

HomeWork 2

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Problem 1 **Solution:**

$$\begin{aligned}
Pr &= \binom{n}{s-1} \left(\frac{1}{N}\right)^{s-1} \left(\frac{N-1}{N}\right)^{N-s} \\
&\leq \frac{n(n-1)(n-2)\dots(n-s+1)}{(s-1)!} * \left(\frac{1}{N}\right)^{s-1} \\
&\leq \frac{n^{s-1}}{N^{s-1} * (s-1)!} \\
&= \frac{\alpha^{s-1}}{(s-1)!}
\end{aligned}$$

Problem 2 **Solution:**

a the probability of a pair entries has same key is $(N/2)^{-2}$.

Thus the the probability of a triple entries has same key is $(N/2)^{-4}$.

The number of all possible triple is $\binom{n}{3} = \frac{n(n-1)(n-2)}{6}$.

Hence $E = (N/2)^{-2} * \binom{n}{3} = \frac{8n(n-1)(n-2)}{3N^4}$.

b

$$\begin{aligned}
Pr[R > t] &\leq E[R]/t \text{ with } t = 1 \\
\rightarrow Pr[R > t] &\leq \frac{8n(n-1)(n-2)}{3N^4} \\
&\leq \frac{8n^3}{3N^4} \\
&\leq \frac{8}{3} \alpha^4 \frac{1}{n} \\
&= O\left(\frac{1}{n}\right)
\end{aligned}$$

Problem 3 **Solution:**

```

def CheckSingle(Input):
    dic = {}
    for item in Input:
        if dic.has_key(item):
            dic[item] += 1
        else:
            dic[item] = 1

```

```
ans = []
for key in dic:
    if dic[key] == 1:
        ans.append(key)
return ans
```

Problem 4 **Solution:**

Cuckoo. Because, using Cuckoo, looking up time is $O(1)$. For two of other algorithm, the worst time could be $O(n)$.