Faztorization of second degree polynomials

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Abstract

Short explanation of a fast method for finding roots of 2nd degree polynomials

1 Method for finding real roots

Let a, b, c, d, $e \in \mathbb{R}$
Then the 2nd. degree polynomial

$$ax^2 + bx + c = 0 \tag{1}$$

have the solutions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{2}$$

thus we can factorize (1) as

$$(x - d)(x - e) = 0 \tag{3}$$

Since

$$(x - d)(x - e) = x^2 - (d + e)x + de \tag{4}$$

where d and e are the solution to the right hand side of equation 2 (assuming (1) has real roots).

Solving (2) by hand often takes time and are in most cases when the roots are real, not necessary.
Instead we can use the following logical deductions:

Start by making sure the coefficient in front of x^2 is one

$$a(x^2 + \frac{b}{a}x + \frac{c}{a}) = 0 \tag{5}$$

So

$$a = 0 \vee (x^2 + \frac{b}{a}x + \frac{c}{a}) = 0 \tag{6}$$

In most cases $a = 1$, and we get

$$x^2 + bx + c = 0 \tag{7}$$

Thus we can ask ourself the question

"What can you multiply in order to get c, and simultaneously add together to get b?"

It's important to note that the numbers searched for in this question should be put into (3) and are **not** the roots.

1.1 Examples

$$x^2 + 6x + 8 = 0 \tag{8}$$

We start by looking for two numbers to multiply in order to get $c = 8$. +4 and +2 are good candidates, and if we add them together we get $b = 6$. Thus

$$(x + 4)(x + 2) = 0 \tag{9}$$

And finding the roots are trivial.

Now let

$$x^2 + 4x - 5 = 0 \tag{10}$$

The first candidates that may come easily to mind for finding c are $-1 * +5$ and $1 * -5$.
Adding +5 together with -1 gives $b = 4$.
Thus

$$(x + 5)(x - 1) = 0 \tag{11}$$

And finding the roots are trivial.

Additionally we can look at

$$x^2 - 9x + 20 = 0 \tag{12}$$

Here we can help ourself by factorizing c which is our first target.
20 = 2*2*5. We need to add some of the factors together to get a negative b, so we se that -4 and -5 gives $b = -9$.
Thus

$$(x - 5)(x - 4) = 0 \tag{13}$$

And finding the roots are trivial.

Going further, we have

$$x^2 - 20x - 69 = 0 \tag{14}$$

Using our technique from (12) we get $c = -2*2*5*3+3*3 = -3*(20+3)$.
We see that -23 and +3 gives $b = -20$.
Thus

$$(x + 3)(x - 23) = 0 \tag{15}$$

And finding the roots are trivial.

Finally, let

$$x^2 - \frac{1}{2}x - \frac{1}{2} = 0 \tag{16}$$

Using our technique from (10) we get $c = 1 * -\frac{1}{2}$ or $c = -1 * \frac{1}{2}$.
We see that -1 added with $\frac{1}{2}$ gives $b = -\frac{1}{2}$.
Thus

$$(x - 1)(x + \frac{1}{2}) = 0 \tag{17}$$

And finding the roots are trivial.

1.2 Exercises

Remember that in the cases where $a \neq 1$, the question regards *what we can multiply in order to get* $\frac{c}{a}$ *and simultaneously add together to get* $\frac{b}{a}$.
Additional interesting problems are given below

$$x^2 - 4 = 0 \tag{18}$$

$$x^2 - 57x = 0 \tag{19}$$

$$x^2 - 15x + 26 = 0 \tag{20}$$

$$x^2 + 14x + 45 = 0 \tag{21}$$

$$x^2 + 10x - 24 = 0 \tag{22}$$

$$x^2 - 13x + 12 = 0 \tag{23}$$

$$x^2 + 3x - 70 = 0 \tag{24}$$

$$x^2 - 12x + 35 = 0 \tag{25}$$

$$2x^2 - x - 21 = 0 \tag{26}$$

$$3x^2 + 2x + \frac{1}{3} = 0 \tag{27}$$

$$2x^2 - 8x - 24 = 0 \tag{28}$$

$$3x^2 - 11x - 4 = 0 \tag{29}$$

$$3x^2 - 14x + 5 = 0 \tag{30}$$

$$8x^2 + 14x - 15 = 0 \tag{31}$$

$$5x^2 - 34x + 24 = 0 \tag{32}$$

$$8x^2 - 47x - 63 = 0 \tag{33}$$

$$6x^2 + 11x - 35 = 0 \tag{34}$$

$$11x^2 + 18x - 7 = 0 \tag{35}$$