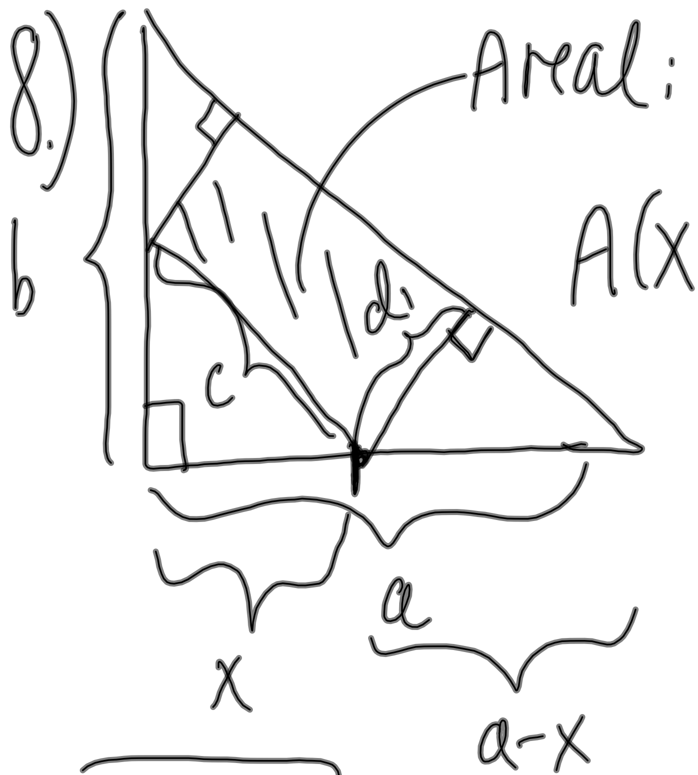
7.1: 8)



Areal:

$$A(x) = c \cdot d$$

$$\frac{c}{x} = \frac{\sqrt{a^2+b^2}}{a} \Rightarrow c = \frac{x}{a}\sqrt{a^2+b^2}$$

$$\frac{d}{a-x} = \frac{b}{\sqrt{a^2+b^2}} \Rightarrow d = (a-x)\,b\,\frac{1}{\sqrt{a^2+b^2}}$$

$$A(x) = c \cdot d = \frac{x}{a}\cancel{\sqrt{a^2+b^2}} \cdot (a-x)\frac{b}{\cancel{\sqrt{a^2+b^2}}}$$

$$A(x) = \frac{(a-x) x b}{a}$$

$$A'(x) = \frac{b}{a}(a-2x) = 0$$

$$\frac{b}{a}(a-2x) = 0$$

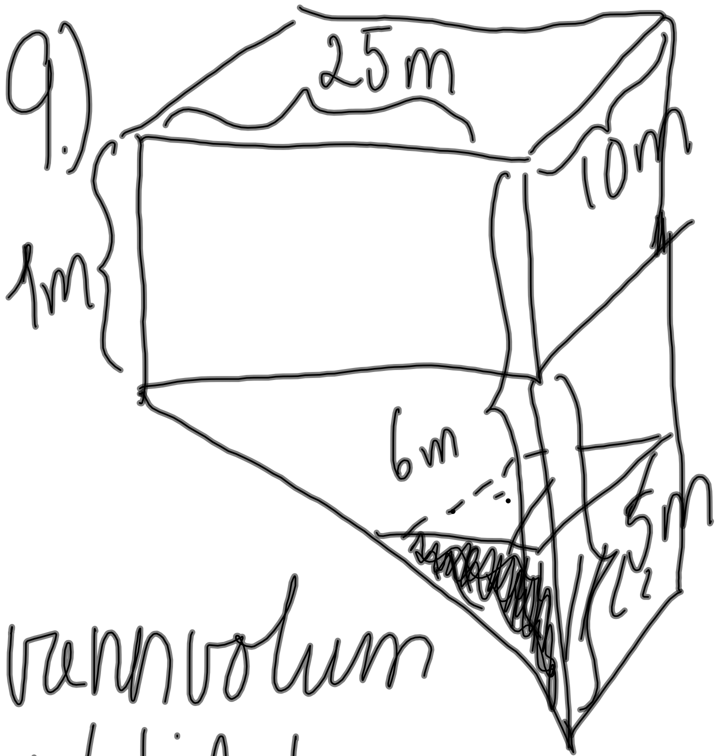
$$a = 2x$$

$$\underline{x = \frac{a}{2}}$$

gjør arealet størst mulig

Max areal: $A(\frac{a}{2})$.

7.2: 9)



$V(t)$ = vannvolum v/ tid t

$h(t)$ = Vannhøyde i den dype enden v/ tid t.

$$V(t) = \frac{h(t) \cdot 5h(t) \cdot 10}{2}$$

$$V(t) = \frac{50}{2} h^2(t)$$

$$V'(t) = \frac{50}{\cancel{2}} \cancel{2} h(t) h'(t)$$

Når vannstanden er 3m:

$\boxed{\begin{array}{l} 2000\,l \\ = 2\,m^3 \end{array}}$ $V'(3) = 2$
$h(3) = 3$

$$\Rightarrow 2 = 50 \cdot 3 \cdot h'(3)$$

$$\underline{h'(3) = \frac{1}{75}\, m/_{min}}$$

7.4: 3.) $f(x) = 2xe^x + 1$.

$$f'(x) = 2e^x + 2xe^x = \underbrace{2}_{>0}\underbrace{e^x}_{>0}\underbrace{(1+x)}_{\substack{>0 \text{ for} \\ x>-1}}$$

$$\Downarrow$$

$f'(x) > 0$ for $x > -1$.

Så siden $f'(x) \geq 0$ for $x \in [-1, \infty)$, så
er f strengt voksende og dermed
lik kun i endepkt.
injektiv på dette området.

La g være f^{-1}: $g'(1)$?

Fra Teorem 7.4.6 er

$$g'(1) = \frac{1}{f'(x)}$$ der x bestemmes fra $f(x) = 1$

Så $f(x) = 1 \iff 2xe^x + 1 = 1 \iff x = 0$.

Dermed er $$g'(1) = \frac{1}{f'(0)} = \frac{1}{2e^0(1+0)} = \underline{\underline{\frac{1}{2}}}$$

$\underset{\sim}{7.5}$: 3) a) $\lim_{x \to 0} x \cot x$ ("$\frac{0}{0}$": L'Hôpital

$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$

$= \lim_{x \to 0} x \frac{\cos x}{\sin x} =$

$= \lim_{x \to 0} \frac{\cos x + x(-\sin x)}{\cos x}$

$= \underline{\underline{1}}$

7.6: 2) b) $(\arctan e^x)'$

$$= \frac{1}{1+(e^x)^2}\, e^x = \frac{e^x}{1+e^{2x}}$$

'Kjerne-regel'

3) e) $\lim\limits_{x \to 0} \frac{\arctan x - x}{x^3}$

$$\underset{\text{"}\frac{0}{0}\text{": L'Hôpital}}{=} \lim_{x \to 0} \frac{\frac{1}{1+x^2} - 1}{3x^2}$$

$$\underset{\text{"}\frac{0}{0}\text{": L'Hôpital}}{=} \lim_{x \to 0} \frac{(-1)(1+x^2)^{-2}\cancel{2x}}{3\,\cancel{6x}} = -\frac{1}{3}$$