Assignment 2
AI1110: Probability and Random Variables

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12.13.4.5 : Question. Find the probability distribution of number of successes in two tosses of die,where a success is defined as

1) number greater than 4
2) six appears on atleast one die

Ans:

1)

X	0	1	2
$\Pr(X)$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

2)

Y	0	1
$\Pr(Y)$	$\frac{25}{36}$	$\frac{11}{36}$

Solution:

1) finding probability distribution for appearance of number greater than 4
   let X :appearance of number greater than 4 on 2 turns , X$\in \{0, 1, 2\}$
   $\Pr(X = x)$: probability of X becoming x , x $\in \{0, 1, 2\}$
   p denotes probability that number greater that 4 appears,$X : bin(n, p)$

$$p = \frac{2}{6}\text{(as there are 2 numbers greater than 4 as outcome of die)} \tag{1}$$

   Using binomial distribution,X:$bin(2, \frac{1}{3})$

$$\Pr(X = i) = \binom{n}{i} \times (p)^i \times (1 - p)^{n-i} \tag{2}$$

$$\text{here } n = 2, p = \frac{1}{3}\text{(from (1))} \tag{3}$$

$$\Pr(X = i) = \binom{2}{i} \times \left(\frac{1}{3}\right)^i \times \left(1 - \frac{1}{3}\right)^{2-i} \tag{4}$$

variable	value	discription
n	2	given in question
p	$\frac{1}{3}$	from (1)

a) finding $\Pr(X = 0)$ values

$$\Pr(X = 0) = \frac{2!}{(0!) \times ((2-0)!)} \times \left(\frac{1}{3}\right)^0 \times \left(1 - \frac{1}{3}\right)^{2-0} \text{(from (4))} \tag{5}$$

$$= \frac{2}{(1) \times (2)} \times (1) \times \left(\frac{2}{3}\right)^2 \tag{6}$$

$$\Pr(X = 0) = \frac{4}{9} \tag{7}$$

b) finding $\Pr(X = 1)$

$$\Pr(X = 1) = \frac{2!}{1! \times (2-1)!} \times \left(\frac{1}{3}\right)^1 \times \left(\frac{2}{3}\right)^{2-1} \text{(from (4))} \tag{8}$$

$$= \frac{2}{(1) \times (1)} \times \left(\frac{1}{3}\right) \times \left(\frac{2}{3}\right) \tag{9}$$

$$\Pr(X = 1) = \frac{4}{9} \tag{10}$$

c) finding $\Pr(X = 2)$

$$\Pr(X = 2) = \frac{2!}{2! \times (2-2)!} \times \left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)^{2-2} \text{(from (4))} \tag{11}$$

$$= \frac{2}{(2) \times (1)} \times \left(\frac{1}{3}\right)^2 \times (1) \tag{12}$$

$$\Pr(X = 2) = \frac{1}{9} \tag{13}$$

2) finding probability distribution for six to appear alteast on one die
let Y : appearence of six on atleast on die , Y$\in \{0, 1\}$
$\Pr(Y = y)$: probability of Y becoming y , y $\in \{0, 1\}$
now,U : appearance of six on die , U$\in \{0, 1, 2\}$
$\Pr(U = u)$: probability of u number of 6s appear , u$\in \{0, 1, 2\}$
p denotes the probability of 6 on one throw of die.$U : bin(n, p)$

$$p = \frac{1}{6} \tag{14}$$

using binomial distribution,$U : bin(2, \frac{1}{6})$

$$\Pr(U = i) = \binom{n}{i} \times (p)^i \times (1-p)^{n-i} \tag{15}$$

variable	value	discription
n	2	
p	$\frac{1}{6}$	from (2)

a) finding $\Pr(Y=0)$

$$\Pr(Y=0) = \Pr(U=0) \tag{16}$$
$$= \frac{2!}{0! \times (2-0)!} \times \left(\frac{1}{6}\right)^0 \times \left(\frac{5}{6}\right)^{2-0} \text{ (from (15))} \tag{17}$$
$$= \frac{2}{1 \times 2}(1) \times \left(\frac{5}{6}\right)^2 \tag{18}$$
$$\Pr(Y=0) = \frac{25}{36} \tag{19}$$

b) finding $\Pr(Y=1)$ we need to find cdf of U before

$$p_U(i) = \Pr(U=0) + \Pr(U=1) + ... + \Pr(U=i) \text{ i} : \{0,1,2\} \tag{20}$$
$$= \binom{n}{0} \times (p)^0 \times (1-p)^n + \binom{n}{1} \times (p)^1 \times (1-p)^{n-1} + ... + \binom{n}{i} \times (p)^i \times (1-p)^{n-i} \tag{21}$$
$$= (1-p)^{n-i} \times (\binom{n}{0} \times (p)^0 \times (1-p)^i + \binom{n}{1} \times (p)^1 \times (1-p)^{i-1} + ... + \binom{n}{i} \times (p)^i \times (1-p)^0) \tag{22}$$

now

$$\Pr(Y=1) = 1 - p_U(0) \tag{23}$$
$$= 1 - \left(1 - \frac{1}{6}\right)^{2-0} \times \left(\binom{n}{0} \times \left(\frac{1}{6}\right)^0 \times \left((1 - \frac{1}{6})\right)^0\right) \text{from (22)} \tag{24}$$
$$= 1 - \left(\frac{5}{6}\right)^2 \tag{25}$$
$$= 1 - \left(\frac{25}{36}\right) \tag{26}$$
$$\Pr(Y=1) = \frac{11}{36} \tag{27}$$