Plenum 18/9

4.3: 1 d, 3 c

4.3: 4 b, 11, 13 c, 18, 19

5.1: 1 e, 3 c, 5 abeg, 6, 7 b, 9 bce

5.2: 1 b, 3 a, 5, 7, 8, 10

4.3: Konvergens av følger

1) d) $\lim_{n \to \infty} \left(\underbrace{\frac{2n^3 - 13}{5n^3 - 4}}_{\mathrm{I}} - \underbrace{\frac{4n^4 + 12}{1 - 5n^4}}_{\mathrm{II}} \right)$

ser om disse konvergerer

I) $\lim_{n \to \infty} \frac{2n^3 - 13}{5n^3 - 4} = \lim_{n \to \infty} \frac{\frac{2n^3 - 13}{n^3}}{\frac{5n^3 - 4}{n^3}}$

deler på høyeste potens av n

$= \lim_{n \to \infty} \frac{2 - \frac{13}{n^3}}{5 - \frac{4}{n^3}} = \frac{2}{5}$

$\frac{13}{n^3} \to 0$, $\frac{4}{n^3} \to 0$

II) $\lim_{n \to \infty} \frac{4n^4 + 12}{1 - 5n^4} = \lim_{n \to \infty} \frac{4 + \frac{12}{n^4}}{\frac{1}{n^4} - 5} = -\frac{4}{5}$

$\frac{12}{n^4} \to 0$, $\frac{1}{n^4} \to 0$

Da er:

$$\lim_{n\to\infty}\left(\frac{2n^3-13}{5n^3-4}-\frac{4n^4+12}{1-5n^4}\right)$$

$$\underset{\text{Setning 4.3.3}}{=}\lim_{n\to\infty}\frac{2n^3-13}{5n^3-4}-\lim_{n\to\infty}\frac{4n^4+12}{1-5n^4}$$

$$=\frac{2}{5}-\left(-\frac{4}{5}\right)=\underline{\underline{\frac{6}{5}}}$$

3) c) $$\lim_{n\to\infty}\left(\sqrt{n^2+n}-n\right)=\lim_{n\to\infty}\frac{\left(\sqrt{n^2+n}-n\right)\left(\sqrt{n^2+n}+n\right)}{\left(\sqrt{n^2+n}+n\right)}$$

$$\underset{\text{3. kvad. set.}}{=}\lim_{n\to\infty}\frac{\cancel{n^2}+n-\cancel{n^2}}{\sqrt{n^2+n}+n}=\lim_{n\to\infty}\frac{n}{\sqrt{n^2+n}+n}$$

$$=\lim_{n\to\infty}\frac{\frac{n}{n}}{\frac{\sqrt{n^2+n}+n}{n}}=\lim_{n\to\infty}\frac{1}{\frac{\sqrt{n^2+n}}{n}+1}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt{\frac{n^2+n}{n^2}}+1}=\lim_{n\to\infty}\frac{1}{\sqrt{1+\frac{1}{n}}+1}$$

$\frac{1}{n}\to 0$

$$\frac{n^2+n}{n^2}=\frac{n^2}{n^2}+\frac{n}{n^2}=1+\frac{1}{n}$$

$$=\frac{1}{\sqrt{1+0}+1}=\underline{\underline{\frac{1}{2}}}$$

4) b) $\underbrace{\lim_{n\to\infty} \frac{2\sin(n)}{n} = 0}$:

La $\varepsilon > 0$. Vil finne $N \in \mathbb{N}$ s.a. for alle $n \geq N$ er:

$$\Big|\underbrace{\frac{2\sin(n)}{n}}_{a_n} - \underbrace{0}_{a}\Big| = \Big|\frac{2\sin(n)}{n}\Big| = \frac{2|\sin(n)|}{n} < \varepsilon$$

$\underline{\text{Merk}}$: $|\sin(n)| \leq 1$ for alle n ($\sin(n) \in [-1, 1]$ for alle n). Derfor er det nok å finne $N \in \mathbb{N}$ s.a.:

$$\frac{2 \cdot 1}{N} = \frac{2}{N} < \varepsilon \quad \left(\text{siden } \frac{2|\sin(n)|}{n} \leq \frac{2 \cdot 1}{n} \text{ for alle } n\right)$$

$$\frac{2}{\varepsilon} < N$$

Velg N til å være det første heltallet større enn $\frac{2}{\varepsilon}$. Da er, for alle $n \geq N$:

$$\Big|\frac{2\sin(n)}{n} - 0\Big| \leq \frac{2}{n} \leq \frac{2}{N} < \frac{2}{\frac{2}{\varepsilon}} = \varepsilon$$

Dermed er $\lim_{n\to\infty} \frac{2\sin(n)}{n} = 0$.

13) c) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = 0$

Vil ha: $\lim_{n\to\infty} \frac{a_n}{b_n}$ er verken 0 eller ∞.

La $\{a_n\} = \{\frac{1}{n}\}$, $\{b_n\} = \{\frac{1}{n}\}$. Da vil

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n} = 0$$

$$\lim_{n\to\infty} b_n = 0$$

Hva med:

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\cancel{\frac{1}{n}}}{\cancel{\frac{1}{n}}} = \lim_{n\to\infty} 1 = \underline{\underline{1}} \notin \{0, \infty\}$$

$$\{c_n\} = \{\frac{2}{n}\}$$

5.1: Kontinuitet

1) e) $f(x) = \frac{\sqrt{x+2}}{\ln|x|}$; antar $f \to \mathbb{R}$ (reell funksjon)

$\sqrt{y}$ er definert for $y \geq 0$ (pga.)

$$y = x + 2 \geq 0$$
$$x \geq -2$$

$\ln|x|$ er definert for alle $x \neq 0$.

$\frac{1}{\ln|x|}$ er definert for alle $x \neq 0$ og s.a. $\ln|x| \neq 0$,
dvs. $x \neq 1$ og $x \neq -1$