Plenum 08/11/13

Integrasjon: 8.6: 1aef, 3, 5c, 7bce, 9, 11ac, 15, 26

9.1: 1abef, 5, 9, 11

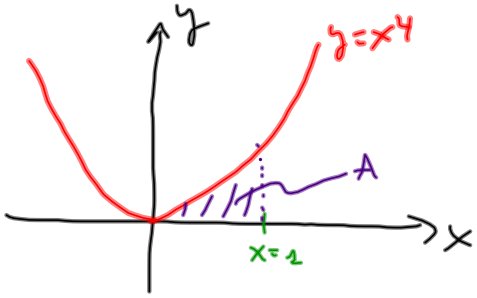
9.2: 1abcd

8.6

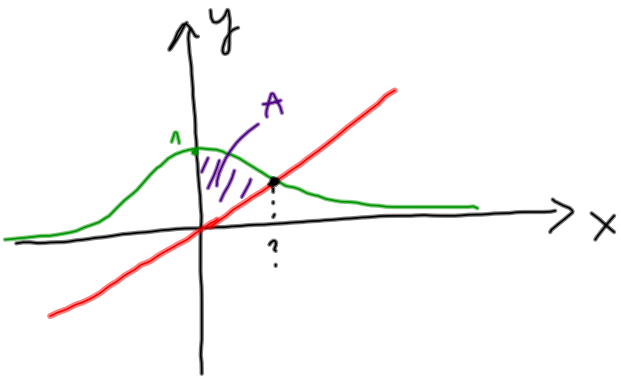
1) Finn arealet av området avgrenset av de oppgitte kurvene

a) $y = x^4$, x-aksen, $x = 1$

$$A = \int_0^1 x^4 dx = \frac{1}{5} \cdot x^5 \Big|_{x=0}^{x=1} = \frac{1}{5}$$



fl) $y = \frac{1}{1+x^2}$, $y = \frac{x}{2}$, y-aksen :



? $\frac{1}{1+x^2} = \frac{x}{2} \Leftrightarrow 2 = x + x^3$

$x^3 + x - 2 = 0$

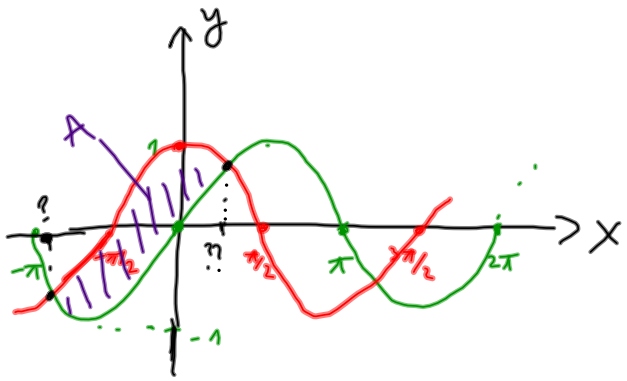
Vi ser at $x = 1$ er en rot.

$$A = \int_0^1 \frac{1}{1+x^2}\,dx - \int_0^1 \frac{x}{2}\,dx = \arctan\Big|_{x=0}^{x=1} - \frac{1}{4}x^2\Big|_{x=0}^{x=1} = \arctan 1 - \underbrace{\arctan 0}_{=0} - \frac{1}{4}$$

$$= \frac{\pi}{4} - \frac{1}{4} = \frac{\pi - 1}{4}$$

$\tan\alpha = \frac{a}{b} = 1$

3) Finne arealet mellom $\sin x$ og $\cos x$.



$\underline{!?}$ $\sin x = \cos x \iff x = \frac{\pi}{4} + k \cdot \pi$

$(\tan x = 1)$ $\quad k \in \mathbb{Z}$

? $x = \frac{\pi}{4} - \pi = \frac{-3\pi}{4}$ $\quad (k=-1)$

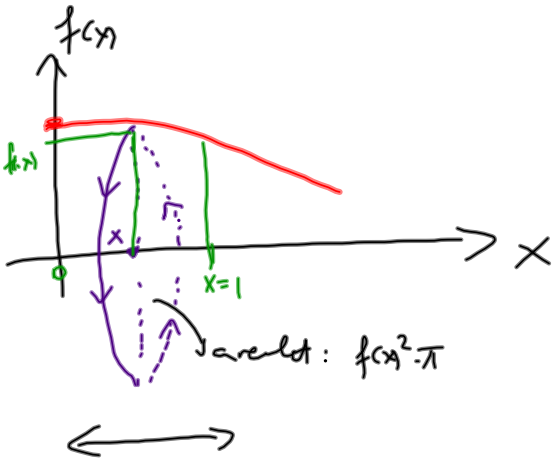
?? $x = \frac{\pi}{4}$ $\quad (k=0)$

$$A = \int_{-3\pi/4}^{\pi/4} \cos x \, dx - \int_{-3\pi/4}^{\pi/4} \sin x \, dx = \sin x \Big|_{x=-3\pi/4}^{x=\pi/4} + \cos x \Big|_{x=-3\pi/4}^{x=\pi/4} = \sin\frac{\pi}{4} - \sin\frac{-3\pi}{4} + \cos\frac{\pi}{4} - \cos\frac{-3\pi}{4}$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = 2\sqrt{2}$$

5) Finn volumet til omdreiningslegemet når vi dreier grafen om x-aksen.

$f(x) = \frac{1}{\sqrt{1+x^2}}$, $x=0$, $x=1$.



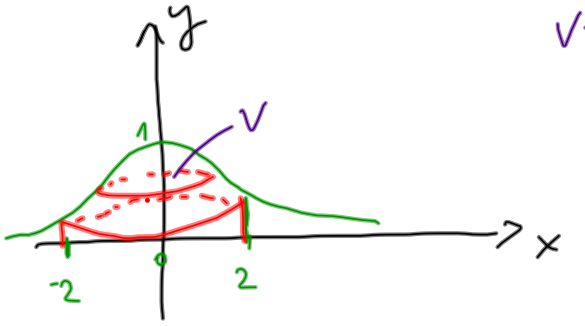
$$V = \int_0^1 \pi \cdot f(x)^2 \, dx = \pi \int_0^1 \frac{1}{1+x^2} \, dx$$

$$= \pi \arctan \Big|_{x=0}^{x=1} = \pi \arctan 1 - \pi \arctan 0$$

$\arctan 1 = \frac{\pi}{4}$, $\arctan 0 = 0$

$$= \pi \cdot \frac{\pi}{4} = \frac{\pi^2}{4}$$

7) c) $y = \frac{1}{1+x^2}$, $x=0$, $x=2$



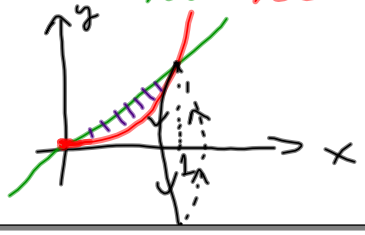
$$V = \int_0^2 2\pi \cdot x \cdot f(x)\,dx = 2\pi \int_0^2 x \cdot \frac{1}{1+x^2}\,dx$$

$$= 2\pi \int_0^2 \frac{x}{1+x^2}\,dx = \pi \int_0^2 \frac{2x}{1+x^2}\,dx = \circledast$$

den deriverte er $2x$

$$\circledast = \pi \cdot \ln(1+x^2)\Big|_{x=0}^{x=2} = \pi \cdot \ln 5 - \pi \ln(1) = \pi \ln 5$$

9) a) $y = x$, $y = x^2$



$\int_a^b \pi f(x)^2\,dx$

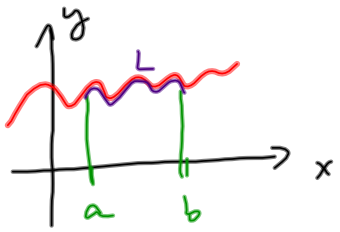
$$V = \int_0^1 \pi (x)^2\,dx - \int_0^1 \pi (x^2)^2\,dx = \pi \frac{x^3}{3}\Big|_0^1 - \pi \frac{x^5}{5}\Big|_0^1$$

$$= \pi \cdot \frac{1}{3} - \pi \cdot \frac{1}{5} = \frac{2\pi}{15}$$ m.

11) Buelengde:

c) $y = \frac{x^2}{2} - \frac{1}{4}\ln x$, $x = 1$, $x = e$.

$f'(x) = x - \frac{1}{4}\cdot\frac{1}{x}$



$$L = \int_a^b \sqrt{1+f'(x)^2}\,dx \overset{\text{8.6.7}}{=} \int_1^e \sqrt{1+\left(x-\frac{1}{4x}\right)^2}\,dx =$$

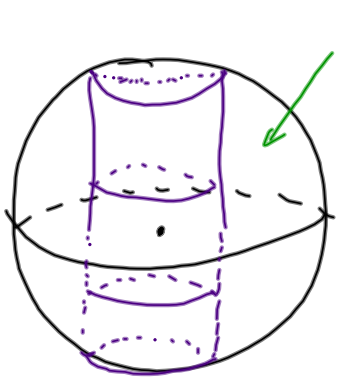
$$= \int_1^e \sqrt{1+x^2-\frac{1}{2}+\frac{1}{16x^2}}\,dx = \int_1^e \sqrt{\frac{1}{2}+x^2+\frac{1}{16x^2}}\,dx$$

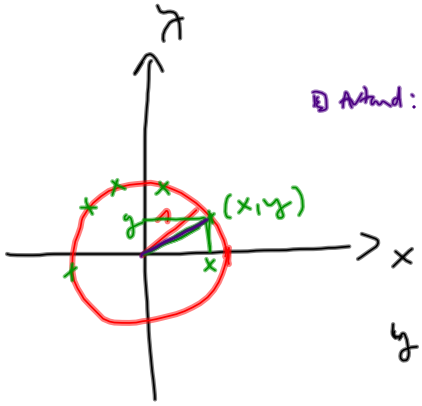
$$= \int_1^e \sqrt{\frac{8x^2+16x^4+1}{16x^2}}\,dx = \int_1^e \sqrt{\frac{(4x^2+1)^2}{16x^2}}\,dx = \int_1^e \frac{4x^2+1}{4x}\,dx$$

$$= \int_1^e \left(x+\frac{1}{4x}\right)dx = \frac{x^2}{2}\Big|_1^e + \frac{1}{4}\cdot\ln x\Big|_1^e = \frac{e^2-1}{2} + \frac{1}{4}\cdot(\ln e - \ln 1) = \frac{e^2-1}{2}+\frac{1}{4}$$

($\ln e = 1$, $\ln 1 = 0$)

15)

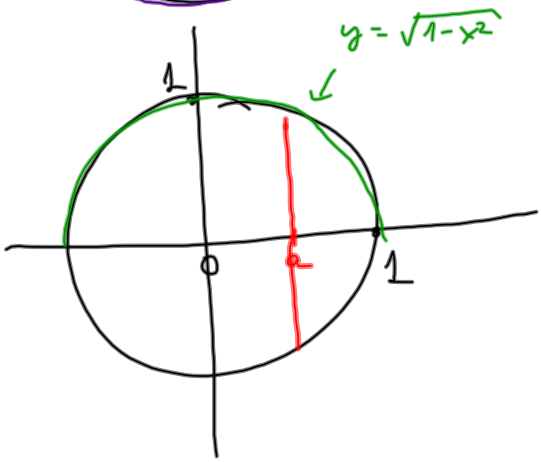




Abstand: $|(x,y)-(0,0)| =$

$= |(x,y)| = x^2+y^2=1$

$y = \pm\sqrt{1-x^2}$



$V = 2\cdot\int_a^1 2\pi x\cdot\underbrace{\sqrt{1-x^2}}_{y}\,dx =$

den deriverte $=-2x$

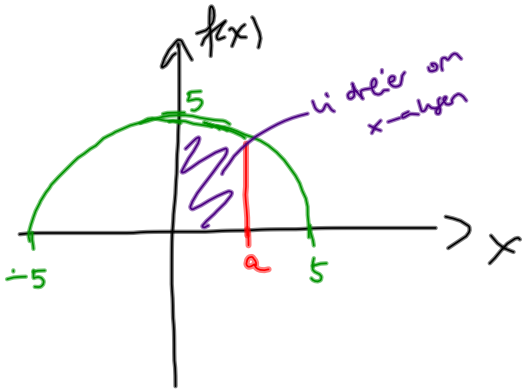
$= -2\pi\int_a^1 -2x\,(1-x^2)^{1/2}\,dx =$

$= -2\pi\,\frac{(1-x^2)^{1/2+1}}{\frac{1}{2}+1}\Big|_{x=a}^{x=1} = \frac{-4\pi}{3}\,(1-x^2)^{3/2}\Big|_{x=a}^{x=1}$

$= \frac{4\pi}{3}\cdot(1-a^2)^{3/2}$

26) La $a \in (0,5)$. Området avgrenset av x-aksen, y-aksen, grafen til $f(x) = \sqrt{25 - x^2}$ og linjen $x = a$ dreies om x-aksen.

a) Volumet til omdreiningslegemet uttrykt ved a.

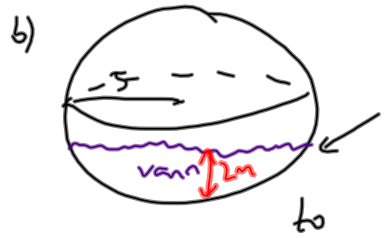


$$V = \int_0^a \pi \cdot f(x)^2 dx = \int_0^a \pi \cdot (25 - x^2)\, dx$$

$$= 25\pi \times \Big|_0^a - \pi \frac{x^3}{3}\Big|_0^a =$$

$$= \pi a \left(25 - \frac{1}{3}a^2\right) \text{ l.e.}$$

b)



akk. $n = t_0$

tømmes vann på 0.5 m³/min.

t_0

- Hvordan forstå avtar vannstanden?

Vi ser på a som funksjon av t

$a(t)$ $\quad a(t_0) = 3$

$$V(t) = \pi \left(25\, a(t) - \frac{a(t)^3}{3}\right)$$

$V'(t) = 25\pi\, a'(t) - a(t)^2 \cdot a'(t)$, så der $a(t) = 3$ er $V'(t) = -\frac{1}{2}$.

$$\frac{1}{2} = \pi\, a'(t) \cdot (25 - 9)$$

$$a'(t) = \frac{1}{2\pi \cdot 16}$$

Så vannhøyden øker med $\frac{1}{32\pi}$ m/min på dette tidspunktet.