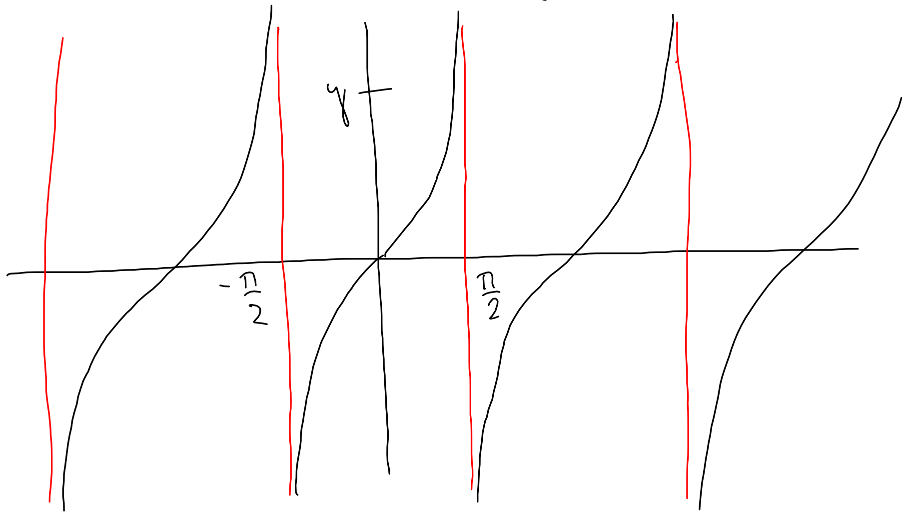
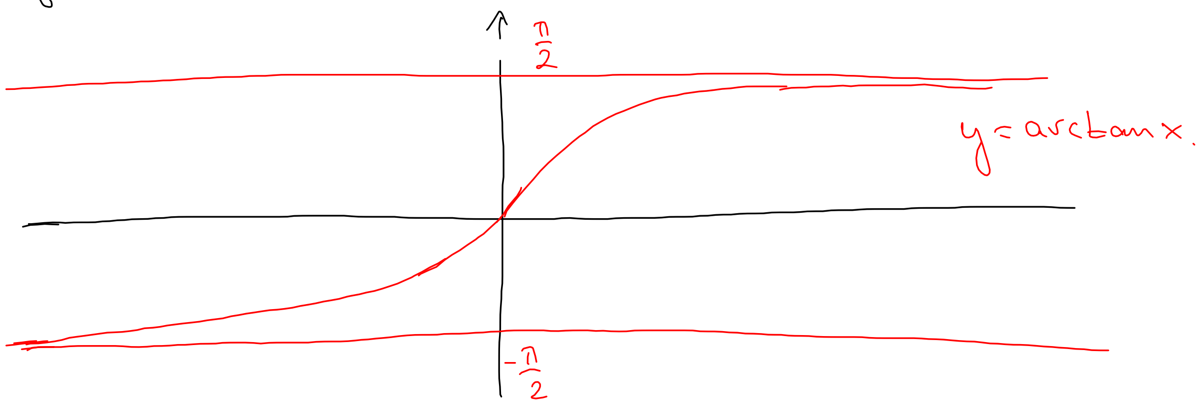
Arcustangens



Definisjon: La $f: (-\frac{\pi}{2}, \frac{\pi}{2}) \to \mathbb{R}$ være gitt ved $f(x) = \tan x$.
Da kalles den omvendte funksjonen $g: \mathbb{R} \to (-\frac{\pi}{2}, \frac{\pi}{2})$ arcustangens og betegnes med arctan, $\tan^{-1}$.



x	$\tan x$
0	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	1
$\frac{\pi}{3}$	$\sqrt{3}$
$x \to \frac{\pi}{2}^-$	$\tan x \to \infty$

x	$\arctan x$
0	0
$\frac{\sqrt{3}}{3}$	$\frac{\pi}{6}$
1	$\frac{\pi}{4}$
$\sqrt{3}$	$\frac{\pi}{3}$
$x \to \infty$	$\arctan x \to \frac{\pi}{2}$

Derivasjon: $\underbrace{\arctan}_{g}$ er den omvendte funksjon til $\underbrace{\tan}_{f}$

Generelt: $g'(y) = \dfrac{1}{f'(x)}$, der $y = f(x) = \tan x$

Vet at $f'(x) = (\tan x)' = \dfrac{1}{\cos^2 x} = \dfrac{\cos^2 x + \sin^2 x}{\cos^2 x} = 1 + \tan^2 x$

Dermed:

$$(\arctan y)' = g'(y) = \frac{1}{f'(x)} = \frac{1}{1+\tan^2 x} = \frac{1}{1+y^2}$$

Altså:

$$\boxed{(\arctan x)' = \frac{1}{1+x^2}}$$

$$\int \frac{1}{1+x^2}\,dx = \arctan x + C$$

Eksempel: Deriver $f(x) = \arctan(\sin x)$

Kjerneregel: $f'(x) = \dfrac{1}{1+\sin^2 x} \cdot \cos x = \dfrac{\cos x}{1+\sin^2 x}$

Eksempel: Finn $\lim\limits_{x\to\infty} \underbrace{x}_{\infty}\underbrace{\left(\frac{\pi}{2} - \arctan x\right)}_{0} = \lim\limits_{x\to\infty} \dfrac{\frac{\pi}{2} - \arctan x}{\frac{1}{x}}$

$$\overset{L'H}{=} \lim_{x\to\infty} \frac{-\frac{1}{1+x^2}\cdot x^2}{-\frac{1}{x^2}\cdot x^2} = \lim_{x\to\infty} \frac{x^2}{1+x^2} \overset{L'H}{=} \lim_{x\to\infty} \frac{2x}{2x} = \underline{\underline{1}}$$