Plenum 26/10

8.3: 1 a, c, e, h, 3 a, b, c, g, 4 e, 7 a, b, 9, 13

8.4: 1 e), 2 a), 3 b), 5

8.3: 1) e) $\int_1^e \frac{1}{x}\,dx = [\ln x]_{x=1}^{e}$

$= \ln e - \ln 1 = 1 - 0 = \underline{\underline{1}}$

3) g) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\frac{1}{\cos^2 x} + \frac{1}{e^{7x}}\right) dx$

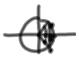
$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{\cos^2 x}\,dx + \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{e^{7x}}\,dx$

$= [\tan x]_{x=-\frac{\pi}{4}}^{\frac{\pi}{4}} + \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^{-7x}\,dx$

$= [\tan x]_{x=-\frac{\pi}{4}}^{\frac{\pi}{4}} + \left[e^{-7x}\left(-\frac{1}{7}\right)\right]_{x=-\frac{\pi}{4}}^{\frac{\pi}{4}}$

$= 1 + 1 + \left(-\frac{1}{7}\right)\left(e^{-\frac{7\pi}{4}} - e^{\frac{7\pi}{4}}\right)$

$= 2 + \frac{e^{\frac{7\pi}{4}} - e^{-\frac{7\pi}{4}}}{7}$



4.) e) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan x\,dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sin x}{\cos x}\,dx = [-\ln(\cos x)]$

$= -\ln(\cos\frac{\pi}{4}) + \ln(\cos(-\frac{\pi}{4}))$

$= \underline{\underline{0}}$

7.) b) $\lim_{x\to\infty} \frac{\int_1^x e^{\frac{1}{t}}\,dt}{x^2} = \lim_{x\to\infty} \frac{\frac{d}{dx}[\int_1^x e^{\frac{1}{t}}\,dt]}{2x}$

"$\frac{\infty}{\infty}$": L'Hôpital

Deriv...

Analysens fund. teorem

$= \lim_{x\to\infty} \frac{e^{\frac{1}{x}}}{2x} = \underline{\underline{0}}$

9) Vil vise: f kont. Vise at fins $c \in (a, b)$ s.a. $\int_a^b f(x)\,dx = f(c)(b-a)$.

Middelverdisetningen for integraler

La $F(x) := \int_d^x f(t)\,dt$ der $d < a$.

Merk: F er veldefinert fordi f er kont., dermed integrerbar.

Da er F kont. og deriverbar; $F'(x) = f(x)$

Analysens fund. teorem

Se på $[a, b]$. Da gir middelverdisetningen at det fins $c \in (a, b)$ s.a.

$F'(c) = \frac{F(b) - F(a)}{b-a}$, dvs.

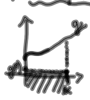
$f(c)(b-a) = \int_d^b f(t)\,dt - \int_d^a f(t)\,dt$

$f(c)(b-a) = \int_a^b f(t)\,dt$ ∎

13.) g: pos., monotont voksende, kont., funksjon på $[0, \infty)$

Def: $h(x) := \int_0^x g(t)\,dt$

a) Vis: h pos. og voks.:



Velg en x:

$h(x) = \int_0^x g(t)\,dt > (x-0)\,g(0) > 0$

Fra def. av integralet

den groveste nedre trappesum (er nedre trappesum pga. g voksende)

h voksende: Anta $z > x$. Vil vise $h(z) > h(x)$.

$h(z) = \int_0^z g(t)\,dt = \int_0^x g(t)\,dt + \int_x^z g(t)\,dt$

$> \int_0^x g(t)\,dt = h(x)$. ∎

b) Vis: $h(x) \le g(x)$ for alle $x \in [0, 1]$:

$h(x) = \int_0^x g(t)\,dt \le (x-0)\,g(x)$ da ...

groveste øvre trappesum

er øvre tr. sum pga. g voksende

$= x\,g(x)$

$\le g(x)$.

$x \in [0, 1]$, g pos.

c) Def. følge $[a_n(x)]_{n=1}^{\infty}$ v/

$a_1(x) = g(x)$

$a_2(x) = \int_0^x g(t)\,dt$

$\Big(a_3(x) = \int_0^x \int_0^t \ldots$

8.4: 1) e) $\int \frac{4}{\sqrt{7-x^2}}\,dx$

$= 4\int \frac{1}{\sqrt{7\left(1-\frac{x^2}{7}\right)}}\,dx = 4\int \frac{1}{\sqrt{7}\sqrt{1-\frac{x^2}{7}}}\,dx$

$= \frac{4}{\sqrt{7}}\int \frac{1}{\sqrt{1-\left(\frac{x}{\sqrt{7}}\right)^2}}\,dx = \frac{4}{\sqrt{7}}\arcsin\left(\frac{x}{\sqrt{7}}\right)\sqrt{7} + C$

$= 4\arcsin\left(\frac{x}{\sqrt{7}}\right) + C$

2) c) $\int e^x \cos(e^x)\,dx = \sin(e^x) + C$

3) b) $\int \sin 2x \,\frac{e^{\cos^2 x}}{e^{\sin^2 x}}\,dx$

$= \int \sin(2x)\, e^{\cos^2 x - \sin^2 x}\,dx$

$= \int \sin(2x)\, e^{\cos 2x}\,dx = -\frac{1}{2}e^{\cos 2x} + C$

5.) $f:(0,\infty)\to\mathbb{R}$, $f(xy)=f(x)+f(y)$ $:(*)$ for alle $x,y\in(0,\infty)$.

f er deriverbar i $x=1$, $f'(1)=k$.

a) Vis $f(1)=0$: Velg $x=y=1$. Da er

$$f(xy)=f(1\cdot1)=f(1)$$

$$\overset{(*)}{=} f(x)+f(y)=f(1)+f(1)=2f(1)$$

$$\Downarrow$$

$$f(1)=2f(1)\Rightarrow \underline{f(1)=0}$$

b) Vis $f(x+h)=f(x)+f(1+\frac{h}{x})$:

$$f(x)+f(1+\tfrac{h}{x})\overset{(*)}{=}f(x(1+\tfrac{h}{x}))$$

$$=f(x+h) \quad \blacksquare$$

Vis $f'(x)=\frac{k}{x}$:

$$f'(x)\underset{\text{Def. av derivert}}{=}\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$$

$$\underset{\text{fra over}}{=}\lim_{h\to0}\frac{\cancel{f(x)}+f(1+\frac{h}{x})-\cancel{f(x)}}{h}$$

$$=\lim_{h\to0}\frac{f(1+\frac{h}{x})}{h}=\lim_{h\to0}\frac{f(1+\frac{h}{x})}{\frac{xh}{x}}$$

$$\underset{\text{x er en konst. i grensa}}{=}\frac{1}{x}\lim_{h\to0}\frac{f(1+\frac{h}{x})}{\frac{h}{x}}$$

$$=\frac{1}{x}\lim_{h\to0}\frac{f(1+\frac{h}{x})-\overbrace{f(1)}^{=0}}{\frac{h}{x}}$$

$$\underset{\text{Sa } y=\frac{h}{x}. \text{ Da vil } h\to0 \Rightarrow y=\frac{h}{x}\to0}{=}\frac{1}{x}\lim_{y\to0}\frac{f(1+y)-f(1)}{y}$$

$$\underset{\text{def. av derivert}}{=}\frac{1}{x}f'(1)=\frac{1}{x}k=\underline{\underline{\frac{k}{x}}} \quad \blacksquare$$

c) $f'(x)=\frac{k}{x} \underset{\text{Analysens fund. teorem}}{\Longrightarrow} f(x)=k\ln x+C$

Men $f(1)=k\ln(1)+C=C$
over

$$f(1)\overset{(a)}{=}0$$

$$\Downarrow$$

$$C=0$$

Dermed er $f(x)=\underline{\underline{k\ln x}}$ $\blacksquare$