

HomeWork 6

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Problem 1 **Solution:**

For every node, we record how many leaves in its subtree.

Then we found the corresponding node of q , the number of leaves is same as the numebr of match postion.

Problem 2 **Solution:**

Assume tree size is n and it is 0-index.

If n is an odd number, $\text{unrank}(\lfloor \frac{n}{2} \rfloor)$ will return the answer.

Otherwise, the median of $\text{unrank}(\lfloor \frac{n}{2} \rfloor)$ and $\text{unrank}(\lfloor \frac{n}{2} \rfloor - 1)$ is the answer.

Problem 3 **Solution:**

$s > t$: when $s > t$, s operations at most change s different memory cells. Thus the lower bound is $\Omega(s \log s)$.

$s < t$: Assume $x = t \bmod s$.

Decompsing t opeations into several different group each with size s and a extra one group with size x . Then for group with size s , it is $\Omega(s \log s)$. For the group with size x , from $s < t$, we konw it is $\Omega(x \log x)$. Thus the final answer is $\lfloor \frac{t}{s} \rfloor * s \log s + x \log x = \Omega(s \log t)$. Therefore it is $\min(s \log s, s \log t)$.

Problem 4 **Solution:**

$$\sum_{x=1}^i \sum_{y=x}^i 2^y = \sum_{x=1}^i x 2^x = (i-1)2^{i+1} + 2$$