RED-BLACK TREE

CSX3003 DATA STRUCTURE AND ALGORITHMS

PROPERTIES

red–black tree

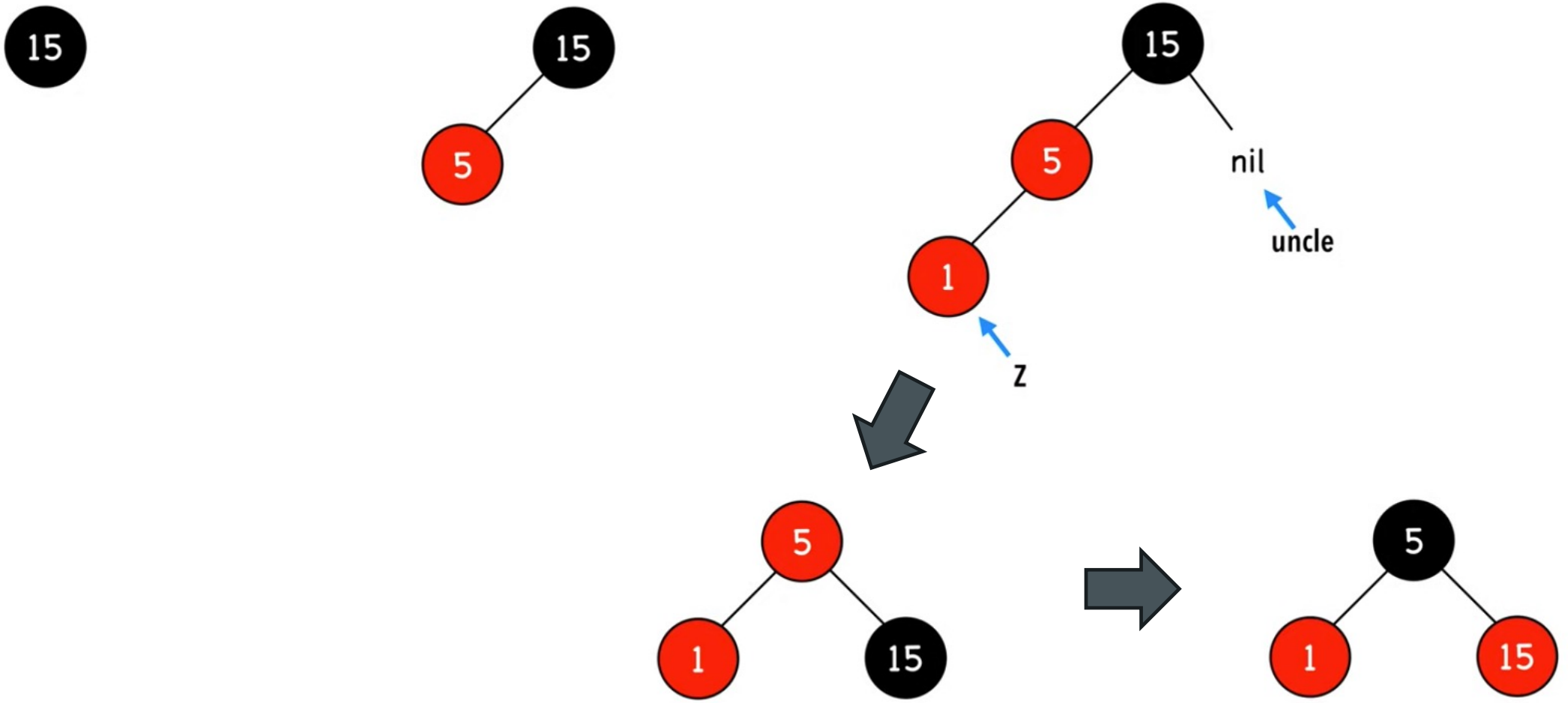
1. A node is either red or black.
2. The root and leaves (NIL) are black.
3. If a node is red, then its children are black.
4. All paths from a node to its NIL descendants contain the same number of black nodes.

INSERTION

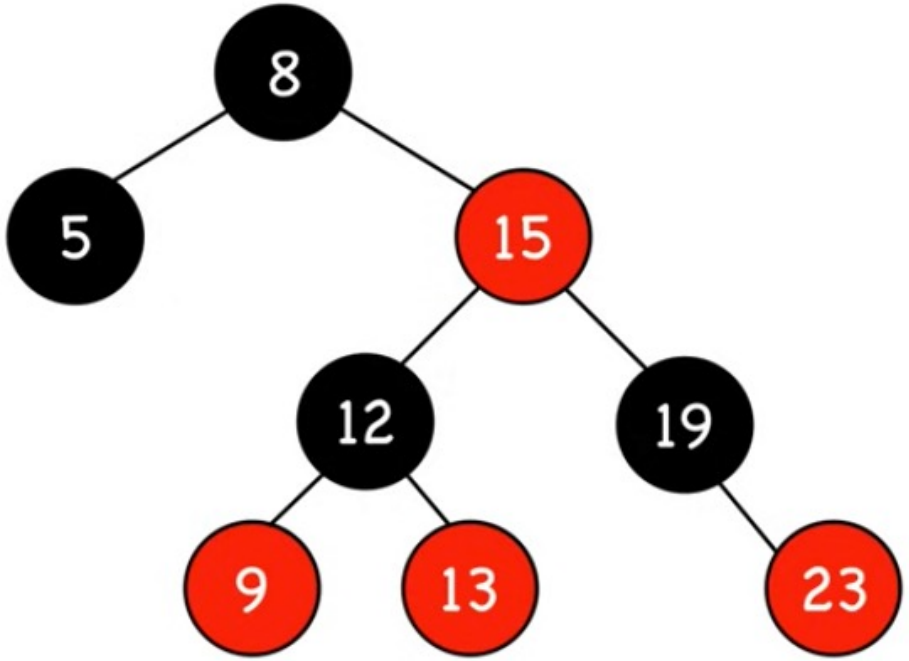
4 scenarios

0. Z = root –> ***color black***
1. Z.uncle = red –> ***recolor***
2. Z.uncle = black (triangle) –> ***rotate Z.parent***
3. Z.uncle = black (line) –> ***rotate Z.grandparent*** *& recolor*

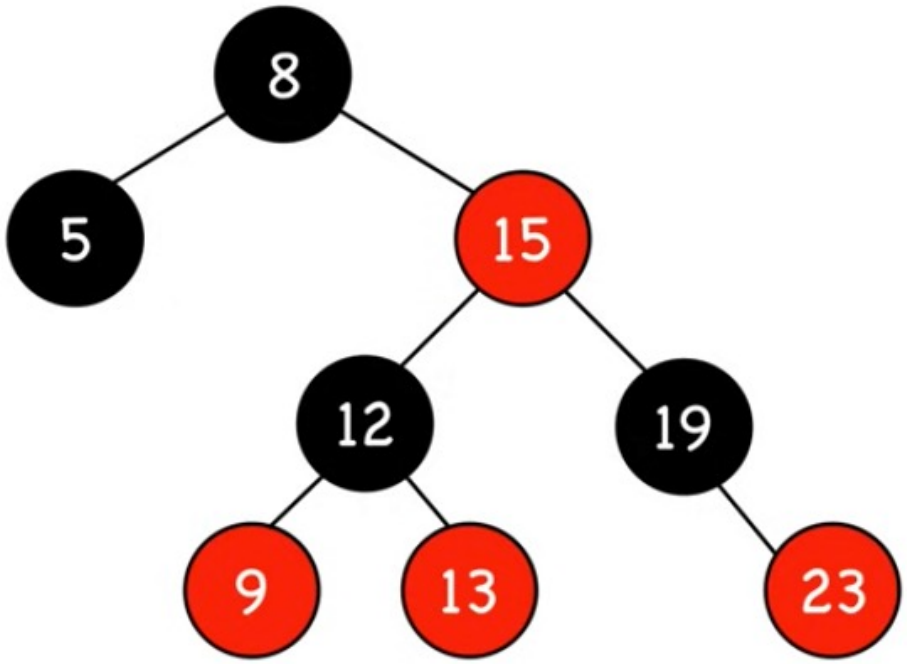
EXAMPLE 1: INSERT 15, 5, 1



EXAMPLE 2: INSERT 10

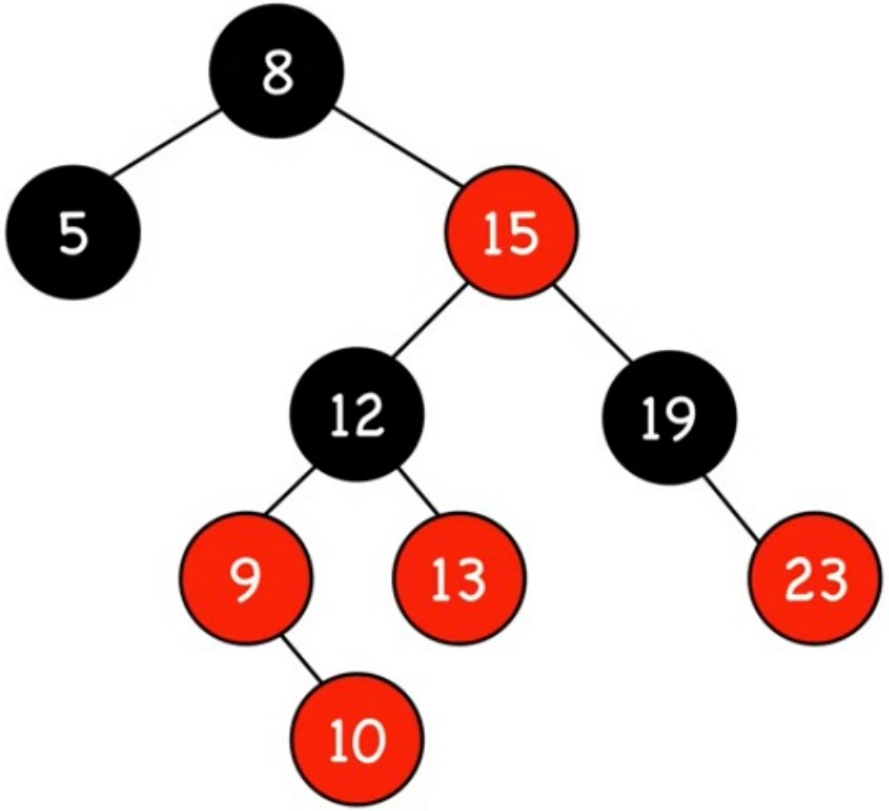


STEP 1: INSERT 10 AND COLOR THE NODE RED

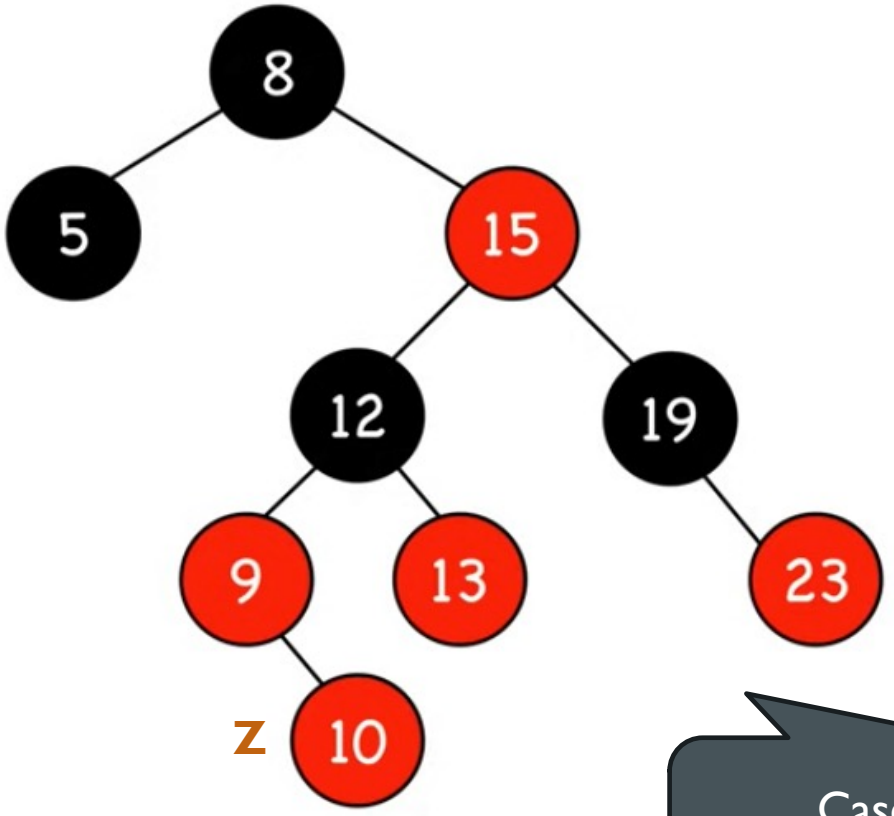


steps

1. Insert **10** and color it red



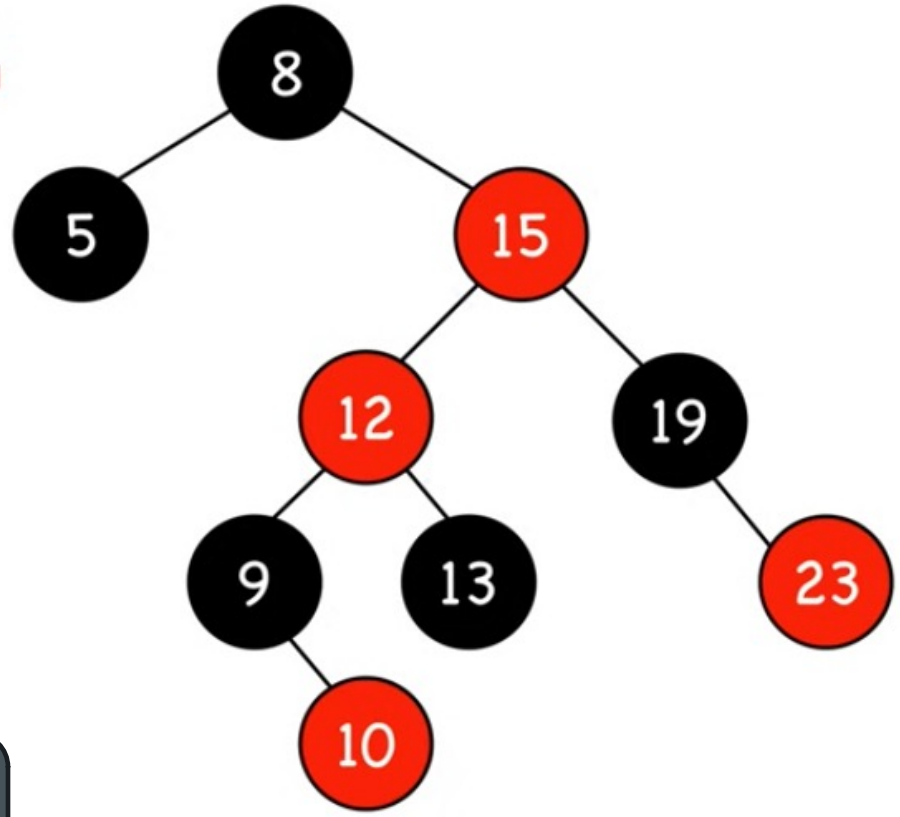
STEP 2: RECOLOR 9 & 13 BLACK AND 12 RED



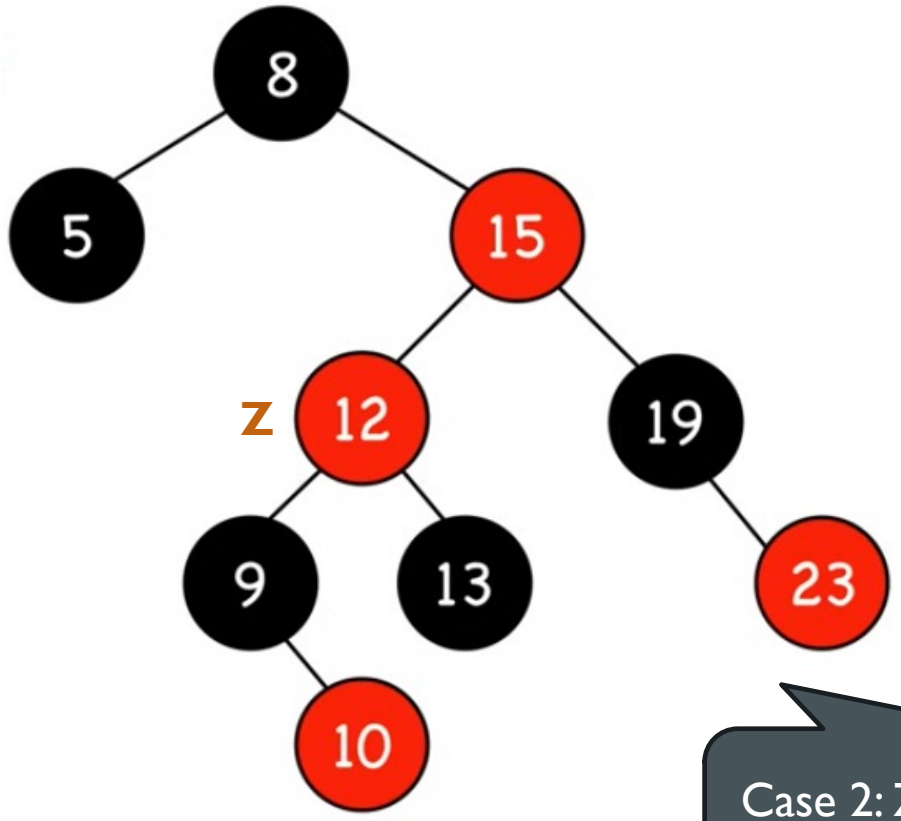
steps

1. Insert **10** and color it red
2. Recolor **9** & **13** black and **12** red

Case 1: Z.uncle = red
→ recolor

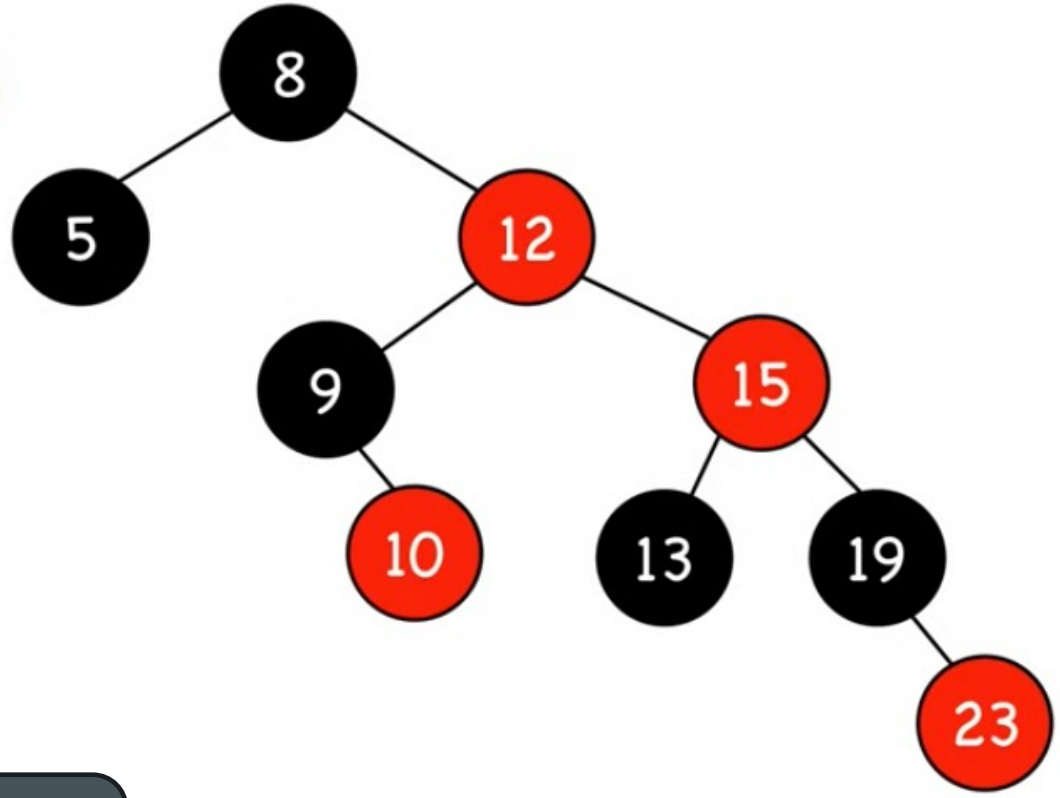


STEP 3: RIGHT ROTATE ON 15



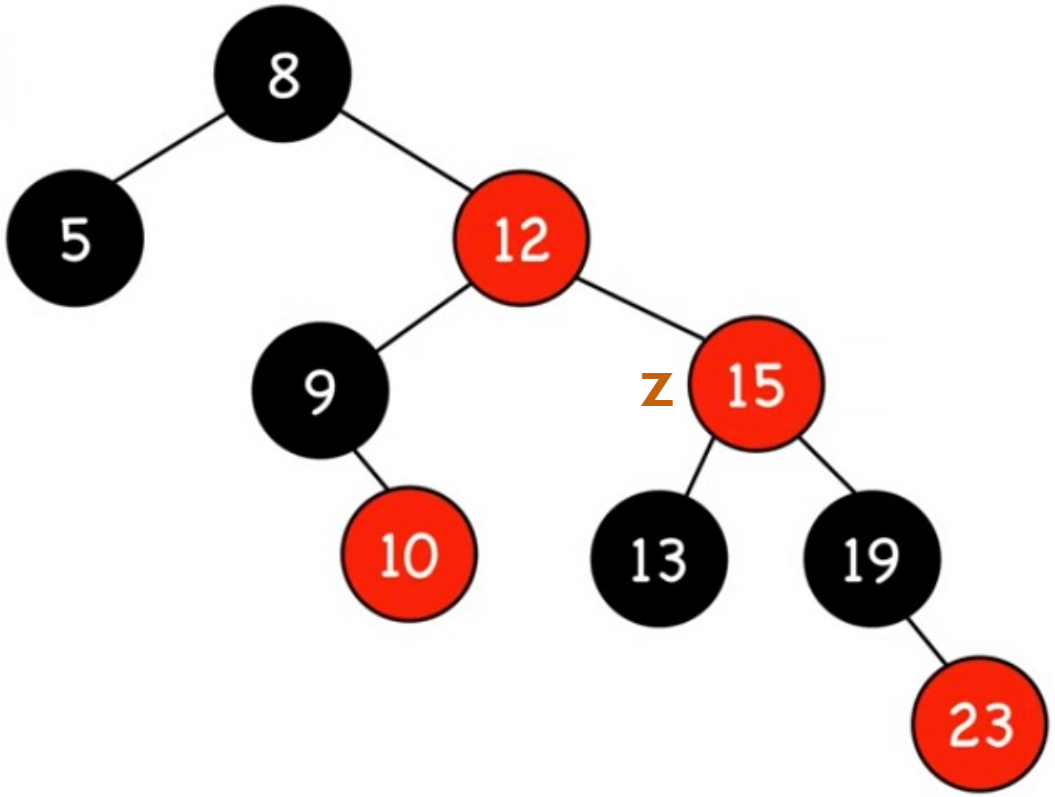
steps

1. Insert **10** and color it red
2. Recolor **9** & **13** black and **12** red
3. Right-rotate on **15**



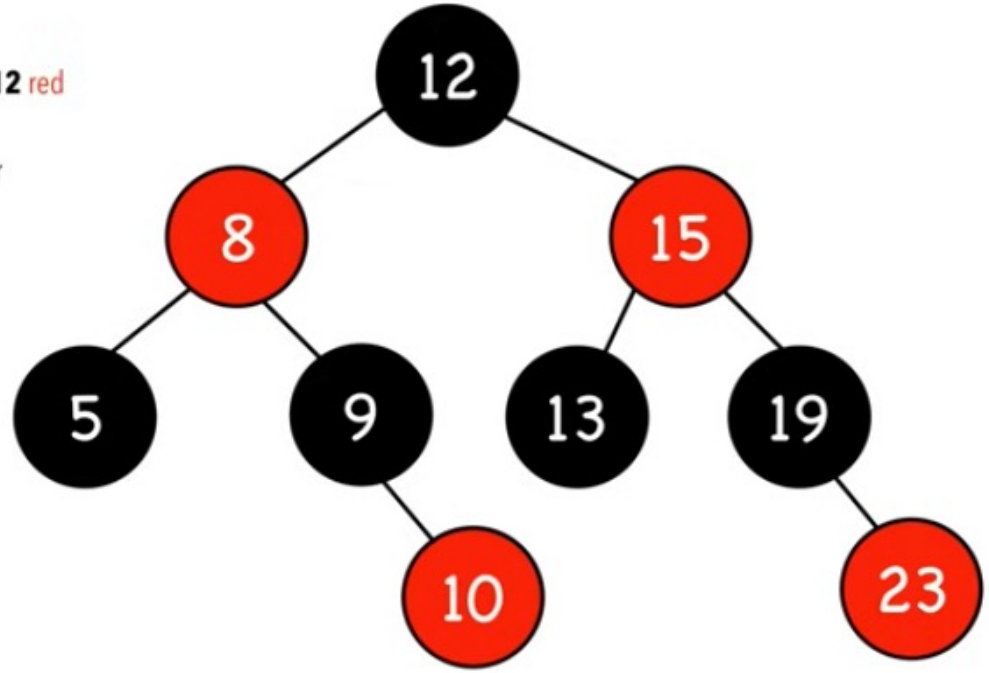
Case 2: Z.uncle = black (triangle)
→ rotate Z.parent

STEP 4: LEFT ROTATE ON 8 AND RECOLOR



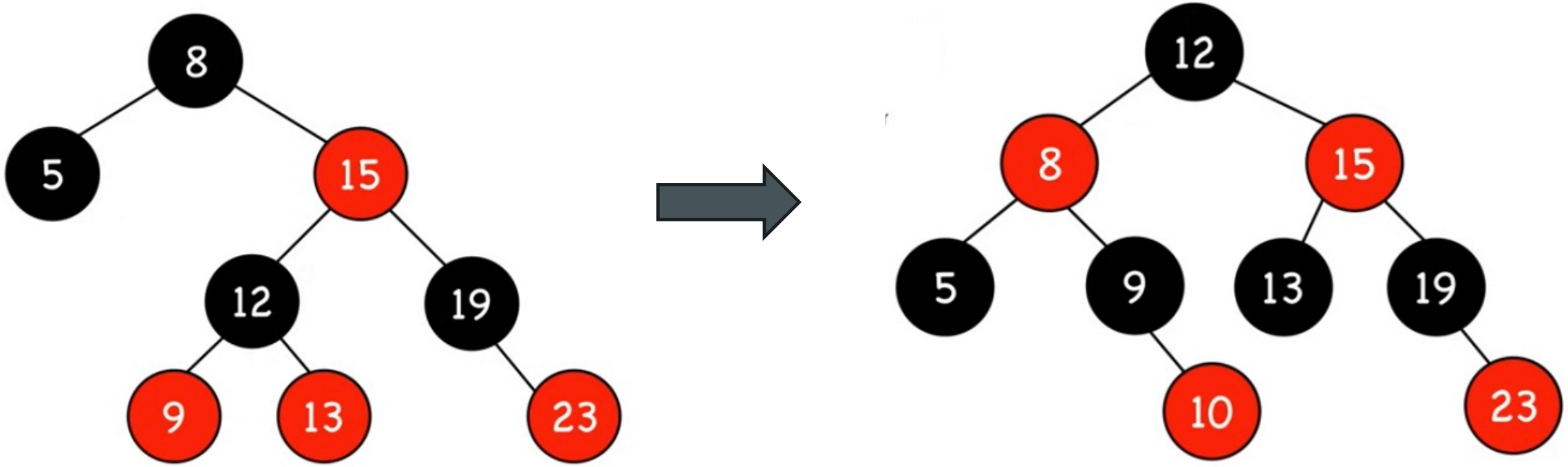
steps

1. Insert **10** and color it red
2. Recolor **9** & **13** black and **12** red
3. Right-rotate on **15**
4. Left-rotate on **8** and recolor



Case 3: Z.uncle = black (line)
→ rotate Z.grandparent
& recolor

FINAL TREE: AFTER INSERTING 10



DELETION

- transplant
  - Move subtree
- delete
  - Delete the node
- delete_fixup
  - Fix red-black violations

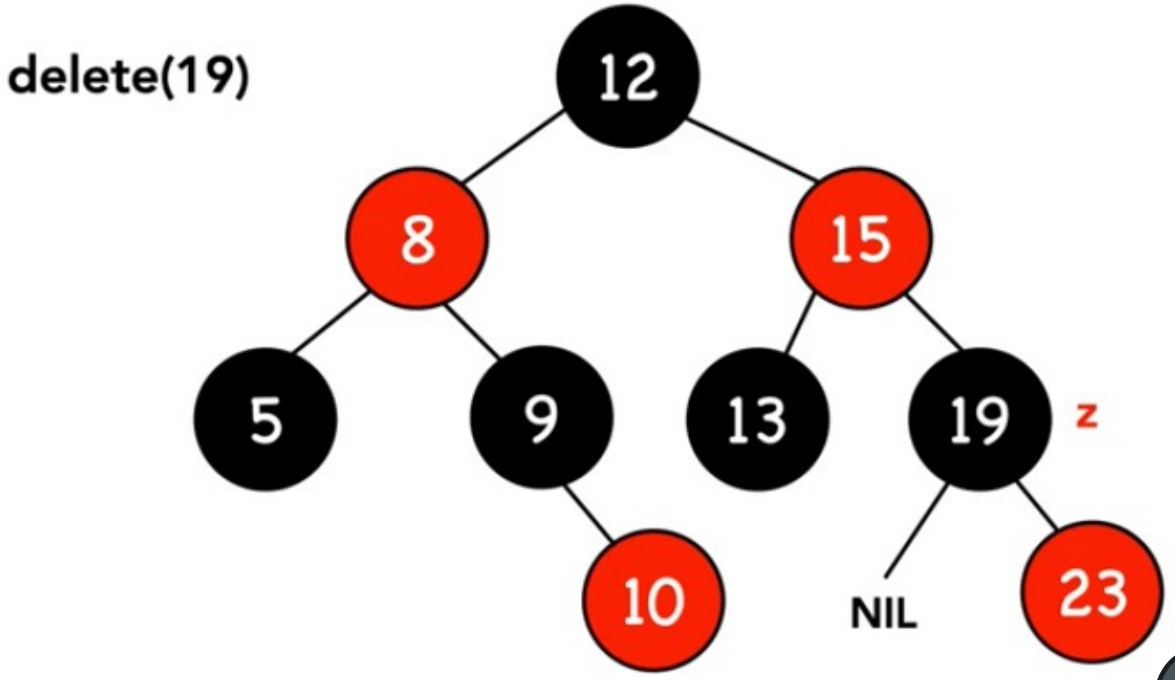
DELETION

3 scenarios

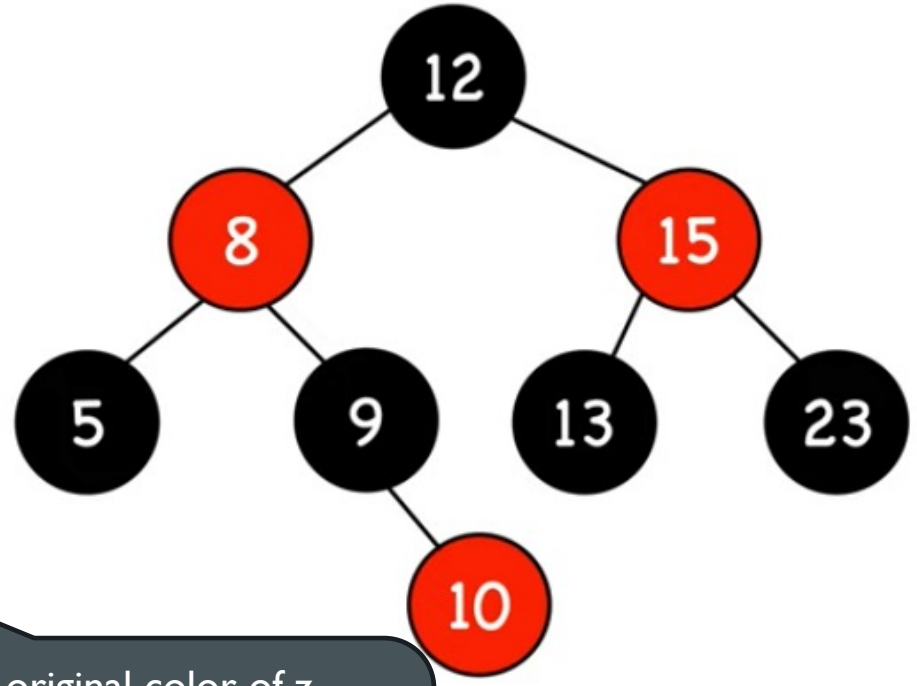
1. **Left** child is NIL
   - save the original color of z
   - transplant(z, x)
   - delete_fixup(x)
   - recolor x with z's original color
2. **Right** child is NIL

3. **Neither** is NIL

EXAMPLE: DELETE 19

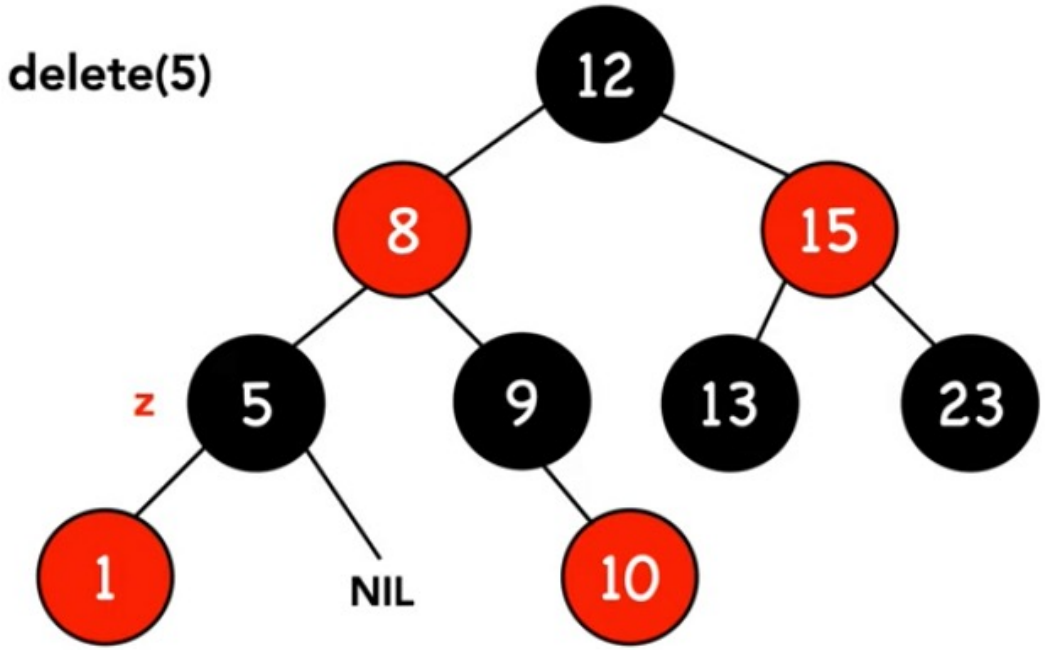


Case 1: Left child is NIL

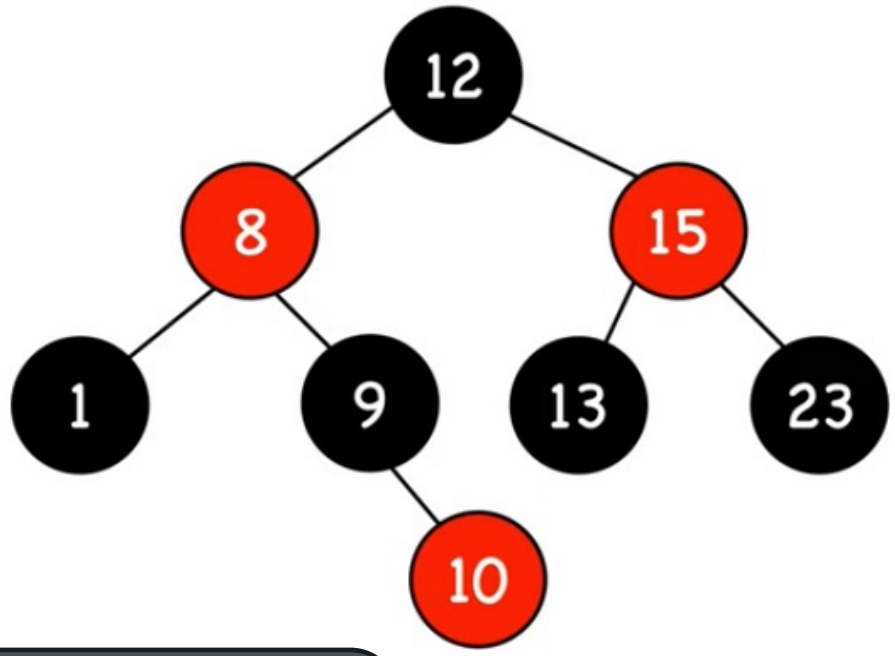


1. save the original color of z
2. x = z.right
3. transplant(z, x)
4. delete_fixup(x)
5. recolor x with z's original color

EXAMPLE: DELETE 5

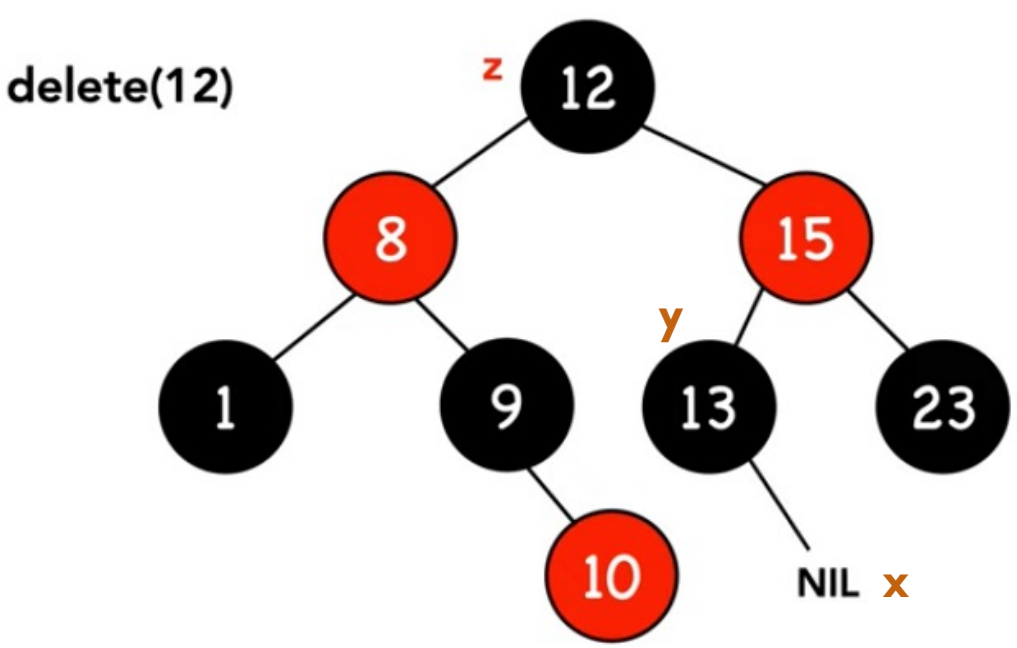


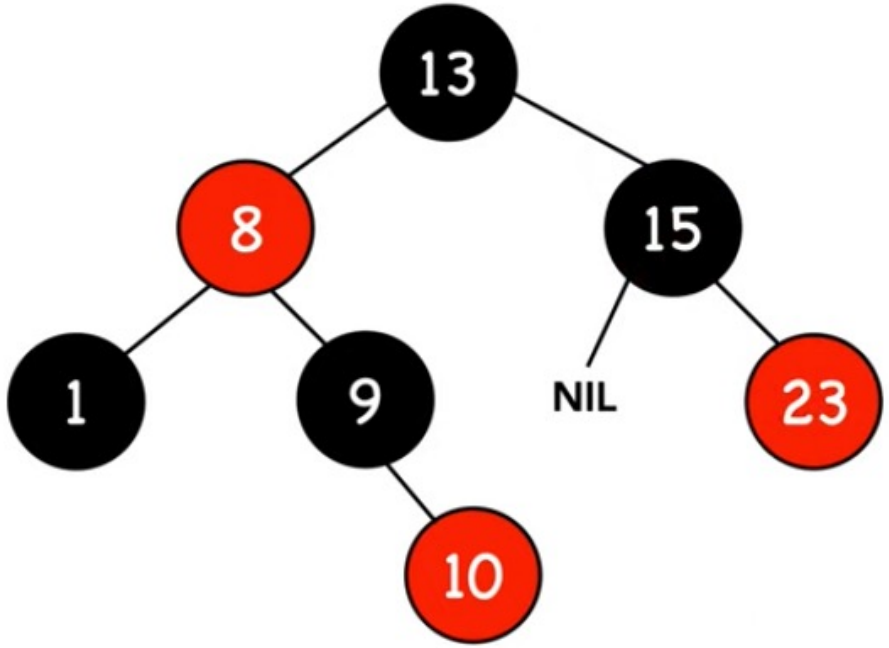
Case 2: Right child is NIL



1. save the original color of z
2. x = z.left
3. transplant(z, x)
4. delete_fixup(x)
5. recolor x with z's original color

EXAMPLE: DELETE 12





Case 3: Neither is NIL