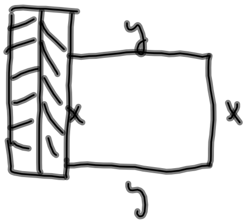
7.1

1.



$$2y + x = 50$$

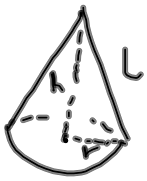
$$A(x) = x \cdot y = x\left(\frac{50-x}{2}\right)$$

$$\frac{d}{dx}A \quad \frac{d}{dx}\left(25x - \frac{1}{2}x^2\right) = 25 - x = 0$$

när $x = 25$

$$A_{max} = A(25) = 25 \cdot \frac{25}{2} = \underline{312.5}$$

5



$$V = \frac{1}{3}\pi r^2 \cdot h$$

$$r^2 = L^2 - h^2 \qquad V(h) = \frac{1}{3}\pi\left(L^2 - h^2\right)h$$

$$h = \sqrt{L^2 - r^2} \qquad V(r) = \frac{1}{3}\pi r^2\sqrt{L^2 - r^2}$$

$$\frac{d}{dh}V(h) = \frac{1}{3}\pi\left(L^2 - 3h^2\right) = 0$$

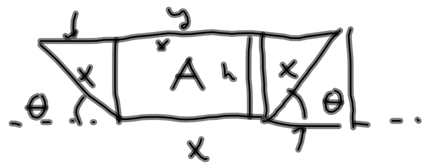
$$h^2 = \frac{1}{3}L^2 \qquad h = L \cdot \frac{1}{\sqrt{3}}$$

$$V_{max} = \frac{1}{3}\pi\left(L^2 - \frac{1}{3}L^2\right)\frac{L}{\sqrt{3}} = \frac{2\pi}{9} \cdot L^2 \cdot \frac{L}{\sqrt{3}} = \underline{18\pi} \cdot \frac{9}{\sqrt{3}}$$

$$= \underline{54\pi\sqrt{3}}$$

okt 20-12:20

7.



$$A = \frac{x+y}{2} \cdot h$$

$$h = x \sin\theta$$

$$y = 2x\cos\theta + x$$

$$A(\theta) = (x + x\cos\theta)\, x \sin\theta = x^2(\sin\theta + \sin\theta\cos\theta)$$

$3x = 60$

$x = 20$

$$= 400(\sin\theta + \sin\theta\cos\theta)$$

$$A'(\theta) = 400(\cos\theta + \cos^2\theta - \sin^2\theta) = 0$$

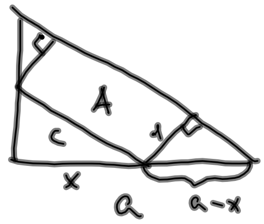
når $\cos\theta + 2\cos^2\theta - 1 = 0$

$$2\cos^2\theta + \cos\theta - 1 = 0$$

$$\cos\theta = \frac{-1 \pm \sqrt{1+8}}{4} = \begin{cases} \frac{1}{2} \\ -1 \end{cases}$$

$\underline{A}_{max}$ når $\cos\theta = \frac{1}{2}$, det vil si $\theta = \underline{\frac{\pi}{3}}$

8.



$$A = c \cdot d$$

$$\frac{c}{x} = \frac{\sqrt{a^2+b^2}}{a} \Rightarrow c = \frac{x}{a}\sqrt{a^2+b^2}$$

$$\frac{d}{a-x} = \frac{b}{\sqrt{a^2+b^2}} \Rightarrow d = (a-x)b \cdot \frac{1}{\sqrt{a^2+b^2}}$$

$$A(x) = \frac{x}{a} \cdot (a-x) \cdot b = \frac{b}{a}(ax - x^2)$$

$$A'(x) = \frac{b}{a}(a - 2x) = 0 \qquad x = \frac{a}{2}.$$

A_{max} når $x = \frac{a}{2}$.

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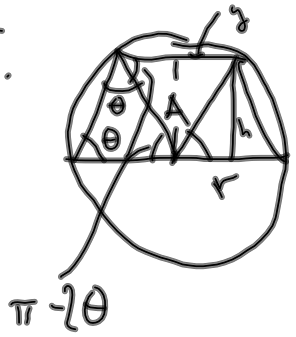
15.



trapes i en sirkel

$$A = \frac{2r+y}{2}\cdot h$$

$$h = r\sin(\pi-2\theta) = r\sin 2\theta$$

$$\frac{y}{2} = r\cos(\pi-2\theta) = -r\cos 2\theta$$

$$A(\theta) = \left(r+(-r\cos 2\theta)\right) r\sin 2\theta$$

$$= r^2\left(\sin 2\theta - \sin 2\theta\cos 2\theta\right)$$

$$A'(\theta) = r^2\left(2\cos 2\theta - \left(2\cos^2 2\theta - 2\sin^2 2\theta\right)\right)$$

$$= 2r^2\left(-2\cos^2 2\theta + \cos 2\theta + 1\right) = 0$$

$$\cos 2\theta = \frac{-1\pm\sqrt{1+8}}{-4} = \begin{cases}-\frac{1}{2}\\ 1\end{cases}$$

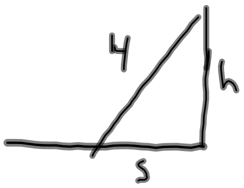
$$\cos 2\theta = -\frac{1}{2} \Rightarrow 2\theta = \frac{2\pi}{3}, \quad \theta = \frac{\pi}{3}$$

$$A = r^2\left(\sin\frac{2\pi}{3} - \sin\frac{2\pi}{3}\cos\frac{2\pi}{3}\right)$$

$$= r^2\left(\frac{1}{2}\sqrt{3} + \frac{1}{2}\sqrt{3}\cdot\frac{1}{2}\right) = \underline{r^2\,\frac{3}{4}\sqrt{3}}$$

7.2 $\quad$ 1, 3, 5, 7, $\underline{9}$, 13

1



$$\frac{ds}{dt} = 0.5$$

$$h^2 + s^2 = 4^2$$

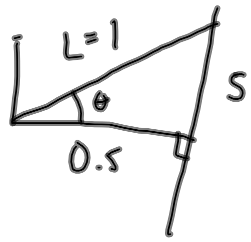
$$h^2 = 4^2 - s^2 \qquad h = \sqrt{16 - s^2}$$

$$\frac{dh}{dt} = \frac{1 \cdot 2s}{2 \cdot \sqrt{16 - s^2}} \frac{ds}{dt} = \frac{s}{\sqrt{16 - s^2}} \cdot \frac{ds}{dt}$$

$$s = 2 \quad \frac{ds}{dt} = 0.5 \quad = \frac{2 \cdot 0.5}{\sqrt{16 - 4}} = \frac{1}{\sqrt{12}}$$

$$= \underline{\frac{1}{2\sqrt{3}}}$$

3



$$\frac{d\theta}{dt} = 4\pi$$

$$s = 0.5 \tan\theta$$

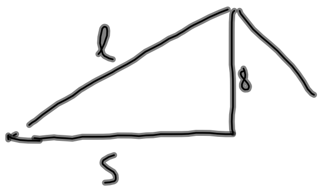
$$\frac{0.5}{L} = \cos\theta = \frac{0.5}{1} = \frac{1}{2}$$

når $\theta = \frac{\pi}{3}$.

$$\frac{ds}{dt} = 0.5 \cdot \frac{1}{\cos^2\theta} \cdot \frac{d\theta}{dt}$$

$$= 0.5 \cdot 4 \cdot 4\pi = \underline{8\pi} \ \text{kms}^{-1}$$

5



$\frac{ds}{dt} = 4 \ ms^{-1}$

$l^2 = 8^2 + s^2$

$l = \sqrt{s^2 + 64}$ $\quad l = 10 \quad s = 6$

$\frac{dl}{dt} = \frac{s}{\sqrt{s^2+64}} \cdot \frac{ds}{dt}$

$\left.\frac{dl}{dt}\right|_{s=6} = \frac{6}{10} \cdot 4 \, ms^{-1} = \underline{2.4 \ ms^{-1}}$

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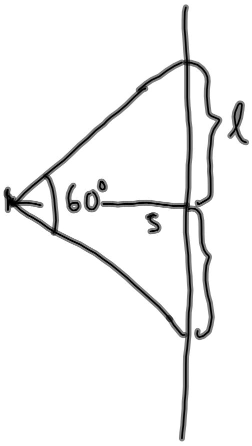
7.



$$\frac{ds}{dt} = -1\ \mathrm{ms^{-1}}$$

$$\frac{l}{s} = \tan 30° = \frac{1}{\sqrt{3}}$$

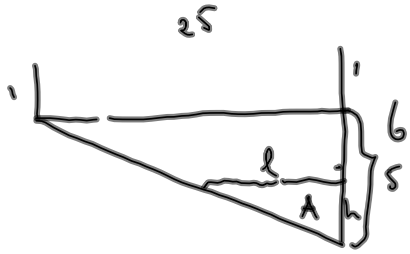
$$l = \frac{1}{\sqrt{3}} \cdot s$$

$$\frac{dl}{dt} = \frac{1}{\sqrt{3}} \cdot \frac{ds}{dt} = -\frac{1}{\sqrt{3}}$$

$$\frac{d\,2l}{dt} = 2\frac{dl}{dt} = -\underline{\frac{2}{\sqrt{3}}}\ \mathrm{ms^{-1}}$$

okt 20-13:27

9



$$\frac{dV}{dt} = 2 \text{ m}^3 \text{ min}^{-1}$$

$$V = 10\,A \quad = 25\,h^2$$

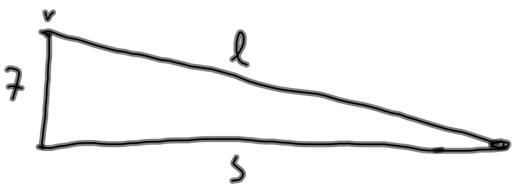
$$\frac{h}{l} = \frac{5}{25} \qquad \underline{A} = \frac{lh}{2} = \underline{\frac{5}{2}h^2}$$

$$l = 5h \qquad \frac{dA}{dt} = \underline{5h \cdot \frac{dh}{dt}}\,, \quad \frac{dV}{dt} = \underline{50h\,\frac{dh}{dt}}$$

$$h = 3 \qquad 2 = 50 \cdot 3 \cdot \frac{dh}{dt}$$

$$\underline{\frac{dh}{dt}} = \underline{\frac{1}{75}} \text{ m min}^{-1}$$

13



$$l^2 = s^2 + 49$$

$$l = \sqrt{s^2+49} \qquad s = \sqrt{l^2-49}$$

$$\frac{dl}{dt} = 30\ ms^{-1} \qquad s = 24 \Rightarrow l = \sqrt{24^2+49}$$

$$= \sqrt{625} = 25$$

$$\frac{ds}{dt} = \frac{l}{\sqrt{l^2-49}} \cdot \frac{dl}{dt} = \frac{25}{24} \cdot 30 = \frac{25 \cdot 5}{4} = \underline{\frac{125}{4}}$$

7.4

5. $f(x) = \tan 2x \qquad -\frac{\pi}{4} < x < \frac{\pi}{4}$

$f'(x) = \frac{2}{\cos^2 2x} > 0$

$g = f^{-1} \qquad g(1) = y \iff f(y) = 1$

$y \overset{?}{=} \frac{\pi}{8}$

$\underline{g'(1)} = \frac{1}{f'(\frac{\pi}{8})} = \frac{1}{\frac{2}{\frac{1}{2}}} = \underline{\underline{\frac{1}{4}}}$

okt 20-13:42

8 $\quad y = g(x)$ er invers til f

$\Rightarrow \quad f(g(x)) = x$

$\frac{d}{dx}: \quad f'(g(x)) \cdot g'(x) = 1 \qquad g'(x) = \frac{1}{f'(g(x))}$

$$f''(g(x)) \cdot g'(x) \cdot g'(x) + f'(g(x)) \cdot g''(x) = 0$$

$$g''(x) = - \frac{f''(g(x)) \, (g'(x))^2}{f'(g(x))}$$

$f(x) = \sin x$

$g''(\frac{1}{2})$

$g(\frac{1}{2}) = \frac{\pi}{6}$

$$= - \frac{f''(g(x)) \, g'(x)}{(f'(g(x)))^2} = - \frac{f''(g(x))}{f'(g(x))^3}$$

$f'(x) = \cos x$

$f''(x) = -\sin x$

$$= - \frac{-\sin\frac{\pi}{6}}{(\cos\frac{\pi}{6})^3} = \frac{\frac{1}{2}}{(\frac{1}{2}\sqrt{3})^3} = \underline{\frac{4}{3\sqrt{3}}}$$

7.5 36

$$\lim_{x \to \frac{\pi}{2}} \frac{\cot x}{\frac{\pi}{2} - x} =$$

$\cos x = \sin \frac{\pi}{2} - x$

$$\frac{\cot x}{\frac{\pi}{2} - x} = \frac{\cos x}{\sin x} \frac{1}{\frac{\pi}{2} - x} = \frac{1}{\sin x} \frac{\sin \frac{\pi}{2} - x}{\frac{\pi}{2} - x}$$

$$\Rightarrow \lim_{x \to \frac{\pi}{2}} \frac{\cot x}{\frac{\pi}{2} - x} = \lim_{x \to \frac{\pi}{2}} \frac{1}{\sin x} \cdot \lim_{x \to \frac{\pi}{2}} \frac{\sin \frac{\pi}{2} - x}{\frac{\pi}{2} - x}$$

$$= 1 \cdot 1 = \underline{\underline{1}}$$

okt 20-13:55

7.1 3b

$$\lim_{x \to 0} \frac{\arcsin x}{\sin 3x}$$

$$= \lim_{x \to 0} \frac{\arcsin x}{\sin 3x} \overset{l'H}{=} \lim_{x \to 0} \frac{\frac{1}{\sqrt{1-x^2}}}{3\cos 3x} = \underline{\underline{\frac{1}{3}}}$$

okt 20-13:58