Assignment 2
AI1110: Probability and Random Variables

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12.13.4.5 : Question. Find the probability distribution of number of successes in two tosses of die,where a success is defined as

1) number greater than 4
2) six appears on atleast one die

Ans:

1)

X	0	1	2
$\Pr(X)$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

2)

Y	0	1
$\Pr(Y)$	$\frac{25}{36}$	$\frac{11}{36}$

Solution:

1) finding probability distribution for appearance of number greater than 4
let X : appearance of number greater than 4 on 2 turns, X $\in \{0, 1, 2\}$
$\Pr(X = x)$: probability of X becoming x,x $\in \{0, 1, 2\}$
p denotes probability that number greater that 4 appears,$X : bin(n, p)$

$$p = \frac{2}{6}(\text{as there are 2 numbers greater than 4 as outcome of die}) \tag{1}$$

Using binomial distribution,X:$bin(2, \frac{1}{3})$

$$\Pr(X = i) = \frac{n!}{(i!).((n-i)!)}(p)^i.(1-p)^{n-i} \tag{2}$$

$$\text{here } n = 2, p = \frac{1}{3}(\text{from (1)}) \tag{3}$$

$$\Pr(X = i) = \frac{2!}{(i!).((2-i)!)}(\frac{1}{3})^i.(1-\frac{1}{3})^{2-i} \tag{4}$$

a) finding $\Pr(X = 0)$

$$\Pr(X = 0) = \frac{2!}{(0!).((2-0)!)}(\frac{1}{3})^0.(1-\frac{1}{3})^{2-0}(\text{from (4)}) \tag{5}$$

$$= \frac{2}{(1).(2)}(1).(\frac{2}{3})^2 \tag{6}$$

$$\Pr(X = 0) = \frac{4}{9} \tag{7}$$

b) finding $\Pr(X = 1)$

$$\Pr(X = 1) = \frac{2!}{(1!).((2-1)!)}(\frac{1}{3})^1.(\frac{2}{3})^{2-1}(\text{from (4)}) \tag{8}$$
$$= \frac{2}{(1).(1)}(\frac{1}{3}).(\frac{2}{3}) \tag{9}$$
$$\Pr(X = 1) = \frac{4}{9} \tag{10}$$

c) finding $\Pr(X = 2)$

$$\Pr(X = 2) = \frac{2!}{(2!).((2-2)!)}(\frac{1}{3})^2.(\frac{2}{3})^{2-2}(\text{from (4)}) \tag{11}$$
$$= \frac{2}{(2).(1)}(\frac{1}{3})^2.(1) \tag{12}$$
$$\Pr(X = 2) = \frac{1}{9} \tag{13}$$

2) finding probability distribution for six to appear alteast on one die
let Y:appearence of six on atleast on die,Y $\in \{0, 1\}$
$\Pr(Y = y)$: probability of Y becoming y,y $\in \{0, 1\}$
let U:appearance of six on die,U$\in \{0, 1, 2\}$
$\Pr(U = i)$: probability of i number of 6s appear on both the tosses. p denotes the probability of 6 on one throw of die.$U : bin(n, p)$

$$p = \frac{1}{6} \tag{14}$$

using binomial distribution,$U : bin(2, \frac{1}{6})$

$$\Pr(U = i) = \frac{n!}{(i!).((n-i)!)}(p)^i.(1-p)^{n-i} \tag{15}$$
$$\text{here } n = 2, p = \frac{1}{6} \tag{16}$$
$$\Pr(U = i) = \frac{2!}{(i!).((2-i)!)}(\frac{1}{6})^i.(1-\frac{1}{6})^{2-i} \tag{17}$$
$$\Pr(U = i) = \frac{2!}{(i!).((2-i)!)}(\frac{1}{6})^i.(\frac{5}{6})^{2-i} \tag{18}$$

a) finding $\Pr(Y = 0)$

$$\Pr(Y = 0) = \Pr(U = 0) \tag{19}$$
$$= \frac{2!}{(0!).((2-0)!)}(\frac{1}{6})^0.(\frac{5}{6})^{2-0}(\text{from (18)}) \tag{20}$$
$$= \frac{2}{1.2}(1).(\frac{5}{6})^2 \tag{21}$$
$$\Pr(Y = 0) = \frac{25}{36} \tag{22}$$

b) finding $\Pr(Y = 1)$

$$\begin{aligned}
\Pr(Y = 1) &= \Pr(U \geq 1) && (23)\\
&= \Pr(U = 1) + \Pr(U = 2) && (24)\\
&= \frac{2!}{(1!).((2-1)!)}(\frac{1}{6})^1.(\frac{5}{6})^{2-1} + \frac{2!}{(2!).((2-2)!)}(\frac{1}{6})^2.(\frac{5}{6})^{2-2}(\text{from } (18)) && (25)\\
&= \frac{2}{(1).(1)}(\frac{1}{6}).(\frac{5}{6}) + \frac{2}{(2).(1)}(\frac{1}{36}).(1) && (26)\\
&= \frac{10}{36} + \frac{1}{36} && (27)\\
\Pr(Y = 1) &= \frac{11}{36} && (28)
\end{aligned}$$