Plenumsregning 31/8

Plenum

- Kristina
- B500, Abel
- kristrd@math.uio.no

1) Les stoffet
2) Gå på forelesning
3) Tenk gjennom det nye.
4) Regn ukeoppgaver
5) Gruppetime
6) Plenum

Komplekse tall

3.1: 1 d), g), 3, 5 a) c), 6, 8, 9

$$\boxed{z = a + ib}$$

3.2: 1, 3 c), 5 b), 7 a), 9, 10, 13, 15

3.3: 1 a), 3 a), 7, 8

3.1: 1) d) $(5+2i)(3+i) = 15 + 5i + 6i - 2$

$$= \underline{\underline{13 + 11i}}$$

g) $$\frac{-5+2i}{5-4i} = \frac{(-5+2i)(5+4i)}{(5-4i)(5+4i)} = \frac{-25-20i+10i-8}{25+16}$$

$$= \frac{-33-10i}{41} = \underline{\underline{-\frac{33}{41} - \frac{10}{41}i}}$$

3.) c) $\overline{-7-8i} = -7+8i$

5.) c) $\dfrac{z-2}{z+1} = 3i$

$$z-2 = 3i(z+1) \cancel{= 3iz+3i}$$

$$z-2 = 3iz + 3i$$

$$z - 3iz = 3i+2$$

$$z(1-3i) = 3i+2$$

$$z = \frac{3i+2}{1-3i} = \frac{(3i+2)(1+3i)}{(1-3i)(1+3i)}$$

$$= \frac{3i-9+2+6i}{1+9} = \frac{-7+9i}{10} = -\frac{7}{10} + \frac{9}{10}i$$

8.) i): $\bar{z} + \bar{w} = \overline{z+w}$:

La $z = a+ib$ og la $w = c+id$. Da er

$$\bar{z} + \bar{w} = \overline{(a+ib)} + \overline{(c+id)} = (a-ib)+(c-id)$$

$$= (a+c) \underline{- i} (b+d)$$

og

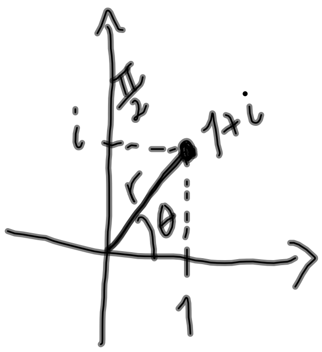
$$\overline{z+w} = \overline{(a+ib)+(c+id)} = \overline{(a+c)+i(b+d)}$$

$$= (a+c) \underline{- i} (b+d)$$

Så dermed er $\bar{z}+\bar{w} = \overline{z+w}$, og dermed er (i) bevist.

3.2:

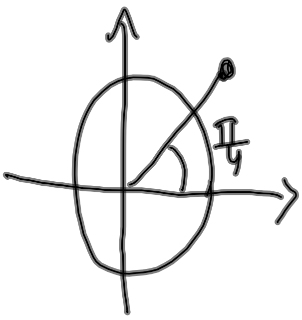
3.) c) $1+i$: $r = \sqrt{1^2 + 1^2} = \underline{\underline{\sqrt{2}}}$



$$\sqrt{2}\sin\theta = 1 \qquad \sin\theta = \frac{1}{\sqrt{2}}$$
$$\Rightarrow$$
$$\sqrt{2}\cos\theta = 1 \qquad \cos\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \frac{\pi}{4} \cancel{(+2k\pi)}$$

$$\theta = \frac{\pi}{4} + 2k\pi$$



$k = 1$

$\frac{\pi}{4} + 2 \cdot 1\pi = \frac{\pi}{4} + 2\pi$

der k er heltall

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2} = 1$
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

5.) b) $r=1$, $\theta=\frac{\pi}{4}$: $z=a+ib$

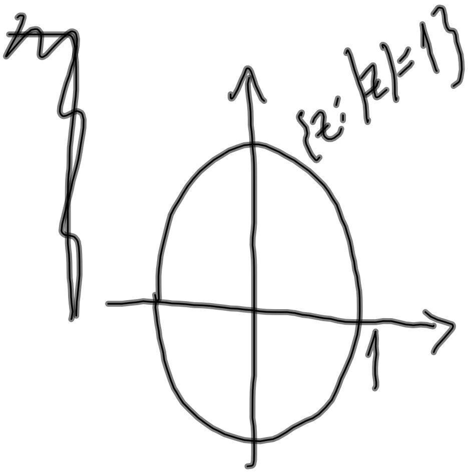
$$a=r\cos\theta=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}}$$

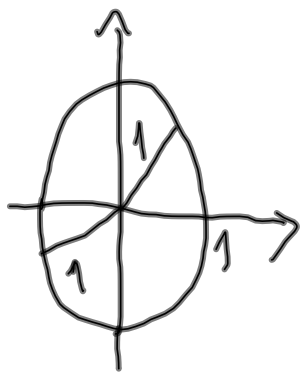
$$b=r\sin\theta=\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$$

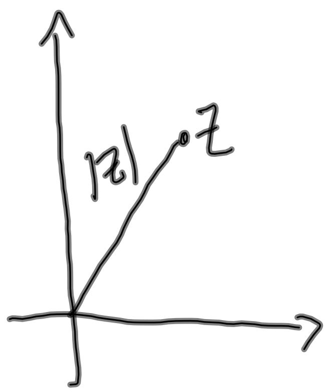
$$z=\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}$$

10.) $\{z : |z|=1\}$ $(\{z \mid |z|=1\})$

$$|z|=|z-0|=1$$







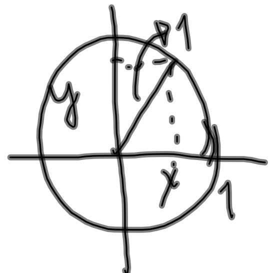
$|z| = 1 \quad , \quad z = \cancel{x} + i\cancel{y}$

$|z| = 1$

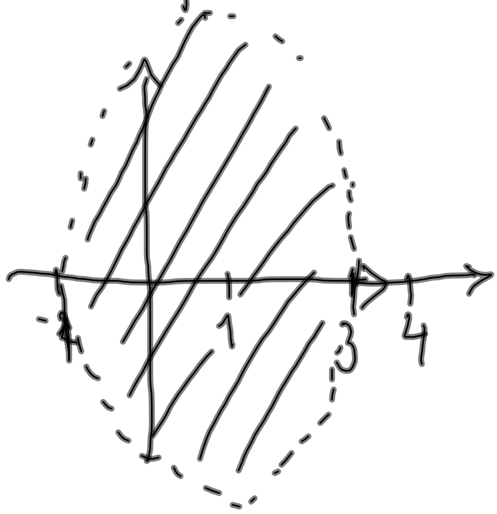
$\sqrt{\cancel{x}^2 + \cancel{y}^2} = 1$

$x^2 + y^2 = 1$

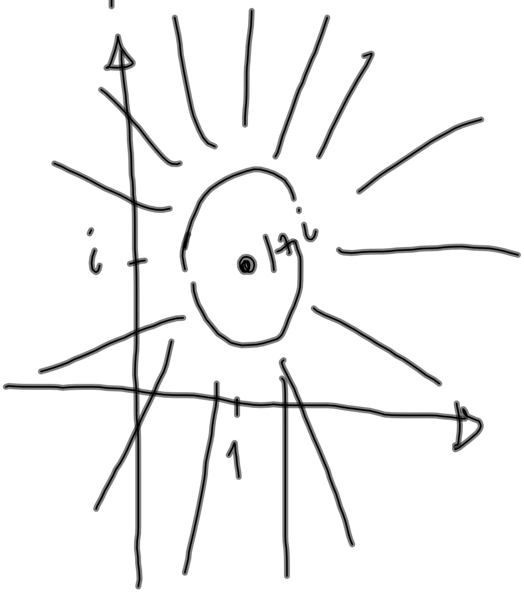
$$\boxed{(x-a)^2 + (y-b)^2 = r^2}$$



b) $\{z : |z - 1| < 2\}$



c) $\{z : |z - (i+1)| \geq \frac{1}{2}\}$



d) $\{z : |z-2| < |z-i+2|\}$

$z = x + iy$

$|z-2| = \sqrt{(x-2)^2+y^2}$, $|z-i+2| = \sqrt{(x+2)^2+(y-1)^2}$

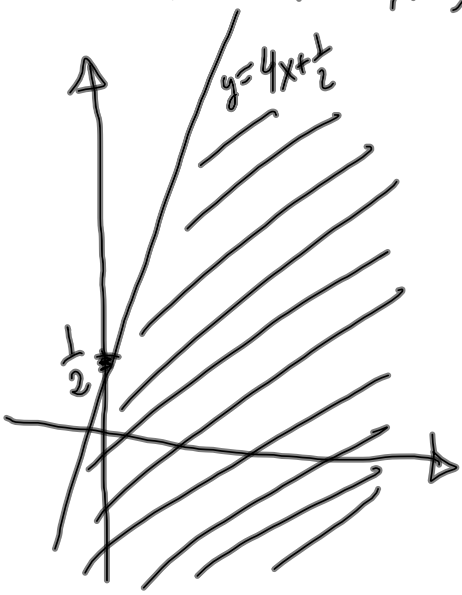
Ulikhet:

$$\sqrt{(x-2)^2+y^2} < \sqrt{(x+2)^2+(y-1)^2}$$

$$\Updownarrow$$

$$(x-2)^2+y^2 < (x+2)^2+(y-1)^2$$

$$\cancel{x^2}-4x+\cancel{4}+\cancel{y^2} < \cancel{x^2}+4x+\cancel{4}+\cancel{y^2}-2y+1$$



$$2y < 8x+1$$

$$y < 4x+\frac{1}{2}$$

(Ikke med linja!)

13.) a) $z = 1 + i\sqrt{3}$, $w = 1 + i$

$$zw = (1-\sqrt{3}) + i(1+\sqrt{3})$$

$$\frac{z}{w} = \frac{1+\sqrt{3}}{2} + i\,\frac{\sqrt{3}-1}{2}$$

b) z: $r_z = 2$, $\theta_z = \frac{\pi}{3} \Rightarrow z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

w: $r_w = \sqrt{2}$, $\theta_w = \frac{\pi}{4} \Rightarrow w = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

c) $\frac{z}{w}$: $r = \frac{r_z}{r_w} = \frac{2}{\sqrt{2}} = \underline{\sqrt{2}}$

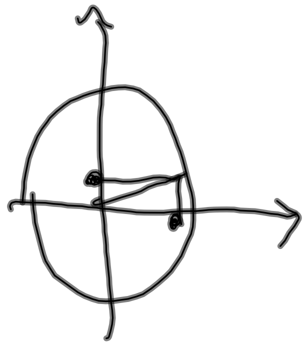
$$\theta = \theta_z - \theta_w = \frac{\pi}{3} - \frac{\pi}{4} = \underline{\frac{\pi}{12}}$$

$$\frac{z}{w} = \sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$$

Vet:

$$\frac{z}{w} = \frac{\sqrt{3}+1}{2} + i\,\frac{\sqrt{3}-1}{2}$$

fra a)



$$\frac{z}{w} = \sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$$

Fra c)

Setter realdel lik realdel og imaginærdel lik imaginærdel:

$$\frac{\sqrt{3}+1}{2} = \sqrt{2}\cos\frac{\pi}{12} \Rightarrow \cos\frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\frac{\sqrt{3}-1}{2} = \sqrt{2}\sin\frac{\pi}{12} \Rightarrow \sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\cos\frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{\sqrt{2}(\sqrt{3}+1)}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

15.) Vil vise: $|z+w|^2 + |z-w|^2 = 2|z|^2 + 2|w|^2$

Bevis:

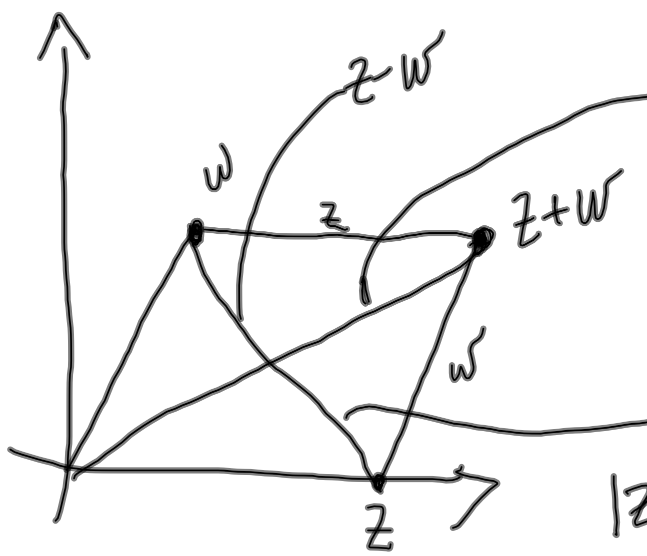
$$|z+w|^2 + |z-w|^2 = (z+w)\overline{(z+w)} + (z-w)\overline{(z-w)}$$

$$= (z+w)(\bar{z}+\bar{w}) + (z-w)(\bar{z}-\bar{w})$$

$$= z\bar{z} + \cancel{z\bar{w}} + \cancel{w\bar{z}} + w\bar{w} + z\bar{z} - \cancel{z\bar{w}} - \cancel{w\bar{z}} + w\bar{w}$$

$$= 2z\bar{z} + 2w\bar{w} = 2|z|^2 + 2|w|^2$$

$\square$



$|z+w|^2 + |z-w|^2$

$|z|^2 + |w|^2 + |w|^2 + |z|^2$

$= 2|z|^2 + 2|w|^2$