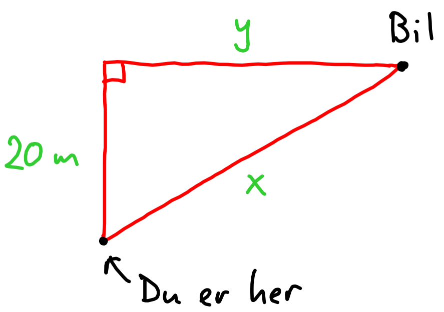
Løsningsforslag oblig 2 Mat1100 høst 2016

Oppgave 1



Pytagoras:

$$x^2 = y^2 + 20^2$$

Deriverer:

$$2x \cdot x'(t) = 2y \cdot y'(t) + 0$$

dvs.

$$y'(t) = \frac{x(t) \cdot x'(t)}{y(t)}$$

Vårt øyeblikk:

$y = 40\ \text{m}$ gir

$$x = \sqrt{20^2 + 40^2}\ \text{m}$$
$$= \sqrt{2000}\ \text{m} = 10\sqrt{20}\ \text{m}$$

$$x'(t) = 105\ \text{km/h}$$

$$= \frac{10\sqrt{20}\ \text{m} \cdot 105\ \text{km/h}}{40\ \text{m}}$$

$$\approx 117\ \text{km/h}$$

Bilens fart er ca 117 km/h

Oppgave 2

a) $$\frac{1}{1+\cot^2 x} = \frac{1}{1+\frac{\cos^2 x}{\sin^2 x}} = \frac{\sin^2 x}{\sin^2 x + \cos^2 x}$$

$$= \frac{\sin^2 x}{1} = \underline{\underline{\sin^2 x}}$$

b) $f(x) = \cot x$ gir $f'(x) = \left(\frac{\cos x}{\sin x}\right)'$

$$= \frac{-\sin x \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = \frac{-1}{\sin^2 x}$$

Siden vi vet at $f(x) = \cot x$ er deriverbar, har vi at

$$(f^{-1})'(f(x)) = \frac{1}{f'(x)}$$

Innsetting gir

$$(f^{-1})'(\cot x) = \frac{1}{\left(\frac{-1}{\sin^2 x}\right)} = -\sin^2 x$$

c) Innsetting av a) i formelen fra b) gir

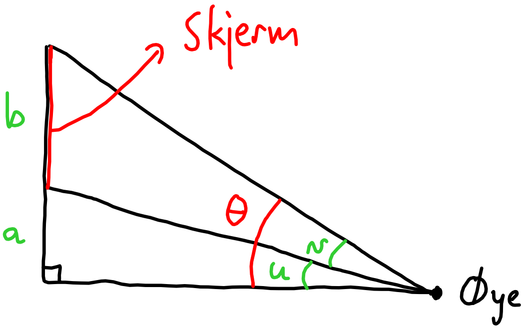
$$(f^{-1})'(\cot x) = -\frac{1}{1+(\cot x)^2}$$

Hvis f^{-1} er arccot, vil alle argumenter for f^{-1} være på formen $\cot x$. Altså kan vi erstatte $\cot x$ med x. Det gir

$$(f^{-1})'(x) = -\frac{1}{1+x^2}, \quad \text{dvs. } \underline{\underline{(\operatorname{arccot} x)' = -\frac{1}{1+x^2}}}$$

Oppgave 3

a)



$\tan u = \frac{a}{x}$, så $u = \arctan \frac{a}{x}$

$\tan \theta = \frac{b+a}{x}$, så $\theta = \arctan \frac{b+a}{x}$

Dermed $v(x) = \theta - u = \arctan\left(\frac{a+b}{x}\right) - \arctan\left(\frac{a}{x}\right)$

b)

$$v'(x) = \frac{1}{1+\left(\frac{a+b}{x}\right)^2} \cdot \left[\frac{-(a+b)}{x^2}\right] - \frac{1}{1+\left(\frac{a}{x}\right)^2} \cdot \left[\frac{-a}{x^2}\right]$$

$$= \frac{a}{x^2+a^2} - \frac{a+b}{x^2+(a+b)^2}$$

c)

$$\lim_{x\to\infty} v(x) = \lim_{x\to\infty}\left[\arctan\left(\frac{a+b}{x}\right) - \arctan\left(\frac{a}{x}\right)\right]$$

$\frac{a+b}{x} \to 0$, $\frac{a}{x} \to 0$

$$= \arctan 0 - \arctan 0 = 0 - 0 = 0$$

d)

$$\lim_{x\to 0^+} v(x) = \lim_{x\to 0^+}\left[\arctan\left(\frac{a+b}{x}\right) - \arctan\left(\frac{a}{x}\right)\right]$$

$\frac{a+b}{x} \to +\infty$, $\frac{a}{x} \to +\infty$

$$= \frac{\pi}{2} - \frac{\pi}{2} = 0$$

(Oppgave 3 forts.)

e) $v'(x) = 0$ gir $\dfrac{a}{x^2+a^2} = \dfrac{a+b}{x^2+(a+b)^2}$

$$ax^2 + a(a+b)^2 = (a+b)(x^2+a^2)$$

$$\cancel{ax^2} + a(a^2+2ab+b^2) = \cancel{ax^2} + bx^2 + a^3 + a^2b$$

$$\cancel{a^3} + 2a^2b + ab^2 = bx^2 + \cancel{a^3} + a^2b$$

$$a^2b + ab^2 = bx^2$$

$$x^2 = ab + a^2 = a(a+b)$$

$$x = \sqrt{a(a+b)} \quad \text{(fordi } x > 0\text{)}$$

Siden $v(x) > 0$ for alle $x \in (0, \infty)$ og

$$\lim_{x \to 0^+} v(x) = \lim_{x \to \infty} v(x) = 0$$

følger at $x = \sqrt{a(a+b)}$ må være globalt maksimumspunkt for $v(x)$ på $(0, \infty)$.

f) Du bør stå i avstand $\sqrt{a(a+b)}$ fra skjermen.

Hvis a er 4 meter og b er 5 meter, blir avstanden

$$\sqrt{4(4+5)} = \sqrt{4} \cdot \sqrt{9} = 2 \cdot 3 = \underline{\underline{6}} \ \text{(meter)}$$