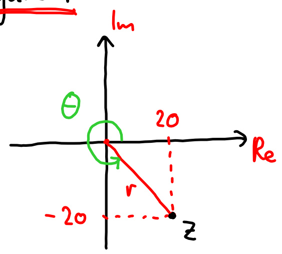
Løsningsforslag oblig 1 Mat 1100 høst 2016

Oppgave 1

a)

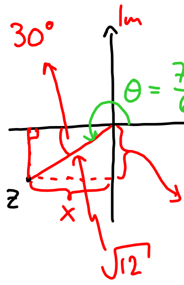


$$r = \sqrt{20^2 + 20^2} = \sqrt{800} = \sqrt{2 \cdot 400}$$
$$= \sqrt{2} \cdot \sqrt{400} = 20\sqrt{2}$$

$$\theta = \frac{7\pi}{4}$$

Så $\underline{z = 20\sqrt{2}\, e^{i(7\pi/4)}}$

b)



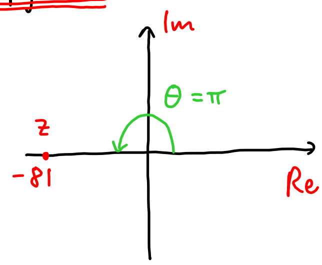
$$\frac{\sqrt{12}}{2} = \frac{\sqrt{12}}{\sqrt{4}} = \sqrt{\frac{12}{4}} = \sqrt{3}$$

$$x^2 + 3 = 12$$
$$x^2 = 9, \text{ dvs. } x = 3.$$

Så $\underline{z = -3 - \sqrt{3}\, i}$

Oppgave 2

a)



$$z = re^{i\theta} = 81 e^{i\pi}$$

Prinsipal rot:

$$w_0 = \sqrt[4]{r}\, e^{i(\theta/4)}$$
$$= \sqrt[4]{81}\, e^{i(\pi/4)} = 3e^{i(\pi/4)}$$

Vi har $w_+ = e^{i(2\pi/4)}$
$$= e^{i(\pi/2)}$$

$$w_1 = w_+ w_0 = e^{i(\pi/2)} \cdot 3 \cdot e^{i(\pi/4)} = 3e^{i\left(\frac{\pi}{2} + \frac{\pi}{4}\right)}$$
$$= 3e^{i(3\pi/4)}$$

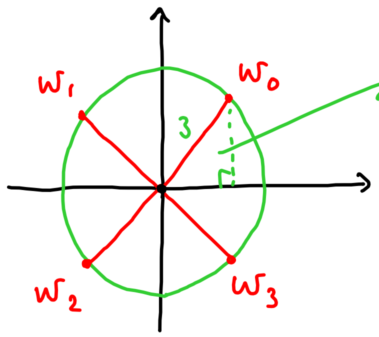
$$w_2 = w_+ w_1 = e^{i(\pi/2)} \cdot 3e^{i(3\pi/4)} = 3e^{i\left(\frac{\pi}{2} + \frac{3\pi}{4}\right)}$$
$$= 3e^{i(5\pi/4)}$$

$$w_3 = w_+ w_2 = e^{i(\pi/2)} \cdot 3e^{i(5\pi/4)} = 3e^{i\left(\frac{\pi}{2} + \frac{5\pi}{4}\right)}$$
$$= 3e^{i(7\pi/4)}$$

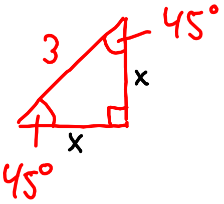
Røttene er $\underline{3e^{i(\pi/4)},\ 3e^{i(3\pi/4)},\ 3e^{i(5\pi/4)},\ 3e^{i(7\pi/4)}}$

(Oppgave 2 forts.)

b)



45/45/90 - trekant med hypotenus 3



$$x^2 + x^2 = 3^2$$
$$2x^2 = 9$$
$$x = \sqrt{\frac{9}{2}} = \frac{3}{\sqrt{2}}$$

Vi ser da at røttene kan skrives

$$\underline{\underline{w_0 = \frac{3}{\sqrt{2}} + \frac{3}{\sqrt{2}}i}} \quad \underline{\underline{w_1 = -\frac{3}{\sqrt{2}} + \frac{3}{\sqrt{2}}i}} \quad \underline{\underline{w_2 = -\frac{3}{\sqrt{2}} - \frac{3}{\sqrt{2}}i}} \quad \underline{\underline{w_3 = \frac{3}{\sqrt{2}} - \frac{3}{\sqrt{2}}i}}$$

Oppgave 3

$$z + zi = \frac{5+i}{8+2i}$$

$$z(1+i) = \frac{5+i}{8+2i}$$

$$z = \frac{5+i}{(8+2i)(1+i)} = \frac{5+i}{8+2i+8i-2} = \frac{5+i}{6+10i}$$

$$= \frac{(5+i)(6-10i)}{(6+10i)(6-10i)} = \frac{30+6i-50i+10}{36+60i-60i+100} = \frac{40-44i}{136}$$

$$= \frac{10-11i}{34} = \underline{\underline{\frac{5}{17} - \frac{11}{34}i}}$$

Oppgave 4

a) $(3+i)^2 = 9 + 6i - 1 = 8 + 6i$

$(3+i)^3 = (8+6i)(3+i) = 24 + 18i + 8i - 6 = 18 + 26i$

$(3+i)^4 = (18+26i)(3+i) = 54 + 78i + 18i - 26 = 28 + 96i$

Dette gir

$$P(3+i) = (28+96i) - 8(18+26i) + 39(8+6i) - 122(3+i) + 170$$
$$= 28 + 96i - 144 - 208i + 312 + 234i - 366 - 122i + 170$$
$$= 0$$

Altså er $z = 3+i$ en rot til $P(z)$

b) Siden $P(z)$ har kun reelle koeffisienter, vet vi at det konjugerte tallet $3-i$ også er en rot. Dermed vet vi at

$$(z-(3+i))\cdot(3-(3-i)) = (z-3-i)\cdot(z-3+i)$$
$$= z^2 - 3z - \cancel{iz} - 3z + 9 + \cancel{3i} + \cancel{iz} - \cancel{3i} + 1$$
$$= z^2 - 6z + 10$$

er en faktor i $P(z)$. Polynomdivisjon:

$$(z^4 - 8z^3 + 39z^2 - 122z + 170) : (z^2 - 6z + 10) = z^2 - 2z + 17$$

$$\begin{array}{l}
\underline{z^4 - 6z^3 + 10z^2} \\
\quad -2z^3 + 29z^2 - 122z + 170 \\
\quad \underline{-2z^3 + 12z^2 + 170} \\
\qquad\qquad 17z^2 - 102z + 170 \\
\qquad\qquad \underline{17z^2 - 102z + 170} \\
\qquad\qquad\qquad\qquad 0
\end{array}$$

Altså: $P(z) = (z^2 - 6z + 10)\cdot(z^2 - 2z + 17)$

(Oppgave 4b) forts.)

For å faktorisere $z^2-2z+17$, setter vi opp

$z^2-2z+17=0$, som gir

$$z=\frac{2\pm\sqrt{4-4\cdot 17}}{2}=\frac{2\pm\sqrt{-64}}{2}=\frac{2\pm\sqrt{(-1)\cdot 64}}{2}$$

$$=\frac{2\pm i\cdot 8}{2}=\begin{cases}1+4i\\1-4i\end{cases}$$

De øvrige røttene til $P(z)$ er $\underline{\underline{3-i}}$, $\underline{\underline{1+4i}}$ og $\underline{\underline{1-4i}}$

c) Kompleks faktorisering:

$$P(z)=\underline{\underline{(z-(3+i))\cdot(z-(3-i))\cdot(z-(1+4i))\cdot(z-(1-4i))}}$$

d) Reell faktorisering:

$$P(z)=\underline{\underline{(z^2-2z+17)\cdot(z^2-6z+10)}}$$

Oppgave 5

a) La galgen ha posisjon g. Første merke er da $m=g+ig$

Andre merke er

$$M=z+i(z-g)+(z-g)$$

Midtpunktet mellom merkene blir da

$$\frac{1}{2}(m+M)=\frac{1}{2}(\cancel{g}+\cancel{ig}+z+iz-\cancel{ig}+z-\cancel{g})=\underline{\underline{z+\frac{1}{2}iz}}$$

b) Start ved østlig palme og gå til vestlig palme, mens du teller antall skritt. Drei så 90° til venstre, og gå halvparten så mange skritt. Da er du der skatten er begravd.