Spla: Open-Source Generalized Sparse Linear Algebra Framework with Vendor-Agnostic GPUs Accelerated Computations

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ABSTRACT

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Scalable high-performance graph analysis is an actual nontrivial challenge. Usage of sparse linear algebra operations as building blocks for graph analysis algorithms, which is a core idea of Graph-BLAS standard, is a promising way to attack it. While it is known that sparse linear algebra operations can be efficiently implemented on GPGPU, full GraphBLAS implementation on GPGPU is a nontrivial task that is almost solved by GraphBLAST project. Though it is shown that utilization of GPGPUs for GraphBLAS implementation significantly improves performance, portability and scalability problems are not solved yet: GraphBLAST uses Nvidia stack and utilizes only one GPGPU. In this work we propose a Spla library that aimed to solve these problems: it uses OpenCL to be portable and designed to utilize multiple GPGPUs. Preliminary evaluation shows that while further optimizations are required, the proposed solution demonstrates performance comparable with GraphBLAST on some tasks. Moreover, our solution on embedded GPU outperforms SuiteSparse:GrpaphBLAS on the respective CPU on some graph analysis tasks.

CCS CONCEPTS

• Mathematics of computing → Graph algorithms; • Computing methodologies → Parallel algorithms; • Computer systems organization

Single instruction, multiple data.

KEYWORDS

graphs, algorithms, graph analysis, sparse linear algebra, Graph-BLAS, GPGPU, OpenCL

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1 INTRODUCTION

Scalable high-performance graph analysis is an actual challenge. There is a big number of ways to attack this challenge [1] and the first promising idea is to utilize general-purpose graphic processing units (GPGPU). Such existing solutions, as CuSha [5] and

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Gunrock [6] show that utilization of GPUs can improve the performance of graph analysis, moreover it is shown that solutions may be scaled to multi-GPU systems. But low flexibility and high complexity of API are problems of these solutions.

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The second promising thing which provides a user-friendly API for high-performance graph analysis algorithms creation is a Graph-BLAS API [4] which provides linear algebra based building blocks to create graph analysis algorithms. The idea of GraphBLAS is based on a well-known fact that linear algebra operations can be efficiently implemented on parallel hardware. Along with that, a graph can be natively represented using matrices: adjacency matrix, incidence matrix, etc. While reference CPU-based implementation of GraphBLAS, SuiteSparse:GraphBLAS [2], demonstrates good performance in real-world tasks, GPU-based implementation is challenging.

One of the challenges in this way is that real data are often sparse, thus underlying matrices and vectors are also sparse, and, as a result, classical dense data structures and respective algorithms are inefficient. So, it is necessary to use advanced data structures and procedures to implement sparse linear algebra, but the efficient implementation of them on GPU is hard due to the irregularity of workload and data access patterns. Though such well-known libraries as cuSPARSE show that sparse linear algebra operations can be efficiently implemented for GPGPU, it is not so trivial to implement GraphBLAS on GPGPU. First of all, it requires generic sparse linear algebra, thus it is impossible just to reuse existing libraries which are almost all specified for operations over floats. The second problem is specific optimizations, such as masking fusion, which can not be natively implemented on top of existing kernels. Nevertheless, there is a number of implementations of GraphBLAS on GPGPU, such as GraphBLAST [10], GBTL [12], which show that GPGPUs utilization can improve the performance of GraphBLAS-based graph analysis solutions. But these solutions are not portable because they are based on Nvidia Cuda stack.

GraphBLAS design issues!!!

To provide portable GPU implementation of GraphBLAS API we developed a SPLA library¹. This library utilizes OpenCL for GPGPU computing to be portable across devices of different vendors. !!!! To sum up, the contribution of this work is the following.

- Design of portable GPU GraphBLAS-like API proposed. Solves some desingn problems !!! Additionally, the proposed design is aimed to simplify library tuning and wrappers for different high-level platforms and languages creation.
- Implemented. !!!!
- Evaluation on such algorithms as breadth-first search (BFS), single source shortest path (SSSP), page rank (PG), and triangles counting (TC), and real-world graphs shows portability across different vendors and promising performance: !!!. Surprisingly, for some problems, the proposed solution

¹Source code available at: https://github.com/SparseLinearAlgebra/spla

on embedded Intel graphic card shows better performance than SuiteSparse:GraphBLAS on the respective CPU. At the same time, the evaluation shows that further optimization is required.

2 BACKGROUND OF STUDY

2.1 Related Work

Related work, existing solutions, systems.

2.2 GraphBLAS

GrpahBLAS API. Discussion of drawbacks of current design and implementation.

2.3 GPU computations

Technologies, problems, challenges, architectures.

3 PROPOSED SOLUTION DESCRIPTION

This section describes the high-level details of the proposed solution. It highlights the design principles, high-level architecture of the solution, data storage representation, operations, and also shows differences from the GraphBLAS API.

3.1 Design Principles

Spla library is designed the way to maximize potential library performance, simplify its implementation and extensions, and to provide the end-user verbose, but expressive interface allowing customization and precise control over operations execution. These ideas are captured in the following principles.

- Optional acceleration. Library is designed in a way, that GPU
 acceleration is fully plugable and optional part. Library can
 perform computations using standard CPU pipeline. If GPU
 acceleration is presented, library can offload a part of a work
 for it. It allows both no-trivial processing of the data on
 the CPU only, as well as possibility to integrate different
 backends in the future.
- User-defined functions. The user can create custom elementby-element functions to parameterize operations. Custom functions can be used for both CPU and GPU execution.
- Predefined scalar data types. The library provides a set
 of built-in scalar data types that have a natural one-toone relationship with native GPU built-in types. Data storage is transparent. The library interprets the data as PODstructures. The user can interpret individual elements as a
 sequence of bytes of a fixed size.
- Hybrid-storage format. The library automates the process of data storage and preprocessing. It supports several data formats, chooses the best one depending on the situation.
- Exportable interface. The library has a C++ interface with an automated reference-counting and with no-templates usage. It can be wrapped by C99 compatible API and exported to other languages, for example, in a form of a Python package.

3.2 Architecture Overview

The general idea of the proposed solution is depicted in Fig. 1. The core of the library and its main part is the CPU, which is the master

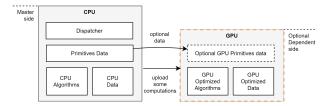


Figure 1: Proposed solution general design idea.

node and controls all calculations. It is responsible for storing data, maintaining a registry with algorithms, and scheduling operations to perform. In this paradigm, the GPU is an optional backend for acceleration, implemented through a special interface. It can optionally store data in a specific format. The CPU can offload the calculation of a part of the operations to the GPU, if the corresponding operation is supported by the given accelerator.

The reason for this is that the CPU and GPU are inherently asymmetric in nature. The end-user uses CPU side API. Thus, some preprocessing on the CPU side must be always done in the majority of cases. In addition, access to data on the GPU and their storage is carried out differently due to the peculiarities of the execution of kernels. Also, VRAM is more expensive and has less capacity than RAM. Therefore, RAM is a cache for VRAM, and data duplication can be neglected. In the end, the explicit separation of the CPU side from the GPU of the backend gives the modularity. This can be used not only to support different GPU technologies, but also to integrate multiple GPUs or distributed processing in the future.

3.3 Data Containers

Library provides general *M-by-N Matrix*, *N Vector* and *Scalar* data containers. Underlying primitive scalar types are specified by *Type* object. Single vector or matrix data is stored in specialized multiformat storage container. An example of the single vector storage is depicted in Fig. 2.

The storage is responsible for keeping data in multiple different formats at the same time. Each format is best suited for a specific type of task and requested on demand. Key-value dictionary suites well frequent insertion, query or deletion operations, when memory usage and response time are critical. Mathematical operations perform better with compacted sequential lists of values since they have more friendly cache behaviour. GPU operations require separate format with a copy of the data resident in VRAM.

Data transformation from one format to another is carried out using a special rules graph shown in Fig. 3. The directed edges in this graph indicate the conversion rule. The graph must the single strongly connected component. An example of the data transformation process is depicted in Fig. 4. For a requested format the best path of convertation is obtained. Currently, the shortest one is used. Weight assignment to rules can potentially be used to prioritize convertations and reduce the cost of transformation for some formats.

Currently, several storage formats are supported. There is dictionary of keys for vector and matrix (DoK), list of coordinates (COO), dense vector, list of lists (LIL) and compressed sparse rows (CSR) matrix formats. Other formats, such as CSC, DCSR, ELL, etc., can be

added to the library by the implementation of formats conversion and by the specialization of operations for a specific format.

3.4 Algebraic Operations

Library provides a number of commonly used operations, such as *vxm*, *mxv*, *mxmT*, *element-wise add*, *assign*, *map*, *reduce*, etc. Other operations can be added on demand. Interface of operations is inspired by GraphBLAS standard. It supports *masking*, parametrization by *binary mult* and *binary add* functions, *select* for filtering and mask application, *unary op* for values transformation, and *descriptor* object for additional operation tweaking.

3.5 Differences with GraphBLAS standard

To be clear, the proposed solution is not an implementation of GraphBLAS C or C++ API. The design of the library uses only the concepts described in the standard. However, in the proposed solution, the signatures and semantics of some of the operations have been changed. The API has been made more verbose and explicit. In particular, the handling of *zero* elements and *masking* are made cleaner for the end user. The library interprets data simply

1. Vec Dok	2. Vec Coo	3. Vec Dense (cpu)	4. Vec Coo	5. Vec Dense
(cpu)	(cpu)		(cl)	(cl)
Fast insertion and query with no memory overhead	with no memory			Simple memory management, fast query, no sparsity, accelerated

Figure 2: Vector primitive storage. Stores the same data potentially in multiple different formats. Some slots can be empty.

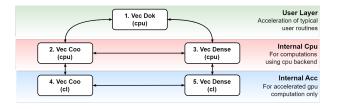


Figure 3: Vector storage transformation graph. The graph defines how data can be obtained from one format in another.

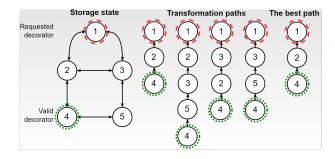


Figure 4: Vector storage transformation process. Green is valid format. Red is requested format. No highlight is currently invalid format.

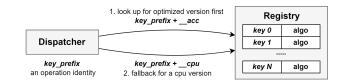


Figure 5: Registry of operation implementations. Keys with special syntax used to fetch required operation in a specific order at runtime.

as collections of bytes, without mathematical semantics and identity elements. Identity element must be explicitly passed by the user where required. It allows to make the memory usage predictable and the result of each operation clear to the end user without internal implicit storage manipulations.

4 IMPLEMENTATION DETAILS

This section describes implementation details of the proposed solution. It highlights key aspects of the core implementation, OpenCL specifics, optimization of particular operations, and high-level optimizations of graph algorithms.

4.1 Core

The implemented library uses the concept of a registry to find operations as shown in Fig.5. A call to a particular operation is stored as a command to be executed later by *Dispatcher*. For each command the special lightweight string key is built depending on type of the operation and arguments passed. This key is used as a regex to get the required implementation of the requested operation. The advantages of the proposed approach are listed below.

- Late binding. The operation call becomes a command. The
 processing of such a command can be configured at run
 time. Changing the acceleration backend can be done without recompilation. Moreover, several backends can be transparently used within a single application.
- Optionality of accelerator. The acceleration backend is free to support only those operations that require it. Fallback implementations will be used automatically for the rest of the operations.
- *Performance tuning.* The key of the command reflects operation type, arguments types, passed user functions types, etc. It can be for *ad-hoc* optimizations. Custom operation implementation with a verbose key can be also stored in the registry. If several operations match the key, the longest key is used, since it is more specific for a particular operation.
- Scheduling. The full list of submitted commands for execution can be examined at runtime. This opens up the possibility for the fusion of some operations, sorting, rearrangement, and any other high-level optimizations that require introspection.

4.2 OpenCL

OpenCL 1.2 is used as the primary API for backend GPU implementation. Header files with C and C++ definitions are supplied with the source code of the project. Official Khronos installable client

driver (ICD) loader bundled within a library to load at runtime particular OpenCL implementation depending on running OS and GPU vendor.

Implementation of sparse linear algebra algorithms for a GPU requires auxiliary libraries for memory management, sorting, reducing, merging, scanning, etc. Nvidia Cuda platform features libraries such as Thrust and Cub. OpenCL lacks such support. All primitives must be implement from a scratch in most cases. What is an extra challenge. Third-party library, such as Boost Compute [7], cannot be used, since it has significant runtime overhead, portability and performance issues, and lack of long term support.

User-defined functions for GPU usage are represented as strings with additional metadata, such as type of parameters, return types, unique id, etc. Source code of particular operations stored in a form of .cl files. Operations implemented with generalization for parameters types and user functions. Their definitions obtained later at runtime in a compilation step through the text pre-processing. Compilation of actual OpenCL kernels is done on demand. All compiled kernels are stored in a cache. Cache key is composed from types of kernel parameters, defines, etc., which identify uniquely a particular variation of a kernel. Key composition is done in O(1). In-place allocation is utilized for a key builder to avoid global heap usage. In order to reduce CPU overhead and keep access to the cache fast, library uses robin hood hashing based hash map.

Custom linear memory allocator implemented in order to reduce the overhead of frequent and small buffer allocations, arising in a time of execution of some operations. Allocator uses sub-buffer mechanism and serves request typically less than 1 MB of size. Otherwise, the general GPU heap is used.

4.3 Linear Algebra Operations

The following primitives are the core of computations: *masked* sparse-vector sparse-matrix product, masked sparse-matrix dense-vector product and masked sparse-matrix sparse-matrix product. Efficient implementation and load balancing of those operations dominate the performance of particular algorithms. The following paragraphs give an insight into these operations implementation in the library.

Masked sparse-vector sparse-matrix product. The implementation is based on the algorithm proposed by Yang et al. [11]. It is a k-way merge based algorithm which suites well for sparse vectors. Our implementation uses custom gather to collect temporary products. Radix sort used to sort products for further reduction. Reduction by key uses parallel prefix scan to carry out final destination of reduced values.

Masked sparse-matrix dense-vector product. The implementation of this operation relies on a classic row-based parallel algorithm. Both scalar and vector versions are implemented to fit better relatively sparse and dense matrix rows.

Masked sparse-matrix sparse-matrix product. The implementation of this algorithm uses the approach proposed by Yang et al. [10]. It is straightforward single-pass row-based and column-based matrices product. Mask is used to estimate the size of the final result to filter out some result of the product.

4.4 Graph Algorithms

The advantage of the linear algebra approach is that graph algorithms can be easily composed of primitive operations using a few lines of code. For preliminary study breadth-first search (BFS), single-source shortest paths (SSSP), page rank (PR) and triangles counting (TC) algorithms were chosen. These are the most commonly evaluated graph algorithms. They allow to test basic operations and key aspects of graph frameworks performance. Implementation details for chosen algorithms are given below.

BFS. It utilizes a number of optimizations described by Yang et al. [9]. It uses masking to filter out already reached vertices, change of direction to switch from sparse from to dense and vice versa, and early exit in mxv operation.

SSSP. This algorithm uses change of direction as well. Also, it employs filtering of unproductive vertices according to Yang et al. [10]. Vertices which do not relax their distance in current iteration are removed from a front of the search. It allows to keep workload moderate.

PR. This algorithm assigns numerical weights to objects in the network depending on their relative relevance. As a key operation it uses mxv operation with a dense vector. For error estimation it uses custom element-wise function with a fusion of subtraction and square operations.

TC. Triangles counting uses masked sparse matrix product [10] and reduction. As an input algorithm accepts a lower triangular component L of an adjacency matrix of the source graph. The result is a count of non-zero values from $B = LL^T \cdot *L$, where $\cdot **$ used for the masking. The second argument is not actually transposed, since row-column based product gives exactly the required effect.

5 EVALUATION

For performance analysis of the proposed solution, we evaluated a few most common graph algorithms using real-world sparse matrix data. As a baseline for comparison we chose LAGraph [8] in connection with SuiteSparse [2] as a CPU tool, Gunrock [6] and GraphBLAST [10] as a Nvidia GPU tools. Also, we tested algorithms on several devices with distinct OpenCL vendors in order to validate the portability of the proposed solution. In general, these evaluation intentions are summarized in the following research questions.

- **RQ1** What is the performance of the proposed solution relative to existing tools for GPU analysis?
- **RQ2** What is the performance of the proposed solution on various devices vendors and OpenCL runtimes?
- **RQ3** What is the performance of the proposed solution on integrated GPU compared to existing CPU tool for analysis?

5.1 Evaluation Setup

For evaluation of RQ1, we use a PC with Ubuntu 20.04 installed, which has 3.40Hz Intel Core i7-6700 4-core CPU, DDR4 64Gb RAM, Intel HD Graphics 530 integrated GPU, and Nvidia GeForce GTX 1070 dedicated GPU with 8Gb on-board VRAM. For evaluation of RQ2, we use a PC with Ubuntu 22.04 installed, which has 4.70Hz AMD Ryzen 9 7900x 12-core CPU, DDR4 128 GB RAM, AMD GFX1036 integrated GPU, and either Intel Apc A770 flux dedicated GPU with 8GB on-board VRAM or AMD Radeon Vega

Frontier Edition dedicated GPU with 16GB on-board VRAM. For evaluation of RQ3, the first PC with Intel CPU and integrated GPU and the second PC with AMD CPU and integrated GPU are used.

Programs using CUDA were compiled with GCC v8.4.0 and Nvidia NVCC v10.1. Release mode and maximum optimization level were enabled for all tested programs. Data loading time, preparation, format transformations, and host-device initial communications are excluded from time measurements. All tests are averaged across 10 runs. The deviation of measurements does not exceed the threshold of 10 percent. Additional warm-up run for each test execution is excluded from measurements.

5.2 Graph Algorithms

For preliminary study breadth-first search (BFS), single-source shortest paths (SSSP), page rank (PR) and triangles counting (TC) algorithms were chosen. Implementation of those algorithms is used from official examples packages of tested libraries with default parameters. Compared tools are allowed to make any optimizations as long as the result remains correct. The graph vertex with index 1 is set as the initial traversal vertex in the algorithms BFS and SSSP for all tested instruments and all tested devices.

5.3 Dataset

Thirteen matrices with graph data were selected from the Sparse Matrix Collection at University of Florida [3]. Information about graphs is summarized in Table 1. The dataset is converted to undirected graphs. Self-loops and duplicated edges are removed. Average, sd and max metrics relate to out degree property of the vertices. For SSSP weights are initialized using pseudo-random generator with uniform [0, 1] distribution of floating-point values.

Graphs are roughly divided into two groups. The first group represents relatively dense graphs, where the number of edges per node is sufficient on average to effectively load the GPU with useful work. The second group represents relatively sparse graphs, where the average vertex degree is below the typical GPU vector register size, and the search depth reaches hundreds of hoops. Graphs are sorted in ascending order by the number of vertices within each group.

5.4 Results Summary

Table 2 presents results of the evaluation and compares the performance of Spla against other Nvidia GPU tools and uses as a baseline LaGraph CPU tool. Table 3 presents result of the portability analysis of the proposed solution. It depicts runtime timings of the proposed solution on discrete GPUs of distinct vendors. Cell left empty with *none* if tested tool failed to analyze graph due to *out of memory* exception.

RQ1. What is the performance of the proposed solution relative to existing tools for GPU analysis?

RQ2. What is the performance of the proposed solution on various devices vendors and OpenCL runtimes?

Table 1: Dataset description.

Graph	Vertices	Edges	Avg	Sd	Max
coAuthorsCit	227.3K	1.6M	7.2	10.6	1.4K
coPapersDBLP	540.5K	30.5M	56.4	66.2	3.3K
amazon2008	735.3K	7.0M	9.6	7.6	1.1K
hollywood2009	1.1M	112.8M	98.9	271.9	11.5K
comOrkut	3.1M	234.4M	76.3	154.8	33.3K
citPatents	3.8M	33.0M	8.8	10.5	793.0
socLiveJournal	4.8M	85.7M	17.7	52.0	20.3K
indochina2004	7.4M	302.0M	40.7	329.6	256.4K
belgiumosm	1.4M	3.1M	2.2	0.5	10.0
roadNetCA	2.0M	5.5M	2.8	1.0	12.0
rggn222s0	4.2M	60.7M	14.5	3.8	36.0
rggn223s0	8.4M	127.0M	15.1	3.9	40.0
roadcentral	14.1M	33.9M	2.4	0.9	8.0

RQ3. What is the performance of the proposed solution on integrated GPU compared to existing CPU tool for analysis?

6 CONCLUSION

We presented a generalized sparse linear algebra framework with vendor-agnostic GPUs accelerated computations.

REFERENCES

- Miguel E. Coimbra, Alexandre P. Francisco, and Luís Veiga. 2021. An analysis
 of the graph processing landscape. *Journal of Big Data* 8, 1 (April 2021). https://doi.org/10.1186/s40537-021-00443-9
- [2] Timothy A. Davis. 2019. Algorithm 1000: SuiteSparse:GraphBLAS: Graph Algorithms in the Language of Sparse Linear Algebra. ACM Trans. Math. Softw. 45, 4, Article 44 (Dec. 2019), 25 pages. https://doi.org/10.1145/3322125
- [3] Timothy A. Davis and Yifan Hu. 2011. The University of Florida Sparse Matrix Collection. ACM Trans. Math. Softw. 38, 1, Article 1 (dec 2011), 25 pages. https://doi.org/10.1145/2049662.2049663
- [4] Jeremy Kepner, Peter Aaltonen, David Bader, Aydin Buluç, Franz Franchetti, John Gilbert, Dylan Hutchison, Manoj Kumar, Andrew Lumsdaine, Henning Meyerhenke, Scott McMillan, Carl Yang, John D. Owens, Marcin Zalewski, Timothy Mattson, and Jose Moreira. 2016. Mathematical foundations of the Graph-BLAS. In 2016 IEEE High Performance Extreme Computing Conference (HPEC). 1–9. https://doi.org/10.1109/HPEC.2016.7761646
- [5] Farzad Khorasani, Keval Vora, Rajiv Gupta, and Laxmi N. Bhuyan. 2014. CuSha: Vertex-Centric Graph Processing on GPUs. In Proceedings of the 23rd International Symposium on High-Performance Parallel and Distributed Computing (Vancouver, BC, Canada) (HPDC '14). Association for Computing Machinery, New York, NY, USA, 239–252. https://doi.org/10.1145/2600212.2600227
- [6] Yuechao Pan, Yangzihao Wang, Yuduo Wu, Carl Yang, and John D. Owens. 2017. Multi-GPU Graph Analytics. In 2017 IEEE International Parallel and Distributed Processing Symposium (IPDPS). 479–490. https://doi.org/10.1109/IPDPS.2017.117
- [7] Jakub Szuppe. 2016. Boost.Compute: A Parallel Computing Library for C++ Based on OpenCL. In Proceedings of the 4th International Workshop on OpenCL (Vienna, Austria) (IWOCL '16). Association for Computing Machinery, New York, NY, USA, Article 15, 39 pages. https://doi.org/10.1145/2909437.2909454
- [8] Gábor Szárnyas, David A. Bader, Timothy A. Davis, James Kitchen, Timothy G. Mattson, Scott McMillan, and Erik Welch. 2021. LAGraph: Linear Algebra, Network Analysis Libraries, and the Study of Graph Algorithms. arXiv:2104.01661 [cs.MS]
- [9] Carl Yang, Aydin Buluc, and John D. Owens. 2018. Implementing Push-Pull Efficiently in GraphBLAS. https://doi.org/10.48550/ARXIV.1804.03327
- [10] Carl Yang, Aydin Buluc, and John D. Owens. 2019. GraphBLAST: A High-Performance Linear Algebra-based Graph Framework on the GPU. arXiv:1908.01407 [cs.DC]
- [11] Carl Yang, Yangzihao Wang, and John D. Owens. 2015. Fast Sparse Matrix and Sparse Vector Multiplication Algorithm on the GPU. In 2015 IEEE International Parallel and Distributed Processing Symposium Workshop. 841–847. https://doi. org/10.1109/IPDPSW.2015.77

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Table 2: Performance comparison of the proposed solution. Time in milliseconds (lower is better).

Dataset GB GR LG SP BFS coAuthorsCit 5.0 1.9 6.3 6.9 coPapersDBLP 19.9 4.5 18.0 11.5 amazon2008 8.3 3.3 20.4 8.1 hollywood2009 64.3 20.3 23.4 20.3 belgiumosm 200.6 84.4 138.0 181.2 roadNetCA116.3 32.4 168.2 101.7 205.0 comOrkut 53.2 none 40.6 citPatents 30.6 41.3 115.9 35.1 rggn222s0 367.3 95.9 1228.1 415.3 socLiveJournal 63.1 61.0 75.5 57.1 indochina2004 33.3 328.7 none 224.6 2790.0 rggn223s0 146.2 754.9 615.3 roadcentral 1383.4 243.8 1951.0 710.2 SSSP coAuthorsCit 14.7 2.1 38.9 10.3 coPapersDBLP 118.6 5.6 92.2 25.7 amazon2008 4.0 90.0 21.7 43.4 hollywood2009 404.3 227.7 57.5 24.6 belgiumosm 650.2 81.1 1359.8 240.9 roadNetCA 509.7 32.4 1149.3 147.9 comOrkut none 219.0 806.5 241.0 citPatents 226.9 49.8 468.5 129.3 rggn222s0 21737.8 101.9 4808.8 865.4 socLiveJournal 346.4 69.2 518.0 189.5 indochina2004 none 40.8 821.9 596.6 rggn223s0 59015.7 161.1 11149.9 1654.8 roadcentral 13724.8 267.0 25703.4 1094.3 PR coAuthorsCit10.0 24.3 3.2 1.6 coPapersDBLP 120.2 297.6 17.6 6.1 amazon2008 5.2 40.6 89.8 5.5 hollywood2009 62.9 559.5 1111.2 32.4 belgiumosm 22.9 9.4 4.4 167.6 roadNetCA 37.7 225.8 19.6 6.6 comOrkut 103.3 2333.6 5239.0 none citPatents 686.1 1487.038.3 27.0 rggn222s0 320.0 45.2 563.5 26.6 socLiveJournal none 445.9 2122.5 112.0 rggn223s0 none 662.7 1155.6 103.4 roadcentral none 408.8 2899.9 172.0 TC coAuthorsCit2.3 2.0 17.3 3.0 coPapersDBLP 105.2 5.3 520.8 128.4 amazon2008 11.2 3.9 73.9 10.8 roadNetCA 6.5 32.4 46.0 7.7 2522.0 comOrkut 1776.9 218.0 23103.8 citPatents 65.5 49.7 675.0 54.5 3886.7 437.8 socLiveJournal 504.3 69.2 rggn222s0 73.2 101.3 484.5 77.7 rggn223s0 151.4 158.9 1040.1 204.2 roadcentral 42.6 259.3 425.3 52.7

GraphBLAST (GB), Gunrock (GR), LaGraph (LG), Spla (SP).

Table 3: Portability of the proposed solution. Time in milliseconds (lower is better).

coPapersDBLP 10.8 14.9 11.5 amazon2008 12.3 12.6 8.1 hollywood2009 15.3 26.7 20.3 belgiumosm 627.5 292.4 181.2 roadNetCA 265.5 298.8 101.7 comOrkut 33.2 63.6 53.2 citPatents 21.0 30.3 35.1 rggn222s0 825.3 1259.7 415.3 socLiveJournal 43.0 85.8 57.1 indochina2004 220.6 573.4 328.7 roadcentral 1864.9 1680.8 710.2 SSSP coAuthorsCit 18.3 10.4 10.3 coAuthorsCit 18.3 10.4 10.3 coAuthorsCit 18.3 10.4 10.3 coAuthorsCit 18.3 10.4 10.3 coAuthorsCit 44.6 56.2 57.5 belgiumosm 1085.9 454.8	Dataset	Intel Arc	AMD Vega	Nvidia Gtx				
coPapersDBLP 10.8 14.9 11.5 amazon2008 12.3 12.6 8.1 hollywood2009 15.3 26.7 20.3 belgiumosm 627.5 292.4 181.2 roadNetCA 265.5 259.8 101.7 comOrkut 33.2 63.6 53.2 citPatents 21.0 30.3 35.1 rggn222s0 825.3 1259.7 415.3 socLiveJournal 43.0 85.8 57.1 indochina2004 220.6 573.4 328.7 rgan223s0 1245.5 2519.6 754.9 roadcentral 1864.9 1680.8 710.2 22.9 27.7 25.7 amazon2008 23.4 22.2 21.7 amazon2008 23.4 22.2 21.7 amazon2008 23.4 22.2 21.7 belgiumosm 1085.9 454.8 240.5 roadNetCA 447.3 422.5		BF	S					
amazon2008 12.3 12.6 8.1 hollywood2009 15.3 26.7 20.3 belgiumosm 627.5 292.4 181.2 roadNetCA 265.5 259.8 101.7 comOrkut 33.2 63.6 53.2 citPatents 21.0 30.3 35.1 rggn222s0 825.3 1259.7 415.3 socLiveJournal 43.0 85.8 57.1 indochina2004 220.6 573.4 328.7 rggn223s0 1245.5 2519.6 754.9 roadcentral 1864.9 1680.8 710.2 SSSP coAuthorsCit 18.3 10.4 10.3 coPapersDBLP 22.9 27.7 25.7 amazon2008 23.4 22.2 21.7 amazon2009 44.6 56.2 57.5 belgiumosm 1085.9 454.8 240.9 roadNetCA 447.3 422.5 147.9 <td< td=""><td></td><td>12.8</td><td>8.3</td><td>6.9</td></td<>		12.8	8.3	6.9				
hollywood2009 15.3 26.7 20.3 belgiumosm 627.5 292.4 181.2 roadNetCA 265.5 259.8 101.7 comOrkut 33.2 63.6 53.2 citPatents 21.0 30.3 35.1 rggn222s0 825.3 1259.7 415.3 socLiveJournal 43.0 85.8 57.1 indochina2004 220.6 573.4 328.7 roadcentral 1864.9 1680.8 710.2	coPapersDBLP	10.8	14.9	11.5				
belgiumosm 627.5 292.4 181.2 roadNetCA 265.5 259.8 101.7 comOrkut 33.2 63.6 53.2 citPatents 21.0 30.3 35.1 rggn222s0 825.3 1259.7 415.3 socLiveJournal 43.0 85.8 57.1 indochina2004 220.6 573.4 328.7 rggn223s0 1245.5 2519.6 754.9 roadcentral 1864.9 1680.8 710.2 SSSP coAuthorsCit 18.3 10.4 10.3 coPapersDBLP 22.9 27.7 25.7 amazon2008 23.4 22.2 21.7 belgiumosm 1085.9 454.8 240.9 roadNetCA 447.3 422.5 147.9 comOrkut 79.7 111.5 241.0 citPatents 49.8 78.4 129.3 rggn222s0 1378.8 924.3 865.4 <t< td=""><td>amazon2008</td><td>12.3</td><td>12.6</td><td>8.1</td></t<>	amazon2008	12.3	12.6	8.1				
roadNetCA	hollywood2009	15.3	26.7	20.3				
comOrkut 33.2 63.6 53.2 citPatents 21.0 30.3 35.1 rggn222s0 825.3 1259.7 415.3 socLiveJournal 43.0 85.8 57.1 indochina2004 220.6 573.4 328.7 rggn223s0 1245.5 2519.6 754.9 roadcentral 1864.9 1680.8 710.2 SSSP coAuthorsCit 18.3 10.4 10.3 coPapersDBLP 22.9 27.7 25.7 amazon2008 23.4 22.2 21.7 amazon2008 23.4 22.2 21.7 belgiumosm 1085.9 454.8 240.9 roadNetCA 447.3 422.5 147.9 comOrkut 79.7 111.5 241.0 citPatents 49.8 78.4 129.3 rggn222s0 1378.8 924.3 865.4 socLiveJournal 82.7 120.7 189.5 <	belgiumosm	627.5	292.4	181.2				
citPatents 21.0 30.3 35.1 rggn222s0 825.3 1259.7 415.3 socLiveJournal 43.0 85.8 57.1 indochina2004 220.6 573.4 328.7 rggn223s0 1245.5 2519.6 754.9 roadcentral 1864.9 1680.8 710.2 SSSP coAuthorsCit 18.3 10.4 10.3 coPapersDBLP 22.9 27.7 25.7 amazon2008 23.4 22.2 21.7 hollywood2009 44.6 56.2 57.5 belgiumosm 1085.9 454.8 240.9 roadNetCA 447.3 422.5 147.9 comOrkut 79.7 111.5 241.0 citPatents 49.8 78.4 129.3 rggn222s0 1378.8 924.3 865.4 socLiveJournal 82.7 120.7 189.5 socLiveJournal 3176.3 2848.8 1094.3	roadNetCA	265.5	259.8	101.7				
rggn222s0 825.3 1259.7 415.3 socLiveJournal 43.0 85.8 57.1 indochina2004 220.6 573.4 328.7 rggn223s0 1245.5 2519.6 754.9 roadcentral 1864.9 1680.8 710.2	comOrkut	33.2	63.6	53.2				
socLiveJournal 43.0 85.8 57.1 indochina2004 220.6 573.4 328.7 rggn223s0 1245.5 2519.6 754.9 roadcentral 1864.9 1680.8 710.2 SSSP coAuthorsCit 18.3 10.4 10.3 coPapersDBLP 22.9 27.7 25.7 amazon2008 23.4 22.2 21.7 belgiumosm 1085.9 454.8 240.9 roadNetCA 447.3 422.5 147.9 comOrkut 79.7 111.5 241.0 citPatents 49.8 78.4 129.3 rggn222s0 1378.8 924.3 865.4 socLiveJournal 82.7 120.7 189.5 indochina2004 366.2 519.0 596.6 rggn223s0 1880.2 1201.4 1654.8 roadcentral 3176.3 2848.8 1094.3 PR <td colsp<="" td=""><td>citPatents</td><td>21.0</td><td>30.3</td><td>35.1</td></td>	<td>citPatents</td> <td>21.0</td> <td>30.3</td> <td>35.1</td>	citPatents	21.0	30.3	35.1			
indochina2004 220.6 573.4 328.7 rggn223s0 1245.5 2519.6 754.9 roadcentral 1864.9 1680.8 710.2 SSSP coAuthorsCit 18.3 10.4 10.3 coPapersDBLP 22.9 27.7 25.7 amazon2008 23.4 22.2 21.7 hollywood2009 44.6 56.2 57.5 belgiumosm 1085.9 454.8 240.9 roadNetCA 447.3 422.5 147.9 comOrkut 79.7 111.5 241.0 citPatents 49.8 78.4 129.3 rggn222s0 1378.8 924.3 865.4 socLiveJournal 82.7 120.7 189.5 indochina2004 366.2 519.0 596.6 rggn223s0 1880.2 120.7 189.5 roadcentral 3176.3 2848.8 1094.3 PR coAuthorsCit 3.	rggn222s0	825.3	1259.7	415.3				
rggn223s0	socLiveJournal	43.0	85.8	57.1				
Toadcentral 1864.9 1680.8 710.2	indochina2004	220.6	573.4	328.7				
SSSP CoAuthorsCit 18.3 10.4 10.3	rggn223s0	1245.5	2519.6	754.9				
coAuthorsCit 18.3 10.4 10.3 coPapersDBLP 22.9 27.7 25.7 amazon2008 23.4 22.2 21.7 hollywood2009 44.6 56.2 57.5 belgiumosm 1085.9 454.8 240.9 roadNetCA 447.3 422.5 147.9 comOrkut 79.7 111.5 241.0 citPatents 49.8 78.4 129.3 rggn222s0 1378.8 924.3 865.4 socLiveJournal 82.7 120.7 189.5 socLiveJournal 366.2 519.0 596.6 rggn223s0 1880.2 1201.4 1654.8 roadcentral 3176.3 2848.8 1094.3 PR coAuthorsCit 3.9 1.0 3.2 coPapersDBLP 5.7 6.1 6.1 amazon2008 25.2 4.0 5.5 hollywood2009 22.6 32.4 32.4 <t< td=""><td>roadcentral</td><td>1864.9</td><td>1680.8</td><td>710.2</td></t<>	roadcentral	1864.9	1680.8	710.2				
coPapersDBLP 22.9 27.7 25.7 amazon2008 23.4 22.2 21.7 hollywood2009 44.6 56.2 57.5 belgiumosm 1085.9 454.8 240.9 roadNetCA 447.3 422.5 147.9 comOrkut 79.7 111.5 241.0 citPatents 49.8 78.4 129.3 rggn222s0 1378.8 924.3 865.4 socLiveJournal 82.7 120.7 189.5 indochina2004 366.2 519.0 596.6 rggn223s0 1880.2 1201.4 1654.8 roadcentral 3176.3 2848.8 1094.3 PR coAuthorsCit 3.9 1.0 3.2 coAuthorsCit 3.9 1.0 3.2 coPapersDBLP 5.7 6.1 6.1 amazon2008 25.2 4.0 5.5 hollywood2009 22.6 32.4 32.4 comOrkut								
amazon2008	coAuthorsCit	18.3	10.4	10.3				
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belgiumosm 1085.9 454.8 240.9 roadNetCA 447.3 422.5 147.9 comOrkut 79.7 111.5 241.0 citPatents 49.8 78.4 129.3 rggn222s0 1378.8 924.3 865.4 socLiveJournal 82.7 120.7 189.5 indochina2004 366.2 519.0 596.6 rggn223s0 1880.2 1201.4 1654.8 roadcentral 3176.3 2848.8 1094.3 PR coAuthorsCit 3.9 1.0 3.2 coPapersDBLP 5.7 6.1 6.1 amazon2008 25.2 4.0 5.5 hollywood2009 22.6 32.4 32.4 belgiumosm 10.2 7.1 9.4 comOrkut 31.9 46.6 103.3 citPatents 12.3 21.3 38.3 rggn222s0 13.4 22.4 26.6 socLiveJou	-	23.4	22.2	21.7				
belgiumosm 1085.9 454.8 240.9 roadNetCA 447.3 422.5 147.9 comOrkut 79.7 111.5 241.0 citPatents 49.8 78.4 129.3 rggn222s0 1378.8 924.3 865.4 socLiveJournal 82.7 120.7 189.5 indochina2004 366.2 519.0 596.6 rggn223s0 1880.2 1201.4 1654.8 roadcentral 3176.3 2848.8 1094.3 PR coAuthorsCit 3.9 1.0 3.2 coPapersDBLP 5.7 6.1 6.1 amazon2008 25.2 4.0 5.5 hollywood2009 22.6 32.4 32.4 belgiumosm 10.2 7.1 9.4 comOrkut 31.9 46.6 103.3 citPatents 12.3 21.3 38.3 rggn222s0 13.4 22.4 26.6 socLiveJou	hollywood2009	44.6	56.2	57.5				
comOrkut 79.7 111.5 241.0 citPatents 49.8 78.4 129.3 rggn222s0 1378.8 924.3 865.4 socLiveJournal 82.7 120.7 189.5 indochina2004 366.2 519.0 596.6 rggn223s0 1880.2 1201.4 1654.8 roadcentral 3176.3 2848.8 1094.3 PR 3.9 1.0 3.2 coPapersDBLP 5.7 6.1 6.1 amazon2008 25.2 4.0 5.5 hollywood2009 22.6 32.4 32.4 belgiumosm 10.2 7.1 9.4 roadNetCA 10.8 15.7 19.6 comOrkut 31.9 46.6 103.3 rggn222s0 13.4 22.4 26.6 socLiveJournal 210.0 64.2 112.0 rggn223s0 38.6 57.2 103.4 roadcentral	•	1085.9	454.8	240.9				
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rggn222s0 1378.8 924.3 865.4 socLiveJournal 82.7 120.7 189.5 indochina2004 366.2 519.0 596.6 rggn223s0 1880.2 1201.4 1654.8 roadcentral 3176.3 2848.8 1094.3	comOrkut	79.7	111.5	241.0				
socLiveJournal 82.7 120.7 189.5 indochina2004 366.2 519.0 596.6 rggn223s0 1880.2 1201.4 1654.8 roadcentral 3176.3 2848.8 1094.3 PR coAuthorsCit 3.9 1.0 3.2 coPapersDBLP 5.7 6.1 6.1 amazon2008 25.2 4.0 5.5 hollywood2009 22.6 32.4 32.4 belgiumosm 10.2 7.1 9.4 comOrkut 31.9 46.6 103.3 citPatents 12.3 21.3 38.3 rggn222s0 13.4 22.4 26.6 socLiveJournal 210.0 64.2 112.0 rggn223s0 38.6 57.2 103.4 roadcentral 57.9 89.6 172.0 TC coAuthorsCit 4.6 2.2 3.0 coPapersDBLP 57.6	citPatents	49.8	78.4	129.3				
socLiveJournal 82.7 120.7 189.5 indochina2004 366.2 519.0 596.6 rggn223s0 1880.2 1201.4 1654.8 roadcentral 3176.3 2848.8 1094.3 PR coAuthorsCit 3.9 1.0 3.2 coPapersDBLP 5.7 6.1 6.1 amazon2008 25.2 4.0 5.5 hollywood2009 22.6 32.4 32.4 belgiumosm 10.2 7.1 9.4 comOrkut 31.9 46.6 103.3 citPatents 12.3 21.3 38.3 rggn222s0 13.4 22.4 26.6 socLiveJournal 210.0 64.2 112.0 rggn223s0 38.6 57.2 103.4 roadcentral 57.9 89.6 172.0 TC coAuthorsCit 4.6 2.2 3.0 coPapersDBLP 57.6	rggn222s0	1378.8	924.3	865.4				
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Distinct devices. Performance in not for comparison.

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Table 4: Integrated GPU mode performance comparison of the proposed solution. Time in milliseconds (lower is better).

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[12] Peter Zhang, Marcin Zalewski, Andrew Lumsdaine, Samantha Misurda, and Scott McMillan. 2016. GBTL-CUDA: Graph Algorithms and Primitives for GPUs. In 2016 IEEE International Parallel and Distributed Processing Symposium Workshops (IPDPSW). 912–920. https://doi.org/10.1109/IPDPSW.2016.185 755

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LaGraph (LG), Spla (SP).