

Assignment 1

AI1110: Probability and Random Variables

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12.13.6.15 : Question. An electronic assembly consists of two subsystems, say A and B. From previous testing procedures, the following probabilities are assumed to be known:

$$\Pr(A \text{ fails}) = 0.20 \quad (1)$$

$$\Pr(B \text{ alone fails}) = 0.15 \quad (2)$$

$$\Pr(A \text{ and } B \text{ fails}) = 0.15 \quad (3)$$

Evaluate the following probabilities

1) $\Pr(A \text{ fails given } B \text{ has failed})$

2) $\Pr(A \text{ fails alone})$

ans:

1)

$$\Pr(A \text{ fails given } B \text{ has failed}) = 0.50 \quad (4)$$

2)

$$\Pr(A \text{ fails alone}) = 0.05 \quad (5)$$

Solution: Given ,

1)

$$\Pr(A') = 0.20 \quad (6)$$

2)

$$\Pr(AB') = 0.15 \quad (7)$$

3)

$$\Pr(A'B') = 0.15 \quad (8)$$

Now to find, $\Pr(A'|B')$ we use

$$\Pr(A'|B') = \frac{\Pr(A'B')}{\Pr(B')} \quad (9)$$

to obtain $\Pr(B')$

$$B'(1) = B'(A + A') \quad (10)$$

$$= B'A + B'A' \text{ since } ((B'A)(B'A')) = 0 \quad (11)$$

$$\Pr(B') = \Pr(AB') + \Pr(A'B') \quad (12)$$

we use,

$$\Pr(B') = \Pr(AB') + \Pr(A'B') \text{ (from (12))} \quad (13)$$

$$= 0.15 + 0.15 \text{ (from (7), (8))} \quad (14)$$

$$\Rightarrow \Pr(B') = 0.30 \quad (15)$$

now to find $\Pr(A'|B')$

$$\Pr(A'|B') = 0.15/0.30 \text{ (from (15), (8), (9))} \quad (16)$$

$$\Rightarrow \Pr(A'|B') = 0.50 \quad (17)$$

similarly, to obtain $\Pr(BA')$

$$BA' + B'A' = BA' + B'A' \quad (18)$$

$$BA' + B'A' = A'(B' + B) \quad (19)$$

$$BA' + B'A' = A' \quad (20)$$

$$A' = BA' + B'A' \text{ since } ((A'B)(B'A')) = 0 \quad (21)$$

$$\Pr(A') = \Pr(BA') + \Pr(A'B') \quad (22)$$

$$\Pr(BA') = \Pr(A') - \Pr(A'B') \quad (23)$$

we use

$$\Pr(BA') = \Pr(A') - \Pr(A'B') \text{ from (23)} \quad (24)$$

$$= 0.20 - 0.15 \text{ from eq:(6), (8)} \quad (25)$$

$$\Rightarrow \Pr(BA') = 0.05 \quad (26)$$