive definite matrix.

[**Marking scheme:** 9 for `mychol.m` + 2 for illustrations (which will be awarded only if the code is correct)] **(11)**

3. Generate x and k by running the following sets of commands. Give full explanations for the values of x and k in both cases. **(2+2 = 4)**

(a)
```
>> x = 1; k = 0;
>> while  1 + x > 1
     x = x/2; k = k+1;
   end
```

(b)
```
>> x = 1; k = 0;
>> while  x + x > x
     x = 2*x; k = k+1;
   end
```