M: $\lim\limits_{x\to 0}\dfrac{x}{\sin x}\;\Big/\;\lim\limits_{x\to 0}\dfrac{1}{\cos x}=1$

$\overset{"\frac{0}{0}"}{\underset{L'Hôpital}{}}$

ä: $\lim\limits_{x\to 0} x\cot x=\left(\lim\limits_{x\to 0}\dfrac{x}{\sin x}\right)^2=1^2=\underline{\underline{1}}$

) $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{\cot x}{\frac{\pi}{2}-x}=\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\frac{\cos x}{\sin x}}{\frac{\pi}{2}-x}$

$=\;\lim\limits_{x\to\frac{\pi}{2}}\dfrac{-\frac{1}{\sin^2 x}}{-1}=\underline{\underline{1}}$

$\overset{"\frac{0}{0}"}{\underset{L'Hôpital}{}}$

7.6: Arcusfunksjonene

Finn eksakte verdier:

1) a) $\arcsin\frac{1}{2}$?

Vet: $\sin\left(\arcsin\frac{1}{2}\right)=\frac{1}{2}\;\overset{\frac{1}{2}}{\underset{\parallel}{}}$ (siden $\sin\frac{\pi}{6}$)

$\Downarrow$

$\arcsin\frac{1}{2}=\underline{\underline{\dfrac{\pi}{6}}}$

(e)) $\arccos\frac{1}{2}$?

Vet: $\cos\left(\arccos\frac{1}{2}\right)=\frac{1}{2}$

$\Downarrow$ (siden $\cos\frac{\pi}{3}=\frac{1}{2}$)

$\arccos\frac{1}{2}=\underline{\underline{\dfrac{\pi}{3}}}$

h) $\arctan\sqrt{3}$?

Vet: $\tan\left(\arctan\sqrt{3}\right)=\sqrt{3}$

$\Downarrow$ (siden $\tan\frac{\pi}{3}=\dfrac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}}=\dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\sqrt{3}$)

$\arctan\sqrt{3}=\underline{\underline{\dfrac{\pi}{3}}}$

2) a) Finn de deriverte:

$$(\arcsin \sqrt{x})' = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{2} x^{-\frac{1}{2}}$$

$$= \frac{1}{2\sqrt{x}\sqrt{1-x}} = \frac{1}{2\sqrt{x(1-x)}}$$

b)

$$(\arctan e^x)' = \frac{1}{1+(e^x)^2} \cdot e^x$$

$$= \frac{e^x}{1+e^{2x}}$$

3) a)

$$\lim_{x\to 0} \frac{\arctan 2x}{x} = \lim_{x\to 0} \frac{\frac{1}{1+(2x)^2} \cdot 2}{1} = \lim_{x\to 0} \frac{2}{1+4x^2}$$

$$\lim_{y\to 0} \arctan y$$
$$= \arctan 0 = 0$$
$$(\text{siden } \tan 0 = 0)$$

$$= 2$$

$$\left[\text{"}\frac{0}{0}\text{"} : L'\text{Hôpital} \right]$$

(b)

$$\lim_{x\to 0} \frac{\arcsin x}{\sin 3x} = \lim_{x\to 0} \frac{\frac{1}{\sqrt{1-x^2}}}{3\cos 3x} = \frac{1}{3}$$

$$\boxed{\begin{array}{l}\arcsin 0 = 0 \\ \text{aldri } \sin 0 = 0\end{array}}$$

$$\text{"}\frac{0}{0}\text{"} : L'\text{Hôpital}$$

$$\boxed{\begin{array}{l}\text{Teller} \to 1 \\ \text{Nevner} \to 3\end{array}}$$

e)

$$\lim_{x\to 0} \frac{\arctan x - x}{x^3} = \lim_{x\to 0} \frac{\frac{1}{1+x^2}-1}{3x^2}$$

$$\text{"}\frac{0}{0}\text{"} : L'\text{Hôpital}$$

$$L'\text{Hôpital}: \text{"}\frac{0}{0}\text{"}$$

$$= \lim_{x\to 0} \frac{-x^2}{3x^2+3x^4} = \lim_{x\to 0} \frac{-2x}{6x+12x^3} = \lim_{x\to 0} \frac{-2}{6+12\cdot 3x^2}$$

$$L'\text{Hôpital}: \text{"}\frac{0}{0}\text{"}$$

$$= -\frac{2}{6} = -\frac{1}{3}$$