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Q2 a

Given, $f(x) = x^3+2x^2+10x-20=0$ —— ①

the function is a continous and Polynomial function.

Using intermediate theorem, checking if the eq^n has solution in interval [1,2].

$f(1) = 1^3+2(1)^2+10(1)-20 = -7$

$f(2) = 2^3+2(2)^3+10(2)-20 = 16$

since $f(1)\,f(2) < 0$, there exists a real root.

Assuming it has 2 real roots in the interval ~~[illegible]~~ [1,2], and real roots a, b such that $f(a) = f(b) = 0$.

As per rolle's theorem, if a function f is continous on the closed interval [a,b) and differentiable on interval (a,b), such that $f(a) = f(b)$, then, $f'(x) = 0$, for some x $a \leq x \leq b$

thus, if the given function $f(x)$, eq^n ① has more than one real roots, in interval [1,2] then $f'(x) = 0$ in the given interval

$$f'(x) = 3x^2 + 4x + 10$$

then the $f'(x)$ has to be zero for some value in the interval $[1,2]$,
However, $f'(x)$ will never be zero, always greater than zero for the given interval $[1,2]$.

Hence, the $f(x) = x^3 + 2x^2 + 10x - 20$ has exactly one real root in $[1,2]$.

Exercise 4:-

Assume function f is a real-valued, defined and continous function on a bounded closed interval $[a,b]$,
then, f is said to be contraction on $[a,b]$ if exists a constant L, such that $0 < L < 1$

$$|g(x) - g(y)| \leq L|x-y| \text{ where } x, y \in [a,b]$$

Given function,

$$g(x) = \frac{20}{x^2 + 2x + 10}$$

Since, the denominator is always greater than zero g can never be less than or equal to zero.

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Differentiating $g(x)$

$$\frac{dg(x)}{dx} = 20 \frac{d}{dx} (x^2+2x+10)^{-1}$$

$$\because \left[\frac{dx^n}{dx} = nx^{n-1}\right]$$

$$= -20 \left[\frac{(2x+2)}{(x^2+2x+10)^2}\right]$$

$$g'(x) = \frac{-40[x+1]}{(x^2+2x+10)^2}$$

$\Rightarrow$ $g'(x)$ is always less than zero & hence can never be zero,

Evaluating $g'(x)$ at 1 & 2,

$$g'(1) = \frac{-40(1+1)}{(1^2+2*1+10)^2} = -0.47$$

$$g'(2) = \frac{-40(2+1)}{(2^2+2*2+10)^2} = -0.37$$

$$-0.47 \leq g'(x) \leq -0.37$$

$$|g'(x)| \leq 0.47$$

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from contraction mapping theorem

$$|g(x)-g(y)| \leq L|x-y|, \quad x y \in [1,2]$$

$$\frac{|g(x)-g(y)|}{|x-y|} \leq L \quad , x y \in [1,2]$$

$$|g'(x)| \leq L$$

$$L \geq 0.47$$

thus,

$$|g(x)-g(y)| \leq 0.47 \, |x-y|$$

where $x, y \in [1,2]$