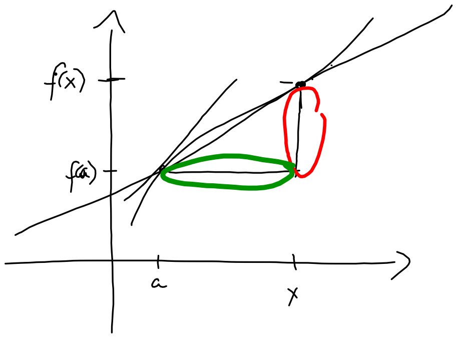
DERIVASJON

stigningstall til sekant $= \dfrac{f(x) - f(a)}{x - a}$

stigningstall til tangent $= \lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

hvis den eksisterer $= f'(a)$

og f er deriverbar i a.



ELEMENTÆRE DERIVASJONSREGLER

$a \in \mathbb{R}$

$D[a] = 0$

$D[x^a] = a x^{a-1}$

$D[a^x] = a^x \ln a \quad a > 0$

$D[e^x] = e^x$

$D[\ln|x|] = \dfrac{1}{x}$

$D[\sin x] = \cos x$

$D[\cos x] = -\sin x$

$D[\tan x] = \dfrac{1}{\cos^2 x}$

$$|x| = \begin{cases} x & \text{når } x \geq 0 \\ -x & \text{når } x < 0 \end{cases}$$

$$\ln|x| = \begin{cases} \ln x & \text{når } x \geq 0 \\ \ln(-x) & \text{når } x < 0 \end{cases}$$

$$D[\ln|x|] = \begin{cases} \dfrac{1}{x} & \text{når } x \geq 0 \\ \dfrac{1}{-x} \cdot (-1) & \text{når } x < 0 \end{cases}$$

↑ Kjerneregel

$$= \frac{1}{x} \quad \text{når } x \in \mathbb{R}$$

SAMMENSATTE

$(cf)'(a) = c \cdot f'(a)$

$(f \pm g)'(a) = f'(a) \pm g'(a)$

$(f \cdot g)'(a) = f'(a) \cdot g(a) + f(a) \cdot g'(a)$

$\left(\dfrac{f}{g}\right)'(a) = \dfrac{f'(a) \cdot g(a) - f(a) \cdot g'(a)}{g(a)^2}$

KJERNEREGEL

$h(x) = f(g(x))$

$= f'(g(x)) \cdot g'(x)$

LOGARITMISK DERIVASJON

$$D[\ln|f(x)|] = \frac{1}{f(x)} \cdot f'(x)$$

$$f'(x) = f(x) \cdot D[\ln|f(x)|]$$

6.1.1. a) $f(x) = \cos x \cdot \sin x$

$f'(x) = -\sin x \cdot \sin x + \cos x \cdot \cos x$

$= -\sin^2 x + \cos^2 x$

b) $f(x) = \dfrac{x}{\cos x} + e^x$

$f'(x) = e^x + \dfrac{1 \cdot \cos x - x(-\sin x)}{\cos^2 x}$

$= e^x + \dfrac{\cos x + x \sin x}{\cos^2 x}$

g) $f(x) = x \cdot \cos(\ln x)$

$= 1 \cdot \cos(\ln x) + x \cdot D[\cos(\ln x)]$

$= \cos(\ln x) + x \cdot \left(-\sin(\ln x) \cdot \frac{1}{x}\right)$

$= \cos(\ln x) - \sin(\ln x)$

$\sqrt{x} = x^{\frac{1}{2}}$

$\frac{1}{x^2} = x^{-2}$

h) $f(x) = \dfrac{\cos(\sqrt{x})}{x^2} = \cos(x^{\frac{1}{2}}) \cdot x^{-2}$

$f'(x) = -\sin(x^{\frac{1}{2}}) \left(\frac{1}{2} \cdot x^{-\frac{1}{2}}\right) \cdot x^{-2} + \cos(x^{\frac{1}{2}}) \cdot (-2x^{-3})$

$= -\frac{1}{2} x^{-\frac{5}{2}} \sin(x^{\frac{1}{2}}) - 2x^{-3} \cos(x^{\frac{1}{2}})$

$= \dfrac{-\sin(x^{\frac{1}{2}})}{2x^{5/2}} - \dfrac{2\cos(x^{\frac{1}{2}})}{x^3}$

6.1.3. a) $f(x) = x^2 \cdot \cos^4 x \cdot e^x$

$f'(x) = f(x) \cdot D\left[\ln|f(x)|\right]$

$\ln a \cdot b = \ln a + \ln b$

$\ln a^n = n \cdot \ln a$

$$\ln|f(x)| = \ln|x^2 \cdot \cos^4 x \cdot e^x|$$
$$= \ln|x^2| + \ln|\cos^4 x| + \ln|e^x|$$
$$= 2\ln|x| + 4\ln|\cos x| + x\ln|e|$$
$$= 2\ln|x| + 4\ln|\cos x| + x$$

$$D\left[\ln|f(x)|\right] = 2 \cdot \frac{1}{x} + 4 \cdot \frac{1}{\cos x} \cdot (-\sin x) + 1$$

$$f'(x) = x^2 \cdot \cos^4 x \cdot e^x \left(\frac{2}{x} - 4\underbrace{\tan x}_{=\frac{\sin x}{\cos x}} + 1\right)$$

6.1.10 $f(x) = \sqrt{x} = x^{\frac{1}{2}}$ $f'(x) = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

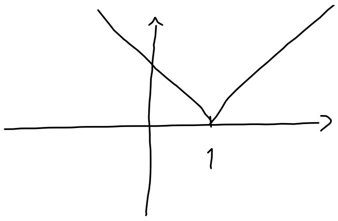
$$= \lim_{h \to 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\ 1}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \underline{\underline{\frac{1}{2\sqrt{x}}}}$$

6.1.11. a) $f(x) = |x-1|$



Skal vise $f(x)$ ikke deriverbar i 1.

Hvis $f'(1)$ eksisterer så er den $\lim\limits_{x\to 1} \dfrac{f(x)-f(1)}{x-1} = \lim\limits_{x\to 1} \dfrac{|x-1|-0}{x-1}$

$$= \lim_{x\to 1} \frac{|x-1|}{x-1}$$

$$|x-1| = \begin{cases} x-1 & \text{når } x-1 \geq 0 \\ -(x-1) & \text{når } x-1 < 0 \end{cases}$$

Se på $\lim\limits_{x\to 1^+} \dfrac{|x-1|}{x-1} = \lim\limits_{x\to 1^+} \dfrac{\cancel{(x-1)}}{\cancel{(x-1)}} = \underline{\underline{1}}$

$\neq$

Se på $\lim\limits_{x\to 1^-} \dfrac{|x-1|}{x-1} = \lim\limits_{x\to 1^-} \dfrac{-\cancel{(x-1)}}{\cancel{(x-1)}} = \underline{\underline{-1}}$

v/ definisjon av abs.verdi.

Så $f'(1)$ eksisterer IKKE!