CS 271 - Introduction to Artificial Intelligence **Fall 2016**

HomeWork 6

Name: *Liangjian Chen*
ID: *#52006933* *November 17, 2016*

Problem 1 **Solution:**

(a) In original expression, x and y could be same. Thus it should be revised as following:

$$\neg\exists x, y, n\ Person(x) \wedge Person(y) \wedge \neg(x = y) \wedge HasSS\#(x, n) \wedge HasSS\#(y, n)$$

(b) Yes, it is correct.

(c) No. In original expression, it said that everyone has every different SSN, which obviously incorrect.

$$\forall x, n\ Person(x) \wedge HasSS\#(x, n) \Rightarrow Digits(n, 9)$$

(d) Assume $SS\#(x)$ means x's social security number.

$$\neg\exists x, y\ Person(x) \wedge Person(y) \wedge (SS\#(x) = SS\#(y))$$

$$SS\#(John) = SS\#(Mary)$$

$$\forall x\ Person(x) \Rightarrow Digits(SS\#(x), 9)$$

Problem 2 **Solution:**

(a) No

(b) $x = A, y = B, z = B$

(c) $x = David, father(x) = George$

(d) $x = g(u) = g(f(v))$

(e) $x = y = z = B$

Problem 3 **Solution:**

$Alpine(x)$ means x is in the Alpine club. $skier(x)$ means x is skier, $climber(x)$ means x is climber. $Like(a, b)$ means a likes b.

$$Alpine(Tony), Alpine(Mike), Alpine(John).$$

$$\forall x, Alpine(x) \Rightarrow skier(x) \vee climber(x)$$

$$\forall x, climber(x) \Rightarrow \neg like(x, Rain)$$

$$\forall x, skier(x) \Rightarrow like(x, snow)$$

$$\forall x, like(John, x) \Rightarrow \neg like(Mike, x)$$

$$\forall x, \neg like(John, x) \Rightarrow like(Mike, x)$$

$$\neg like(John, rain)$$

$$\neg like(John, snow)$$

Then, question is $\exists x, Alpine(x) \wedge climber(x) \wedge \neg skier(x)$.
By negating the answer term, we obtain $\forall x, \neg Alpine(x) \vee \neg climber(x) \vee skier(x)$.
Convert the original expression to CNF form plus the answer term we obtain.

$$Alpine(Tony), Alpine(Mike), Alpine(John). \tag{1}$$
$$\neg Alpine(x_1) \vee skier(x_1) \vee climber(x_1) \tag{2}$$
$$\neg climber(x_2) \vee \neg like(x_2, Rain) \tag{3}$$
$$\neg skier(x_3) \vee like(x_3, snow) \tag{4}$$
$$\neg like(John, x_4) \vee \neg like(Mike, x_4) \tag{5}$$
$$like(John, x_5) \vee \neg like(Mike, x_5) \tag{6}$$
$$\neg like(John, rain) \tag{7}$$
$$\neg like(John, snow) \tag{8}$$
$$\neg Alpine(x_6) \vee \neg climber(x_6) \vee skier(x_6) \tag{9}$$
$$\neg skier(John) \vee like(John, snow)\{4, x_3/John\} \tag{10}$$
$$\neg skier(John)\{8, 10\} \tag{11}$$
$$climber(John)\{1, 2, 11\} \tag{12}$$
$$Alpine(John) \wedge climber(John) \wedge \neg siker(John)\{1, 11, 12\} \tag{13}$$
$$\neg Alpine(John) \vee \neg climber(John) \vee skier(John)\{9, x_6/John\} \tag{14}$$
$$Empty\{13, 14\} \tag{15}$$

substitute x_6 by $John$, we can find it contradict with (13), and it show John is the answer.

Problem 4 **Solution:**

(a) Yes.

(b) No.

(c) No.

Problem 5 **Solution:**

(a)

$$\forall x, Horse(x) \Rightarrow mammals(x)$$
$$\forall x, Cow(x) \Rightarrow mammals(x)$$
$$\forall x, Sheep(x) \Rightarrow mammals(x)$$

(b)

$$\forall x, y, Pig(y) \wedge offspring(x, y) \Rightarrow Pig(x)$$

(c)

$$Pig(Bluebeard)$$

(d)

$$parent(Bluebeard, Charlies)$$

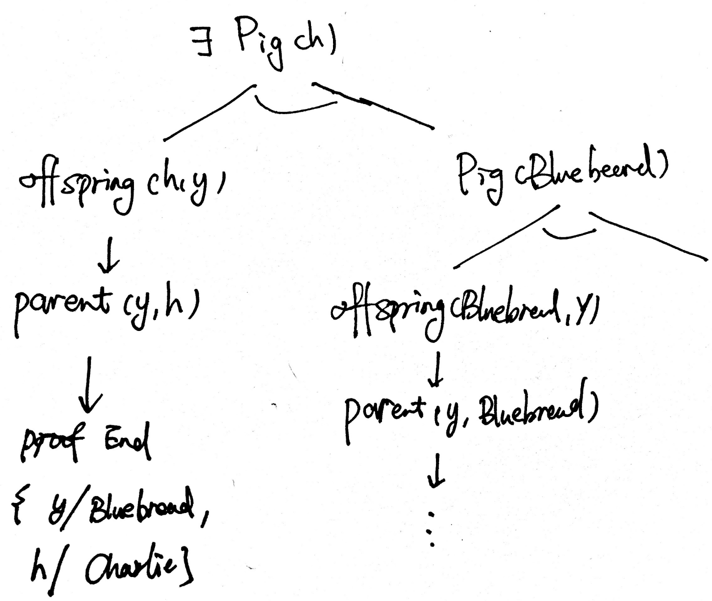
(e)

$$\forall x, y, parent(x, y) \Leftrightarrow offspring(y, x)$$

(f)

$$\forall x, mammals(x) \Rightarrow \exists y, parent(y, x)$$

Problem 6 **Solution:**



Problem 7 **Solution:**

(a)

$$((\exists x)[P(x)] \vee (\exists x)[Q(x)]) \Rightarrow (\exists y)[P(y) \vee Q(y)]$$
$$\neg((\exists x)[P(x)] \vee (\exists x)[Q(x)]) \vee (\exists y)[P(y) \vee Q(y)]$$
$$((\forall x)[\neg P(x)] \wedge (\forall x)[\neg Q(x)]) \vee P(Y) \vee Q(Y)$$
$$(\neg P(x_1) \wedge \neg Q(x_1)) \vee P(Y) \vee Q(Y)$$
$$(\neg P(x_1) \vee P(Y) \vee Q(Y)) \wedge (\neg Q(x_1) \vee P(Y) \vee Q(Y))$$

(b)

$$(\forall x)[P(x)] \Rightarrow (\exists z)[(\forall x)[Q(x,z)] \vee (\forall x)[R(x,y,z)]]$$
$$\neg(\forall x)[P(x)] \vee (\exists z)[(\forall x)[Q(x,z)] \vee (\forall x)[R(x,y,z)]]$$
$$(\exists x)[\neg P(x)] \vee (\exists z)[(\forall x)[Q(x,z)] \vee (\forall x)[R(x,y,z)]]$$
$$\neg P(X) \vee [Q(x_1,Z) \vee R(x_2,y,Z)$$

(c)

$$(\forall x)[P(x) \Rightarrow Q(x,y)] \Rightarrow ((\exists y)[P(y)] \wedge (\exists v)[Q(y,v)])$$
$$\neg(\forall x)[\neg P(x) \vee Q(x,y)] \vee ((\exists y)[P(y)] \wedge (\exists v)[Q(y,v)])$$
$$(\exists x)[P(x) \wedge \neg Q(x,y)] \vee (P(Y) \wedge Q(y,V))$$
$$(P(X) \wedge \neg Q(X,y)) \vee (P(Y) \wedge Q(y,V))$$
$$(P(X) \vee P(Y)) \wedge (P(X) \vee Q(y,V)) \wedge (\neg Q(X,y) \vee P(Y)) \wedge (\neg Q(X,y) \vee Q(y,V))$$

Problem 8 **Solution:**

(a)

$$(\exists x)[Blue(x) \wedge Push(x)] \Rightarrow (\forall y)[\neg Push(y) \Rightarrow Green(y)]$$
$$(\forall x)[(Blue(x) \wedge \neg Green(x)) \vee (\neg Blue(x) \wedge Green(x))]$$
$$(\exists x)[\neg Push(x)] \Rightarrow (\forall y)[Push(y) \Rightarrow Blue(y)]$$
$$Push(01)$$
$$\neg Push(02)$$

(b)

$$\neg Blue(x_1) \vee \neg Push(x_1) \vee Push(y_1) \vee Green(y_1)$$
$$(Blue(x_2) \vee Green(x_2)) \wedge (\neg Blue(x_2) \vee \neg Green(x_2))$$
$$Push(x_3) \vee \neg Push(y_2) \vee Blue(y_2)$$
$$Push(01)$$
$$\neg Push(02)$$

(c) answer term $(\exists x)Green(x)$.

Negating the answer term, obtain $\neg Green(x_4)$.

$$\neg Blue(x_1) \vee \neg Push(x_1) \vee Push(y_1) \vee Green(y_1) \tag{16}$$
$$(Blue(x_2) \vee Green(x_2)) \wedge (\neg Blue(x_2) \vee \neg Green(x_2)) \tag{17}$$
$$Push(x_3) \vee \neg Push(y_2) \vee Blue(y_2) \tag{18}$$
$$Push(01) \tag{19}$$
$$\neg Push(02) \tag{20}$$
$$\neg Green(x_4) \tag{21}$$
$$Push(02) \vee \neg Push(01) \vee Blue(01)\{18, x_3/02, y_2/01\} \tag{22}$$
$$Blue(01) \text{ (17,18,19,20)} \tag{23}$$
$$\neg Blue(01) \vee \neg Push(01) \vee Push(02) \vee Green(02)\{14, x_1/01, y1/02\} \tag{24}$$
$$Green(02)\{17, 18, 21, 22, \} \tag{25}$$
$$\neg Green(02)\{19, x_4/02\} \tag{26}$$
$$Empty\{23, 24\} \tag{27}$$

Thus, answer has been proofed.