The Harmonic Oscillator with Modified Damping

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Abstract

In this paper we will be analyzing the long-term behavior and different effects of dampening levels applied to a harmonic oscillator. We will look at systems with linear, non-linear, and a special case of dampening. To have a better understanding of how these systems behave corresponding their coefficients and initial conditions.

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# Introduction

When working with a spring system we can vary the immediate and long-term behavior of the system by varying the damping coefficient on that system. In this paper we will analyze the different outcomes produced on our system with linear, nonlinear, and other forms of dampening. This analysis can serve as a broad basis for understanding some of the behaviors associated with spring force and dampening.

The damped harmonic oscillator is a very significant mathematical model which has real world applications in mass-spring systems, RLC circuits, and blood glucose regulatory systems.(Paul Blanchard, 183). Effectively understanding the affects of dampening on the system allows one to have better control in working with and understanding the system. In this paper we will analyze the different affects of linear, non-linear, and a special case of dampening on our system. We will interpret how the different models correspond in a broader sense and how the models vary between each other.

**Model**

In this model we will be working with variations of the equation:

Where *m* represents our mass, *b* will represent our damping coefficient and *k* will represent our spring system. We will study the equation with the following cases

|  |  |  |  |
| --- | --- | --- | --- |
| *Mass* | *Spring Constant* | *Dampening* | *Alpha* |
| 2 | 6 | 3 | 5 |
| 2 | 5 | 2 | 3 |

We will analyze three sperate equations under these conditions:

Eq 1:

Eq2:

Eq3:

The methods used for analyzing the system will be to decompose and rewrite the system as a first order system by introducing a new variable.

For example, Eq1 will be rewritten as follows:

For our graphs we will be using a python graphing programs(included with this paper) using libraries from matplot, numpy, and tinktor.

### Analysis.

Both instances which will be analyzed have the same mass. It seems that the differences we will see associated in our systems will be most dependent on our *spring constant*  and *Dampening*. In our first case we will have more dampening with a more tense spring. This shows us that the system will have a higher resistance but a stronger spring force. In our second case we have less dampening but more spring force. These different scenarios should allow for the analysis of whether the *k* or *b* will have the most effect on the behaviors of the system

Chart

Description automatically generated

Figure : Mass = 2 , Spring Constant = 6, Dampening = 3

Our first figure shows us that this system tends to oscillate around the origin while gradually sinking towards the origin. It seems that despite the initial position all solutions tend to equilibrium.

Chart

Description automatically generated

Figure : Mass = 2, Spring Constant = 5, Dampening = 2

It seems that there is no change in the qualitative behavior of our graph with the varying of our constants over this Eq1. It seems that solutions will descend towards zero with a period which is gradually decreasing.

#### Chart Description automatically generated

Figure : Mass = 2 , Spring K = 3, Dampening = 3

Moving from Eq1 to Eq2 we begin to see the distinction between nonlinear and linear damping. While it seems that this system still tends to equilibrium and towards the origin, we can easily notice the non-symmetries and magnitude fluctuations of this graph which are indicative of the non-linear behavior.

#### Chart Description automatically generated

Figure : Mass = 2, Spring K = 6, Dampening = 3, Alpha = 5

We notice a large jump in the qualitative behavior of our system moving into Eq3. It seems in this scenario. It seems that depending on the initial conditions the system can tend to zero or rather negative or positive infinity. This behavior is interesting as in all previous cases we saw that our system would tend to infinity.

##### **Discussion**

It seems that the overall behavior of our first two systems is periodic with fluctuations around the origin tending towards zero. It is intriguing to see the scenario where our system could tend towards infinity. It seems that our Eq 3 corresponds to a system where the first derivative or the velocity of the system is dependent on its own position with alpha acting as a resistant force. It seems that this system could correspond to one where there is some external force acting on the system where that force is a function of position. It seems that given the right initial conditions the system will still fall back into equilibrium, however the possibility that the system could tend towards infinity shows that perhaps this sort of system can be unstable. Understanding this sort of variation within the system can be critical in working with systems which are of this form. It would be good to do more research understanding the long-term behavior of Eq. 3 and its dependencies on initial conditions. The overall change in the behavior of our systems with the chosen coefficients was negligible. It would be interesting to examine the behavior of the systems with a broader scope of coefficients to truly grasp the fluctuations and dependencies within the system.