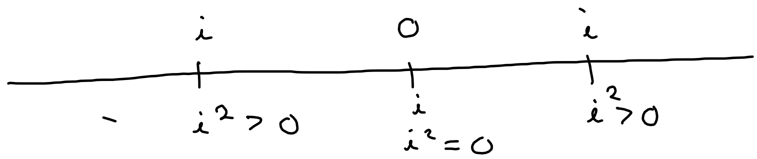
Komplekse tall

Hva er $\sqrt{-1}$? Hvis $i = \sqrt{-1}$, så $i^2 = -1$. Sånne tall finnes ikke på tallinjen



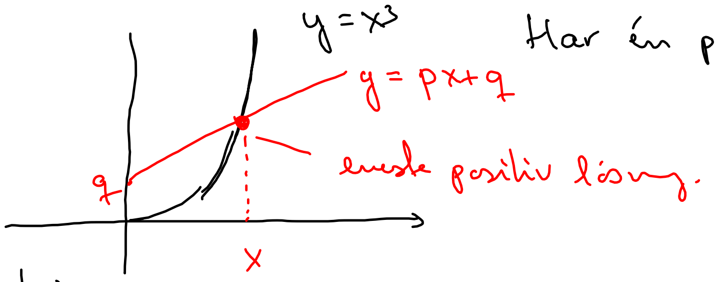
Behov for å regne med $\sqrt{-1}$ $\quad b^2 < 4ac$

$$ax^2 + bx + c = 0, \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

ingen løsninger

Tredjegradsligninger: $\quad x^3 = \underline{px + q}, \; p, q > 0$

Har én positiv rot



Løsning:

$$x = \sqrt[3]{\sqrt{-\left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2} + \frac{q}{2}} - \sqrt[3]{\sqrt{-\left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2} - \frac{q}{2}}$$

Eksempel: $\boxed{x^3 = 15x + 4}$ $\quad p = 15, q = 4.$

$$x = \underbrace{\sqrt[3]{-11\sqrt{-1} + 2} - \sqrt[3]{-11\sqrt{-1} - 2}}_{} \overset{?}{=} 4$$

formell regning gir 4.

$\underline{x = 4}$: VS $4^3 = 64$

HS $15 \cdot 4 + 4 = 64$

Reell problem $\quad$ imaginært $\quad$ Reell løsning

$x^3 = 15x + 4$ $\qquad\qquad$ $x = 4$

Komplekse tall formelt

Anta at $i = \sqrt{-1}$ finnes og studer konsekvensene:

$z = a + ib$ komplekst tall $a, b \in \mathbb{R}$

realdelen til z — imaginærdelen til z

Addisjon

$z = a + ib$, $w = c + id$

$$z + w = a + ib + c + id = \underbrace{(a+c)}_{\text{realdel}} + i\underbrace{(b+d)}_{\text{imaginær}}$$

Subtraksjon:

$$z - w = a + ib - (c + id) = \underbrace{(a-c)}_{\text{realdel}} + i\underbrace{(b-d)}_{\text{imaginærdel}}$$

Multiplikasjon

$$zw = (a+ib)\cdot(c+id) = ac + iad + ibc + i^2 bd$$

-1

$$= \underbrace{ac - bd}_{\text{realdel}} + i\underbrace{(ad+bc)}_{\text{imaginærdel}}$$

Division: $\dfrac{z}{w} = \dfrac{a+ib}{c+id} = \dfrac{(a+ib)(c-id)}{(c+id)(c-id)}$

$$= \frac{ac - iad + ibc \overbrace{- i^2}^{+} bd}{c^2 \underbrace{- i^2}_{+} d^2} = \frac{(ac+bd)+i(bc-ad)}{c^2+d^2}$$

$$= \underbrace{\frac{ac+bd}{c^2+d^2}}_{\text{realdel}} + i\,\underbrace{\frac{bc-ad}{c^2+d^2}}_{\text{imaginærdel}}$$

Eksempel: $z = 3+2i$, $w = \underbrace{1-4i} = 1+(-4)i$

$$zw = (3+2i)(1-4i) = 3\cdot 1 - 3\cdot 4i + 2i \underbrace{- 8 \underbrace{i^2}_{=-1}}_{+8}$$

$$= 11 - 10i = 11 + (-10)i$$

$$\frac{z}{w} = \frac{(3+2i)(1+4i)}{(1-4i)(1+4i)} = \frac{\overset{-8}{3} + 12i + 2i \overset{+8}{+ 8i^2}}{1^2 \underbrace{- 4^2 \cdot i^2}_{+4^2}}$$

$$= \frac{-5+14i}{17} = \underline{\underline{-\frac{5}{17} + \frac{14}{17}i}}$$