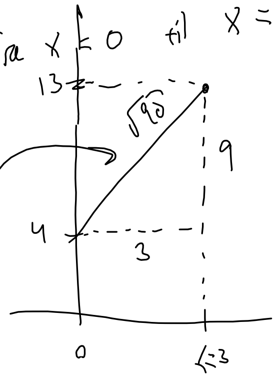
8.6.11 a)

$y = 3x + 4$ fra $x = 0$ til $x = 3$

$[3x + 4]' = 3$

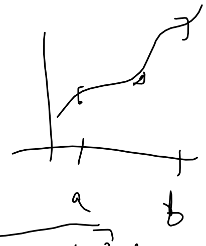
vil finne lengden på denne



$$L = \int_0^3 \sqrt{1 + f'(t)^2}\, dt$$

$$= \int_0^3 \sqrt{1 + 3^2}\, dt = \int_0^3 \sqrt{10}\, dt = \underline{\underline{\sqrt{10} \cdot 3}}$$

Buelengde



$$L = \int_a^b \sqrt{1 + f'(t)^2}\, dt$$

b)

$f(x) = \frac{x^2}{2} - \frac{1}{4} \ln x$ fra $x = 1$ til $x = e$ (tegn denne)

$f'(x) = x - \frac{1}{4}\frac{1}{x}$

$$f'(x)^2 = x^2 - 2 \cdot x \cdot \frac{1}{4}\frac{1}{x} + \frac{1}{16}\frac{1}{x^2}$$

$$= x^2 - \frac{1}{2} + \frac{1}{16}\frac{1}{x^2}$$

Da er

$$L = \int_1^e \sqrt{\underbrace{x^2 + \frac{1}{2} + \frac{1}{16}\frac{1}{x^2}}_{\left(x + \frac{1}{4}\frac{1}{x}\right)^2}}\, dx = \int_1^e x + \frac{1}{4}\frac{1}{x} = \left[\frac{1}{2}x^2 + \frac{1}{4}\ln x\right]_1^e$$

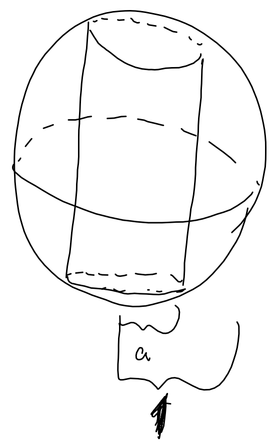
$$= \frac{1}{2}e^2 + \underbrace{\frac{1}{4} - \frac{1}{2}} = \underline{\frac{1}{2}e^2 - \frac{1}{4}}$$

8.6.15

Kule m/ radius 1

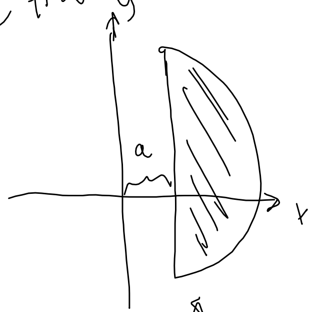
borer sylinder-formet hull.

Sylinderen har radius $a < 1$.



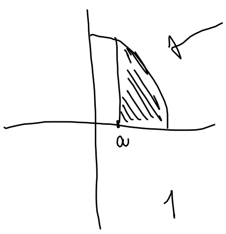
Finn Volum.

Ser denne fra siden:



Parentes: Roter denne rundt y-aksen, og vi får figuren over.

Ikke en funksjonsgraf!



Roter heller denne rundt y-aksen

$y = \sqrt{1-x^2}$

Området legemet vi får da er halve svaret.

$$\frac{V}{2} = \int_a^1 2\pi x \cdot f(x)\, dx = \pi \int_a^1 2x\sqrt{1-x^2}\, dx$$

$$= -\pi \int_a^1 \underbrace{-2x}_{u'}\underbrace{\sqrt{1-x^2}}_{u}\, dx = \pi \int_1^a \underbrace{-2x}_{u'}\underbrace{\sqrt{1-x^2}}_{u}\, dx$$

$u = 1-x^2$

$du = -2x\, dx$

$u(1) = 1-1 = 0$

$u(a) = 1-a^2$

$$= \pi \int_0^{1-a^2} \sqrt{u}\, du = \pi \left[\frac{2}{3} u^{\frac{3}{2}}\right]_0^{1-a^2}$$

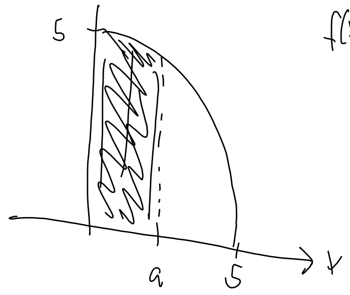
$$= \frac{2\pi}{3}\left(1-a^2\right)^{3/2}$$

Så $V = \underline{\underline{\frac{4\pi}{3}(1-a^2)^{3/2}}}$

8.6.26 $a \in [0,5]$

$f(x) = \sqrt{25 - x^2}$

Finn volumet av omdr. legemet rundt x-aksen.



a)

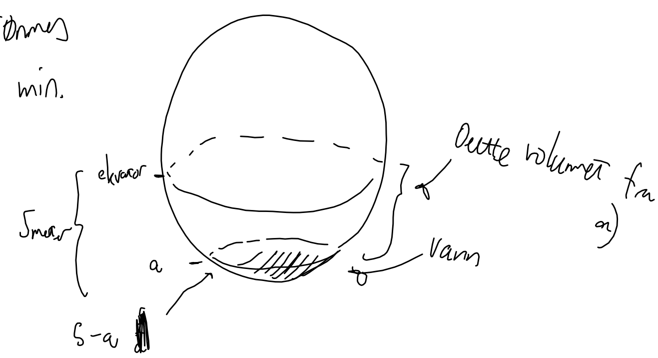
$$V(a) = \pi \int_0^a f(x)^2 \, dx$$

$$= \pi \int_0^a 25 - x^2 \, dx = \pi \left[25x - \frac{1}{3} x^3 \right]_0^a$$

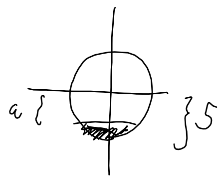
$$= \pi \left(25a - \frac{1}{3} a^3 \right)$$

b) Når vanndybden er 2 meter tømmes tanken m/ $\frac{1}{2}$ kubikkm pr min.

Hvor fort avtar vanndybden?



Volum luft v/ h meter vann $= V(5-h)$



Høyden h avhenger av tiden t, så skriver $h(t)$.

Så volum v/ tiden t er $V(5-h(t))$.

Vi skal finne $h'(t)$ når det er 2 m/ vann.

Har at $V(t)' = V(5-h(t))' = -h'(t) \cdot V'(5-h(t))$

$$h'(t) = \frac{-V(t)'}{V'(\underbrace{5-h(t)}_{3})} = \frac{-\frac{1}{2}\frac{m^3}{min}}{16\pi} = \frac{-1}{32\pi}$$

$$V'(a) = \pi(25 - a^2)$$

$$V'(3) = \pi(25 - 9) = 16\pi$$

Kjerneregel $\dfrac{\partial h}{\partial t} = \dfrac{\frac{\partial V}{\partial t}}{\frac{\partial V}{\partial a}}$

høyde-endring

volum-endr pr tid

volum-endr pr meter