Delvis integrasjon

11) $$\int \frac{x^2 \arctan x}{1+x^2}\,dx = uv - \int v \cdot u' = (x-\arctan x)\arctan x - \underbrace{\int \frac{x-\arctan x}{1+x^2}\,dx}_{=\ \mathrm{I}}$$

2

$u = \arctan x \quad u' = \dfrac{1}{1+x^2}$

$v' = \dfrac{x^2}{1+x^2} \quad v = x - \arctan x$

1

Først, finner vi v:

$$v = \int \frac{x^2}{1+x^2}\,dx = \int dx - \int \frac{dx}{1+x^2} = x - \arctan x.$$

$$\begin{array}{r|l} x^2 & x^2+1 \\ -\ x^2+1 & 1 \\ \hline 0-1 & \end{array} \Rightarrow \frac{x^2}{1+x^2} = 1 - \frac{1}{1+x^2}$$

$$I = \int \frac{x \arctan x}{1+x^2}\,dx = \int \frac{x}{1+x^2}\,dx - \int \frac{\arctan x}{1+x^2}\,dx$$

$$= \frac{1}{2} \cdot \int \frac{2x}{1+x^2}\,dx - \int \arctan x \cdot \frac{1}{1+x^2}\,dx$$

$$= \frac{1}{2} \cdot \ln(1+x^2) - \frac{(\arctan x)^2}{2} + C.$$

til sammen:

$$\int \frac{x^2 \arctan x}{1+x^2}\,dx = \arctan x\,(x - \arctan x) - \frac{1}{2}\ln(1+x^2) + \frac{(\arctan x)^2}{2} + C$$

$$= x \cdot \arctan x - \frac{1}{2}\arctan^2 x - \frac{1}{2}\ln(1+x^2) + C$$

9.2. Substitusjon

2) b) $\int \frac{\sqrt{x}}{1+x}\,dx = \int \frac{u}{1+u^2}\cdot 2u\,du = \int \frac{2u^2}{1+u^2}\,du =$

$u = \sqrt{x} \leadsto x = u^2$

$dx = 2u\,du$

$= 2u - 2\arctan u + C$

$= 2\sqrt{x} - 2\arctan\sqrt{x} + C$

shift tilbake!

nov 8-09:50

g) $\int \cos(\ln x)\, dx = \int \cos(u)\, e^u\, du = (*)$

$u = \ln x \rightsquigarrow x = e^u$

$dx = e^u\, du.$

$w = \cos(u) \qquad w' = -\sin(u)$

$v' = e^u \qquad v = e^u$

$(*) = \cos u \cdot e^u + \int \sin u \cdot e^u\, du = \cos u\, e^u + \sin u\, e^u - \int \cos u \cdot e^u\, du$

$w = \sin u \qquad w' = \cos u$

$v' = e^u \qquad v = e^u$

$$\int \cos u\, e^u\, du = \frac{1}{2}\left(\cos u\, e^u + \sin u\, e^u\right) + C = \frac{1}{2} x \left(\cos(\ln x) + \sin(\ln x)\right) + C$$

skift tilbake