f har <u>ingen</u> asymptoter.

10.) $\underline{f(x) = x^2 (e^{\frac{1}{x}} - 1)}$; <u>Vertikale asymptoter</u>

f er kontinuerlig overalt hvor $x \neq 0 \Rightarrow$
ingen andre mulige vertikale asymptoter.

$$\lim_{x \to 0^{\pm}} \underset{\downarrow}{x^2} \underset{\downarrow}{(e^{\frac{1}{x}} - 1)} = \lim_{x \to 0^{\pm}} \frac{\overset{\to \infty}{e^{\frac{1}{x}} - 1}}{\underset{\searrow \, \infty}{\frac{1}{x^2}}}$$

$$= \lim_{x \to 0^{+}} \frac{e^{\frac{1}{x}} (-\frac{1}{x^2})}{-2\frac{1}{x^3}} = \frac{1}{2} \lim_{x \to 0^{\pm}} \frac{e^{\frac{1}{x}}}{\frac{1}{x}}$$

$\boxed{\text{"} \frac{\infty}{\infty} \text{"} : \text{L'Hôpital}}$

$\frac{1}{x^2} = x^{-2}$

$$= \frac{1}{2} \lim_{x \to 0^{\pm}} \frac{e^{\frac{1}{x}} (-\frac{1}{x^2})}{-\frac{1}{x^2}}$$

$\boxed{\text{"} \frac{\infty}{\infty} \text{"} : \text{L'Hôpital}}$

$$= \begin{cases} \infty & , \ x \to 0^{+} \\ 0 & , \ x \to 0^{-} \end{cases}$$

$\Rightarrow f$ har en vertikal asymptote når $x \to 0^{+}$
(men ikke når $x \to 0^{-}$).

Skråasymptoter:

i) $\lim_{x \to \pm\infty} \frac{f(x)}{x} = \lim_{x \to \pm\infty} \overset{\nearrow \infty \quad \nearrow 0}{x (e^{\frac{1}{x}} - 1)}$

$= \lim_{x \to \infty} \frac{e^{\frac{1}{x}} - 1}{\frac{1}{x}} = \lim_{x \to \pm\infty} \frac{e^{\frac{1}{x}} (-\frac{1}{x^2})}{-\frac{1}{x^2}}$

$\boxed{\text{"} \frac{0}{0} \text{"} : \text{L'H}}$

$= \underline{1}$

ii) $\lim\limits_{x \to \pm\infty} \left(\underbrace{x^2(e^{\frac{1}{x}}-1)}_{\infty \cdot 0} - \underbrace{x}_{\infty} \right)$

$= \lim\limits_{x \to \pm\infty} \underbrace{x}_{\infty} \left(\underbrace{x(e^{\frac{1}{x}}-1)-1}_{\downarrow \ (\text{fra } i))} \right)$

$\downarrow 0$

$= \lim\limits_{x \to \pm\infty} \dfrac{x(e^{\frac{1}{x}}-1)-1}{\frac{1}{x}} = \lim\limits_{x \to \pm\infty} \dfrac{e^{\frac{1}{x}}-1+x\,e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}}$

$\overset{\text{"}\frac{0}{0}\text{"}: \ L'H}{}$

$= \lim\limits_{x \to \pm\infty} \dfrac{e^{\frac{1}{x}}-1-\frac{e^{\frac{1}{x}}}{x^2}}{-\frac{1}{x^2}}$

$= \lim\limits_{x \to \pm\infty} -\left(\underbrace{x^2 e^{\frac{1}{x}}}_{\downarrow} + x^2 - \underbrace{e^{\frac{1}{x}}}_{\downarrow} \right)$

$\qquad\qquad\qquad \infty \qquad\qquad 0$

$= \lim\limits_{x \to \pm\infty} \left(\underbrace{x^2(e^{\frac{1}{x}}-1)}_{} + 0 \right)$

$= -\lim\limits_{x \to \pm\infty} \underbrace{x^2}_{\infty} \underbrace{(e^{\frac{1}{x}}-1)}_{0} = -\lim\limits_{x \to \pm\infty} \dfrac{\overset{\nearrow 0}{e^{\frac{1}{x}}-1}}{\frac{1}{x^2} \searrow 0}$

$\qquad\qquad\qquad\qquad\qquad\qquad \overset{\text{(somige side)}}{}$

$= -\lim\limits_{x \to \pm\infty} \frac{1}{2} e^{\frac{1}{x}} = \underline{0}$

$\overset{\text{"}\frac{0}{0}\text{"}: \ L'H}{} \ \Rightarrow \ y = 1 \cdot x + 0 = x \quad$ er en skrå-

$\boxed{6.5.5}$ asymptote for f (når $x \to \pm\infty$)