Assignment 2
AI1110: Probability and Random Variables

Ch.Kushwanth
AI22BTECH11006

12.13.4.5 : Question. Find the probability distribution of number of successes in two tosses of die,where a success is defined as

1) number greater than 4
2) six appears on atleast one die

Ans:

1)

X	0	1	2
$\Pr(X)$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

2)

Y	0	1
$\Pr(Y)$	$\frac{25}{36}$	$\frac{11}{36}$

Solution: let, A: outcome of first throw of die,A$\in \{1,2,3,4,5,6\}$
B: outcome of first throw of die,B$\in \{1,2,3,4,5,6\}$
we know that

$$\Pr(A=i)=\frac{1}{6}\ , i\in\{1,2,3,4,5,6\} \tag{1}$$

$$\Pr(B=j)=\frac{1}{6}\ , j\in\{1,2,3,4,5,6\} \tag{2}$$

since both A,B are independent

$$\Pr(A,B)=\Pr(A)\,.\,\Pr(B) \tag{3}$$

1) finding probability distribution for appearance of number greater than 4
   let X : appearance of number greater than 4 on 2 turns, X $\in \{0,1,2\}$
   $\Pr(X=x)$: probability of X becoming x,x $\in \{0,1,2\}$
   a) finding $\Pr(X=0)$

$$\Pr(X=0)=\Pr(A\le 4,B\le 4) \tag{4}$$

$$=\Pr(A\le 4)\,.\,\Pr(B\le 4)\,(\text{from (3)}) \tag{5}$$

$$ \tag{6}$$

$$\Pr(A\le 4)=\Pr(A=1)+\Pr(A=2)+\Pr(A=3)+\Pr(A=4) \tag{7}$$

$$=\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}(\text{from (1)}) \tag{8}$$

$$\Pr(A\le 4)=\frac{4}{6} \tag{9}$$

similarly

$$\Pr(B \leq 4) = \Pr(B = 1) + \Pr(B = 2) + \Pr(B = 3) + \Pr(B = 4) \tag{10}$$
$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} (\text{from } (1)) \tag{11}$$
$$\Pr(B \leq 4) = \frac{4}{6} \tag{12}$$

now

$$\Pr(X = 0) = \frac{4}{6} * \frac{4}{6} (\text{from}(5), (12), (9)) \tag{13}$$
$$\Pr(X = 0) = \frac{4}{9} \tag{14}$$

b) finding $\Pr(X = 1)$

$$\Pr(X = 1) = \Pr(A > 4) + \Pr(B > 4) - 2.\Pr(A > 4, B > 4) \tag{15}$$
$$= \Pr(A > 4) + \Pr(B > 4) - 2.\Pr(A > 4).\Pr(B > 4) (\text{from } (3)) \tag{16}$$
$$\tag{17}$$
$$\Pr(A > 4) = \Pr(A = 5) + \Pr(A = 6) \tag{18}$$
$$= \frac{1}{6} + \frac{1}{6} (\text{from } (1)) \tag{19}$$
$$\Pr(A > 4) = \frac{2}{6} \tag{20}$$

similarly,

$$\Pr(B > 4) = \Pr(B = 5) + \Pr(B = 6) \tag{21}$$
$$= \frac{1}{6} + \frac{1}{6} (\text{from } (2)) \tag{22}$$
$$\Pr(B > 4) = \frac{2}{6} \tag{23}$$

now,

$$\Pr(X = 1) = \Pr(A > 4) + \Pr(B > 4) - 2.\Pr(A > 4, B > 4) \tag{24}$$
$$= \frac{2}{6} + \frac{2}{6} - 2.\frac{2}{6}.\frac{2}{6} \text{ (from}(20)), (23)) \tag{25}$$
$$\Pr(X = 1) = \frac{4}{9} \tag{26}$$

c) finding $\Pr(X = 2)$

$$\Pr(X = 2) = \Pr(A > 4, B > 4) \tag{27}$$
$$= \Pr(A > 4).\Pr(B > 4) \text{ (from}(3)) \tag{28}$$
$$= \frac{2}{6}.\frac{2}{6} \text{ (from}(20)) \tag{29}$$
$$\Pr(X = 2) = \frac{1}{9} \tag{30}$$

2) finding probability distribution for six to appear alteast on one die
let Y:appearence of six on atleast on die,Y $\in \{0, 1\}$
$\Pr(Y = y)$: probability of Y becoming y,y $\in \{0, 1\}$

a) finding $\Pr(Y=0)$

$$\Pr(Y=0) = \Pr(A<6, B<6) \tag{31}$$
$$= \Pr(A<6)\,.\,\Pr(B<6)\,(\text{from } (3)) \tag{32}$$
$$\tag{33}$$
$$\Pr(A<6) = \Pr(A=1) + \Pr(A=2) + \Pr(A=3) + \Pr(A=4) + \Pr(A=5) \tag{34}$$
$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}(\text{from } (1)) \tag{35}$$
$$\Pr(A<6) = \frac{5}{6} \tag{36}$$

similarly,

$$\Pr(B<6) = \Pr(B=1) + \Pr(B=2) + \Pr(B=3) + \Pr(B=4) + \Pr(B=5) \tag{37}$$
$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}(\text{from } (2)) \tag{38}$$
$$\Pr(B<6) = \frac{5}{6} \tag{39}$$

now,

$$\Pr(Y=0) = \Pr(A<6)\,.\,\Pr(B<6) \tag{40}$$
$$= \frac{5}{6}.\frac{5}{6}(\text{from } (36), (39)) \tag{41}$$
$$\Pr(Y=0) = \frac{25}{36} \tag{42}$$

b) finding $\Pr(Y=1)$

$$\Pr(Y=1) = \Pr(A<6, B=6) + \Pr(A=6, B<6) + \Pr(A=6, B=6)\ \ (\text{from } (3)) \tag{43}$$
$$= \Pr(A<6)\,.\,\Pr(B=6) + \Pr(A=6)\,.\,\Pr(B<6) + \Pr(A=6)\,.\,\Pr(B=6) \tag{44}$$
$$= \frac{5}{6}.\frac{1}{6} + \frac{1}{6}.\frac{5}{6} + \frac{1}{6}.\frac{1}{6}(\text{from } (1), (2), (36), (39)) \tag{45}$$
$$\Pr(Y=1) = \frac{11}{36} \tag{46}$$