Assignment 2
AI1110: Probability and Random Variables

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12.13.4.5 : Question. Find the probability distribution of number of successes in two tosses of die,where a success is defined as

1) number greater than 4
2) six appears on atleast one die

Ans:

1)

X	0	1	2
$\Pr(X)$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

2)

Y	0	1
$\Pr(Y)$	$\frac{25}{36}$	$\frac{11}{36}$

Solution: defining variables used in solution

variable	what it denotes	domain of variable
X	appearance of number greater than 4 on 2 turns	X$\in \{0,1,2\}$
Y	appearence of six on atleast on die	Y$\in \{0,1\}$
U	appearance of six on die	U$\in \{0,1,2\}$

defining probabilities related to random variables

varibles	random variable probability	what it denotes	domain of microvariable
X	$\Pr(X = x)$	probability of X becoming x	x $\in \{0,1,2\}$
Y	$\Pr(Y = y)$	probability of Y becoming y	y $\in \{0,1\}$
U	$\Pr(U = u)$	probability of u number of 6s appear	u$\in \{0,1,2\}$

1) finding probability distribution for appearance of number greater than 4
   p denotes probability that number greater that 4 appears,$X : bin(n, p)$

$$p = \frac{2}{6}\text{(as there are 2 numbers greater than 4 as outcome of die)} \tag{1}$$

Using binomial distribution,X:$bin(2, \frac{1}{3})$

$$\Pr(X = i) = \binom{n}{i} \times (p)^i \times (1-p)^{n-i} \tag{2}$$

$$\text{here } n = 2, p = \frac{1}{3}\text{(from (1))} \tag{3}$$

$$\Pr(X = i) = \binom{2}{i} \times \left(\frac{1}{3}\right)^i \times \left(1 - \frac{1}{3}\right)^{2-i} \tag{4}$$

a) finding $\Pr(X=0)$

$$\Pr(X=0) = \frac{2!}{(0!)\times((2-0)!)} \times \left(\frac{1}{3}\right)^0 \times \left(1-\frac{1}{3}\right)^{2-0} \text{ (from (4))} \tag{5}$$

$$= \frac{2}{(1)\times(2)} \times (1) \times \left(\frac{2}{3}\right)^2 \tag{6}$$

$$\Pr(X=0) = \frac{4}{9} \tag{7}$$

b) finding $\Pr(X=1)$

$$\Pr(X=1) = \frac{2!}{1!\times(2-1)!} \times \left(\frac{1}{3}\right)^1 \times \left(\frac{2}{3}\right)^{2-1} \text{ (from (4))} \tag{8}$$

$$= \frac{2}{(1)\times(1)} \times \left(\frac{1}{3}\right) \times \left(\frac{2}{3}\right) \tag{9}$$

$$\Pr(X=1) = \frac{4}{9} \tag{10}$$

c) finding $\Pr(X=2)$

$$\Pr(X=2) = \frac{2!}{2!\times(2-2)!} \times \left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)^{2-2} \text{ (from (4))} \tag{11}$$

$$= \frac{2}{(2)\times(1)} \times \left(\frac{1}{3}\right)^2 \times (1) \tag{12}$$

$$\Pr(X=2) = \frac{1}{9} \tag{13}$$

2) finding probability distribution for six to appear alteast on one die
p denotes the probability of 6 on one throw of die.$U : bin(n,p)$

$$p = \frac{1}{6} \tag{14}$$

using binomial distribution,$U : bin(2, \frac{1}{6})$

$$\Pr(U=i) = \binom{n}{i} \times (p)^i \times (1-p)^{n-i} \tag{15}$$

$$\text{here } n = 2, p = \frac{1}{6} \tag{16}$$

$$\Pr(U=i) = \binom{2}{i} \times \left(\frac{1}{6}\right)^i \times \left(1-\frac{1}{6}\right)^{2-i} \tag{17}$$

$$\Pr(U=i) = \frac{2!}{i!\times(2-i)!} \times \left(\frac{1}{6}\right)^i \times \left(\frac{5}{6}\right)^{2-i} \tag{18}$$

a) finding $\Pr(Y=0)$

$$\Pr(Y=0) = \Pr(U=0) \tag{19}$$

$$= \frac{2!}{0!\times(2-0)!} \times \left(\frac{1}{6}\right)^0 \times \left(\frac{5}{6}\right)^{2-0} \text{ (from (18))} \tag{20}$$

$$= \frac{2}{1\times 2}(1) \times \left(\frac{5}{6}\right)^2 \tag{21}$$

$$\Pr(Y=0) = \frac{25}{36} \tag{22}$$

b) finding $\Pr(Y = 1)$

$$\Pr(Y = 1) = p_U(1) \tag{23}$$
$$= \Pr(U = 1) + \Pr(U = 2) \tag{24}$$
$$= \frac{2!}{1! \times (2-1)!} \times \left(\frac{1}{6}\right)^1 \times \left(\frac{5}{6}\right)^{2-1} + \frac{2!}{2! \times (2-2)!} \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^{2-2} \text{ (from (18))} \tag{25}$$
$$= \frac{2}{(1) \times (1)} \times \left(\frac{1}{6}\right) \times \left(\frac{5}{6}\right) + \frac{2}{(2) \times (1)} \times \left(\frac{1}{36}\right) \times (1) \tag{26}$$
$$= \frac{5}{36} + \frac{5}{36} + \frac{1}{36} \tag{27}$$
$$\Pr(Y = 1) = \frac{11}{36} \tag{28}$$