9.3.5 f $\displaystyle\int \frac{3x^2+x}{(x-1)(x+1)^2}\,dx$

$$\frac{3x^2+x}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$3x^2+x = A(x+1)^2 + B(x+1)(x-1) + C(x-1)$$

Sett $x=1$

$4 = 4A + 0 + 0$

Så $A = 1$

$x=-1$

$2 = C(-2) \Rightarrow$ så $C = -1$.

Sett $x=0$

$0 = A - B - C$

$= 1 - B + 1$

$B = 2$

Så $$\frac{3x^2+x}{(x-1)(x+1)^2} = \frac{1}{x-1} + \frac{2}{x+1} - \frac{1}{(x+1)^2}$$

Alle ledd enkle å integrere.

Så $$\int \frac{3x^2+x}{(x-1)(x+1)^2}\,dx = \ln|x-1| + 2\ln|x+1| + \frac{1}{x+1} + C$$

nov 25-08:12

9.3.21 a) $I = \int \frac{u+2}{u^2+2u+5}\,du$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Er irredusibel om $b^2-4ac<0$:

$4-4\cdot 1\cdot 5<0$

Så ingen reelle røtter.

Derivert til nevner er $2u+2$.

Merk at $\frac{u+2}{u^2+2u+5} = \frac{1}{2}\frac{2u+4}{u^2+2u+5} = \frac{1}{2}\frac{\overbrace{2u+2}^{f'}+2}{u^2+2u+5}$

$= \frac{1}{2}\frac{2u+2}{u^2+2u+5} + \frac{1}{u^2+2u+5}$

$(ab)^2 = a^2 \cdot b^2$

Husk

$(\arctan x)' = \frac{1}{1+x^2}$

Så $I = \frac{1}{2}\int \underbrace{\frac{2u+2}{u^2+2u+5}}\,du + \int \frac{1}{u^2+2u+5}\,du$

$v = u^2+2u+5$
$dv = 2u+2\,du$

$= \frac{1}{2}\int \frac{1}{v}\,dv + \int \frac{1}{u^2+2u+1+4}\,du$

$\frac{1}{4}(u+1)^2 = \left(\frac{1}{2}\right)^2(u+1)^2 = \left(\frac{u+1}{2}\right)^2$

$= \frac{1}{2}\ln|v| + \int \frac{1}{4+(u+1)^2}\,du = \frac{1}{2}\ln|v| + \int \frac{1/4}{1+\left(\frac{u+1}{2}\right)^2}\,du$

$= \frac{1}{2}\ln|v| + \frac{1}{4}\int \frac{1}{1+\left(\frac{u+1}{2}\right)^2}\,du$

Sett $v = \frac{u+1}{2}$ $\quad dv = \frac{1}{2}\,du$

$du = 2\,dv$

Så $= \frac{1}{2}\ln|v| + \frac{1}{4}\int \frac{2\,dv}{1+v^2}$

$= \frac{1}{2}\ln|v| + \frac{1}{2}\arctan v = \frac{1}{2}\ln|u^2+2u+5| + \frac{1}{2}\arctan\left(\frac{u+1}{2}\right) + C$

nov 25-08:30

B) Finn A, B, C s.a.

$$\frac{1}{u(u^2+2u+5)} = \frac{A}{u} + \frac{Bu+C}{u^2+2u+5}.$$

$$1 = A(u^2+2u+5) + (Bu+C)u$$

$$= u^2(A+B) + u(2A+C) + 5A$$

$A + B = 0$ $\quad B = -\frac{1}{5}$

$2A + C = 0$ $\quad C = -\frac{2}{5}$

$5A = 1 \Rightarrow A = \frac{1}{5}$

$$= \frac{1}{5}\frac{1}{u} - \frac{1}{5}\frac{u+2}{u^2+2u+5}$$

nov 25-08:41

c) $\int \frac{\tan x}{\cos^2 x + 2\cos x + 5}\,dx$ $\qquad \tan x = \frac{\sin x}{\cos x}$

$= -\int \frac{-\sin x}{\cos x(\cos^2 x + 2\cos x + 5)}\,dx$

Setter $u = \cos x$
så er $du = -\sin x\,dx$

$= -\int \frac{1}{u(u^2+2u+5)}\,du$

(*)

$= -\int \frac{1}{5}\frac{1}{u} - \frac{1}{5}\underbrace{\frac{u+2}{u^2+2u+5}}_{\text{gjør denne først, kall den } I'}\,du$

$\frac{f'}{f} = \frac{2u+2}{u^2+2u+5} + \frac{2}{u^2+2u+5}$

$I' = \int \frac{u+2}{u^2+2u+5}\,du = \frac{1}{2}\int \frac{2u+2+2}{u^2+2u+5}\,du$

$= \frac{1}{2}\ln(u^2+2u+5) + \int \frac{1}{u^2+2u+5}\,du$

$= - \text{''} + \int \frac{1}{(u^2+2u+1)+4}\,du$

$= - \text{''} \frac{1}{4}\int \frac{1}{1+\left(\frac{u+1}{2}\right)^2}\,du$

Sett $v = \frac{u+1}{2}$ så $dv = \frac{1}{2}du$, $du = 2\,dv$

$= - \text{''} + \frac{1}{2}\arctan\left(\frac{u+1}{2}\right) + C$

(*) $= -\frac{1}{5}\ln|u| + \frac{1}{10}\ln(u^2+2u+5) + \frac{1}{10}\arctan\left(\frac{u+1}{2}\right) + C$

$u = \cos x$ (skriv inn)

9.3.25 a) V.a. $2+i$ er en rot i $\underbrace{z^3 - 11z + 30}_{P} = 0$.

Metode 1: Sett inn $2+i$ og håp på null....

Bedre: Husk 1: Om z er en rot av P hvor P har reelle koeffisienter, så er også $\bar{z}$ en rot.

Husk 2: Om α er en av P så er $z-\alpha$ en faktor i P.

$(z-(2+i))(z-(2-i))$ er en faktor i P.

Så vi kan skrive om

$$(z-(2+i))(z-(2-i)) = z^2 - z(2-i+2+i) + 5$$

$$f = z^2 - 4z + 5$$

Husk: $\alpha\bar{\alpha} = |\alpha|^2$

Så skriver om $f \mid P$:

$$z^3 - 11z + 20 : z^2 - 4z + 5 = z + 4$$
$$-(z^3 - 4z^2 + 5z)$$
$$4z^2 - 16z + 20$$
$$4z^2 - 16z + 20$$
$$0$$

Konkl. m/ at

$$P = (z^2 - 4z + 5)(z + 4)$$

b) $\int \frac{10x+3}{x^3 - 11x + 20}\,dx$ — Bruk

31 $\int 1 \cdot \ln(x^2 + 2x + 10)\,dx$

H!!T $uv - \int uv'$

$= x\ln(x^2+2x+10) - \int x \cdot \frac{2x+2}{x^2+2x+10}\,dx$

Bruk at: $\frac{2x^2+2x}{x^2+2x+10} = \frac{1}{2}\,\frac{x^2+x}{x^2+2x+10}$

$= \frac{1}{2}\,\frac{x^2+2x+10-x-10}{x^2+2x+10}$

$= \frac{1}{2}\left(1 - \frac{x+10}{x^2+2x+10}\right)$

Løs denne

$\int \frac{x+10}{x^2+2x+10} = \frac{1}{2}\,\frac{2x+20}{x^2+2x+10} = \frac{1}{2}\,\frac{2x+2+18}{x^2+2x+10} = \frac{1}{2}\,\frac{2x+2}{x^2+2x+10} + \frac{9}{x^2+2x+10}$

$\underbrace{x^2+2x+10}_{t}$, $\underbrace{\frac{9}{x^2+2x+10}}_{\arctan(\sim)}$