

HomeWork 5

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Problem 1 **Solution:**

- (a) True: $1 * 4 = 4(B = \text{True}, A = \text{True})$.
False: $(2 * 2 - 1) * 4 = 12$
- (b) False: 1. $(A = \text{True} \text{ or } B = \text{True} \text{ or } C = \text{True} \text{ or } D = \text{True})$.
True: $16 - 1 = 15$.
- (c) True: 0
False: 16

Problem 2 **Solution:**

Set: Locate(a,b): a is at b' house.

Statement: $\neg \text{locate}(\text{car}, \text{Fred}) \rightarrow \text{locate}(\text{car}, \text{John})$
 $\neg \text{locate}(\text{car}, \text{John}) \rightarrow \text{locate}(\text{car}, \text{Fred})$

Constraints: No, I can not.

Problem 3 **Solution:**

First, it is obvious that a must be true, otherwise for any logical expression s $s \wedge \neg a = \text{False}$ i.e. it is unsatisfied.

a	P	Q	$P \vee a$	$Q \vee \neg a$
1	0	0	1	0
1	0	1	1	1
1	1	0	1	0
1	1	1	1	1

From the table, we can conclude, for any logical expression P, Q , $P \vee a = 1, Q \vee \neg a = Q$. Thus the unit resolution is sound.

Problem 4 **Solution:**

$$\begin{aligned}
 & \neg[(P \vee \neg Q) \rightarrow R] \rightarrow (P \wedge Q) \\
 \Leftrightarrow & \neg[\neg(P \vee \neg Q) \vee R] \rightarrow (P \wedge Q) \\
 \Leftrightarrow & \neg[\neg(\neg(P \vee \neg Q) \vee R) \vee (P \wedge Q)] \\
 \Leftrightarrow & (\neg P \vee R) \wedge (Q \vee R) \wedge (\neg P \vee \neg Q)
 \end{aligned}$$

Problem 5 **Solution:**

$$\begin{aligned}
r_{i,j} &= \neg q_{i,1} \wedge \neg q_{i,2} \wedge \dots \wedge \neg q_{i,j-1} \wedge q_{i,j} \wedge \neg q_{i,j+1} \wedge \dots \wedge \neg q_{i,n} \\
R_i &= r_{i,1} \vee r_{i,2} \vee \dots \vee r_{i,n} \\
Row &= R_0 \wedge R_1 \wedge \dots \wedge R_n
\end{aligned}$$

$$\begin{aligned}
c_{i,j} &= \neg q_{1,j} \wedge \neg q_{2,j} \wedge \dots \wedge \neg q_{i-1,j} \wedge q_{i,j} \wedge \neg q_{i+1,j} \wedge \dots \wedge \neg q_{n,j} \\
C_i &= c_{1,i} \vee c_{2,i} \vee \dots \vee c_{n,i} \\
Col &= C_1 \wedge C_2 \wedge \dots \wedge C_n
\end{aligned}$$

$$\begin{aligned}
xd_{i,j}^1 &= \neg q_{1,j-i+1} \wedge \neg q_{2,j} \wedge \dots \wedge \neg q_{i-1,j-1} \wedge q_{i,j} \wedge \neg q_{i+1,j+1} \wedge \dots \wedge \neg q_{n,i-j+m} (i \leq j) \\
xd_{i,j}^1 &= \neg q_{i-j+1,1} \wedge \neg q_{2,j} \wedge \dots \wedge \neg q_{i-1,j-1} \wedge q_{i,j} \wedge \neg q_{i+1,j+1} \wedge \dots \wedge \neg q_{n,j-i+n} (i > j) \\
xD_i &= \bigvee_{x-y=i} xd_{x,y} \\
xDiagnol &= xD_{-m} \wedge xD_{-m+1} \wedge \dots \wedge xD_n
\end{aligned}$$

$$\begin{aligned}
yd_{i,j} &= \neg q_{1,i+j-1} \wedge \neg q_{2,i+j-2} \wedge \dots \wedge \neg q_{i-1,j+1} \wedge q_{i,j} \wedge \neg q_{i+1,j-1} \wedge \dots \wedge \neg q_{i+j-1,1} (i \leq j) \\
yd_{i,j} &= \neg q_{i+j-m,1} \wedge \neg q_{i+j-m+1,m-1} \wedge \dots \wedge \neg q_{i-1,j+1} \wedge q_{i,j} \wedge \neg q_{i+1,j-1} \wedge \dots \wedge \neg q_{n,m-i-j} (i > j) \\
yD_i &= \bigvee_{x+y=i} d_{x,y} \\
yDiagnol &= D_2 \wedge D_3 \wedge \dots \wedge D_{n+m} \\
\text{Final answer is } &Row \wedge Col \wedge xDiagnol \wedge yDiagnol.
\end{aligned}$$

Problem 6 **Solution:**

(a) Table is shown below:

P	Q	R	$P \wedge (Q \wedge R)$	$(P \wedge Q) \wedge R$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

(b) Table is shown below:

P	Q	R	$P \wedge (Q \vee R)$	$(P \wedge Q) \vee (P \wedge R)$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

(c) Table is shown below:

P	Q	$\neg(P \wedge Q)$	$\neg P \vee \neg Q$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

(d) Table is shown below:

P	Q	$P \leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \wedge \neg Q)$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	1	1

Problem 7 **Solution:**

- (a) valid, $\neg Smoke \vee Smoke$.
- (b) Neither
- (c) Neither $\neg(\neg Smoke \vee Fire) \vee (Smoke \vee \neg Fire)$
 $(Smoke \wedge \neg Fire) \vee Smoke \vee \neg Fire$
 $Smoke \vee \neg Fire$
- (d) valid, $Smoke \vee Fire \vee \neg Fire = Smoke \vee True = True$
- (e) valid, $\neg Smoke \vee \neg Heat \vee Fire \leftrightarrow \neg Smoke \vee \neg Heat \vee Fire$. Left part and right part are same.
- (f) valid $Big \vee Dumb \vee \neg Dumb \vee Big = True \vee Big = True$.

Problem 8 **Solution:**

Prove $KB \wedge \neg Q$ is unsatisfied. KB is:

$\{\neg P \vee Q, \neg L \vee \neg M \vee P, \neg B \vee \neg L \vee M, \neg A \vee \neg P \vee L, \neg A \vee \neg B \vee L, A, B, \neg Q\}$

step 1 unit clause $A = True, B = True, Q = False$
 $\{\neg P, \neg L \vee \neg M \vee P, \neg L \vee M, \neg P \vee L, L\}$

step 2 unit clause $L = True, P = False$
 $\{\neg M, M\}$
 It is unsatisfied, thus problem solved.