Assignment 2
AI1110: Probability and Random Variables

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12.13.4.5 : Question. Find the probability distribution of number of successes in two tosses of die,where a success is defined as

1) number greater than 4
2) six appears on atleast one die

Ans:

1)

X	0	1	2
$\Pr(X)$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

2)

Y	0	1
$\Pr(Y)$	$\frac{25}{36}$	$\frac{11}{36}$

Solution:

1) finding probability distribution for appearance of number greater than 4
let X :appearance of number greater than 4 on 2 turns , X$\in \{0, 1, 2\}$
$\Pr(X = x)$: probability of X becoming x , x $\in \{0, 1, 2\}$
p denotes probability that number greater that 4 appears,$X : bin(n, p)$

$$p = \frac{2}{6}(\text{as there are 2 numbers greater than 4 as outcome of die}) \tag{1}$$

Using binomial distribution,X:$bin(2, \frac{1}{3})$

$$\Pr(X = i) = \binom{n}{i} \times (p)^i \times (1 - p)^{n-i} \tag{2}$$

$$\text{here } n = 2, p = \frac{1}{3}(\text{from (1)}) \tag{3}$$

$$\Pr(X = i) = \binom{2}{i} \times \left(\frac{1}{3}\right)^i \times \left(1 - \frac{1}{3}\right)^{2-i} \tag{4}$$

variable	n	p	$\Pr(X = i)$
X	2	$\frac{1}{3}$	$\binom{2}{i} \times \left(\frac{1}{3}\right)^i \times \left(\frac{2}{3}\right)^{2-i}$

X=i	$\Pr(X=i)= \binom{2}{i} \times \left(\frac{1}{3}\right)^{i} \times \left(\frac{2}{3}\right)^{2-i}$	$\Pr(X=i)$
0	$\frac{2!}{(0!)\times((2-0)!)} \times \left(\frac{1}{3}\right)^{0} \times \left(\frac{2}{3}\right)^{2-0} = \frac{2}{(1)\times(2)} \times (1) \times \left(\frac{2}{3}\right)^{2}$	$\frac{4}{9}$
1	$\frac{2!}{1!\times(2-1)!} \times \left(\frac{1}{3}\right)^{1} \times \left(\frac{2}{3}\right)^{2-1} = \frac{2}{(1)\times(2)} \times (1) \times \left(\frac{2}{3}\right)^{2}$	$\frac{4}{9}$
2	$\frac{2!}{2!\times(2-2)!} \times \left(\frac{1}{3}\right)^{2} \times \left(\frac{2}{3}\right)^{2-2} = \frac{2}{(2)\times(1)} \times \left(\frac{1}{3}\right)^{2} \times (1)$	$\frac{1}{9}$

2) finding probability distribution for six to appear alteast on one die
let Y : appearence of six on atleast on die , Y$\in \{0,1\}$
$\Pr(Y=y)$: probability of Y becoming y , y $\in \{0,1\}$
now,U : appearance of six on die , U$\in \{0,1,2\}$
$\Pr(U=u)$: probability of u number of 6s appear , u$\in \{0,1,2\}$
p denotes the probability of 6 on one throw of die.$U : bin(n,p)$

$$p = \frac{1}{6} \tag{5}$$

using binomial distribution,$U : bin(2,\frac{1}{6})$

$$\Pr(U=i) = \binom{n}{i} \times (p)^{i} \times (1-p)^{n-i} \tag{6}$$

variable	n	p	$\Pr(U=i)$
X	2	$\frac{1}{6}$	$\binom{2}{i} \times \left(\frac{1}{6}\right)^{i} \times \left(\frac{5}{6}\right)^{2-i}$

U=i	$\Pr(U=i)= \binom{2}{i} \times \left(\frac{1}{6}\right)^{i} \times \left(\frac{5}{6}\right)^{2-i}$	$\Pr(U=i)$
0	$\frac{2!}{(0!)\times((2-0)!)} \times \left(\frac{1}{6}\right)^{0} \times \left(\frac{5}{6}\right)^{2-0} E = \frac{2}{1\times2}(1) \times \left(\frac{5}{6}\right)^{2}$	$\frac{25}{36}$
1	$\frac{2!}{1!\times(2-1)!} \times \left(\frac{1}{6}\right)^{1} \times \left(\frac{5}{6}\right)^{2-1} = \frac{2}{(1)\times(2)} \times (1) \times \left(\frac{5}{6}\right)^{2}$	$\frac{10}{36}$
2	$\frac{2!}{2!\times(2-2)!} \times \left(\frac{1}{6}\right)^{2} \times \left(\frac{5}{6}\right)^{2-2} = \frac{2}{(2)\times(1)} \times \left(\frac{1}{6}\right)^{2} \times (1)$	$\frac{1}{36}$

now to find $\Pr(Y=i)$

i	$\Pr(Y=i)$ interms of $\Pr(U=i)$	$\Pr(Y=i)$
0	$\Pr(U=0)$	$\frac{25}{36}$
1	$p_U(1) = \Pr(U=1) + \Pr(U=2)$	$\frac{10}{36} + \frac{1}{36} = \frac{11}{36}$