Assignment 2
AI1110: Probability and Random Variables

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12.13.4.5 : Question. Find the probability distribution of number of successes in two tosses of die,where a success is defined as

1) number greater than 4
2) six appears on atleast one die

Ans:

1)

X	0	1	2
$\Pr(X)$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

2)

Y	0	1
$\Pr(Y)$	$\frac{25}{36}$	$\frac{11}{36}$

Solution: let, Z_1: outcome of first throw of die, $Z_1 \in \{1, 2, 3, 4, 5, 6\}$
Z_2: outcome of first throw of die, $Z_2 \in \{1, 2, 3, 4, 5, 6\}$
we know that

$$\Pr\left(Z_j = i\right) = \frac{1}{6}\ , i \in \{1, 2, 3, 4, 5, 6\} \text{ and } j \in \{1, 2\} \tag{1}$$

since both A,B are independent

$$\Pr(Z_1, Z_2) = \Pr(Z_1) . \Pr(Z_2) \tag{2}$$

1) finding probability distribution for appearance of number greater than 4
   let X : appearance of number greater than 4 on 2 turns, X $\in \{0, 1, 2\}$
   $\Pr(X = x)$: probability of X becoming x,x $\in \{0, 1, 2\}$ probability that number greater that 4 appears

$$p = \frac{2}{6}(\text{as there are 2 numbers greater than 4 as outcome of die}) \tag{3}$$

   Using binomial distribution,

$$\Pr(X = i) = \frac{n!}{(i!).((n-i)!)}(p)^i.(1-p)^{n-i} \tag{4}$$

$$\text{here } n = 2, p = \frac{1}{3}(\text{from (3)}) \tag{5}$$

$$\Pr(X = i) = \frac{2!}{(i!).((2-i)!)}(\frac{1}{3})^i.(1-\frac{1}{3})^{2-i} \tag{6}$$

a) finding $\Pr(X=0)$

$$\Pr(X=0) = \frac{2!}{(0!).((2-0)!)}(\frac{1}{3})^0.(1-\frac{1}{3})^{2-0}(\text{from (6)}) \tag{7}$$
$$= \frac{2}{(1).(2)}(1).(\frac{2}{3})^2 \tag{8}$$
$$\Pr(X=0) = \frac{4}{9} \tag{9}$$

b) finding $\Pr(X=1)$

$$\Pr(X=1) = \frac{2!}{(1!).((2-1)!)}(\frac{1}{3})^1.(\frac{2}{3})^{2-1}(\text{from (6)}) \tag{10}$$
$$= \frac{2}{(1).(1)}(\frac{1}{3}).(\frac{2}{3}) \tag{11}$$
$$\Pr(X=1) = \frac{4}{9} \tag{12}$$

c) finding $\Pr(X=2)$

$$\Pr(X=2) = \frac{2!}{(2!).((2-2)!)}(\frac{1}{3})^2.(\frac{2}{3})^{2-2}(\text{from (6)}) \tag{13}$$
$$= \frac{2}{(2).(1)}(\frac{1}{3})^2.(1) \tag{14}$$
$$\Pr(X=2) = \frac{1}{9} \tag{15}$$

2) finding probability distribution for six to appear alteast on one die
let Y:appearence of six on atleast on die,Y $\in \{0,1\}$
$\Pr(Y=y)$: probability of Y becoming y,y $\in \{0,1\}$
let $\Pr(U=i)$: probability of i number of 6s appear on both the tosses. p denotes the probability of 6 on one throw of die

$$\implies p = \frac{1}{6} \tag{16}$$

using binomial distribution,

$$\Pr(U=i) = \frac{n!}{(i!).((n-i)!)}(p)^i.(1-p)^{n-i} \tag{17}$$
$$\text{here } n=2, p=\frac{1}{6} \tag{18}$$
$$\Pr(U=i) = \frac{2!}{(i!).((2-i)!)}(\frac{1}{6})^i.(1-\frac{1}{6})^{2-i} \tag{19}$$
$$\Pr(U=i) = \frac{2!}{(i!).((2-i)!)}(\frac{1}{6})^i.(\frac{5}{6})^{2-i} \tag{20}$$

a) finding $\Pr(Y=0)$

$$\Pr(Y=0) = \Pr(U=0) \tag{21}$$
$$= \frac{2!}{(0!).((2-0)!)}(\frac{1}{6})^0.(\frac{5}{6})^{2-0}(\text{from (20)}) \tag{22}$$
$$= \frac{2}{1.2}(1).(\frac{5}{6})^2 \tag{23}$$
$$\Pr(Y=0) = \frac{25}{36} \tag{24}$$

b) finding $\Pr(Y = 1)$

$$\Pr(Y = 1) = \Pr(U = 1) + \Pr(U = 2) \tag{25}$$

$$= \frac{2!}{(1!).((2-1)!)}(\frac{1}{6})^1.(\frac{5}{6})^{2-1} + \frac{2!}{(2!).((2-2)!)}(\frac{1}{6})^2.(\frac{5}{6})^{2-2}\text{(from (20))} \tag{26}$$

$$= \frac{2}{(1).(1)}(\frac{1}{6}).(\frac{5}{6}) + \frac{2}{(2).(1)}(\frac{1}{36}).(1) \tag{27}$$

$$= \frac{10}{36} + \frac{1}{36} \tag{28}$$

$$\Pr(Y = 1) = \frac{11}{36} \tag{29}$$