CS 260 Fundamentals of the Design and Analysis of Algorithms **Fall 2016**

HomeWork 6

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In this homework, $build(x, y, c)$ means build a edge from x to y which capacity c. s is the source, t is the sink.

Problem 6 **Solution:**

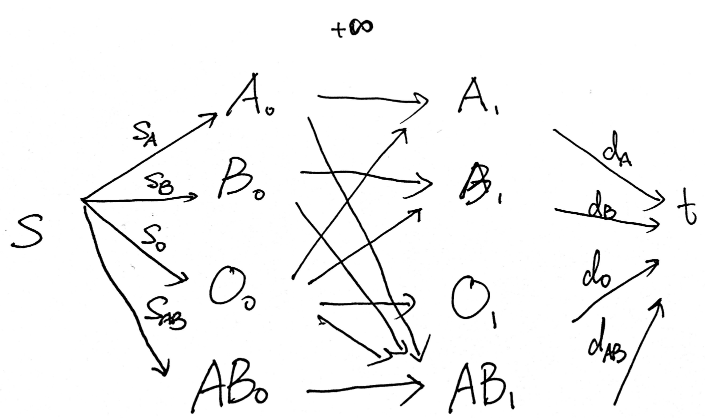
Consider a bipartite graph, left part is fixture, right part is switch. For every pair of fixture and switch, draw a line segment between these two point, and test whether it intersects with any boundary segment. If it does not intersect with any boundary then we draw an edge between these two points.

Next, run the bipartite graph matching algorithm. If the result is n then it is possible to make such an arrangement. Otherwise it does not.

Problem 7 **Solution:**

for every client c_i, $build(s, c_i, 1)$. for every base stationb_i $build(b_i, t, L)$. for every pair of clientc_i and base station b_j. If c_i can connect to b_j then $build(c_i, b_j, 1)$. Then run the maximum flow algorithm. If the result is same as the number of clients, then all clients can be assigned to a base station.

Problem 8 **Solution:**



(a)

Run maximum flow algorithm on above graph. If $d_A + d_B + d_O + d_{AB} =$ Maximum flow then it is sufficient.

(b) Maximum is 99. All B,AB patient will be served and one of A patient or one of O patient could not get the blood.

Explain: the supply of A and O is 86, but the demand of A and O is 87. So at least a patient of type A or O can not be served.

Problem 9 **Solution:**

for every injured peoplep_i, $build(s, p_i, 1)$, for every hospital h_i, $build(h_i, t, \lceil \frac{n}{k} \rceil)$. Then for every pair of peoplep_i and hospitalh_j, $build(p_i, h_j, 1)$. If the result is n, then it is possible.

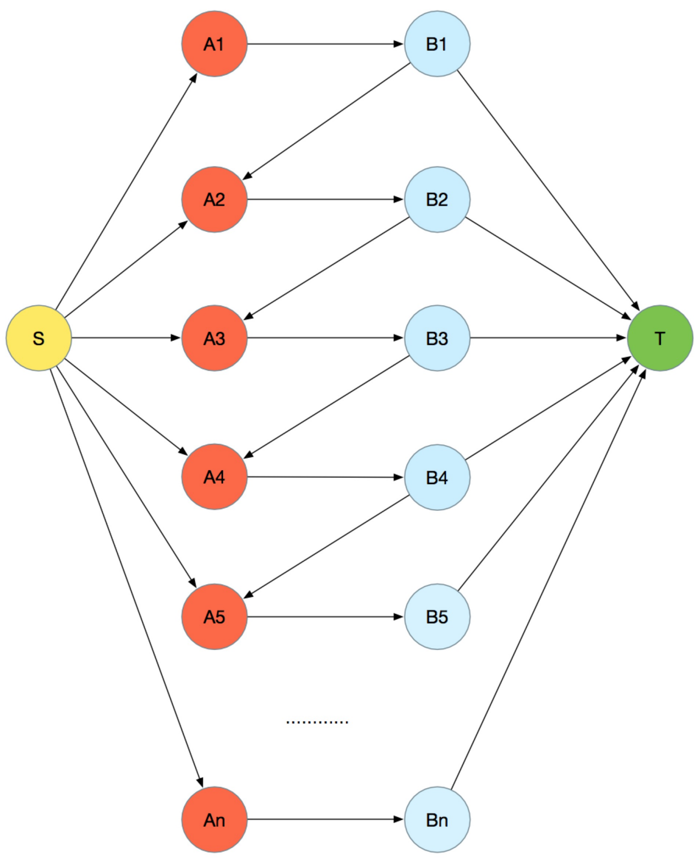
Problem 10 **Solution:**

Assume that e^* connects u and v. If e^* is not in the cut, then the maximum flow does not change.

Otherwise, in the residual network. Think about now we set t as a new source and s as a new sink. Then finding a path from t to v, and a path u to s. and reduce all the flow in this path by one.

Problem 11 **Solution:**

No, it is not.For any positive integer number n, construct the follow network(all edges' capacity is 1).



In this network, apparently the maximum flow is n. But if we find the augment path $s \rightarrow A_1 \rightarrow B_1 \rightarrow A_2 \rightarrow B_2 \rightarrow ... \rightarrow A_n \rightarrow B_n \rightarrow t$ first. Then we can not find any other augment path if we use the algorithm mentioned in the question. Thus the ratio is $\frac{1}{n}$. Because n could be any larger positive number, so there is no such b.

Problem 24 **Solution:**

Run the maximum flow algorithm in the original network. Then in the residual network, if one edge is not full, called it valid. Starting from the s, find the node set that can be reached from s through valid edge and call it set $\mathcal{S}$. From t find the node set that can be reached to the t through valid edge call it $\mathcal{T}$. If $|\mathcal{S}|+|\mathcal{T}|= n$ then, it is a unique cut. Otherwise it is not.