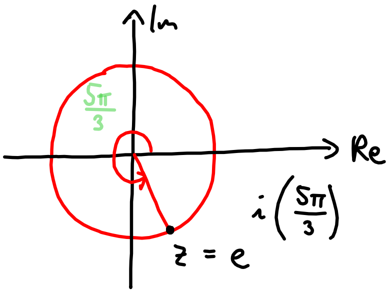
Løsningsforslag utsatt eksamen Mat 1100 h16

Del 1: DAEBC EEBBE

Oppgave 11



$$w_0 = e^{i\left(\frac{5\pi}{3}/5\right)} = e^{i(\pi/3)} = e^{i(5\pi/15)}$$

$$w_+ = e^{i(2\pi/5)} = e^{i(6\pi/15)}$$

$$w_1 = w_+ w_0 = e^{i(6\pi/15)} \cdot e^{i(5\pi/15)} = e^{i(11\pi/15)}$$

$$w_2 = w_+ w_1 = e^{i(6\pi/15)} \cdot e^{i(11\pi/15)} = e^{i(17\pi/15)}$$

$w_3 = w_+ w_2 = e^{i(23\pi/15)}$ tilsvarende

$w_4 = w_+ w_3 = e^{i(29\pi/15)}$ tilsvarende

Røttene er $e^{i(5\pi/15)}, e^{i(11\pi/15)}, e^{i(17\pi/15)}, e^{i(23\pi/15)}, e^{i(29\pi/15)}$

Oppgave 12

Hvis vi kaller determinanten $D(x)$, får vi

$$D(x) = x \cdot \begin{vmatrix} x & x \\ 1 & 1 \end{vmatrix} - 1 \cdot \begin{vmatrix} 1 & x \\ 1 & 1 \end{vmatrix} + x^3 \cdot \begin{vmatrix} 1 & x \\ 1 & 1 \end{vmatrix}$$

$$= x(x-x) - 1 \cdot (1-x) + x^3(1-x)$$

$$= -1 + x + x^3 - x^4$$

$$D'(x) = 1 + 3x^2 - 4x^3$$

(forts neste side)

(Oppg. 12 forts.)

$D'(x) = 0$ gir $4x^3 - 3x^2 - 1 = 0$

Dette er en 3.gradslikning, så vi gjetter på løsninger.

$x = 1$ passer

Altså er $(x-1)$ en faktor i $4x^3 - 3x^2 - 1$.

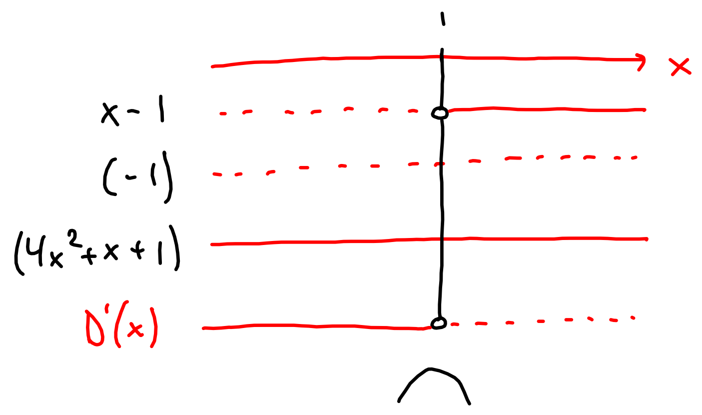
Polynomdivisjon:

$$
\begin{array}{l}
(4x^3 - 3x^2 - 1) : (x-1) = 4x^2 + x + 1 \\
\underline{4x^3 - 4x^2} \\
\qquad x^2 - 1 \\
\qquad \underline{x^2 - x} \\
\qquad\qquad x - 1
\end{array}
$$

Så $D'(x) = -(4x^3 - 3x^2 - 1) = -(4x^2 + x + 1)\cdot(x-1)$

$4x^2 + x + 1 = 0$ gir $x = \dfrac{-1 \pm \sqrt{1 - 16}}{2}$ (ingen reelle løsninger)

Dermed får vi denne fortegnslinjen:



Verdien til determinanten er størst for $x = 1$

Oppgave 13

a) $$\int \frac{\cos x}{\sin^3 x}\,dx = \int \frac{\cancel{\cos x}}{u^3}\cdot\frac{1}{\cancel{\cos x}}\,du = \int u^{-3}\,du$$

$u = \sin x$ $\quad \frac{du}{dx} = \cos x$

$dx = \frac{1}{\cos x}\,du$

$$= \frac{1}{-2}u^{-2} + C$$

$$= -\frac{1}{2}\frac{1}{\sin^2 x} + C$$

b) $$\int x\cdot\frac{\cos x}{\sin^3 x}\,dx = -\frac{1}{2}x\,\frac{1}{\sin^2 x} + \frac{1}{2}\int\frac{1}{\sin^2 x}\,dx$$

Delvis $\quad F(x) = x \qquad G'(x) = \frac{\cos x}{\sin^3 x}$

$F'(x) = 1 \qquad G(x) \overset{a)}{=} -\frac{1}{2}\frac{1}{\sin^2 x}$

$$= -\frac{1}{2}x\frac{1}{\sin^2 x} - \frac{1}{2}\cot x + C$$

Oppgave 14

a) $$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \arctan\Big(\underbrace{\tfrac{1}{x}}_{+\infty}\Big) = \frac{\pi}{2} = f(0)$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\Big[\arctan\Big(\underbrace{\tfrac{1}{x}}_{-\infty}\Big) + \pi\Big]$$

$$= -\frac{\pi}{2} + \pi = \frac{\pi}{2} = f(0)$$

Altså er f kontinuerlig i $x = 0$.

(forts. neste side)

(Oppg. 14 forts.)

b) $$f'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$$

Vi har

$$\lim_{h\to 0^+} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^+} \frac{\arctan\left(\overset{+\infty}{\frac{1}{h}}\right)-\frac{\pi}{2}}{h}$$

$$\overset{\left[\frac{0}{0}\right]}{=} \lim_{h\to 0^+} \frac{\frac{1}{1+\left(\frac{1}{h}\right)^2}\cdot\left(-\frac{1}{h^2}\right)-0}{1}$$

$$= \lim_{h\to 0^+} -\frac{1}{h^2+1} = \underline{-1}$$

Vi har også

$$\lim_{h\to 0^-} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^-} \frac{\pi+\arctan\left(\overset{-\infty}{\frac{1}{h}}\right)-\frac{\pi}{2}}{h}$$

$$\overset{\left[\frac{0}{0}\right]}{=} \lim_{h\to 0^-} \frac{\frac{1}{1+(1/h)^2}\cdot\left(-\frac{1}{h^2}\right)}{1}$$

$$= \lim_{h\to 0^-} \frac{-1}{h^2+1} = -1$$

Ergo $$f'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \underline{\underline{-1}}$$

<u>f er deriverbar i $x=0$</u>

(forts. neste side)

(Oppg. 12 forts.)

c) Volum av omdreiningslegemet er

$$V = 2\pi \int_1^{\sqrt{3}} x\, f(x)\, dx$$

$$= 2\pi \int_1^{\sqrt{3}} x \arctan\left(\frac{1}{x}\right) dx$$

$$= 2\pi \int_{\pi/4}^{\pi/6} \frac{\cos u}{\sin u} \cdot u \cdot \left(-\frac{1}{\sin^2 u}\, du\right)$$

$u = \arctan\left(\frac{1}{x}\right)$, $\tan u = \frac{1}{x}$, $x = \frac{1}{\tan u} = \frac{\cos u}{\sin u}$

$dx = -\frac{1}{\sin^2 u}\, du$ $\qquad$ $x = 1$ gir $\tan u = 1$, $u = \pi/4$

$x = \sqrt{3}$ gir $\tan u = 1/\sqrt{3}$, $u = \pi/6$

$$= 2\pi \int_{\pi/6}^{\pi/4} u \cdot \frac{\cos u}{\sin^3 u}\, du$$

13 b)

$$= 2\pi \left[-\frac{x}{2\sin^2 x} - \frac{1}{2}\,\frac{\cos x}{\sin x} \right]_{\pi/6}^{\pi/4}$$

$$= 2\pi \left[-\frac{\pi/4}{2 \cdot \frac{1}{2}} - \frac{1}{2}\,\frac{1/\sqrt{2}}{1/\sqrt{2}} + \frac{\pi/6}{2 \cdot \frac{1}{4}} + \frac{1}{2}\,\frac{\sqrt{3}/2}{1/2} \right]$$

$$= 2\pi \left[-\frac{\pi}{4} - \frac{1}{2} + \frac{\pi}{3} + \frac{\sqrt{3}}{2} \right]$$

$$= 2\pi \left[\frac{\pi}{12} - \frac{1}{2} + \frac{\sqrt{3}}{2} \right] = \underline{\underline{\frac{\pi^2}{6} + (\sqrt{3} - 1)\pi}}$$