

# Spla: Open-Source Generalized Sparse Linear Algebra Framework with Vendor-Agnostic GPUs Accelerated Computations

Anonymous Author(s)

## ABSTRACT

Scalable high-performance graph analysis is an actual nontrivial challenge. Usage of sparse linear algebra operations as building blocks for graph analysis algorithms, which is a core idea of GraphBLAS standard, is a promising way to attack it. While it is known that sparse linear algebra operations can be efficiently implemented on GPU, full GraphBLAS implementation on GPU is a nontrivial task that is almost solved by GraphBLAST project. Though it is shown that utilization of GPUs for GraphBLAS implementation significantly improves performance, portability problem is not solved yet: GraphBLAST uses Nvidia Cuda stack. Moreover, while GraphBLAS is stable and mature, it has some limitations discussed by John R. Gilbert at HPEC GraphBLAS BoF. In this work we propose Spla library that aimed to solve some of these problems. The API of the library provides runtime information and introspection. Its implementation streamlines storage management, implicit zeros and masking semantics, and provides GPUs accelerated computations. Evaluation shows that while further optimizations are required, the proposed solution demonstrates performance comparable with GraphBLAST, outperforming it up to 36 times in some cases, remaining portable across different GPUs vendors. Moreover, our solution on integrated GPU outperforms SuiteSparse:GrpahBLAS on the respective CPU on some graph analysis tasks.

## CCS CONCEPTS

• **Mathematics of computing** → **Graph algorithms**; • **Computing methodologies** → **Parallel algorithms**; • **Computer systems organization** → **Single instruction, multiple data**.

## KEYWORDS

graphs, algorithms, graph analysis, sparse linear algebra, GraphBLAS, GPGPU, OpenCL

## ACM Reference Format:

Anonymous Author(s). 2018. Spla: Open-Source Generalized Sparse Linear Algebra Framework with Vendor-Agnostic GPUs Accelerated Computations. In *Proceedings of Make sure to enter the correct conference title from your rights confirmation email (Conference acronym 'XX)*. ACM, New York, NY, USA, 10 pages. <https://doi.org/XXXXXXX.XXXXXXX>

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. Copyrights for components of this work owned by others than ACM must be honored. Abstracting with credit is permitted. To copy otherwise, or republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee. Request permissions from [permissions@acm.org](mailto:permissions@acm.org).

Conference acronym 'XX, June 03–05, 2018, Woodstock, NY

© 2018 Association for Computing Machinery.

ACM ISBN 978-1-4503-XXXX-X/18/06...\$15.00

<https://doi.org/XXXXXXX.XXXXXXX>

## 1 INTRODUCTION

Scalable high-performance graph analysis is an actual challenge. There is a big number of ways to attack this challenge [3] and the first promising idea is to utilize general-purpose graphic processing units (GPGPU). Such existing solutions, as CuSha [8] and Gunrock [10] show that utilization of GPUs can improve the performance of graph analysis, moreover it is shown that solutions may be scaled to multi-GPU systems. But low flexibility and high complexity of API are problems of these solutions.

The second promising thing which provides a user-friendly API for high-performance graph analysis algorithms creation is a GraphBLAS API [7] which provides linear algebra based building blocks to create graph analysis algorithms. The idea of GraphBLAS is based on a well-known fact that linear algebra operations can be efficiently implemented on parallel hardware. Along with that, a graph can be natively represented using matrices: adjacency matrix, incidence matrix, etc. While reference CPU-based implementation of GraphBLAS, SuiteSparse [4], demonstrates good performance in real-world tasks, GPU-based implementation is challenging.

One of the challenges in this way is that real data are often sparse, thus underlying matrices and vectors are also sparse, and, as a result, classical dense data structures and respective algorithms are inefficient. So, it is necessary to use advanced data structures and procedures to implement sparse linear algebra, but the efficient implementation of them on GPU is hard due to the irregularity of workload and data access patterns. Though such well-known libraries as cuSPARSE show that sparse linear algebra operations can be efficiently implemented for GPGPU, it is not so trivial to implement GraphBLAS on GPGPU. First of all, it requires *generalized* sparse linear algebra, thus it is impossible just to reuse existing libraries which are almost all specified for operations over floats. The second problem is specific optimizations, such as masking fusion, which can not be natively implemented on top of existing kernels. Nevertheless, there is a number of implementations of GraphBLAS on GPU, such as GraphBLAST [16], GBTL [19], which show that GPGPUs utilization can improve the performance of GraphBLAS-based graph analysis solutions. But these solutions are not portable because they are based on Nvidia Cuda stack.

Although GraphBLAS is solid and mature standard with a number of implementation, it has limitations and shortcomings discussed in a talk given by John R. Gilbert [6]. Some of them are lack of interoperability and introspection, what is an obstacle on the way of GraphBLAS integration into real-world data analysis pipelines. Implicit zeroes mechanism and masking, which uses mix of engineering and math, leads to unpredictable memory usage in some cases, keeping API complex for both implementation and usage.

To provide portable GPU implementation of GraphBLAS API we developed a *Spla* library<sup>1</sup>. This library utilizes OpenCL for GPGPU

<sup>1</sup>Source code of Spla library available at: <https://anonymous.link>

computing to be portable across devices of different vendors and aimed to solve some GraphBLAS limitations. To sum up, the contribution of this work is the following.

- Design of the library with GraphBLAS-inspired API proposed. The proposed solution addresses some GraphBLAS limitations, such as lack of introspection, implicit zeroes mechanism and ambiguous masking. Also, the core of the library is configurable, so GPU acceleration can be plugged in for some operations with a little effort.
- The proposed design implemented in C++ using OpenCL to provide GPU acceleration of some operations. Such linear algebra operations as matrix-vector multiplication for both dense and sparse vector, masked matrix-matrix multiplication, implemented on GPU. Totally, Spla provides all operations required to implement GPU-accelerated versions of breadth-first search (BFS), single source shortest path (SSSP), page rank (PR), and triangles counting (TC), that also are implemented.
- Evaluation on BFS, SSSP, PR, and TC, and real-world graphs shows portability across different vendors and promising performance: proposed solution consistently outperforms SuiteSparse, reaching up to 25 times speedup on some graphs, and shows performance comparable with GraphBLAST, archiving up to 36 times speedup in some cases. Surprisingly, for some problems, the proposed solution on integrated Intel GPU shows better performance than SuiteSparse on the respective CPU. At the same time, the evaluation shows that further optimization is required.

## 2 BACKGROUND OF STUDY

This section provides a brief overview of existing solutions for graph analysis on both CPU and GPU, and also describes the concepts of the GraphBLAS standard, highlighting some of its shortcomings and limitations on the way to a full-fledged GPU implementation.

### 2.1 Related Work

There is a number of graph processing frameworks for a both CPU and GPU analysis. A greater survey of such frameworks is done by Batari et al. [1] and Shi et al. [12]. From an conceptual perspective, this frameworks can be roughly divided into three categories: vertex-centric, edge-centric, linear algebra-based.

*Vertex-centric.* Introduces in Pregel [9] framework, vertex-centric model is based on a parallel processing of a graph vertices. The whole process is divided into iterations, where active vertices do a actual processing and send messages.

*Edge-centric.* Edge-centric model focuses on edges processing rather than vertices. It is first introduced in X-Stream [11] project. This model streams edges of the graphs and allows to effectively process them.

*Linear-algebra based.* Linear algebra based frameworks for a graph processing originate from a CombinationalBLAS [2] project. Linear algebra approach relies on the fact, that the graph traversal can be represented as matrix-vector multiplication. The graph analysis community has formalized this method in a GraphBLAS [7] standard, which provides linear algebra primitives in a form of C API. This standard has a number of implementations, including

the reference implementation SuiteSparse [4], GBTL [19], Huawei GraphBLAS implementation [18], GraphBLAST [16], etc.

### 2.2 GraphBLAS concepts

GraphBLAS standard [7] is a mathematical notation translated into a C API. This standard provides linear algebra building blocks for the implementation of graph algorithms in terms of operations over matrices and vectors. Essential parts of this standard are described below.

*Data containers.* Primary data containers in this standard are general M by N matrix and M vector of values, as well as a scalar value. Containers are parameterised by the type of stored elements. As an example, matrix can be used to represent the adjacency matrix of the graph. Vector can be used to store a set of active vertices for traversal purposes.

*Algebraic structure.* Algebraic structures are called semiring and monoid, where two or one operation is provided respectively with some semantic requirements. These structures are adapted for containers, so their mathematical properties differ a bit from those, which are stated in classical algebra. These structures define the element-wise operations, which work with elements in vector and matrix containers.

*Operations.* GraphBLAS provides a number of commonly used linear algebra operations, such as *mxv* and *mxm*, element-wise multiplication, etc. Also, there are some extra operations, such as filtering, selection using predicate, reduction of matrix to vector or of vector to scalar, etc.

*Programming constructs.* GraphBLAS provides extra objects, which are required for practical algorithms implementation, such a mask. Masking allows to use structure of matrix or vector to filter result and reduce amount of computations.

*Algorithms.* Using GraphBLAS constructs it is possible with little effort to write generalized graph analysis algorithms, such as breadth-first search, single-source shortest path, triangles counting, connected components, etc.

### 2.3 GraphBLAS limitations

Although GraphBLAS is mature standard with a number of implementation, it has some limitations and shortcomings, discussed in a talk given by Gilbert [6]. Some of them are explained in the next paragraphs.

*C-oriented.* GraphBLAS standard is initially oriented on a C99 compatible API due to some technical and performance considerations. Standard does not considers any high-level built-in package for a more streamlined work with API.

*Lack of interoperability.* GraphBLAS declares opaque objects with are hidden from the user structure. It is not possible to somehow extend or interact with an existing standard implementation. However, practical tasks may required integration of existing formats, storage into a library for practical analysis.

*Little introspection.* GraphBLAS declares a very limited functionality to inspect structure, state, type, behaviour, performance, correctness, progress of library primitives and operations. It is not feasible to build production-ready data-analysis platform without these features, which are common for all modern DBMS.

*Implicit zeros.* GraphBLAS standard tries to use a mix of math and engineering concepts to address the values storage model. As the result, this model is too complex and not obvious for both mathematicians and programmers. Thus, inaccurate storage manipulations may cause a sufficient memory usage increase in your application even if you precisely follow the standard.

*Masking.* GraphBLAS standard provides an ability to apply a mask to filter out result matrices or vectors. However, rules for selecting values from a mask are implicit and rely on selecting raw zero values, like in a C program. This mechanism is not configurable.

*Templates usage.* There is a number of libraries which implement GraphBLAS in a form of C++ interface. These libraries heavily rely on a template meta programming for a generalization of a processed data. This approach simplifies implementation of the library. Auxiliary code is generated by the compiler. However, template-based approach requires the whole project recompilation for each executable and for any change of a user code.

*GPU support.* GraphBLAS has no fully-featured implementation with GPU support. The primary reason for this is the complexity of the standard. There is a number of attempts to adopt GraphBLAS for a GPU analysis. But, most of them are focused only on Nvidia platform, what limits the portability of the potential solution.

### 3 PROPOSED SOLUTION DESCRIPTION

This section describes the high-level details of the proposed solution. It highlights the design principles, high-level architecture of the solution, data storage representation, operations, and also shows differences from the GraphBLAS API.

#### 3.1 Design Principles

Spla library is designed the way to maximize potential library performance, simplify its implementation and extensions, and to provide the end-user verbose, but expressive interface allowing customization and precise control over operations execution. These ideas are captured in the following principles.

- *Optional acceleration.* Library is designed in a way, that GPU acceleration is fully pluggable and optional part. Library can perform computations using standard CPU pipeline. If GPU acceleration is presented, library can offload a part of a work for it. It allows both non-trivial processing of the data on the CPU only, as well as possibility to integrate different backends in the future.
- *User-defined functions.* The user can create custom element-by-element functions to parameterize operations. Custom functions can be used for both CPU and GPU execution.
- *Predefined scalar data types.* The library provides a set of built-in scalar data types that have a natural one-to-one relationship with native GPU built-in types. Data storage is transparent. The library interprets the data as POD-structures. The user can interpret individual elements as a sequence of bytes of a fixed size.
- *Hybrid-storage format.* The library automates the process of data storage and preprocessing. It supports several data formats, chooses the best one depending on the situation.
- *Exportable interface.* The library has a C++ interface with an automated reference-counting and with no-templates usage.

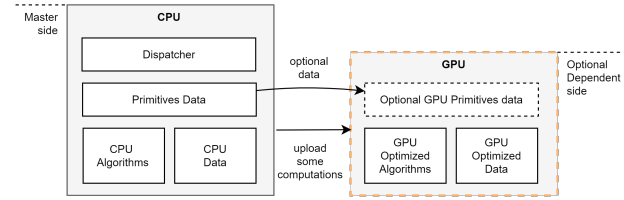


Figure 1: Proposed solution general design idea.

It can be wrapped by C99 compatible API and exported to other languages, for example, in a form of a Python package.

- *Introspection.* Each library class instantiates into a first-class object. Such objects can be saved, manipulated, passed as arguments and returned as function results. Parameterization types of containers can be inspected, as well as declared user functions.

#### 3.2 Architecture Overview

The general idea of the proposed solution is depicted in Fig. 1. The core of the library and its main part is the CPU, which is the master node and controls all calculations. It is responsible for storing data, maintaining a registry with algorithms, and scheduling operations to perform. In this paradigm, the GPU is an optional backend for acceleration, implemented through a special interface. It can optionally store data in a specific format. The CPU can offload the calculation of a part of the operations to the GPU, if the corresponding operation is supported by the given accelerator.

The reason for this is that the CPU and GPU are inherently asymmetric in nature. The end-user uses CPU side API. Thus, some preprocessing on the CPU side must be always done in the majority of cases. In addition, access to data on the GPU and their storage is carried out differently due to the peculiarities of the execution of kernels. Also, VRAM is more expensive and has less capacity than RAM. Therefore, RAM is a cache for VRAM, and data duplication can be neglected. In the end, the explicit separation of the CPU side from the GPU of the backend gives the modularity. This can be used not only to support different GPU technologies, but also to integrate multiple GPUs or distributed processing in the future.

#### 3.3 Data Containers

Library provides general *M-by-N Matrix*, *N Vector* and *Scalar* data containers. Underlying primitive scalar types are specified by *Type* object. Single vector or matrix data is stored in specialized multi-format storage container. An example of the single vector storage is depicted in Fig. 2.

The storage is responsible for keeping data in multiple different formats at the same time. Each format is best suited for a specific type of task and requested on demand. Key-value dictionary suites well frequent insertion, query or deletion operations, when memory usage and response time are critical. Mathematical operations perform better with compacted sequential lists of values since they have more friendly cache behaviour. GPU operations require separate format with a copy of the data resident in VRAM.

Data transformation from one format to another is carried out using a special rules graph shown in Fig. 3. The directed edges in



this graph indicate conversion rules. The graph must be the single strongly connected component. An example of the data transformation process is depicted in Fig. 4. For a requested format the best path of conversion is obtained. Currently, the shortest one is used. Weight assignment to rules can potentially be used to prioritize conversions and reduce the cost of transformation for some formats.

Currently, several storage formats are supported. There is dictionary of keys for vector and matrix (DoK), list of coordinates (COO), dense vector, list of lists (LIL) and compressed sparse rows (CSR) matrix formats. Other formats, such as CSC, DCSR, ELL, etc., can be added to the library by the implementation of formats conversion and by the specialization of operations for a specific format.

### 3.4 Algebraic Operations

Library provides a number of commonly used operations, such as *vxm*, *mxv*, *mxmT*, *element-wise add*, *assign*, *map*, *reduce*, etc. Other operations can be added on demand. Interface of operations is inspired by GraphBLAS standard. It supports *masking*, parametrization by *binary mult* and *binary add* functions, *select* for filtering and mask application, *unary op* for values transformation, and *descriptor* object for additional operation tweaking.

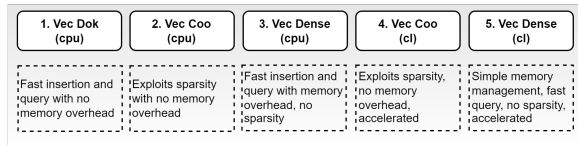


Figure 2: Vector primitive storage holds the same data potentially in multiple different formats. Some slots can be empty.

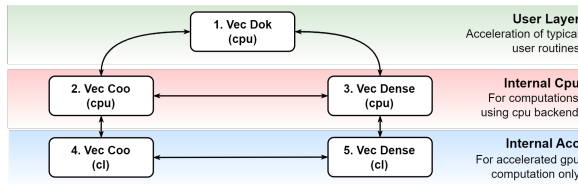


Figure 3: Vector storage transformation graph. The graph defines how data can be obtained from one format in another.

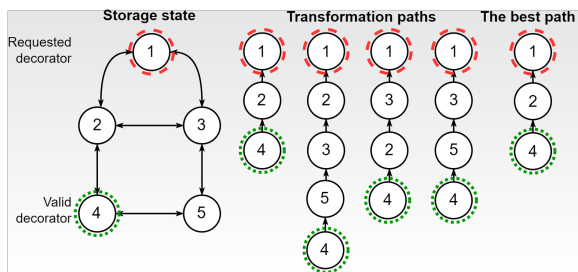


Figure 4: Vector storage transformation process. Green is valid format. Red is requested format. No highlight is currently invalid format.

### 3.5 Differences with GraphBLAS standard

To be clear, the proposed solution is not an implementation of GraphBLAS C or C++ API. The design of the library uses only the concepts described in the standard. However, in the proposed solution, the signatures and semantics of some of the operations have been changed. The API has been made more verbose and explicit. In particular, the handling of *zero* elements and *masking* are made cleaner for the end user. The library interprets data simply as collections of bytes, without mathematical semantics and identity elements. Identity element must be explicitly passed by the user where required. It allows to make the memory usage predictable and the result of each operation clear to the end user without internal implicit storage manipulations.

## 4 IMPLEMENTATION DETAILS

This section describes implementation details of the proposed solution. It highlights key aspects of the core implementation, OpenCL specifics, optimization of particular operations, and high-level optimizations of graph algorithms.

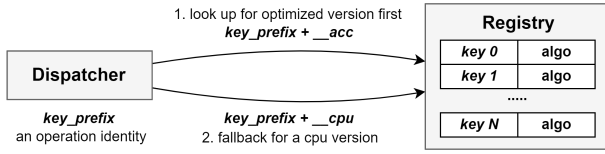
### 4.1 Core

The implemented library uses the concept of a registry to find operations as shown in Fig.5. A call to a particular operation is stored as a command to be executed later by *Dispatcher*. For each command the special lightweight string key is built depending on type of the operation and arguments passed. This key is used as a regex to get the required implementation of the requested operation. The advantages of the proposed approach are listed below.

- *Late binding*. The operation call becomes a command. The processing of such a command can be configured at runtime. Changing the acceleration backend can be done without recompilation. Moreover, several backends can be transparently used within a single application.
- *Optionality of accelerator*. The acceleration backend is free to support only those operations that require it. Fallback implementations will be used automatically for the rest of the operations.
- *Performance tuning*. The key of the command reflects operation type, arguments types, passed user functions types, etc. It can be used for *ad-hoc* optimizations. Custom operation implementation with a verbose key can be also stored in the registry. If several operations match the key, the longest key is used, since it is more specific for a particular operation.
- *Scheduling*. The full list of submitted commands for execution can be examined at runtime. This opens up the possibility for the fusion of some operations, sorting, rearrangement, and any other high-level optimizations that require introspection.

### 4.2 OpenCL

OpenCL 1.2 is used as the primary API for backend GPU implementation. Header files with C and C++ definitions are supplied with the source code of the project. Official Khronos installable client driver (ICD) loader within a library to load at runtime



**Figure 5: Registry of operation implementations. Keys with special syntax used to fetch required operation in a specific order at runtime.**

particular OpenCL implementation depending on running OS and GPU vendor.

Implementation of sparse linear algebra algorithms for a GPU requires auxiliary libraries for memory management, sorting, reducing, merging, scanning, etc. Nvidia Cuda platform features libraries such as Thrust and Cub. OpenCL lacks such support. All primitives must be implement from a scratch in most cases. What is an extra challenge. Third-party library, such as Boost Compute [13], cannot be used, since it has significant runtime overhead, portability and performance issues, and lack of long term support.

User-defined functions for GPU usage are represented as strings with additional metadata, such as type of parameters, return types, unique id, etc. Source code of particular operations stored in a form of .cl files. Operations implemented with generalization for parameters types and user functions. Their definitions obtained later at runtime in a compilation step through the text pre-processing. Compilation of actual OpenCL kernels is done on demand. All compiled kernels are stored in a cache. Cache key is composed from types of kernel parameters, defines, etc., which identify uniquely a particular variation of a kernel. Key composition is done in  $O(1)$ . In-place allocation is utilized for a key builder to avoid global heap usage. In order to reduce CPU overhead and keep access to the cache fast, library uses robin hood hashing based hash map.

Custom linear memory allocator implemented in order to reduce the overhead of frequent and small buffer allocations, arising in a time of execution of some operations. Allocator uses sub-buffer mechanism and serves request typically less than 1 MB of size. Otherwise, the general GPU heap is used.

### 4.3 Linear Algebra Operations

The following primitives are the core of computations: *masked sparse-vector sparse-matrix product*, *masked sparse-matrix dense-vector product* and *masked sparse-matrix sparse-matrix product*. Efficient implementation and load balancing of those operations dominate the performance of particular algorithms. The following paragraphs give an insight into these operations implementation in the library.

*Masked sparse-vector sparse-matrix product.* The implementation is based on the algorithm proposed by Yang et al. [17]. It is a  $k$ -way merge based algorithm which suites well for sparse vectors. Our implementation uses custom gather to collect temporary products. Radix sort used to sort products for further reduction. Reduction by key uses parallel prefix scan to carry out final destination of reduced values.

*Masked sparse-matrix dense-vector product.* The implementation of this operation relies on a classic row-based parallel algorithm. Both scalar and vector versions are implemented to fit better relatively sparse and dense matrix rows.

*Masked sparse-matrix sparse-matrix product.* The implementation of this algorithm uses the approach proposed by Yang et al. [16]. It is straightforward single-pass row-major and column-major matrix product. Mask is used to estimate the size of the final result to filter out some result of the product.

### 4.4 Graph Algorithms

The advantage of the linear algebra approach is that graph algorithms can be easily composed of primitive operations using a few lines of code. For preliminary study breadth-first search (BFS), single-source shortest paths (SSSP), page rank (PR) and triangles counting (TC) algorithms were chosen. These are the most commonly evaluated graph algorithms. They allow to test basic operations and key aspects of graph frameworks performance. Implementation details for chosen algorithms are given below.

*BFS.* It utilizes a number of optimizations described by Yang et al. [15]. It uses masking to filter out already reached vertices, change of direction (push-pull) to switch from sparse from to dense and vice versa, and early exit in *mxv* operation.

*SSSP.* This algorithm uses change of direction as well. Also, it employs filtering of unproductive vertices according to Yang et al. [16]. Vertices which do not relax their distance in current iteration are removed from a front of the search. It allows to keep workload moderate.

*PR.* This algorithm assigns numerical weights to objects in the network depending on their relative relevance. As a key operation it uses *mxv* operation with a dense vector. For error estimation it uses custom element-wise function with a fusion of subtraction and square operations.

*TC.* Triangles counting uses masked sparse matrix product [16] and reduction. As an input algorithm accepts a lower triangular component  $L$  of an adjacency matrix of the source graph. The result is a count of non-zero values from  $B = LL^T \cdot * L$ , where  $*$  used for the masking. The second argument is not actually transposed, since row-column based product gives exactly the required effect.

## 5 EVALUATION

For performance analysis of the proposed solution, we evaluated a few most common graph algorithms using real-world sparse matrix data. As a baseline for comparison we chose LAGraph [14] in connection with SuiteSparse [4] as a CPU tool, Gunrock [10] and GraphBLAST [16] as a Nvidia GPU tools. Also, we tested algorithms on several devices with distinct OpenCL vendors in order to validate the portability of the proposed solution. In general, these evaluation intentions are summarized in the following research questions.

- RQ1** What is the performance of the proposed solution relative to existing tools for GPU analysis?
- RQ2** What is the performance of the proposed solution on various devices vendors and OpenCL runtimes?
- RQ3** What is the performance of the proposed solution on integrated GPU compared to existing CPU tool for analysis?

## 5.1 Evaluation Setup

**For evaluation of RQ1**, we use a PC with Ubuntu 20.04 installed, which has 3.40Hz Intel Core i7-6700 4-core CPU, DDR4 64Gb RAM, Intel HD Graphics 530 integrated GPU, and Nvidia GeForce GTX 1070 dedicated GPU with 8Gb on-board VRAM. **For evaluation of RQ2**, we use a PC with Ubuntu 22.04 installed, which has 4.70Hz AMD Ryzen 9 7900x 12-core CPU, DDR4 128 GB RAM, AMD GFX1036 integrated GPU, and either Intel Arc A770 flux dedicated GPU with 8Gb on-board VRAM or AMD Radeon Vega Frontier Edition dedicated GPU with 16Gb on-board VRAM. **For evaluation of RQ3**, the first PC with Intel CPU and integrated GPU and the second PC with AMD CPU and integrated GPU are used.

Programs were compiled with GCC v9.4. Programs using CUDA were compiled with GCC v8.4 and Nvidia NVCC v10.1. Release mode and maximum optimizations level were enabled for all tested programs. Data loading time, preparation, format transformations, and host-device initial communications are excluded from time measurements. All tests are averaged across 10 runs. The deviation of measurements does not exceed the threshold of 10 percent. Additional warm-up run for each test execution is excluded from measurements.

## 5.2 Graph Algorithms

For preliminary study *breadth-first search* (BFS), *single-source shortest paths* (SSSP), *page rank* (PR) and *triangles counting* (TC) algorithms were chosen. Implementation of those algorithms for competitors is used from official source code repositories with default parameters. Compared tools are allowed to make any optimizations as long as the result remains correct. The graph vertex with index 1 is set as the initial traversal vertex in the algorithms BFS and SSSP for all tested instruments and all tested devices.

## 5.3 Dataset

Thirteen matrices with graph data were selected from the Sparse Matrix Collection at University of Florida [5]. Information about graphs is summarized in Table 1. The dataset is converted to undirected graphs. Self-loops and duplicated edges are removed. Average, sd and max metrics relate to out degree property of the vertices. For SSSP weights are initialized using pseudo-random generator with uniform  $[0, 1]$  distribution of floating-point values.

Graphs are roughly divided into two groups. The first group represents relatively dense graphs, where the number of edges per node is sufficient on average to effectively load the GPU with useful work. The second group represents relatively sparse graphs, where the average vertex degree is below the typical GPU vector register size, and the search depth reaches hundreds of hops. Graphs are sorted in ascending order by the number of vertices within each group.

## 5.4 Results Summary

Fig. 6 presents results of the evaluation and compares the performance of Spla against other Nvidia GPU tools and uses as a baseline LaGraph CPU tool. Fig. 7 presents result of the portability analysis of the proposed solution. It shows performance of the proposed solution on discrete GPUs of distinct vendors. Fig. 8 present result

Table 1: Dataset description.

| Graph          | Vertices | Edges  | Avg  | Sd    | Max    |
|----------------|----------|--------|------|-------|--------|
| coAuthorsCit   | 227.3K   | 1.6M   | 7.2  | 10.6  | 1.4K   |
| coPapersDBLP   | 540.5K   | 30.5M  | 56.4 | 66.2  | 3.3K   |
| amazon2008     | 735.3K   | 7.0M   | 9.6  | 7.6   | 1.1K   |
| hollywood2009  | 1.1M     | 112.8M | 98.9 | 271.9 | 11.5K  |
| comOrkut       | 3.1M     | 234.4M | 76.3 | 154.8 | 33.3K  |
| citPatents     | 3.8M     | 33.0M  | 8.8  | 10.5  | 793.0  |
| socLiveJournal | 4.8M     | 85.7M  | 17.7 | 52.0  | 20.3K  |
| indochina2004  | 7.4M     | 302.0M | 40.7 | 329.6 | 256.4K |
| belgiumosm     | 1.4M     | 3.1M   | 2.2  | 0.5   | 10.0   |
| roadNetCA      | 2.0M     | 5.5M   | 2.8  | 1.0   | 12.0   |
| rggn222s0      | 4.2M     | 60.7M  | 14.5 | 3.8   | 36.0   |
| rggn223s0      | 8.4M     | 127.0M | 15.1 | 3.9   | 40.0   |
| roadcentral    | 14.1M    | 33.9M  | 2.4  | 0.9   | 8.0    |

of per-device comparison of Spla library running on integrated GPU and CPU LaGraph running on the same chip.

The absolute results of the performance study are available in the appendix section. Cells left empty with *none* if tested tool failed to analyze graph due to *out of memory* exception.

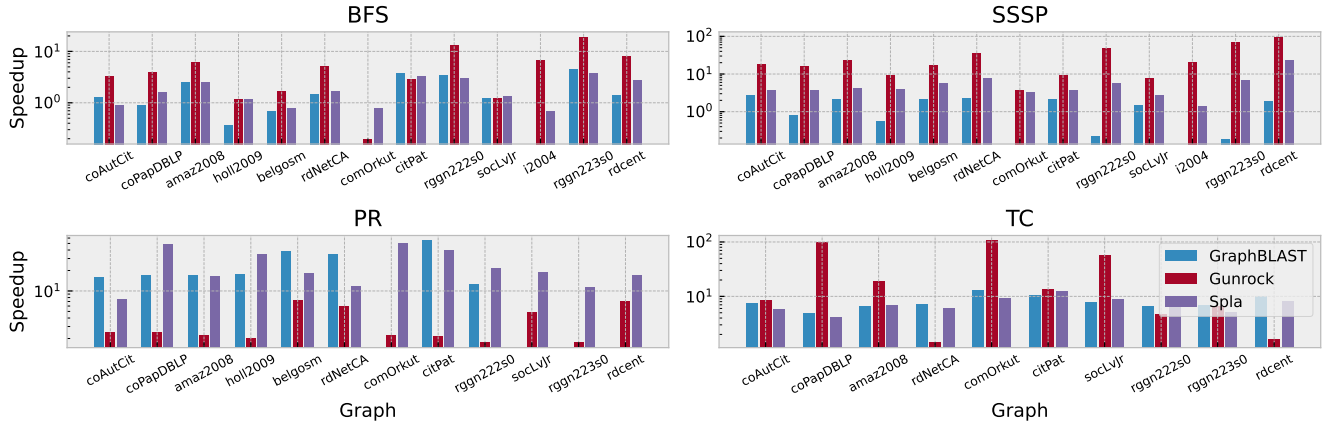
*RQ1. What is the performance of the proposed solution relative to existing tools for GPU analysis?*

In general, Spla shows very acceptable performance in all algorithms, running with comparable speed to its nearest competitor, GraphBLAST. Also proposed library does not suffer from memory issues on some large graphs such as *indochina*, *orkut* and *rggn23*. Spla is consistently several times faster than LaGraph, overcoming it up to 12× in some cases. Gunrock, the fastest GPU framework for analysis, dominates the overall performance and only suffers in a PR algorithm.

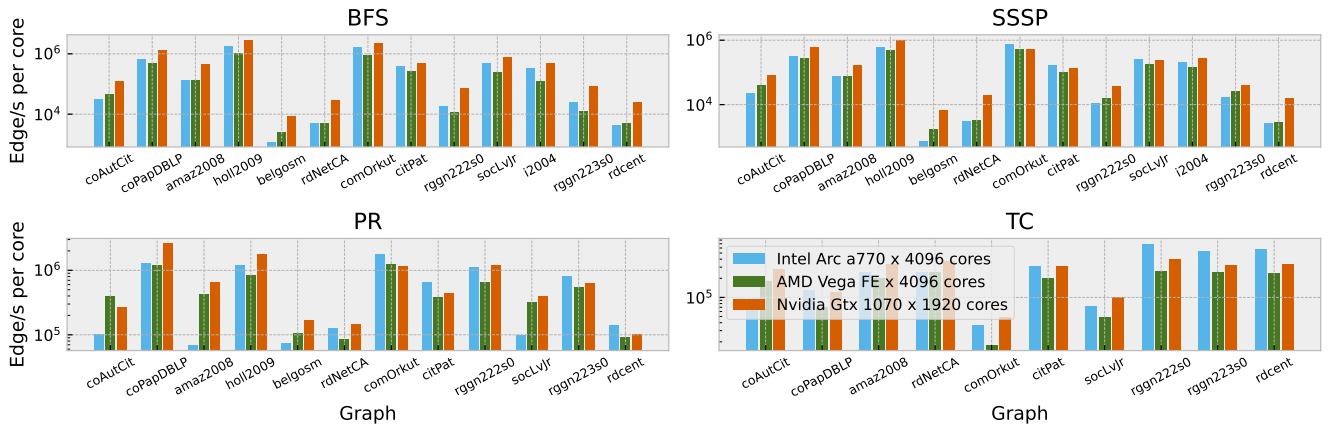
Taking a closer look at Fig. 6, Spla-based BFS shows comparable to GraphBLAST performance in most cases. Spla has good speed at relatively dense graphs with high vertex degree and small depth of the search, what allows to saturate GPU with a work better. However, the performance degrades in network and road graphs with small front of the search and large diameter, what cause a lot of iterations. Thus, both Spla and GraphBLAST suffer from the overhead of kernel launches and relatively small amount of the work for a GPU. SSSP shares with BFS the same picture in general. However, Spla behaves here slightly better than GraphBLAST, running up to 36× faster at some extreme cases.

For PR, Spla and GraphBLAST show the best performance, except cases with GraphBLAST memory issues. Both tools are faster than Gunrock in average reaching up to 20× and more relative speedup. This performance can be motivated by the usage of *mxv* operation as a core primitive for pull-updates, which is computationally intensive and has good work load balance compared to Gunrock push-updates. However, Spla suffers a bit in case of lower-degree graphs due to lack of more precise balance for small matrix rows.

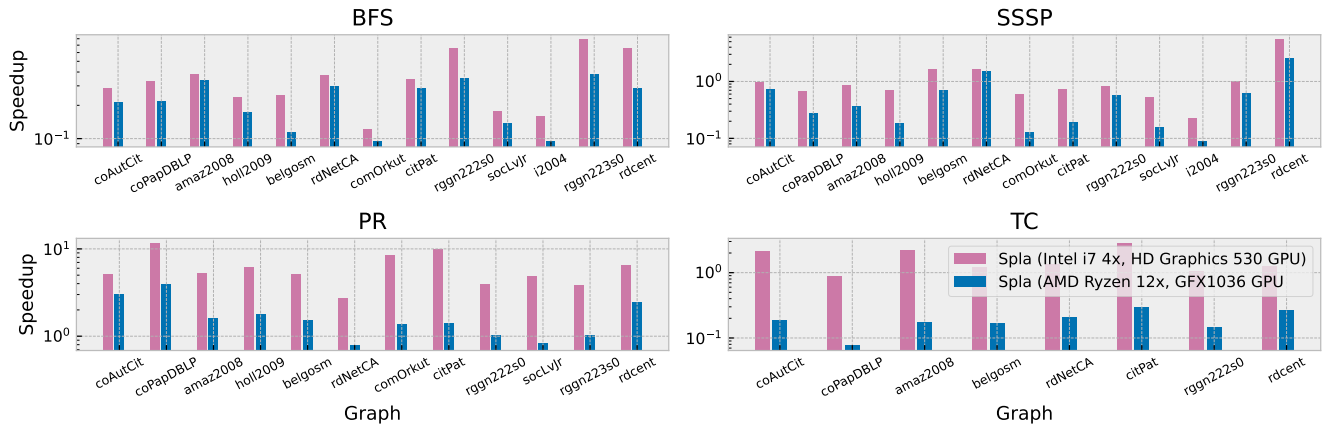
Finally, Gunrock dominates performance in TC as well, except two sparse road graphs where it has significant performance drop



**Figure 6: Performance of Spla library and GPU tools on the same device compared to LaGraph. Logarithmic scale is used.**



**Figure 7: Performance of Spla library on different devices relative to the number of compute cores. Logarithmic scale is used.**



**Figure 8: Performance of Spla library on integrated GPU compared to LaGraph on the same chip. Logarithmic scale is used.**



down. Spla and GraphBLAST have comparable results. However, GraphBLAST slightly faster nearly in all runs. Both tools use the same approach for *mxmT* implementation. However, Spla may encounter some OpenCL overhead or lack of precise performance tuning.

*RQ2. What is the performance of the proposed solution on various devices vendors and OpenCL runtimes?*

Spla successfully launches and works on the GPU of distinct vendors, including Intel, AMD and Nvidia. It shows promising performance and demonstrated scalability in relation to the number of computing cores. Fig. 7 depicts the edge/s throughput per a GPU core for all devices. This metric is quite predictable for the same graphs. This can be seen if one takes into account the overall shape of the figures for BFS, SSSP and PR as a whole.

In general, Spla on Nvidia shows better average performance, especially for sparser graphs with smaller degree per row. Nvidia OpenCL driver features faster memory allocations and has less overhead on a frequent kernel launches. Spla on Intel runtime lags a bit behind Nvidia, but performs better in some PR and TC cases. Spla performance on AMD is acceptable. However, better tuning and further polishing are required.

*RQ3. What is the performance of the proposed solution on integrated GPU compared to existing CPU tool for analysis?*

Result of detailed comparison are shown in Fig. 8. This figure depict Spla relative to LaGraph speedup on the same chip, where Spla is running on integrated GPU part and LaGraph is running on multi-core CPU part.

In general, LaGraph shows better performance for both CPUs, especially on a new powerful AMD Ryzen with 12 cores. The difference in a speed is extremely dramatic in BFS and SSSP algorithms. For a PR algorithm the picture is slightly better. Spla shows up to 10× speedup. PR algorithm tends to be more computationally intensive, so difference to BFS and SSSP is reasonable. For TC Spla performs better only for Intel device, having in some cases conservative 2× speedup.

## 6 CONCLUSION

We presented Spla, generalized sparse linear algebra framework with vendor-agnostic GPUs accelerated computations. The evaluation of the proposed solutions for some real-world graph data in four different algorithms shows, that OpenCL-based solution has a promising performance, comparable to analogs, has acceptable scalability on devices of different GPU vendors, and, surprisingly, has a speedup in some cases when compared with highly-optimized CPU library on some integrated GPUs. All in all, there are still a plenty of research questions and directions for improvement. Some of them are listed below.

- *Performance tuning.* There is a still space for optimizations. Better workload balancing must be done. More optimized algorithms must be implemented. Also, library performance must be improved on AMD and Intel devices.

- *Operations.* Additional linear algebra operations must be implemented as well as useful subroutines for filtering, joining, loading, saving data, and other manipulations involved in typical graphs analysis.
- *Graph streaming.* The next important direction of the study is streaming of data from CPU to GPU. Modern GPUs have a limited VRAM. Even high-end devices allow only a moderate portion of the memory to be addressed by the kernel at the same time. Thus, manual streaming of the data from CPU to GPU is required in order to support analysis of extremely large graphs, which count billions of edges to process.
- *Multi-GPU.* Finally, scaling of the library to multiple GPUs must be implemented. Potentially, it allows to increase overall throughput and speedup processing of really dense graph. In connection with a streaming, it can be an ultimate solution for a large real-world graphs analysis.

## REFERENCES

- [1] Omar Batarfi, Radwa El Shawi, Ayman G. Fayoumi, Reza Nouri, Seyed-Mehdi Reza Beheshti, Ahmed Barnawi, and Sherif Sakr. 2015. Large Scale Graph Processing Systems: Survey and an Experimental Evaluation. *Cluster Computing* 18, 3 (sep 2015), 1189–1213. <https://doi.org/10.1007/s10586-015-0472-6>
- [2] Aydın Buluç and John R Gilbert. 2011. The Combinatorial BLAS: Design, Implementation, and Applications. *Int. J. High Perform. Comput. Appl.* 25, 4 (nov 2011), 496–509. <https://doi.org/10.1177/1094342011403516>
- [3] Miguel E. Coimbra, Alexandre P. Francisco, and Luís Veiga. 2021. An analysis of the graph processing landscape. *Journal of Big Data* 8, 1 (April 2021). <https://doi.org/10.1186/s40537-021-00443-9>
- [4] Timothy A. Davis. 2019. Algorithm 1000: SuiteSparse:GraphBLAS: Graph Algorithms in the Language of Sparse Linear Algebra. *ACM Trans. Math. Softw.* 45, 4, Article 44 (Dec. 2019), 25 pages. <https://doi.org/10.1145/3322125>
- [5] Timothy A. Davis and Yifan Hu. 2011. The University of Florida Sparse Matrix Collection. *ACM Trans. Math. Softw.* 38, 1, Article 1 (dec 2011), 25 pages. <https://doi.org/10.1145/2049662.2049663>
- [6] John R. Gilbert. 2022. What did the GraphBLAS get wrong? *HPEC GraphBLAS BoF* (2022). <https://sites.cs.ucsb.edu/~gilbert/talks/talks.htm>
- [7] Jeremy Kepner, Peter Aaltonen, David Bader, Aydın Buluç, Franz Franchetti, John Gilbert, Dylan Hutchison, Manoj Kumar, Andrew Lumsdaine, Henning Meyerhenke, Scott McMillan, Carl Yang, John D. Owens, Marcin Zalewski, Timothy Mattson, and Jose Moreira. 2016. Mathematical foundations of the GraphBLAS. In *2016 IEEE High Performance Extreme Computing Conference (HPEC)*. 1–9. <https://doi.org/10.1109/HPEC.2016.7761646>
- [8] Farzad Khorasani, Kevall Vora, Rajiv Gupta, and Laxmi N. Bhuyan. 2014. CuSha: Vertex-Centric Graph Processing on GPUs. In *Proceedings of the 23rd International Symposium on High-Performance Parallel and Distributed Computing (Vancouver, BC, Canada) (HPDC '14)*. Association for Computing Machinery, New York, NY, USA, 239–252. <https://doi.org/10.1145/2600212.2600227>
- [9] Grzegorz Malewicz, Matthew H. Austern, Aart J.C. Bik, James C. Dehnert, Ilan Horn, Naty Leiser, and Grzegorz Czajkowski. 2010. Pregel: A System for Large-Scale Graph Processing. In *Proceedings of the 2010 ACM SIGMOD International Conference on Management of Data (Indianapolis, Indiana, USA) (SIGMOD '10)*. Association for Computing Machinery, New York, NY, USA, 135–146. <https://doi.org/10.1145/1807167.1807184>
- [10] Yuechao Pan, Yangzihao Wang, Yuduo Wu, Carl Yang, and John D. Owens. 2017. Multi-GPU Graph Analytics. In *2017 IEEE International Parallel and Distributed Processing Symposium (IPDPS)*. 479–490. <https://doi.org/10.1109/IPDPS.2017.117>
- [11] Amitabha Roy, Ivo Mihailovic, and Willy Zwaenepoel. 2013. X-Stream: Edge-Centric Graph Processing Using Streaming Partitions. In *Proceedings of the Twenty-Fourth ACM Symposium on Operating Systems Principles (Farmington, Pennsylvania) (SOSP '13)*. Association for Computing Machinery, New York, NY, USA, 472–488. <https://doi.org/10.1145/2517349.2522740>
- [12] Xuanhua Shi, Zhigao Zheng, Yongluan Zhou, Hai Jin, Ligang He, Bo Liu, and Qiang-Sheng Hua. 2018. Graph Processing on GPUs: A Survey. *ACM Comput. Surv.* 50, 6, Article 81 (jan 2018), 35 pages. <https://doi.org/10.1145/3128571>
- [13] Jakub Szuppe. 2016. Boost.Compute: A Parallel Computing Library for C++ Based on OpenCL. In *Proceedings of the 4th International Workshop on OpenCL (Vienna, Austria) (IWOCCL '16)*. Association for Computing Machinery, New York, NY, USA, Article 15, 39 pages. <https://doi.org/10.1145/2909437.2909454>
- [14] Gábor Szármay, David A. Bader, Timothy A. Davis, James Kitchen, Timothy G. Mattson, Scott McMillan, and Erik Welch. 2021. LaGraph: Linear



**Table 2: Performance comparison of the proposed solution. Time in milliseconds (lower is better).**

| Dataset        | GB      | GR     | LG      | SP     |
|----------------|---------|--------|---------|--------|
| BFS            |         |        |         |        |
| coAuthorsCit   | 5.0     | 1.9    | 6.3     | 6.9    |
| coPapersDBLP   | 19.9    | 4.5    | 18.0    | 11.5   |
| amazon2008     | 8.3     | 3.3    | 20.4    | 8.1    |
| hollywood2009  | 64.3    | 20.3   | 23.4    | 20.3   |
| belgiumosm     | 200.6   | 84.4   | 138.0   | 181.2  |
| roadNetCA      | 116.3   | 32.4   | 168.2   | 101.7  |
| comOrkut       | none    | 205.0  | 40.6    | 53.2   |
| citPatents     | 30.6    | 41.3   | 115.9   | 35.1   |
| rggn222s0      | 367.3   | 95.9   | 1228.1  | 415.3  |
| socLiveJournal | 63.1    | 61.0   | 75.5    | 57.1   |
| indochina2004  | none    | 33.3   | 224.6   | 328.7  |
| rggn223s0      | 615.3   | 146.2  | 2790.0  | 754.9  |
| roadcentral    | 1383.4  | 243.8  | 1951.0  | 710.2  |
| SSSP           |         |        |         |        |
| coAuthorsCit   | 14.7    | 2.1    | 38.9    | 10.3   |
| coPapersDBLP   | 118.6   | 5.6    | 92.2    | 25.7   |
| amazon2008     | 43.4    | 4.0    | 90.0    | 21.7   |
| hollywood2009  | 404.3   | 24.6   | 227.7   | 57.5   |
| belgiumosm     | 650.2   | 81.1   | 1359.8  | 240.9  |
| roadNetCA      | 509.7   | 32.4   | 1149.3  | 147.9  |
| comOrkut       | none    | 219.0  | 806.5   | 241.0  |
| citPatents     | 226.9   | 49.8   | 468.5   | 129.3  |
| rggn222s0      | 21737.8 | 101.9  | 4808.8  | 865.4  |
| socLiveJournal | 346.4   | 69.2   | 518.0   | 189.5  |
| indochina2004  | none    | 40.8   | 821.9   | 596.6  |
| rggn223s0      | 59015.7 | 161.1  | 11149.9 | 1654.8 |
| roadcentral    | 13724.8 | 267.0  | 25703.4 | 1094.3 |
| PR             |         |        |         |        |
| coAuthorsCit   | 1.6     | 10.0   | 24.3    | 3.2    |
| coPapersDBLP   | 17.6    | 120.2  | 297.6   | 6.1    |
| amazon2008     | 5.2     | 40.6   | 89.8    | 5.5    |
| hollywood2009  | 62.9    | 559.5  | 1111.2  | 32.4   |
| belgiumosm     | 4.4     | 22.9   | 167.6   | 9.4    |
| roadNetCA      | 6.6     | 37.7   | 225.8   | 19.6   |
| comOrkut       | none    | 2333.6 | 5239.0  | 103.3  |
| citPatents     | 27.0    | 686.1  | 1487.0  | 38.3   |
| rggn222s0      | 45.2    | 320.0  | 563.5   | 26.6   |
| socLiveJournal | none    | 445.9  | 2122.5  | 112.0  |
| rggn223s0      | none    | 662.7  | 1155.6  | 103.4  |
| roadcentral    | none    | 408.8  | 2899.9  | 172.0  |
| TC             |         |        |         |        |
| coAuthorsCit   | 2.3     | 2.0    | 17.3    | 3.0    |
| coPapersDBLP   | 105.2   | 5.3    | 520.8   | 128.4  |
| amazon2008     | 11.2    | 3.9    | 73.9    | 10.8   |
| roadNetCA      | 6.5     | 32.4   | 46.0    | 7.7    |
| comOrkut       | 1776.9  | 218.0  | 23103.8 | 2522.0 |
| citPatents     | 65.5    | 49.7   | 675.0   | 54.5   |
| socLiveJournal | 504.3   | 69.2   | 3886.7  | 437.8  |
| rggn222s0      | 73.2    | 101.3  | 484.5   | 77.7   |
| rggn223s0      | 151.4   | 158.9  | 1040.1  | 204.2  |
| roadcentral    | 42.6    | 259.3  | 425.3   | 52.7   |

GraphBLAST (GB), Gunrock (GR), LaGraph (LG), Spla (SP).

- Algebra, Network Analysis Libraries, and the Study of Graph Algorithms. arXiv:2104.01661 [cs.MS]
- [15] Carl Yang, Aydin Buluc, and John D. Owens. 2018. Implementing Push-Pull Efficiently in GraphBLAS. <https://doi.org/10.48550/ARXIV.1804.03327>
- [16] Carl Yang, Aydin Buluc, and John D. Owens. 2019. GraphBLAST: A High-Performance Linear Algebra-based Graph Framework on the GPU. arXiv:1908.01407 [cs.DC]
- [17] Carl Yang, Yangzihao Wang, and John D. Owens. 2015. Fast Sparse Matrix and Sparse Vector Multiplication Algorithm on the GPU. In *2015 IEEE International Parallel and Distributed Processing Symposium Workshop*. 841–847. <https://doi.org/10.1109/IPDPSW.2015.77>
- [18] A. N. Yzelman, D. Di Nardo, J. M. Nash, and W. J. Suijlen. 2020. A C++ GraphBLAS: specification, implementation, parallelisation, and evaluation. (2020). <http://albert-jan.yzelman.net/PDFs/yzelman20.pdf> Preprint.
- [19] Peter Zhang, Marcin Zalewski, Andrew Lumsdaine, Samantha Misurda, and Scott McMillan. 2016. GBTL-CUDA: Graph Algorithms and Primitives for GPUs. In *2016 IEEE International Parallel and Distributed Processing Symposium Workshops (IPDPSW)*. 912–920. <https://doi.org/10.1109/IPDPSW.2016.185>

## A DETAILED EVALUATION RESULTS

Evaluation results TODO

**Table 4: Integrated GPU mode performance comparison of the proposed solution. Time in milliseconds (lower is better).**

| Dataset        | Intel   |         | AMD    |        |
|----------------|---------|---------|--------|--------|
|                | LG      | SP      | LG     | SP     |
| BFS            |         |         |        |        |
| coAuthorsCit   | 7.5     | 26.3    | 3.9    | 18.2   |
| coPapersDBLP   | 18.7    | 57.3    | 12.0   | 54.9   |
| amazon2008     | 24.6    | 65.0    | 13.5   | 40.0   |
| hollywood2009  | 23.8    | 100.1   | 14.8   | 86.6   |
| belgiumosm     | 131.4   | 536.0   | 60.0   | 527.6  |
| roadNetCA      | 173.2   | 461.8   | 100.8  | 339.7  |
| comOrkut       | 41.6    | 341.4   | 25.2   | 269.4  |
| citPatents     | 126.9   | 371.6   | 61.3   | 217.7  |
| rggn222s0      | 1288.0  | 1959.9  | 644.6  | 1821.7 |
| socLiveJournal | 75.0    | 429.8   | 41.6   | 301.6  |
| indochina2004  | 228.5   | 1424.8  | 137.0  | 1445.1 |
| rggn223s0      | 2850.8  | 3647.2  | 1403.9 | 3701.3 |
| roadcentral    | 2087.8  | 3196.3  | 767.2  | 2670.3 |
| SSSP           |         |         |        |        |
| coAuthorsCit   | 40.5    | 42.5    | 29.2   | 40.5   |
| coPapersDBLP   | 92.9    | 141.8   | 48.9   | 181.6  |
| amazon2008     | 97.4    | 114.4   | 48.3   | 131.3  |
| hollywood2009  | 236.7   | 337.9   | 93.8   | 507.4  |
| belgiumosm     | 1383.2  | 854.3   | 588.9  | 845.7  |
| roadNetCA      | 1174.2  | 721.7   | 712.7  | 482.9  |
| comOrkut       | 822.9   | 1420.5  | 214.8  | 1699.5 |
| citPatents     | 488.3   | 669.4   | 171.4  | 897.3  |
| rggn222s0      | 4919.1  | 5928.3  | 2845.6 | 4952.9 |
| socLiveJournal | 534.7   | 1007.7  | 185.3  | 1205.1 |
| indochina2004  | 837.1   | 3708.3  | 345.5  | 3971.8 |
| rggn223s0      | 11375.6 | 11567.8 | 6099.6 | 9899.7 |
| roadcentral    | 26314.1 | 4887.0  | 7867.2 | 3102.0 |
| PR             |         |         |        |        |
| coAuthorsCit   | 25.3    | 5.0     | 17.6   | 5.9    |
| coPapersDBLP   | 302.3   | 26.2    | 154.5  | 39.0   |
| amazon2008     | 93.0    | 17.5    | 36.0   | 22.4   |
| hollywood2009  | 1109.8  | 179.9   | 531.7  | 300.7  |
| belgiumosm     | 178.9   | 35.0    | 45.1   | 29.4   |
| roadNetCA      | 236.9   | 86.9    | 67.6   | 86.2   |
| comOrkut       | 4458.5  | 531.9   | 959.6  | 701.4  |
| citPatents     | 1559.9  | 159.8   | 277.4  | 195.7  |
| rggn222s0      | 576.7   | 145.9   | 275.1  | 270.2  |
| socLiveJournal | 2181.0  | 449.7   | 520.5  | 630.9  |
| rggn223s0      | 1187.0  | 309.3   | 617.2  | 605.3  |
| roadcentral    | 2995.8  | 461.4   | 993.7  | 409.8  |
| TC             |         |         |        |        |
| coAuthorsCit   | 17.3    | 8.3     | 5.2    | 28.3   |
| coPapersDBLP   | 534.1   | 604.2   | 129.4  | 1682.3 |
| amazon2008     | 75.4    | 34.5    | 22.2   | 126.6  |
| belgiumosm     | 28.1    | 23.4    | 11.3   | 67.8   |
| roadNetCA      | 47.7    | 35.2    | 21.5   | 105.6  |
| citPatents     | 693.1   | 247.6   | 170.5  | 589.3  |
| rggn222s0      | 495.2   | 481.3   | 177.7  | 1218.1 |
| roadcentral    | 438.8   | 355.8   | 176.6  | 679.7  |

LaGraph (LG), Spla (SP).

**Table 3: Portability of the proposed solution. Time in milliseconds (lower is better).**

| Dataset        | Intel Arc | AMD Vega | Nvidia Gtx |
|----------------|-----------|----------|------------|
| BFS            |           |          |            |
| coAuthorsCit   | 12.8      | 8.3      | 6.9        |
| coPapersDBLP   | 10.8      | 14.9     | 11.5       |
| amazon2008     | 12.3      | 12.6     | 8.1        |
| hollywood2009  | 15.3      | 26.7     | 20.3       |
| belgiumosm     | 627.5     | 292.4    | 181.2      |
| roadNetCA      | 265.5     | 259.8    | 101.7      |
| comOrkut       | 33.2      | 63.6     | 53.2       |
| citPatents     | 21.0      | 30.3     | 35.1       |
| rggn222s0      | 825.3     | 1259.7   | 415.3      |
| socLiveJournal | 43.0      | 85.8     | 57.1       |
| indochina2004  | 220.6     | 573.4    | 328.7      |
| rggn223s0      | 1245.5    | 2519.6   | 754.9      |
| roadcentral    | 1864.9    | 1680.8   | 710.2      |
| SSSP           |           |          |            |
| coAuthorsCit   | 18.3      | 10.4     | 10.3       |
| coPapersDBLP   | 22.9      | 27.7     | 25.7       |
| amazon2008     | 23.4      | 22.2     | 21.7       |
| hollywood2009  | 44.6      | 56.2     | 57.5       |
| belgiumosm     | 1085.9    | 454.8    | 240.9      |
| roadNetCA      | 447.3     | 422.5    | 147.9      |
| comOrkut       | 79.7      | 111.5    | 241.0      |
| citPatents     | 49.8      | 78.4     | 129.3      |
| rggn222s0      | 1378.8    | 924.3    | 865.4      |
| socLiveJournal | 82.7      | 120.7    | 189.5      |
| indochina2004  | 366.2     | 519.0    | 596.6      |
| rggn223s0      | 1880.2    | 1201.4   | 1654.8     |
| roadcentral    | 3176.3    | 2848.8   | 1094.3     |
| PR             |           |          |            |
| coAuthorsCit   | 3.9       | 1.0      | 3.2        |
| coPapersDBLP   | 5.7       | 6.1      | 6.1        |
| amazon2008     | 25.2      | 4.0      | 5.5        |
| hollywood2009  | 22.6      | 32.4     | 32.4       |
| belgiumosm     | 10.2      | 7.1      | 9.4        |
| roadNetCA      | 10.8      | 15.7     | 19.6       |
| comOrkut       | 31.9      | 46.6     | 103.3      |
| citPatents     | 12.3      | 21.3     | 38.3       |
| rggn222s0      | 13.4      | 22.4     | 26.6       |
| socLiveJournal | 210.0     | 64.2     | 112.0      |
| rggn223s0      | 38.6      | 57.2     | 103.4      |
| roadcentral    | 57.9      | 89.6     | 172.0      |
| TC             |           |          |            |
| coAuthorsCit   | 4.6       | 2.2      | 3.0        |
| coPapersDBLP   | 57.6      | 106.2    | 128.4      |
| amazon2008     | 6.9       | 8.5      | 10.8       |
| roadNetCA      | 5.4       | 5.4      | 7.7        |
| comOrkut       | 1533.5    | 3267.6   | 2522.0     |
| citPatents     | 25.9      | 39.8     | 54.5       |
| socLiveJournal | 280.6     | 420.3    | 437.8      |
| rggn222s0      | 21.0      | 57.8     | 77.7       |
| rggn223s0      | 56.7      | 123.2    | 204.2      |
| roadcentral    | 14.5      | 34.6     | 52.7       |

Distinct devices. Performance in not for comparison.