Assignment 2
AI1110: Probability and Random Variables

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12.13.4.5 : Question. Find the probability distribution of number of successes in two tosses of die,where a success is defined as

1) number greater than 4
2) six appears on atleast one die

Ans:

1)

X	0	1	2
$\Pr(X)$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

2)

Y	0	1
$\Pr(Y)$	$\frac{25}{36}$	$\frac{11}{36}$

Solution: let, Z_1: outcome of first throw of die, $Z_1 \in \{1,2,3,4,5,6\}$
Z_2: outcome of first throw of die, $Z_2 \in \{1,2,3,4,5,6\}$
we know that

$$\Pr\left(Z_j = i\right) = \frac{1}{6}\ , i \in \{1,2,3,4,5,6\} \text{ and } j \in \{1,2\} \tag{1}$$

since both A,B are independent

$$\Pr(Z_1, Z_2) = \Pr(Z_1)\,.\,\Pr(Z_2) \tag{2}$$

1) finding probability distribution for appearance of number greater than 4
   let X : appearance of number greater than 4 on 2 turns, X $\in \{0,1,2\}$
   $\Pr(X = x)$: probability of X becoming x,x $\in \{0,1,2\}$
   a) finding $\Pr(X = 0)$

$$\Pr(X = 0) = \Pr(Z_1 \le 4, Z_2 \le 4) \tag{3}$$
$$= \Pr(Z_1 \le 4)\,.\,\Pr(Z_2 \le 4)\ (\text{from } (2)) \tag{4}$$
$$\tag{5}$$
$$\Pr(Z_i \le 4) = \Pr(Z_i = 1) + \Pr(Z_i = 2) + \Pr(Z_i = 3) + \Pr(Z_i = 4)\,, i \in \{1,2\} \tag{6}$$
$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} (\text{from } (1)) \tag{7}$$
$$\Pr(Z_i \le 4) = \frac{4}{6}, i \in \{1,2\} \tag{8}$$

now

$$\Pr(X=0) = \frac{4}{6} * \frac{4}{6} (\text{from}(4),(8)) \tag{9}$$

$$\Pr(X=0) = \frac{4}{9} \tag{10}$$

b) finding $\Pr(X=1)$

$$\Pr(X=1) = \Pr(Z_1>4) + \Pr(Z_2>4) - 2.\Pr(Z_1>4, Z_2>4) \tag{11}$$

$$= \Pr(Z_1>4) + \Pr(Z_2>4) - 2.\Pr(Z_1>4).\Pr(Z_2>4) \,(\text{from } (2)) \tag{12}$$

$$\Pr(Z_i>4) = \Pr(Z_i=5) + \Pr(Z_i=6), i \in \{1,2\} \tag{13}$$

$$= \frac{1}{6} + \frac{1}{6} (\text{from } (1)) \tag{14}$$

$$\Pr(Z_i>4) = \frac{2}{6}, i \in \{1,2\} \tag{15}$$

now,

$$\Pr(X=1) = \Pr(Z_1>4) + \Pr(Z_2>4) - 2.\Pr(Z_1>4, Z_2>4) \tag{16}$$

$$= \frac{2}{6} + \frac{2}{6} - 2.\frac{2}{6}.\frac{2}{6} \text{ (from(15))} \tag{17}$$

$$\Pr(X=1) = \frac{4}{9} \tag{18}$$

c) finding $\Pr(X=2)$

$$\Pr(X=2) = \Pr(Z_1>4, Z_2>4) \tag{19}$$

$$= \Pr(Z_1>4).\Pr(Z_2>4) \text{ (from(2))} \tag{20}$$

$$= \frac{2}{6}.\frac{2}{6} \text{ (from(15))} \tag{21}$$

$$\Pr(X=2) = \frac{1}{9} \tag{22}$$

2) finding probability distribution for six to appear alteast on one die
let Y:appearence of six on atleast on die,Y $\in \{0,1\}$
$\Pr(Y=y)$: probability of Y becoming y,y $\in \{0,1\}$

a) finding $\Pr(Y=0)$

$$\Pr(Y=0) = \Pr(Z_1<6, Z_2<6) \tag{23}$$

$$= \Pr(Z_1<6).\Pr(Z_2<6) \,(\text{from } (2)) \tag{24}$$

$$\Pr(Z_i<6) = \Pr(Z_i=1) + \Pr(Z_i=2) + \Pr(Z_i=3) + \Pr(Z_i=4) + \Pr(Z_i=5), i \in \{1,2\} \tag{25}$$

$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} (\text{from } (1)) \tag{26}$$

$$\Pr(Z_i<6) = \frac{5}{6}, i \in \{1,2\} \tag{27}$$

now,

$$\Pr(Y=0) = \Pr(Z_1<6).\Pr(Z_2<6) \tag{28}$$

$$= \frac{5}{6}.\frac{5}{6} (\text{from } (27)) \tag{29}$$

$$\Pr(Y=0) = \frac{25}{36} \tag{30}$$

b) finding $\Pr(Y = 1)$

$$\Pr(Y = 1) = \Pr(Z_1 < 6, Z_2 = 6) + \Pr(Z_1 = 6, Z_2 < 6) + \Pr(Z_1 = 6, Z_2 = 6) \text{ (from (2))} \quad (31)$$

$$= \Pr(Z_1 < 6) \,.\, \Pr(Z_2 = 6) + \Pr(Z_1 = 6) \,.\, \Pr(Z_2 < 6) + \Pr(Z_1 = 6) \,.\, \Pr(Z_2 = 6) \quad (32)$$

$$= \frac{5}{6}.\frac{1}{6} + \frac{1}{6}.\frac{5}{6} + \frac{1}{6}.\frac{1}{6} \text{(from (1), (27))} \quad (33)$$

$$\Pr(Y = 1) = \frac{11}{36} \quad (34)$$