MAE 552 Heuristic Optimization

HOMEWORK #3

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1. INTRODUCTION

Problem given is to design the following three major optimization algorithms:

- Particle Swarm Optimization
- Bayesian Optimization
- Ant Colony Optimization

2. Coding for Particle Swarm Optimization

2.1: Coding Implementation

The PSO algorithm has been implemented using Python class and functions. The following are the parameter values set during implementation:

- Population size (Number of particles): 50
- Number of decision variables: 3
- Local search coefficient (localc): 1.6
- Global search coefficient (globalc): 1
- Inertial weight coefficient (alpha): 0.5
- Maximum iterations: 50

2.2: Implementing the code and solving the benchmark problems

Here, the objective functions mentioned in the problem statement have been implemented. The objective functions are as follows: De Jong's function 1, Rosenbrock's function , Rastrigin's function, Griewangk's function, Schwefel's function, Ackley's function, and Michalewicz's function.

The results and observations for all functions are as follows:

2.2.1: De Jong's function 1 (Bounds: [-5.12, 5.12])

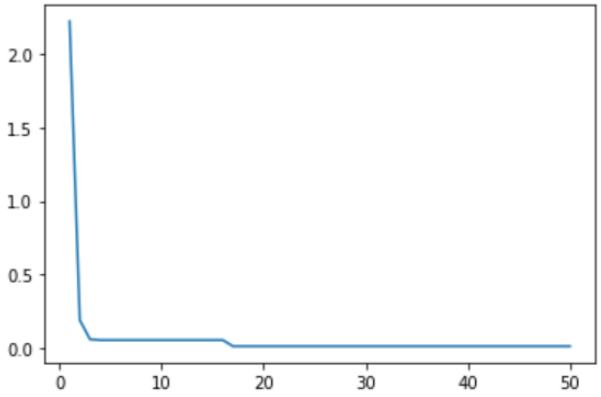
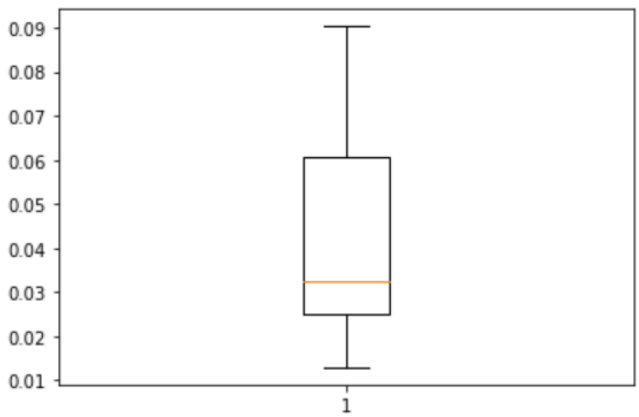


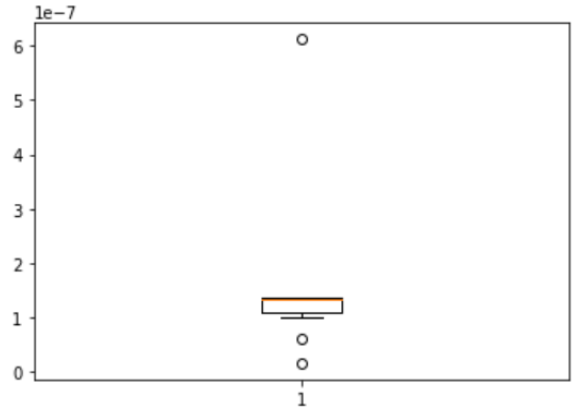
Figure 1: De Jong's Convergence

```
 DeJong function activated for 10 runs.
Global best in run 1 : 0.10176879139026553 Global best position in run 1 : [-0.27343577  0.0004977   0.16432109]
Global best in run 2 : 0.02703101143108667 Global best position in run 2 : [-0.05615966  0.06511535 -0.14013242]
Global best in run 3 : 0.02562370544768027 Global best position in run 3 : [0.04321949 0.14068146 0.06296433]
Global best in run 4 : 0.06059956672003834 Global best position in run 4 : [ 0.20108119  0.13372944 -0.04777407]
Global best in run 5 : 0.12760618012324254 Global best position in run 5 : [-0.13055327  0.32058187  0.08825693]
Global best in run 6 : 0.01883861909053784 Global best position in run 6 : [ 0.00036401  0.13400265 -0.0296947 ]
Global best in run 7 : 0.029983828414916317 Global best position in run 7 : [-0.11031692 -0.09494145 -0.09380899]
Global best in run 8 : 0.023626328481123834 Global best position in run 8 : [-0.03171117 -0.1248369  -0.08388372]
Global best in run 9 : 0.007280043344525382 Global best position in run 9 : [ 0.01491858 -0.04442667  0.07130042]
Global best in run 10 : 0.026796805644543984 Global best position in run 10 : [ 0.11943444 -0.10611571 -0.03566056]
Number of decision variables: 3
Best value of Global best across 10 Runs:  0.007280043344525382
Mean value of Global best across 10 Runs:  0.044915488008796065
Standard deviation value of Global best across 10 Runs:  0.03756190301290961
```

Figure 2: Best values in 10 runs



PSO box plot



GA box plot

Figure 3: Box plots for De Jong's function

2.2.2: Rosenbrock's function (Bounds: [-2.048, 2.048])

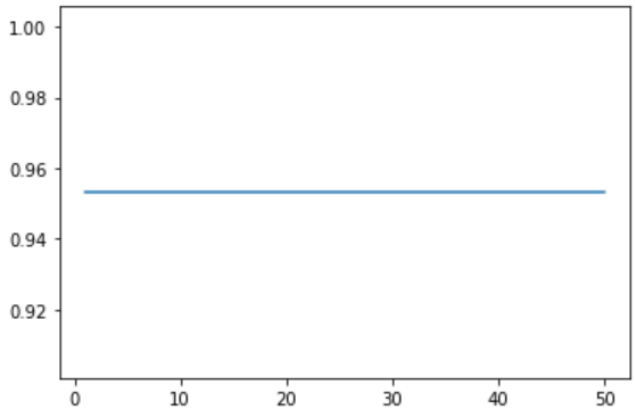
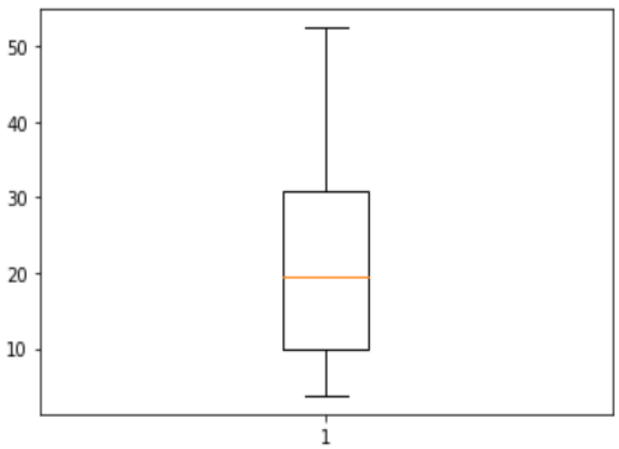


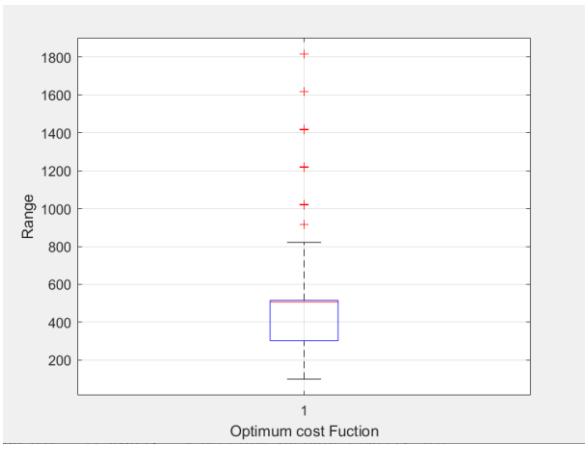
Figure 4: Rosenbrock's convergence

```
 Rosenbrock function activated for 10 runs.
Global best in run 1 : 52.47897179917541 Global best position in run 1 : [-0.6594717  -0.23637199 -0.12120197]
Global best in run 2 : 14.195225046059887 Global best position in run 2 : [-0.43504098  0.28050458  0.40709816]
Global best in run 3 : 6.953937036472851 Global best position in run 3 : [-0.03281861  0.08084739 -0.20336965]
Global best in run 4 : 31.186687866811344 Global best position in run 4 : [-0.99312071  0.76912293  0.11779113]
Global best in run 5 : 8.608827620513992 Global best position in run 5 : [1.09150806 1.22409742 1.2078417 ]
Global best in run 6 : 36.38476715258647 Global best position in run 6 : [ 0.53814599 -0.15103784  0.41562665]
Global best in run 7 : 29.693481461692002 Global best position in run 7 : [0.85596378 0.65229075 0.96312331]
Global best in run 8 : 16.29904350380528 Global best position in run 8 : [1.30777425 1.30896536 1.70703699]
Global best in run 9 : 22.526124252738995 Global best position in run 9 : [1.14324902 1.19254867 1.88215275]
Global best in run 10 : 3.7638772723029126 Global best position in run 10 : [-0.27848909 -0.02192703  0.03135427]
Number of decision variables: 3
Best value of Global best across 10 Runs:  3.7638772723029126
Mean value of Global best across 10 Runs:  22.20909430121592
Standard deviation value of Global best across 10 Runs:  14.529256137140354
```

Figure 5: Best values in 10 runs



PSO box plot



GA package box plot

Figure 6: Box plots for Rosenbrock's function

2.2.3: Rastrigin's function (Bounds: [-5.12, 5.12])

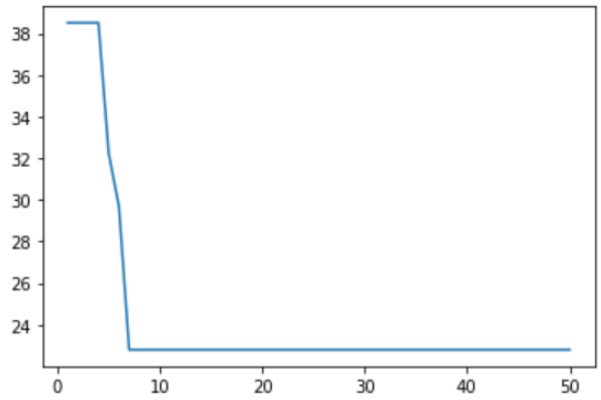
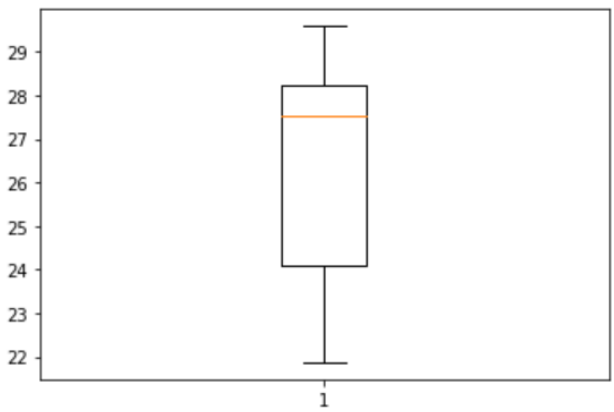


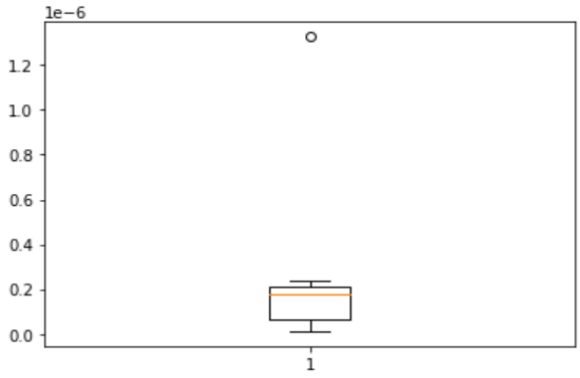
Figure 7: Rastrigin's convergence

```
 Rastrigin function activated for 10 runs.
Global best in run 1 : 28.168143214148056 Global best position in run 1 : [0.10608211 0.88949061 1.09451544]
Global best in run 2 : 23.235169252988676 Global best position in run 2 : [1.05026986 0.03802142 0.95234088]
Global best in run 3 : 34.7040268149429 Global best position in run 3 : [ 2.14057498  2.09518112 -0.04169539]
Global best in run 4 : 29.587048003209855 Global best position in run 4 : [-1.53662312e-03  1.97595933e+00 -2.08014408e+00]
Global best in run 5 : 27.39092359108743 Global best position in run 5 : [ 0.14117606  0.14019496 -0.01295243]
Global best in run 6 : 21.87171513999074 Global best position in run 6 : [ 1.0490211  -0.01184341  0.03716887]
Global best in run 7 : 26.611309130982725 Global best position in run 7 : [-0.11532021 -0.02152303  1.98143622]
Global best in run 8 : 23.099701493184845 Global best position in run 8 : [ 1.00892001  0.06933209 -0.96907087]
Global best in run 9 : 28.25936026087814 Global best position in run 9 : [-1.90876065 -0.0099045  -0.12618805]
Global best in run 10 : 27.633725521840677 Global best position in run 10 : [-0.93245561  0.93227262  0.86541807]
Number of decision variables: 3
Best value of Global best across 10 Runs:  21.87171513999074
Mean value of Global best across 10 Runs:  27.0561122423254
Standard deviation value of Global best across 10 Runs:  3.5405508331104016
```

Figure 8: Best values in 10 runs



PSO box plot



GA box plot

Figure 9: Box plots for Rastrigin's function

2.2.4: Griewangk's function (Bounds: [-600, 600])

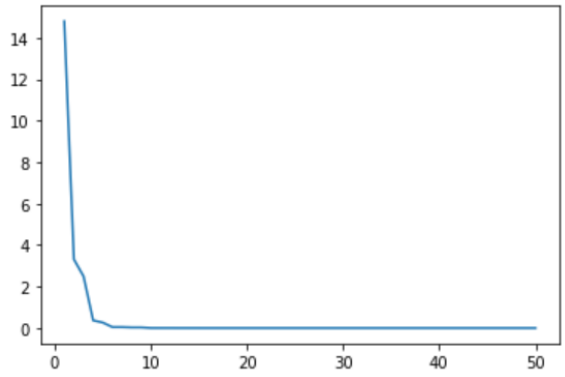
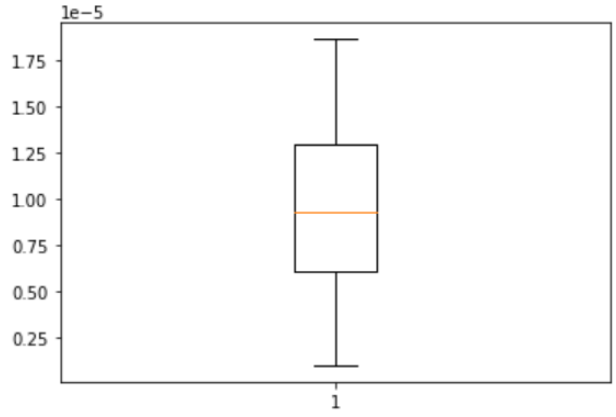


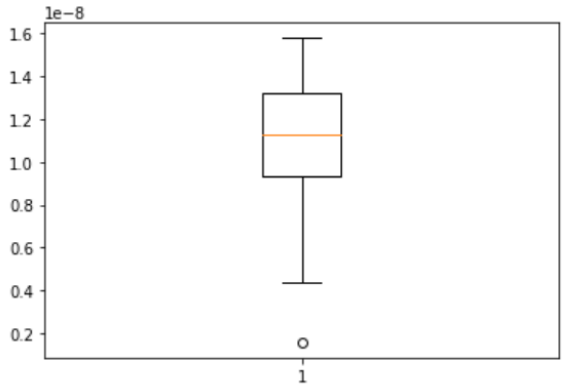
Figure 10: Griewangk's convergence

```
 Griewangk function activated for 10 runs.
Global best in run 1 : 5.704797257639128e-06 Global best position in run 1 : [0.01083812 0.02731622 0.14817405]
Global best in run 2 : 1.0638021482426104e-05 Global best position in run 2 : [-0.09906741 -0.07104863 -0.1664026 ]
Global best in run 3 : 7.929265863983726e-06 Global best position in run 3 : [-0.06148601 -0.1557472  -0.06065759]
Global best in run 4 : 1.7098514718969935e-05 Global best position in run 4 : [-0.24765187  0.08402642 -0.00147399]
Global best in run 5 : 1.0168168865821092e-06 Global best position in run 5 : [ 0.00545289 -0.05546093  0.03100998]
Global best in run 6 : 3.016146410321009e-06 Global best position in run 6 : [-0.04436202 -0.04375114  0.09045681]
Global best in run 7 : 1.2987329963501039e-05 Global best position in run 7 : [ 0.16030102 -0.02823676 -0.15954808]
Global best in run 8 : 7.075978654725497e-06 Global best position in run 8 : [-0.05333242  0.09534752  0.12793912]
Global best in run 9 : 1.2846739213976242e-05 Global best position in run 9 : [-0.18981137 -0.12221399  0.02055101]
Global best in run 10 : 1.8634294140985333e-05 Global best position in run 10 : [ 0.23604194 -0.05682371  0.12486972]
Number of decision variables: 3
Best value of Global best across 10 Runs:  1.0168168865821092e-06
Mean value of Global best across 10 Runs:  9.694790459311012e-06
Standard deviation value of Global best across 10 Runs:  5.497289794666149e-06
```

Figure 11: Best values in 10 runs



PSO box plot



GA box plot

Figure 12: Box plots for Griewangk's function

2.2.5: Schwefel's function (Bounds: [-500, 500])

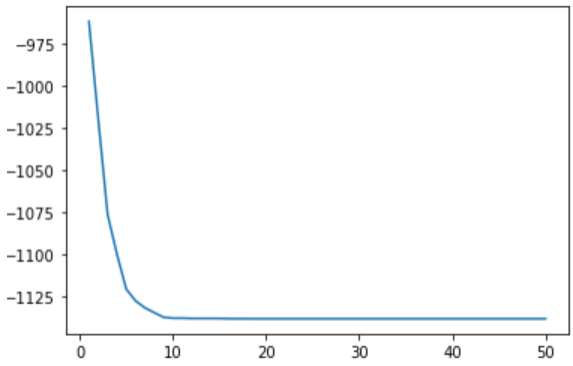
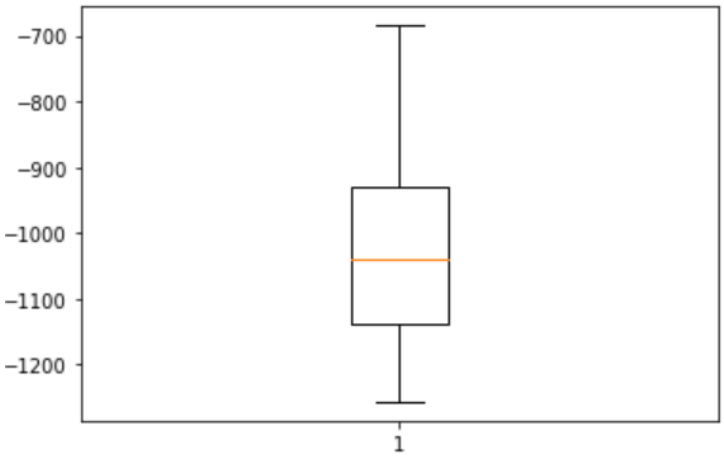


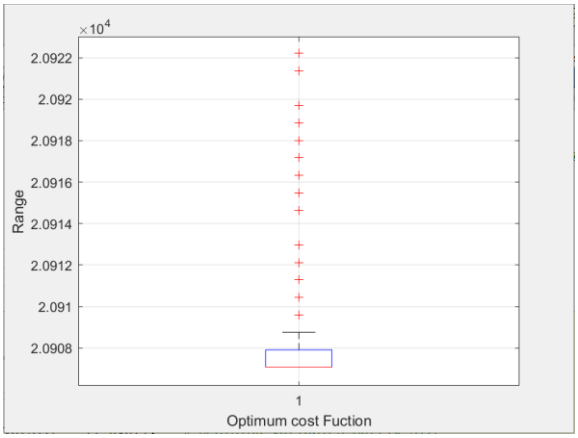
Figure 13: Schwefel's convergence

```
 Schwefel function activated for 10 runs.
Global best in run 1 : -1039.8043485146286 Global best position in run 1 : [204. 421. 421.]
Global best in run 2 : -1138.508810436308 Global best position in run 2 : [ 421.01108326  421.         -302.42892587]
Global best in run 3 : -1020.06866772371 Global best position in run 3 : [-302.53875114  421.         -302.6833254 ]
Global best in run 4 : -901.5985478473782 Global best position in run 4 : [421.         65.6695585 421.        ]
Global best in run 5 : -1256.948292048433 Global best position in run 5 : [421. 421. 421.]
Global best in run 6 : -1138.5093594567034 Global best position in run 6 : [ 420.90391016 -302.47513673  421.        ]
Global best in run 7 : -684.4565554973358 Global best position in run 7 : [203.75640482 420.80930712  65.46553509]
Global best in run 8 : -1039.8074800339823 Global best position in run 8 : [203.79295159 421.         420.86602805]
Global best in run 9 : -723.9637685095333 Global best position in run 9 : [-302.44439567 -124.87850367 -302.57951987]
Global best in run 10 : -1138.510079566624 Global best position in run 10 : [-302.52791429  421.          421.        ]
Number of decision variables: 3
Best value of Global best across 10 Runs:  -1256.948292048433
Mean value of Global best across 10 Runs:  -1008.2175909634637
Standard deviation value of Global best across 10 Runs:  176.6125696100542
```

Figure 14: Best values for 10 runs



PSO box plot



GA package box plot

Figure 15: Box plots for Schwefel's function

2.2.6: Ackley's function (Bounds: [-1, 1])

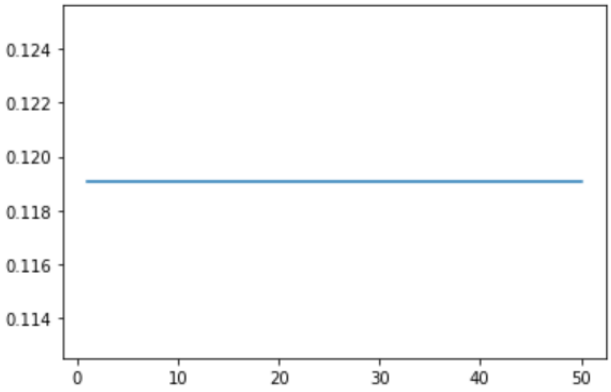
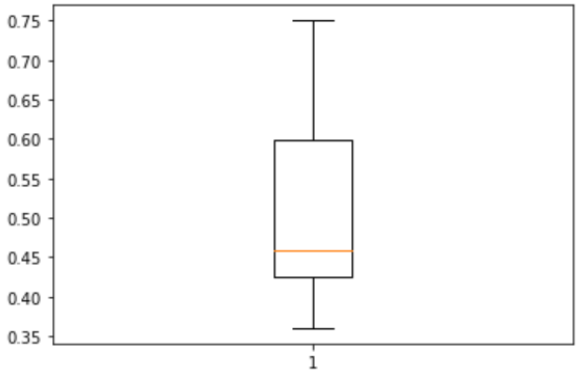


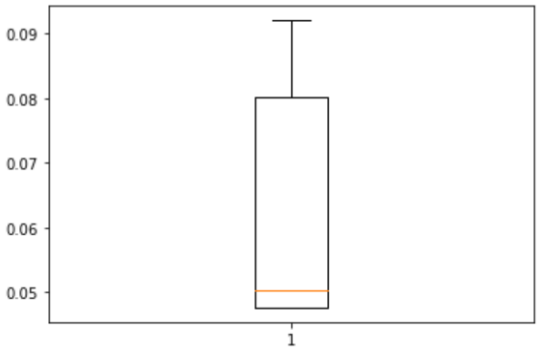
Figure 16: Ackley's Convergence

```
 Ackley function activated for 10 runs.
Global best in run 1 : 0.8599281584324641 Global best position in run 1 : [-0.20292243  0.12949605  0.19037284]
Global best in run 2 : 0.4762303109678112 Global best position in run 2 : [ 0.04967162 -0.15039557  0.09054527]
Global best in run 3 : 0.4396660063363176 Global best position in run 3 : [-0.16722229 -0.02752142 -0.0108869 ]
Global best in run 4 : 0.6267690700599613 Global best position in run 4 : [ 0.12457515  0.1058812  -0.1659562 ]
Global best in run 5 : 0.7504979565622176 Global best position in run 5 : [ 0.2229224   0.13736127 -0.0769533 ]
Global best in run 6 : 0.42543575806535516 Global best position in run 6 : [-0.03918685  0.16008483  0.00179524]
Global best in run 7 : 0.5144557375818946 Global best position in run 7 : [ 0.08253393 -0.174458    0.03163241]
Global best in run 8 : 0.3600688691637477 Global best position in run 8 : [-0.076234    0.06812893 -0.09781552]
Global best in run 9 : 0.42616964392863155 Global best position in run 9 : [ 0.02308146  0.14300974 -0.07907309]
Global best in run 10 : 0.1473312654222494 Global best position in run 10 : [-0.0012418  -0.05018947  0.03480359]
Number of decision variables: 3
Best value of Global best across 10 Runs:  0.1473312654222494
Mean value of Global best across 10 Runs:  0.502655277652065
Standard deviation value of Global best across 10 Runs:  0.1917493833732025
```

Figure 17: Best values in 10 runs



PSO box plot



GA box plot

Figure 18: Box plot for Ackley's function

2.2.7: Michalewicz's function (Bounds: [0, pi])

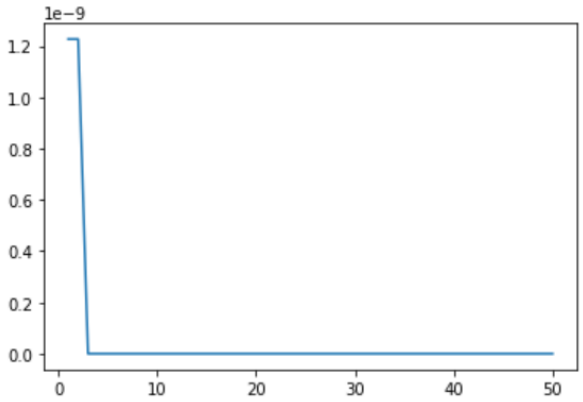
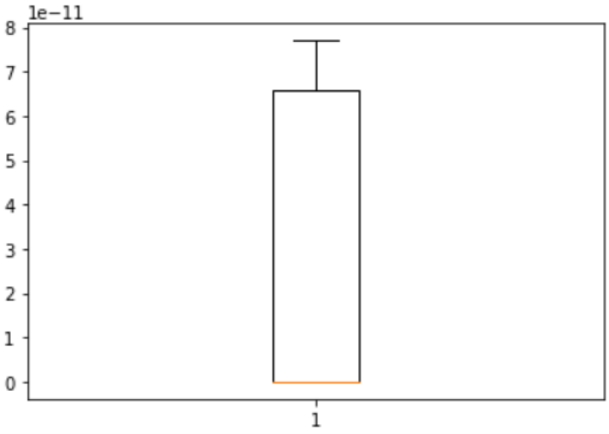


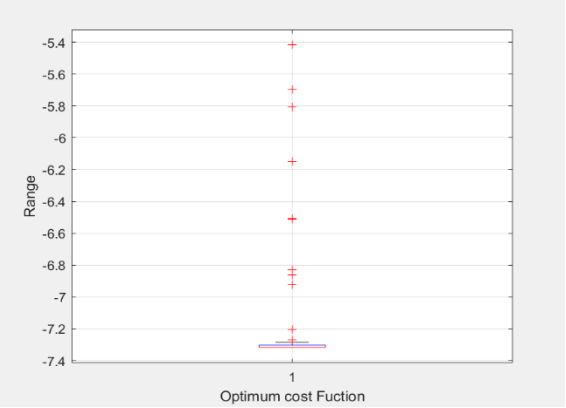
Figure 19: Michalewicz's Convergence

```
 Michalewicz function activated for 10 runs.
Global best in run 1 : 2.713305902145059e-29 Global best position in run 1 : [3.14159265 3.09795833 3.14159265]
Global best in run 2 : 1.3230341877536384e-13 Global best position in run 2 : [0.19079391 2.89575777 0.20249623]
Global best in run 3 : 7.701257069402153e-11 Global best position in run 3 : [2.89894944 0.33619671 3.0549303 ]
Global best in run 4 : 6.873863677244713e-20 Global best position in run 4 : [0.12232477 0.10622784 0.02924544]
Global best in run 5 : 0.0 Global best position in run 5 : [0.         0.         3.14159265]
Global best in run 6 : 3.160754401716387e-11 Global best position in run 6 : [0.32174289 0.09660465 2.97593973]
Global best in run 7 : 1.3347692614385764e-23 Global best position in run 7 : [0.0815107  3.14159265 0.        ]
Global best in run 8 : 1.0628019196032663e-06 Global best position in run 8 : [2.67067602 0.54447359 3.14159265]
Global best in run 9 : 8.576211393425942e-09 Global best position in run 9 : [0.28209301 2.7174396  2.76992829]
Global best in run 10 : 0.0 Global best position in run 10 : [0. 0. 0.]
Number of decision variables: 3
Best value of Global best across 10 Runs:  0.0
Mean value of Global best across 10 Runs:  1.0714868834148907e-07
Standard deviation value of Global best across 10 Runs:  3.1856130720965266e-07
```

Figure 20: Best values for 10 runs



PSO box plot



GA package box plot

Figure 21: Box plots for Michalewicz's function

3. Bayesian Optimization

3.1: Coding implementation

The Bayesian Optimization code is implemented to solve the single objective continuous optimization problem given by the code testfunc_HW4. This problem contains two design variables with their bounds as: $x1 \in [-3,3]$; $x2 \in [-2,2]$. The MATLAB inbuilt function "bayesopt" is used for this implementation.

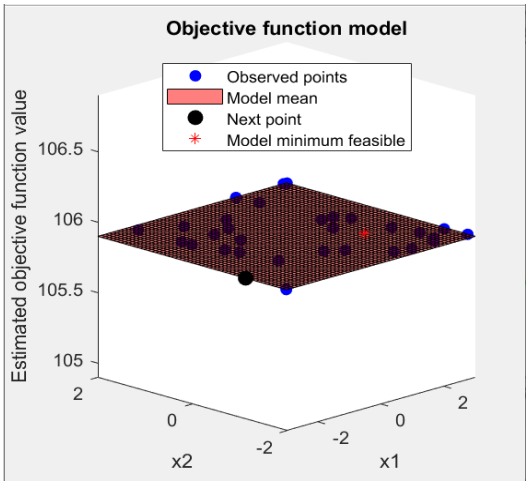


Figure 22: Estimated objective function value

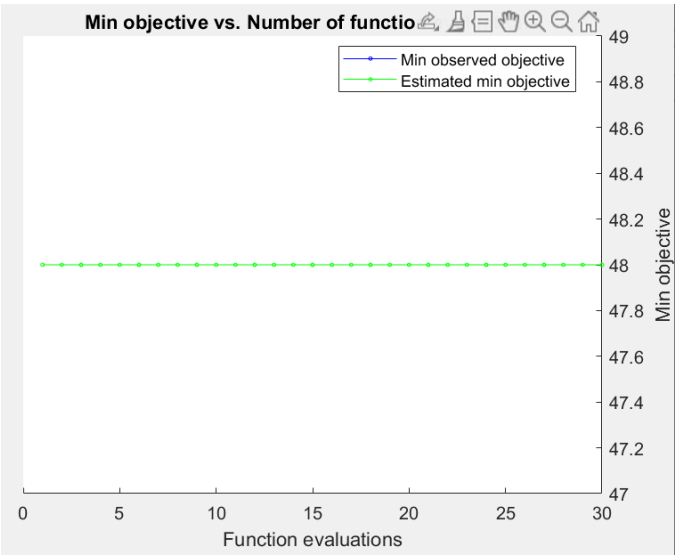


Figure 23: Convergence history of BO

```
Optimization completed.
MaxObjectiveEvaluations of 30 reached.
Total function evaluations: 30
Total elapsed time: 20.3541 seconds
Total objective function evaluation time: 0.038963

Best observed feasible point:
      x1          x2
    _______    ________

    -2.6137    -0.27596

Observed objective function value = 3.7333
Estimated objective function value = 3.7333
Function evaluation time = 0.013651

Best estimated feasible point (according to models):
      x1         x2
    _______    _______

    0.64187    0.74691

Estimated objective function value = 3.7333
Estimated function evaluation time = 0.00069586
```

Figure 24: BO Optimum feasible points and function values

4. Coding for Ant Colony Optimization

4.1: Coding Implementation for ACO

The following were the parameters set during implementation:

- Number of ants: 100
- Pheromone exponent parameter (alpha): 0.85
- Heuristic information parameter (beta): 1.7
- Pheromone evaporation rate (rho): 0.6
- Pheromone reward factor (Q): 100
- Maximum iterations: 100
- Initial pheromone on paths between all cities: 10^{-6}
- Number of runs: 10

There are 52 cities for which the optimum path is to be generated. As per the values provided in the text file Berlin_52.txt, the coordinates of the cities are taken, and the code is implemented using ACO algorithm.

The following two plot shows the implemented code output as the best path and the optimum path provided as reference. The best path has been obtained by implementing and experimenting with the parameter multiple times.

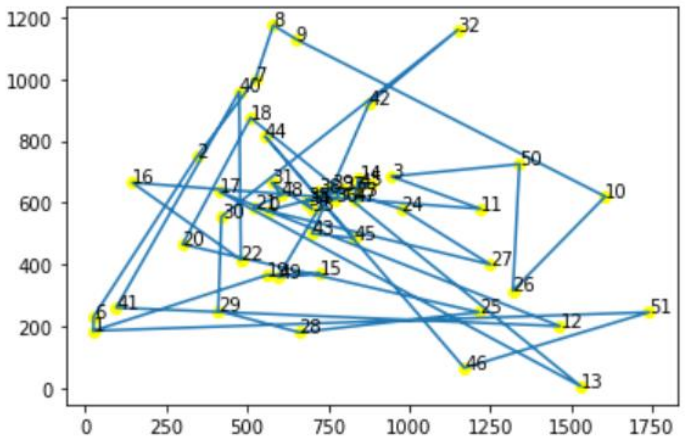


Figure 25: Implemented best path

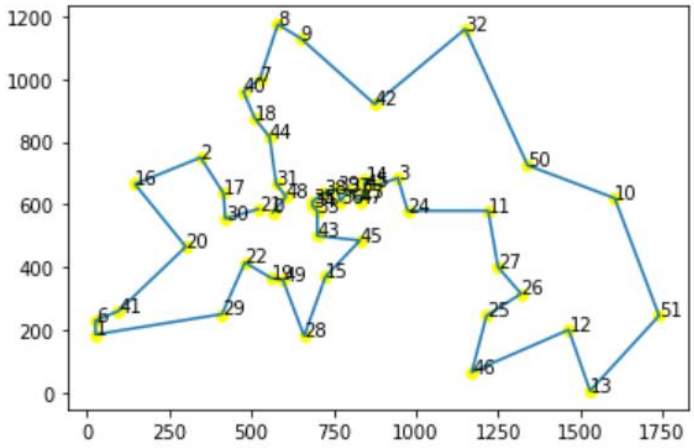


Figure 26: Best Optimal path

From the above plots, it can be inferred that there can be improvement in the implemented code by changing some logic and experimenting with the parameters.

- Best route obtained across 10 runs is as follows:

```
Best route taken by an ant across all runs [ 0.  5. 14.  4. 15.  3.  6.  8.  9. 31. 33. 34. 35. 38.  7. 21.  1. 10.
 11. 16.  2. 12. 13. 17. 18. 50. 19. 49. 22. 48. 20. 30. 23. 47. 24. 25.
 27. 26. 28. 51. 29. 32. 42. 36. 39. 37. 40. 41. 43. 44. 45. 46.  1.]
```

- Best Cost obtained across 10 runs is:

```
Best cost among all runs 14680.119692121321
```

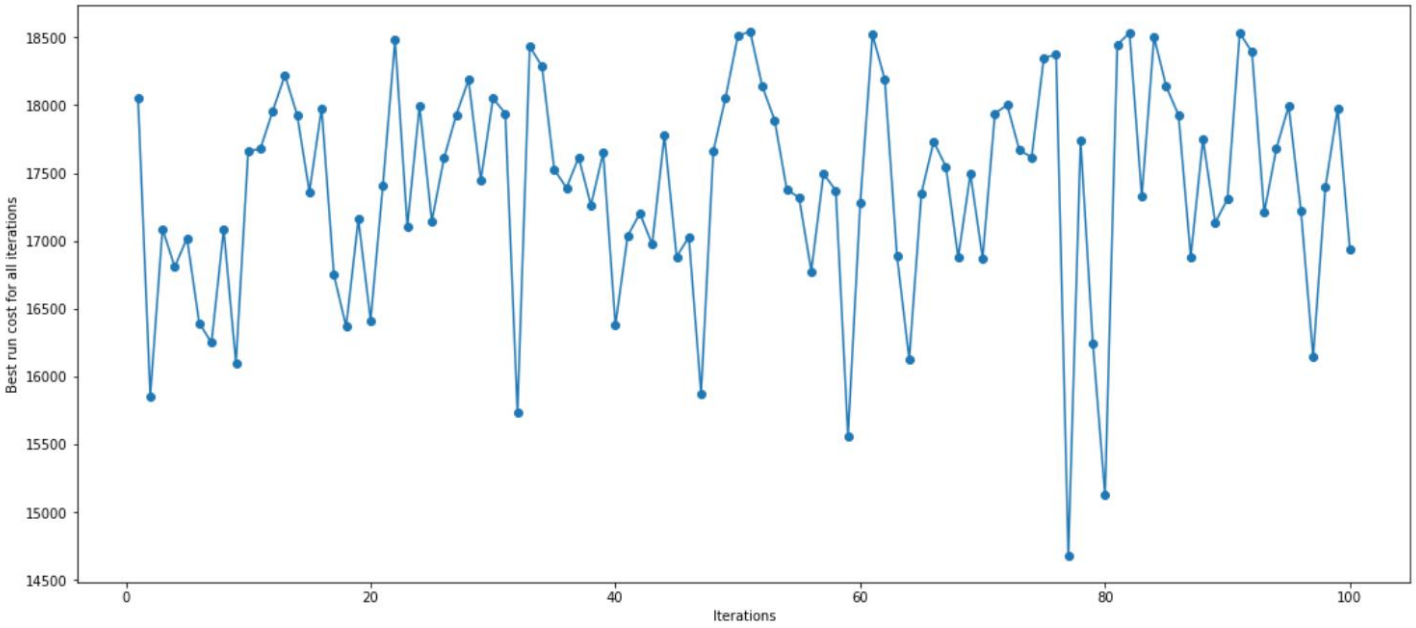
- Best Costs for all 10 runs are as follows:

```
Best Cost for Run 1 : 14844.168109237122

Best Cost for Run 2 : 15812.091249466966

Best Cost for Run 3 : 14963.914808573369

Best Cost for Run 4 : 15510.196170567433

Best Cost for Run 5 : 15017.678202067938

Best Cost for Run 6 : 14784.42638199111

Best Cost for Run 7 : 15056.34615569919

Best Cost for Run 8 : 14998.081474480288

Best Cost for Run 9 : 14892.444577914464

Best Cost for Run 10 : 14680.119692121321
```

- Mean and Standard deviation of the costs across all runs is as follows:

```
Mean of all the best costs in all runs 15055.94668221192
Standard Deviation of all the best costs in all runs 328.31880702099704
```

- The convergence of number of iterations and cost in the best representative run is as follows:



5. Conclusion

The three important optimization algorithms are implemented in this assignment. The PSO algorithm is a very efficient and strong algorithm as its computational cost is significantly less than Binary genetic algorithm. The results turn out to be better than the Binary GA as well.

For Ant colony optimization, Berlin 52 problem is solved which is a Travelling salesman problem. The ACO is a very robust algorithm which tends to give best results even if the problem to be solved is complex.