

HomeWork 6

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Problem 1 **Solution:**

- (a) In original expression,
- x
- and
- y
- could be same. Thus it should be revised as following:

$$\neg \exists x, y, n \text{ Person}(x) \wedge \text{Person}(y) \wedge \neg(x = y) \wedge \text{HasSS}\#(x, n) \wedge \text{HasSS}\#(y, n)$$

- (b) Yes, it is correct.

- (c) No. In original expression, it said that everyone has every different SSN, which obviously incorrect.

$$\forall x, n \text{ Person}(x) \wedge \text{HasSS}\#(x, n) \Rightarrow \text{Digits}(n, 9)$$

- (d) Assume
- $\text{SS}\#(x)$
- means
- x
- 's social security number.

$$\neg \exists x, y \text{ Person}(x) \wedge \text{Person}(y) \wedge (\text{SS}\#(x) = \text{SS}\#(y))$$

$$\text{SS}\#(\text{John}) = \text{SS}\#(\text{Mary})$$

$$\forall x \text{ Person}(x) \Rightarrow \text{Digits}(\text{SS}\#(x), 9)$$

Problem 2 **Solution:**

- (a) No

- (b)
- $x = A, y = B, z = B$

- (c)
- $x = \text{David}, \text{father}(x) = \text{George}$

- (d)
- $x = g(u) = g(f(v))$

- (e)
- $x = y = z = B$

Problem 3 **Solution:**

$\text{Alpine}(x)$ means x is in the Alpine club. $\text{skier}(x)$ means x is skier, $\text{climber}(x)$ means x is climber. $\text{Like}(a, b)$ means a likes b .

$$\text{Alpine}(\text{Tony}), \text{Alpine}(\text{Mike}), \text{Alpine}(\text{John}).$$

$$\forall x, \text{Alpine}(x) \Rightarrow \text{skier}(x) \vee \text{climber}(x)$$

$$\forall x, \text{climber}(x) \Rightarrow \neg \text{like}(x, \text{Rain})$$

$$\forall x, \text{skier}(x) \Rightarrow \text{like}(x, \text{snow})$$

$$\forall x, \text{like}(\text{John}, x) \Rightarrow \neg \text{like}(\text{Mike}, x)$$

$$\forall x, \neg \text{like}(\text{John}, x) \Rightarrow \text{like}(\text{Mike}, x)$$

$$\neg \text{like}(\text{John}, \text{rain})$$

$$\neg \text{like}(\text{John}, \text{snow})$$

Then, question is $\exists x, \text{Alpine}(x) \wedge \text{climber}(x) \wedge \neg \text{skier}(x)$.

By negating the answer term, we obtain $\forall x, \neg \text{Alpine}(x) \vee \neg \text{climber}(x) \vee \text{skier}(x)$.

Convert the original expression to CNF form plus the answer term we obtain.

$$\text{Alpine}(\text{Tony}), \text{Alpine}(\text{Mike}), \text{Alpine}(\text{John}). \quad (1)$$

$$\neg \text{Alpine}(x_1) \vee \text{skier}(x_1) \vee \text{climber}(x_1) \quad (2)$$

$$\neg \text{climber}(x_2) \vee \neg \text{like}(x_2, \text{Rain}) \quad (3)$$

$$\neg \text{skier}(x_3) \vee \text{like}(x_3, \text{snow}) \quad (4)$$

$$\neg \text{like}(\text{John}, x_4) \vee \neg \text{like}(\text{Mike}, x_4) \quad (5)$$

$$\text{like}(\text{John}, x_5) \vee \neg \text{like}(\text{Mike}, x_5) \quad (6)$$

$$\neg \text{like}(\text{John}, \text{rain}) \quad (7)$$

$$\neg \text{like}(\text{John}, \text{snow}) \quad (8)$$

$$\neg \text{Alpine}(x_6) \vee \neg \text{climber}(x_6) \vee \text{skier}(x_6) \quad (9)$$

$$\neg \text{skier}(\text{John}) \vee \text{like}(\text{John}, \text{snow}) \{4, x_3/\text{John}\} \quad (10)$$

$$\neg \text{skier}(\text{John}) \{8, 10\} \quad (11)$$

$$\text{climber}(\text{John}) \{1, 2, 11\} \quad (12)$$

$$\text{Alpine}(\text{John}) \wedge \text{climber}(\text{John}) \wedge \neg \text{skier}(\text{John}) \{1, 11, 12\} \quad (13)$$

$$\neg \text{Alpine}(\text{John}) \vee \neg \text{climber}(\text{John}) \vee \text{skier}(\text{John}) \{9, x_6/\text{John}\} \quad (14)$$

$$\text{Empty}\{13, 14\} \quad (15)$$

substitute x_6 by *John*, we can find it contradict with (13), and it show John is the answer.

Problem 4 **Solution:**

(a) Yes.

(b) No.

(c) No.

Problem 5 **Solution:**

(a)

$$\forall x, \text{Horse}(x) \Rightarrow \text{mammals}(x)$$

$$\forall x, \text{Cow}(x) \Rightarrow \text{mammals}(x)$$

$$\forall x, \text{Sheep}(x) \Rightarrow \text{mammals}(x)$$

(b)

$$\forall x, y, \text{Pig}(y) \wedge \text{offspring}(x, y) \Rightarrow \text{Pig}(x)$$

(c)

$$\text{Pig}(\text{Bluebeard})$$

(d)

$$\text{parent}(\text{Bluebeard}, \text{Charlies})$$

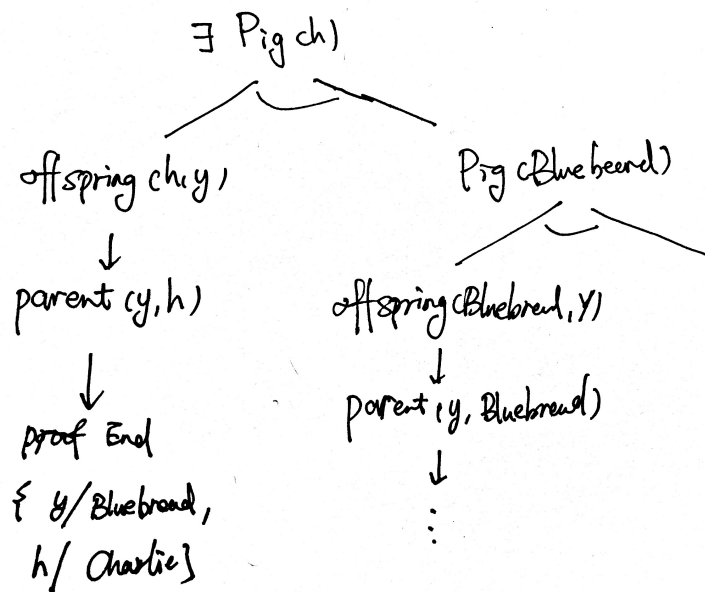
(e)

$$\forall x, y, \text{parent}(x, y) \Leftrightarrow \text{offspring}(y, x)$$

(f)

$$\forall x, \text{mammals}(x) \Rightarrow \exists y, \text{parent}(y, x)$$

Problem 6 **Solution:**



Problem 7 **Solution:**

(a)

$$\begin{aligned} & ((\exists x)[P(x)] \vee (\exists x)[Q(x)]) \Rightarrow (\exists y)[P(y) \vee Q(y)] \\ & \neg((\exists x)[P(x)] \vee (\exists x)[Q(x)]) \vee (\exists y)[P(y) \vee Q(y)] \\ & ((\forall x)[P(x)] \wedge (\forall x)[Q(x)]) \vee P(Y) \vee Q(Y) \\ & (\neg P(x_1) \wedge \neg Q(x_1)) \vee P(Y) \vee Q(Y) \\ & (\neg P(x_1) \vee P(Y) \vee Q(Y)) \wedge (\neg Q(x_1) \vee P(Y) \vee Q(Y)) \end{aligned}$$

(b)

$$\begin{aligned} & (\exists x)[P(x)] \Rightarrow (\exists z)[(\forall x)[Q(x, z)] \vee (\forall x)[R(x, y, z)]] \\ & \neg(\exists x)[P(x)] \vee (\exists z)[(\forall x)[Q(x, z)] \vee (\forall x)[R(x, y, z)]] \\ & (\forall x)[\neg P(x)] \vee (\exists z)[(\forall x)[Q(x, z)] \vee (\forall x)[R(x, y, z)]] \\ & \neg P(x_1) \vee [Q(x_2, Z) \vee R(x_3, y, Z)] \end{aligned}$$

(c)

$$\begin{aligned} & (\forall x)[P(x) \Rightarrow Q(x, y)] \Rightarrow ((\exists y)[P(y)] \wedge (\exists v)[Q(y, v)]) \\ & \neg(\forall x)[\neg P(x) \vee Q(x, y)] \vee ((\exists y)[P(y)] \wedge (\exists v)[Q(y, v)]) \\ & (\exists x)[P(x) \wedge \neg Q(x, y)] \vee (P(Y) \wedge Q(y, V)) \\ & (P(X) \wedge \neg Q(X, y)) \vee (P(Y) \wedge Q(y, V)) \\ & (P(X) \vee P(Y)) \wedge (P(X) \vee Q(y, V)) \wedge (\neg Q(X, y) \vee P(Y)) \wedge (\neg Q(X, y) \vee Q(y, V)) \end{aligned}$$

Problem 8 **Solution:**

(a)

$$\begin{aligned} & (\exists x)[Blue(x) \wedge Push(x)] \Rightarrow (\forall y)[\neg Push(y) \Rightarrow Green(y)] \\ & (\forall x)[(Blue(x) \wedge \neg Green(x)) \vee (\neg Blue(x) \wedge Green(x))] \\ & (\exists x)[\neg Push(x)] \Rightarrow (\forall y)[Push(y) \Rightarrow Blue(y)] \\ & Push(01) \\ & \neg Push(02) \end{aligned}$$

(b)

$$\begin{aligned} & \neg Blue(x_1) \vee \neg Push(x_1) \vee Push(y_1) \vee Green(y_1) \\ & (Blue(x_2) \vee Green(x_2)) \wedge (\neg Blue(x_2) \vee \neg Green(x_2)) \\ & Push(x_3) \vee \neg Push(y_2) \vee Blue(y_2) \\ & Push(01) \\ & \neg Push(02) \end{aligned}$$

(c) answer term $(\exists x)Green(x)$.

Negating the answer term, obtain $\neg Green(x_4)$.

$$\neg Blue(x_1) \vee \neg Push(x_1) \vee Push(y_1) \vee Green(y_1) \quad (16)$$

$$(Blue(x_2) \vee Green(x_2)) \wedge (\neg Blue(x_2) \vee \neg Green(x_2)) \quad (17)$$

$$Push(x_3) \vee \neg Push(y_2) \vee Blue(y_2) \quad (18)$$

$$Push(01) \quad (19)$$

$$\neg Push(02) \quad (20)$$

$$\neg Green(x_4) \quad (21)$$

$$Push(02) \vee \neg Push(01) \vee Blue(01)\{16, x_3/02, y_2/01\} \quad (22)$$

$$Blue(01) \text{ (17,18,19,20)} \quad (23)$$

$$\neg Blue(01) \vee \neg Push(01) \vee Push(02) \vee Green(02)\{14, x_1/01, y_1/02\} \quad (24)$$

$$Green(02)\{17, 18, 21, 22, \} \quad (25)$$

$$\neg Green(02)\{19, x_4/02\} \quad (26)$$

$$Empty\{23, 24\} \quad (27)$$

Thus, answer has been proofed.