CS 261 - Data Structure **Spring 2017**

HomeWork 5

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Problem 1 **Solution:**

there would be one tree with 64 nodes.

Problem 2 **Solution:**

Let's say the potential function is $f(n) = cn \log n$.

Insert operation: $\log n + f(n+1) - f(n) = \log n + c(n+1)\log n + 1 - cn \log n = O(\log n)$

Delete operation: $\log n + f(n-1) - f(n) = \log n + c(n-1)\log n - 1 - cn \log n = O(1)$

Problem 3 **Solution:**

For all even number n, n-element heap has a element with one child.

The child is lay on right most position of the last layer. This element would be on the top of his child.

Problem 4 **Solution:**

Assume n,m is the number of nodes and edges in a graph. When applying Dijkstra with k-ary heap. The time complexity is $O(m \log_k n + nk \log_k n)$

Now, $n = 2^d, m = d2^{d-1}$, let's say $f(k) = d2^{d-1}\log_k 2^d + 2^d k \log_k 2^d$

$$
\begin{aligned}
f(k) &= d2^{d-1}\log_k 2^d + 2^d k \log_k 2^d \\
&= d2^{d-1}\frac{\log_2 2^d}{\log_2 k} + 2^d k \frac{\log_2 2^d}{\log_2 k} \\
&= \frac{d^2 2^{d-1} + kd2^d}{\log_2 k}
\end{aligned}
$$

Take deravative of $f(k)$

$$f'(k) = \frac{d2^d \log_2 k - \frac{(d^2 2^{d-1} + kd2^d)}{k \ln 2}}{\log_2^2 k}$$

assume $f'(k) = 0$, obtain that

$$\frac{d}{2} = k \ln k - k$$

the solution of k would give us the best bound

But what is the best bound? Appartently if we choose $k = d/2$, we can obtain a bound of $O(\frac{d^2 2^d}{\log d})$. Now I am going to approve this bound is same as the bound of best k. To achieve that, we need prove the following:

for a constant C and the best k_0 which follows $\frac{d}{2} = k_0 \ln k_0 - k_0$, the following inequlity should holds:

$$\frac{d^2 2^{d-1} + k_0 d 2^d}{\log_2 k_0} \geq C \frac{d^2 2^d}{\log_2 d}$$

$$C \leq \min_d \left(\frac{\log d}{\log k_0} * \frac{d + 2k_0}{2d} \right)$$

$$\frac{\log d}{\log k_0} * \frac{d + 2k_0}{2d} \geq \frac{\log d}{\log k_0} * \frac{1}{2} \geq \frac{1}{2}$$

Thus there is a constant $C < \frac{1}{2}$ to achieve this inequlity. Now we can say the best bound is indeed $O(\frac{d^2 2^d}{\log d})$.