4.3.4. c) $\lim_{n\to\infty} \frac{n+\frac{1}{2}}{3n+2} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{n\to\infty} \frac{\frac{1}{n}(n+\frac{1}{2})}{\frac{1}{n}(3n+2)}$

$= \lim_{n\to\infty} \frac{(1+\frac{1}{2n})}{(3+\frac{2}{n})} = \underline{\underline{\frac{1}{3}}}$

DEF: Gitt $\varepsilon > 0$. Vi har $|a_n - a| = \left|\frac{n+\frac{1}{2}}{3n+2} - \frac{1}{3}\right|$

$= \left|\frac{3(n+\frac{1}{2}) - (3n+2)}{3(3n+2)}\right|$

$= \left|\frac{\cancel{3n} + \frac{3}{2} - \cancel{3n} - 2}{3(3n+2)}\right|$

$= \left|\frac{-\frac{1}{2}}{3(3n+2)}\right|$

$= \left|\frac{1}{6(3n+2)}\right| = \left|\frac{1}{18n+12}\right|$ $\quad 18n+12 > 18n$

$< \frac{1}{18n}$.

$\boxed{\text{Så hvis vi nå velger } \frac{1}{18N} < \varepsilon, \text{ dvs. } N > \frac{1}{18\varepsilon}}$

Så er $< \frac{1}{18 \cdot \frac{1}{18\varepsilon}} = \varepsilon$ for alle $n \geq N$.

Så $a_n \to \frac{1}{3}$ når $n \to \infty$.

4.3.3. c) $\lim_{n\to\infty} \boxed{\sqrt{n^2+n} - n}$ "$\infty - \infty$"

$$= \lim_{n\to\infty} \frac{(\sqrt{n^2+n} - \underline{n})(\sqrt{n^2+n} + \underline{n})}{(\sqrt{n^2+n} + n)}$$

$(a+b)(a-b)$
$= a^2 - b^2$

$$= \lim_{n\to\infty} \frac{\cancel{n^2} + n - \cancel{n^2}}{(\sqrt{n^2+n} + n)}$$

$$= \lim_{n\to\infty} \frac{n}{(\sqrt{n^2+n} + n)} = \lim_{n\to\infty} \frac{\frac{1}{n} \cdot n}{\frac{1}{n}(\sqrt{n^2+n} + n)}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{\frac{1}{n^2}(n^2+n)} + \frac{n}{n}}$$

$$\frac{1}{n}\sqrt{n^2+n} = \sqrt{\frac{1}{n^2}} \cdot \sqrt{n^2+n}$$
$$= \sqrt{\frac{1}{n^2}(n^2+n)}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n}} + 1}$$

$$= \frac{1}{2}$$