CS 260 Fundamentals of the Design and Analysis of Algorithms **Fall 2016**

HomeWork 8

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In this homework, $build(x, y, c)$ means build a edge from x to y which capacity c. s is the source, t is the sink.

Problem 1 **Solution:**

(a) Yes, interval scheduling is a P-problem, so it can be solved even without the access of black box of Vertex Cover.

(b) No, interval scheduling is a P-problem, it would implied P=NP;

Problem 2 **Solution:**

For any given graph G, construct a customer for each node, a product for each edge. Then ask diverse set black box. Any diverse set of size k will implied an independent set of size k in graph G.Thus$Independent\ Set \leq_p Diverse\ Set$. Thus it is a NPC problem.

Problem 3 **Solution:**

Consider every counselors as a set of his/her qualified sports. Then it is a set cover problem which is NPC hard.

Problem 4 **Solution:**

(a) It is a NP hard problem. For any given graph G, construct a processor for each node, a resource for each edge. Then ask diverse set black box. Any diverse set of size k will implied an independent set of size k in graph G.Thus$Independent\ Set \leq_p Resource\ Reservation$. Thus it is a NPC problem.

(b) It is a P problem, enumerate every combination of 2 processor and check whether it is compatible.

(c) It is a maximum bipartite graph matching. Node in left part represent people, and right part represent equipment. Any processor would link between the corresponding node. The maximum match is the maximum sub set of processor could be found if any two of them are compatible.

(d) Same in (a), still NPC.

Problem 5 **Solution:**

For any given graph G, construct a a_i for each node, a b_i for each edge and b_i contains the two end points of this edge. Then ask hitting set black box. Any hitting set of size k will implied an vertex cover set of size k in graph G.Thus$Vetex\ Cover \leq_p Hitting\ Set$. Thus it is a NPC problem.

Problem 6 **Solution:**

For any given graph G, construct a x_i for each node, a clauses for each edge and this clauses is the disjoint of two end points' variable. Then ask hitting set black box. Any hitting set of size k will implied an vertex cover set of size k in graph G.Thus$Vetex\ Cover \leq_p Monotone\ Satisfiability\ with\ Few\ True\ Variables$. Thus it is a NPC problem.

Problem 7 **Solution:**

For any 3-D matching problem, we add one dimension and the forth dimension set has enough element to extend all triple in collection C by adding an number and non of the forth dimension is same. Then we can ask this new question to 4-D dimension matching black box. Any set of size k in 4-D problem will implied a solution of size k in 3-D problem by simply omitting the last dimension. Thus 3-$D\ Matching \leq_p$ 4-$D\ Matching$.