9.1.11'

$$I = \int \frac{x^2 \arctan x}{1+x^2}\,dx$$

$v$ $u'$

$$\int u'v = uv - \int uv'$$

$$u' = \frac{x^2}{1+x^2} \qquad u = \int \frac{x^2}{1+x^2}\,dx = \int \frac{1+x^2-1}{1+x^2}\,dx$$

$$\left(I = uv - \int uv'\right) \qquad = \int 1 - \frac{1}{1+x^2}\,dx = \underline{x - \arctan x}$$

$$I = (x - \arctan x)\cdot \arctan x - \int (x - \arctan x)\cdot \frac{1}{1+x^2}\,dx$$

$$= (x - \arctan x)\arctan x - \underbrace{\int \frac{x}{1+x^2}\,dx}_{I_1} + \underbrace{\int \frac{\arctan x}{1+x^2}\,dx}_{I_2}$$

$$\int_a^b u'(t)\,v(t)\,dt = \Big[u(t)v(t)\Big]_a^b - \int_a^b u(t)\,v'(t)\,dt$$

Merk $\left[f(t)^2\right]' = 2f(t)f'(t)$

Har $I_1 = \frac{1}{2}\int \frac{2x}{1+x^2}\,dx = \frac{1}{2}\ln(1+x^2)$

$$I_2 = \int \underbrace{\frac{1}{1+x^2}}_{f'} \cdot \underbrace{\arctan x}_{f}\,dx = \frac{1}{2}\int \underset{2}{2}\,\underset{f'}{\frac{1}{1+x^2}}\,\underset{f}{\arctan x}\,dx = \frac{1}{2}(\arctan x)^2 + C$$

Så $I = x\arctan x - (\arctan x)^2 - \frac{1}{2}\ln(1+x^2) + \frac{1}{2}(\arctan x)^2 + C$

$$= x\arctan x - \frac{1}{2}(\arctan x)^2 - \frac{1}{2}\ln(1+x^2) + C$$

9.2.1

c) $I = \int \frac{x}{\sqrt{x}+1}\,dx$

$u = \sqrt{x}$

eller $\underbrace{u = \sqrt{x} + 1}$

$\Leftrightarrow \sqrt{x} = u - 1$

$\Leftrightarrow x = (u-1)^2 = u^2 - 2u + 1$

$dx = 2u\,du - 2\,du$

$\boxed{dx = (2u-2)\,du}$

$I = \int \frac{u^2-2u+1}{u}(2u-2)\,du = 2\int \frac{(u^2-2u+1)(u-1)}{u}\,du$

$= 2\int \frac{(u-1)^3}{u}\,du = 2\int \frac{u^3 - 3u^2 + 3u - 1}{u}\,du$

$= 2\int u^2 - 3u + 3 - \frac{1}{u}\,du = 2\left[\frac{1}{3}u^3 - \frac{3}{2}u^2 + 3u - \ln u + C\right]$

$= \frac{2}{3}u^3 - 3u^2 + 6u - 2\ln u + C'$

$= \frac{2}{3}(\sqrt{x}+1)^3 - 3(\sqrt{x}+1)^2 + 6(\sqrt{x}+1) - 2\ln(\sqrt{x}+1) + C'$

$$\int_0^3 \arctan\sqrt{x}\,dx \qquad u=\sqrt{x},\quad u^2 = x,\quad 2u\,du = dx$$

$u=\sqrt{x}$ $\Downarrow$ nye grenser: $u(0)=0 \quad u(3)=\sqrt{3}$

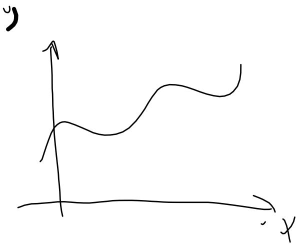
$$= 2\int_0^{\sqrt{3}} \arctan u \cdot u\,du$$

$$= 2\left[\left[\arctan u \cdot \frac{1}{2}u^2\right]_0^{\sqrt{3}} - \int_0^{\sqrt{3}} \frac{1}{1+u^2}\cdot\frac{1}{2}u^2\,du\right]$$

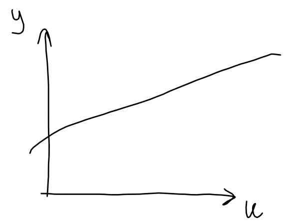
$$= 2\left[\frac{3}{2}\cdot\frac{\pi}{3} - \frac{1}{2}\int_0^{\sqrt{3}} \frac{u^2}{1+u^2}\,du\right] = \pi - \int_0^{\sqrt{3}} 1 - \frac{1}{1+u^2}\,du$$

$$= \pi - \Big[u - \arctan u\Big]_0^{\sqrt{3}} = \pi - \left(\sqrt{3} - \frac{\pi}{3}\right)$$

$$= \frac{4\pi}{3} - \sqrt{3} \quad //$$




$u = f(x)$




$x + iy$ $\qquad re^{i\theta}$

$$z = \underbrace{x^2 + y^2}_{r^2} = r^2$$