

Assignment 2

AI1110: Probability and Random Variables

Ch.Kushwanth
AI22BTECH11006

12.13.4.5 : Question. Find the probability distribution of number of successes in two tosses of die, where a success is defined as

- 1) number greater than 4
- 2) six appears on atleast one die

Ans:

| | | | | |
|----|--------|---------------|---------------|---------------|
| 1) | X | 0 | 1 | 2 |
| | Pr (X) | $\frac{4}{9}$ | $\frac{4}{9}$ | $\frac{1}{9}$ |

| | | | |
|----|--------|-----------------|-----------------|
| 2) | Y | 0 | 1 |
| | Pr (Y) | $\frac{25}{36}$ | $\frac{11}{36}$ |

Solution:

- 1) finding probability distribution for appearance of number greater than 4

let X : appearance of number greater than 4 on 2 turns , $X \in \{0, 1, 2\}$

$\Pr(X = x)$: probability of X becoming x , $x \in \{0, 1, 2\}$

p denotes probability that number greater than 4 appears, $X : \text{bin}(n, p)$

$$p = \frac{2}{6} \text{ (as there are 2 numbers greater than 4 as outcome of die)} \quad (1)$$

Using binomial distribution, $X : \text{bin}(2, \frac{1}{3})$

$$\Pr(X = i) = \binom{n}{i} \times (p)^i \times (1 - p)^{n-i} \quad (2)$$

$$\text{here } n = 2, p = \frac{1}{3} \text{ (from (1))} \quad (3)$$

$$\Pr(X = i) = \binom{2}{i} \times \left(\frac{1}{3}\right)^i \times \left(1 - \frac{1}{3}\right)^{2-i} \quad (4)$$

| variable | value | discription |
|--------------|--|-------------|
| n | 2 | |
| p | $\frac{1}{3}$ | from (1) |
| $\Pr(X = i)$ | $\binom{2}{i} \times \left(\frac{1}{3}\right)^i \times \left(\frac{2}{3}\right)^{2-i}$ | from (4) |

a) finding $\Pr(X = 0)$ values

$$\Pr(X = 0) = \frac{2!}{(0!) \times ((2 - 0)!) } \times \left(\frac{1}{3}\right)^0 \times \left(1 - \frac{1}{3}\right)^{2-0} \quad (\text{from (4)}) \quad (5)$$

$$= \frac{2}{(1) \times (2)} \times (1) \times \left(\frac{2}{3}\right)^2 \quad (6)$$

$$\Pr(X = 0) = \frac{4}{9} \quad (7)$$

b) finding $\Pr(X = 1)$

$$\Pr(X = 1) = \frac{2!}{1! \times (2 - 1)!} \times \left(\frac{1}{3}\right)^1 \times \left(\frac{2}{3}\right)^{2-1} \quad (\text{from (4)}) \quad (8)$$

$$= \frac{2}{(1) \times (1)} \times \left(\frac{1}{3}\right) \times \left(\frac{2}{3}\right) \quad (9)$$

$$\Pr(X = 1) = \frac{4}{9} \quad (10)$$

c) finding $\Pr(X = 2)$

$$\Pr(X = 2) = \frac{2!}{2! \times (2 - 2)!} \times \left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)^{2-2} \quad (\text{from (4)}) \quad (11)$$

$$= \frac{2}{(2) \times (1)} \times \left(\frac{1}{3}\right)^2 \times (1) \quad (12)$$

$$\Pr(X = 2) = \frac{1}{9} \quad (13)$$

2) finding probability distribution for six to appear atleast on one die

let Y : appearance of six on atleast on die , $Y \in \{0, 1\}$

$\Pr(Y = y)$: probability of Y becoming y , $y \in \{0, 1\}$

now, U : appearance of six on die , $U \in \{0, 1, 2\}$

$\Pr(U = u)$: probability of u number of 6s appear , $u \in \{0, 1, 2\}$

p denotes the probability of 6 on one throw of die. $U : \text{bin}(n, p)$

$$p = \frac{1}{6} \quad (14)$$

using binomial distribution, $U : \text{bin}(2, \frac{1}{6})$

$$\Pr(U = i) = \binom{n}{i} \times (p)^i \times (1 - p)^{n-i} \quad (15)$$

| variable | value | discription |
|--------------|--|-------------|
| n | 2 | |
| p | $\frac{1}{6}$ | from (2) |
| $\Pr(U = i)$ | $\binom{2}{i} \times \left(\frac{1}{6}\right)^i \times \left(\frac{5}{6}\right)^{2-i}$ | from (15) |

a) finding $\Pr(Y = 0)$

$$\Pr(Y = 0) = \Pr(U = 0) \quad (16)$$

$$= \frac{2!}{0! \times (2 - 0)!} \times \left(\frac{1}{6}\right)^0 \times \left(\frac{5}{6}\right)^{2-0} \quad (\text{from (15)}) \quad (17)$$

$$= \frac{2}{1 \times 2} (1) \times \left(\frac{5}{6}\right)^2 \quad (18)$$

$$\Pr(Y = 0) = \frac{25}{36} \quad (19)$$

b) finding $\Pr(Y = 1)$

$$\Pr(Y = 1) = p_U(1) \quad (20)$$

$$= \Pr(U = 1) + \Pr(U = 2) \quad (21)$$

$$= \frac{2!}{1! \times (2 - 1)!} \times \left(\frac{1}{6}\right)^1 \times \left(\frac{5}{6}\right)^{2-1} + \frac{2!}{2! \times (2 - 2)!} \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^{2-2} \quad (\text{from (15)}) \quad (22)$$

$$= \frac{2}{(1) \times (1)} \times \left(\frac{1}{6}\right) \times \left(\frac{5}{6}\right) + \frac{2}{(2) \times (1)} \times \left(\frac{1}{36}\right) \times (1) \quad (23)$$

$$= \frac{5}{36} + \frac{5}{36} + \frac{1}{36} \quad (24)$$

$$\Pr(Y = 1) = \frac{11}{36} \quad (25)$$