CS 271 - Introduction to Artificial Intelligence **Fall 2016**

HomeWork 6

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Problem 1 **Solution:**

Define cost function as how many constrains are violated. At beginning the cost is 4.

(a) In original expression, x and y could be same. Thus it should be revised as following:

$$\neg\exists x, y, n\ Person(x) \wedge Person(y) \wedge \neg(x = y) \wedge HasSS\#(x, n) \wedge HasSS\#(y, n)$$

(b) Yes, it is correct.

(c) No. In original expression, it said that everyone has every different SSN, which obviously incorrect.

$$\forall x, n\ Person(x) \wedge HasSS\#(x, n) \Rightarrow Digits(n, 9)$$

(d) Assume $SS\#(x)$ means x's social security number.

$$\neg\exists x, y\ Person(x) \wedge Person(y) \wedge (SS\#(x) = SS\#(y))$$

$$SS\#(John) = SS\#(Mary)$$

$$\forall x\ Person(x) \Rightarrow Digits(SS\#(x), 9)$$

Problem 2 **Solution:**

(a) No

(b) $x = A, y = B, z = B$

(c) $x = David, father(x) = George$

(d) $x = g(u) = g(f(v))$

(e) $x = y = z = B$

Problem 3 **Solution:**

$Alpine(x)$ means x is in the Alpine club. $skier(x)$ means x is skier, $climber(x)$ means x is climber. $Like(a, b)$ means a likes b.

$$
\begin{aligned}
&Alpine(Tony), Alpine(Mike), Alpine(John).\\
&\forall x, Alpine(x) \Rightarrow skier(x) \vee climber(x)\\
&\forall x, climber(x) \Rightarrow \neg like(x, Rain)\\
&\forall x, skier(x) \Rightarrow like(x, snow)\\
&\forall x, like(John, x) \Rightarrow \neg like(Mike, x)\\
&\forall x, \neg like(John, x) \Rightarrow like(Mike, x)\\
&\neg like(John, rain)\\
&\neg like(John, snow)
\end{aligned}
$$

Then, question is $\exists x, Alpine(x) \wedge climber(x) \wedge \neg skier(x)$.
By negating the answer term, we obtain $\forall x, \neg Alpine(x) \vee \neg climber(x) \vee skier(x)$.
Convert the original expression to CNF form plus the answer term we obtain.

$$
\begin{aligned}
&Alpine(Tony), Alpine(Mike), Alpine(John). && (1)\\
&\neg Alpine(x_1) \vee skier(x_1) \vee climber(x_1) && (2)\\
&\neg climber(x_2) \vee \neg like(x_2, Rain) && (3)\\
&\neg skier(x_3) \vee like(x_3, snow) && (4)\\
&\neg like(John, x_4) \vee \neg like(Mike, x_4) && (5)\\
&like(John, x_5) \vee \neg like(Mike, x_5) && (6)\\
&\neg like(John, rain) && (7)\\
&\neg like(John, snow) && (8)\\
&\neg Alpine(x_6) \vee \neg climber(x_6) \vee skier(x_6) && (9)\\
&\neg skier(John) && (10)\\
&climber(John) && (11)\\
&Alpine(John) \wedge climber(John) \wedge \neg siker(John) && (12)\\
&\neg(\neg Alpine(John) \vee \neg climber(John) \vee skier(John)) && (13)
\end{aligned}
$$

substitute x_6 by $John$, we can find it contradict with (13), and it show John is the answer.

Problem 4 **Solution:**

(a) Yes.

(b) No.

(c) No.

Problem 5 **Solution:**

(a)

$$\forall x, Horse(x) \Rightarrow mammals(x)$$

$$\forall x, Cow(x) \Rightarrow mammals(x)$$

$$\forall x, Sheep(x) \Rightarrow mammals(x)$$

(b)

$$\forall x, y, Pig(y) \wedge offspring(x, y) \Rightarrow Pig(x)$$

(c)

$$Pig(Bluebeard)$$

(d)

$$parent(Bluebeard, Charlies)$$

(e)

$$\forall x, y, parent(x, y) = offspring(y, x)$$

(f)

$$\forall x, mammals(x) \Rightarrow \exists y, parent(y, x)$$

Problem 6 **Solution:**

Problem 7 **Solution:**

(a)

$$\begin{aligned}
&((\exists x)[P(x)] \vee (\exists x)[Q(x)]) \Rightarrow (\exists y)[P(y) \vee Q(y)] \\
&\neg((\exists x)[P(x)] \vee (\exists x)[Q(x)]) \vee (\exists y)[P(y) \vee Q(y)] \\
&((\forall x)[P(x)] \wedge (\forall x)[Q(x)]) \vee P(Y) \vee Q(Y) \\
&(\neg P(x_1) \wedge \neg Q(x_1)) \vee P(Y) \vee Q(Y) \\
&(\neg P(x_1) \vee P(Y) \vee Q(Y)) \wedge (\neg Q(x_1) \vee P(Y) \vee Q(Y))
\end{aligned}$$

(b)

$$\begin{aligned}
&(\exists x)[P(x)] \Rightarrow (\exists z)[(\forall x)[Q(x, z)] \vee (\forall x)[R(x, y, z)]] \\
&\neg(\exists x)[P(x)] \vee (\exists z)[(\forall x)[Q(x, z)] \vee (\forall x)[R(x, y, z)]] \\
&(\forall x)[\neg P(x)] \vee (\exists z)[(\forall x)[Q(x, z)] \vee (\forall x)[R(x, y, z)]] \\
&\neg P(x_1) \vee [Q(x_2, Z) \vee R(x_3, y, Z)
\end{aligned}$$

(c)

$$\begin{aligned}
&(\forall x)[P(x) \Rightarrow Q(x, y)] \Rightarrow ((\exists y)[P(y)] \wedge (\exists v)[Q(y, v)]) \\
&\neg(\forall x)[\neg P(x) \vee Q(x, y)] \vee ((\exists y)[P(y)] \wedge (\exists v)[Q(y, v)]) \\
&(\exists x)[P(x) \wedge \neg Q(x, y)] \vee (P(Y) \wedge Q(y, V)) \\
&(P(X) \wedge \neg Q(X, y)) \vee (P(Y) \wedge Q(y, V)) \\
&(P(X) \vee P(Y)) \wedge (P(X) \vee Q(y, V)) \wedge (\neg Q(X, y) \vee P(Y)) \wedge (\neg Q(X, y) \vee Q(y, V))
\end{aligned}$$

Problem 8 **Solution:**

(a)

$$
\begin{aligned}
&(\exists x)[Blue(x) \wedge Push(x)] \Rightarrow (\forall y)[\neg Push(y) \Rightarrow Green(y)] \\
&(\forall x)[(Blue(x) \wedge \neg Green(x)) \vee (\neg Blue(x) \wedge Green(x))] \\
&(\exists x)[\neg Push(x)] \Rightarrow (\forall y)[Push(y) \Rightarrow Blue(y)] \\
&Push(01) \\
&\neg Push(02)
\end{aligned}
$$

(b)

$$
\begin{aligned}
&\neg Blue(x_1) \vee \neg Push(x_1) \vee Push(y_1) \vee Green(y_1) \\
&(Blue(x_2) \vee Green(x_2)) \wedge (\neg Blue(x_2) \vee \neg Green(x_2)) \\
&Push(x_3) \vee \neg Push(y_2) \vee Blue(y_2) \\
&Push(01) \\
&\neg Push(02)
\end{aligned}
$$

(c) answer term $(\exists x)Green(x)$.
Negating the answer term, obtain $\neg Green(x_4)$.

$$\neg Blue(x_1) \vee \neg Push(x_1) \vee Push(y_1) \vee Green(y_1) \tag{14}$$
$$(Blue(x_2) \vee Green(x_2)) \wedge (\neg Blue(x_2) \vee \neg Green(x_2)) \tag{15}$$
$$Push(x_3) \vee \neg Push(y_2) \vee Blue(y_2) \tag{16}$$
$$Push(01) \tag{17}$$
$$\neg Push(02) \tag{18}$$
$$\neg Green(x_4) \tag{19}$$
$$Push(02) \vee \neg Push(01) \vee Blue(01)\{16, x_3/02, y_2/01\} \tag{20}$$
$$Blue(01) \text{ (17,18,19,20)} \tag{21}$$
$$\neg Blue(01) \vee \neg Push(01) \vee Push(02) \vee Green(02)\{14, x_1/01, y1/02\} \tag{22}$$
$$Green(02)\{17, 18, 21, 22, \} \tag{23}$$
$$\neg Green(02)\{19, x_4/02\} \tag{24}$$
$$Empty\{23, 24\} \tag{25}$$