INTEGRASJONSTEKNIKKER

DELVIS INTEGRASJON

- PRODUKTREGEL BAKLENGS

$$(u\cdot v)' = u'\cdot v + u\cdot v'$$

$$\int (u\cdot v)'dx = \int u'v + u\cdot v' dx$$

$$u\cdot v = \int u'v dx + \int uv' dx$$

$$\int u\cdot v' dx = u\cdot v - \int u'v dx$$

$$\boxed{\begin{matrix} u = & u' = \\ v' = & v = \end{matrix}}$$

- Gjenta flere ganger
- $\cdot 1$ er en faktor (v')
- $+ C$

SUBSTITUSJON

- KJERNEREGEL BAKLENGS
- Sammensatt funksjon + derivert av kjerne
- Vi kan "invertere substitusjonen"
- V/ bestemte integraler: BYTT GRENSER

DELVIS:

9.1.5.

$$\int \frac{\ln(x^2)}{x^2}\,dx$$

$u = \ln(x^2)$ $\quad u' = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$

$v' = x^{-2}$ $\quad v = -x^{-1}$

$$= \int \underbrace{\ln(x^2)}_{u} \cdot \underbrace{x^{-2}}_{v'}\,dx$$

$$= -x^{-1}\ln(x^2) - \int -x^{-1} \cdot \frac{2}{x}\,dx$$

$$= -\frac{\ln(x^2)}{x} + 2\int x^{-2}\,dx$$

$$= -\frac{\ln(x^2)}{x} + 2(-x^{-1}) + C$$

$$= -\frac{\ln(x^2)}{x} - \frac{2}{x} + C$$

$$= -\frac{2\ln x}{x} - \frac{2}{x} + C$$

$$= -\frac{2}{x}(\ln x + 1) + C$$

9.1.9. $\int 1 \cdot \sin(\ln x)\,dx$

$= x\sin(\ln x) - \int \frac{\cos(\ln x)}{\cancel{x}} \cdot \cancel{x}\,dx$

$= x\sin(\ln x) - \int 1\cos(\ln x)\,dx$

$= x\sin(\ln x) - \left[x\cos(\ln x) - \int \frac{-\sin(\ln x)}{x} \cdot x\,dx\right]$

$= x\sin(\ln x) - x\cos(\ln x) - \int \sin(\ln x)\,dx$

$2\int \sin(\ln x)\,dx = x\sin(\ln x) - x\cos(\ln x) + C$

$\int \sin(\ln x)\,dx = \frac{x}{2}\left(\sin(\ln x) - \cos(\ln x)\right) + C$

DELVIS I

$u = \sin(\ln x)$ $u' = \cos(\ln x) \cdot \frac{1}{x}$

$v' = 1$ $v = x$ $(\int 1\,dx)$

DELVIS II

$u = \cos(\ln x)$ $u' = -\sin(\ln x) \cdot \frac{1}{x}$

$v' = 1$ $v = x$

9.1.11. $\int \left(\frac{x^2}{1+x^2}\right) \cdot \arctan x \, dx$

$u = \arctan x \qquad u' = \frac{1}{1+x^2}$

$v' = \frac{x^2}{1+x^2}$

$$v = \int \frac{x^2 + 0}{1+x^2}\,dx = \int \frac{1+x^2-1}{1+x^2}\,dx$$
$$= \int \frac{1+x^2}{1+x^2}\,dx - \int \frac{1}{1+x^2}\,dx$$
$$= \int 1\,dx - \arctan x$$
$$= x - \arctan x$$

$$= x\arctan x - \arctan^2 x - \int \frac{x - \arctan x}{1+x^2}\,dx$$

$$\int \frac{x}{1+x^2}\,dx - \int \frac{\arctan x}{1+x^2}\,dx$$

$s = 1+x^2$ $\qquad$ $t = \arctan x$

$ds = 2x\,dx$ $\qquad$ $dt = \frac{1}{1+x^2}\,dx$

$\frac{1}{2} ds = x\,dx$

$$= \frac{1}{2}\int s^{-1}\,ds - \int t\,dt$$
$$= \frac{1}{2}\ln|s| - \frac{1}{2}t^2 + C$$
$$= \frac{1}{2}\ln|1+x^2| - \frac{1}{2}\arctan^2 x + C$$

$$= x\arctan x - \arctan^2 x - \frac{1}{2}\ln|1+x^2| + \frac{1}{2}\arctan^2 x + C'$$
$$= x\arctan x - \frac{1}{2}\arctan^2 x - \frac{1}{2}\ln|1+x^2| + C'$$

9.2.1. a) $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

$= \int \frac{\sin u}{\cancel{u}} \cdot 2\cancel{u}\,du$

$= 2\int \sin u\,du$

$= -2\cos u + C$

$= -2\cos\sqrt{x} + C$

$u = \sqrt{x}$

$u'(x) \rightarrow du = \frac{1}{2\sqrt{x}}\,dx$

$u = \sqrt{x}$

$u^2 = x$

$x'(u) \rightarrow 2u = \frac{dx}{du}$

$2u\,du = dx$

9.2.1. e) $\int e^{\sqrt{x}}\,dx$

$$= 2\int s e^{s}\,ds$$

$$= 2\left[\underline{s}e^{s} - \int \underline{e^{s}}\,ds\right]$$

$$= 2\left[se^{s} - e^{s} + C\right]$$

$$= 2\left(\sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}}\right) + C$$

$$= 2e^{\sqrt{x}}\left(\sqrt{x} - 1\right) + C$$

$$\boxed{\begin{aligned} s &= \sqrt{x} \\ s^2 &= x \\ 2s &= \frac{dx}{ds} \\ 2s\,ds &= dx \end{aligned}}$$

DELVIS:

$u = s$	$u' = 1$
$v' = e^{s}$	$v = e^{s}$

9.2.15.

$$\int_0^{\sqrt{3}} \frac{1+x}{\sqrt{4-x^2}}\,dx$$

$$= \int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\,dx + \int_0^{\sqrt{3}} \frac{x}{\sqrt{4-x^2}}\,dx$$

$$= \int_0^{\sqrt{3}} \frac{1}{2\sqrt{1-\left(\frac{x}{2}\right)^2}}\,dx + \int_0^{\sqrt{3}} \frac{x}{\sqrt{4-x^2}}\,dx$$

$$= \int_0^{\frac{\sqrt{3}}{2}} \frac{du}{\sqrt{1-u^2}} + \int_4^1 \left(-\frac{1}{2}\right)\frac{dt}{\sqrt{t}}$$

$$= \Big[\arcsin u\Big]_0^{\frac{\sqrt{3}}{2}} - \frac{1}{2}\left[\frac{1}{-\frac{1}{2}+1} t^{-\frac{1}{2}+1}\right]_4^1$$

$$= \frac{\pi}{3} - 0 - \frac{1}{2}\left[2t^{\frac{1}{2}}\right]_4^1$$

$$= \frac{\pi}{3} - (1-2)$$

$$= \underline{\frac{\pi}{3} + 1}$$

$$\sqrt{4-x^2} = \sqrt{4\left(1-\frac{x^2}{4}\right)} = 2\sqrt{1-\left(\frac{x}{2}\right)^2}$$

$u = \frac{x}{2}$

$du = \frac{1}{2}dx$

Nye grenser

$\frac{\sqrt{3}}{2}$

0

$$\boxed{\begin{array}{ll} x=0 & u=0 \\ x=\sqrt{3} & u=\frac{\sqrt{3}}{2} \end{array}}$$

$t = 4-x^2$

$dt = -2x\,dx$

$-\frac{1}{2}dt = x\,dx$

$$\boxed{\begin{array}{ll} x=0 & t=4 \\ x=\sqrt{3} & t=1 \end{array}}$$

DELBRØKSOPPSPALTNING. $\int \frac{P(x)}{Q(x)} dx$ ← POLYNOMER

$\deg Q(x) > \deg P(x)$

ELLERS: Bruk polynomdiv.

- Vi vet at $Q(x)$ kan faktoriseres over $\mathbb{R}$.

I $(x-r_i)^{n_i}$ 1. gradsuttrykk (n_i = multiplisitet)

II $(x^2+a_jx+b_j)^{m_j}$ 2. gradsuttrykk (m_j = multiplisitet)

Pass på koeffisient foran x og x^2

I $\quad$ a) $\frac{C_1}{x-r_i}$ + b) $\frac{C_2}{(x-r_i)^2} + \dots + \frac{C_{n_i}}{(x-r_i)^{n_i}}$

II $\quad$ a) $\frac{A_1x+B_1}{x^2+a_jx+b_j}$ + b) $\frac{A_2x+B_2}{(x^2+a_jx+b_j)^2} + \dots + \frac{A_{m_j}x+B_{m_j}}{(x^2+a_jx+b_j)^{m_j}}$

INTEGRALER:

I a): $C_1 \ln|x-r_i|$

I b): $\frac{C_2}{1-n}(x-r_i)^{1-n}$

II a) $m=1$

① $A_1=0 \quad B_1\int \frac{1}{x^2+a_jx+b_j}dx$ — $\frac{1}{1+u^2}$ fullføre kvadrat arctan ~~ex~~

② $\int \frac{A_1x+B_1}{x^2+a_jx+b_j}dx$ – få telleren "lik" den deriverte av nevneren.
– du får ett integral med substitusjon ($\ln(x^2\dots)$) og ett som II a) ①.

II b) $m_j>1$ REKURSJONSFORMEL

$$I_m = \int \frac{du}{(1+u^2)^m} = \frac{1}{2(m-1)}\frac{u}{(1+u^2)^{m-1}} + \frac{2m-3}{2(m-1)} I_{m-1}$$

9.3.5. g) $\int \frac{-x^2+2x-1}{(x+1)(x^2+1)}\,dx$

$$\frac{C}{x+1} + \frac{Ax+B}{x^2+1} = \frac{-x^2+2x-1}{(x+1)(x^2+1)}$$

$$\frac{C(x^2+1)+(Ax+B)(x+1)}{(x+1)(x^2+1)} = \frac{-x^2+2x-1}{(x+1)(x^2+1)}$$

SE PÅ TELLERNE:

$$Cx^2 + C + Ax^2 + Ax + Bx + B = -x^2 + 2x - 1$$

$$\begin{bmatrix} 1 & 0 & 1 & -1 \\ 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$C + A = -1$ $\qquad A = -1 - C$ $\qquad A = 1$

$A + B = 2$ $\qquad (-1-C) + B = 2$ $\qquad B = 2 + 1 + C = 3 + C$

$C + B = -1$ $\qquad C + (3 + C) = -1$ $\qquad B = 1$

$2C = -4$

$C = -2$

$$= \int \frac{-2}{x+1}\,dx + \int \frac{x+1}{x^2+1}\,dx$$

I a) $\qquad$ II a) 2

$$= -2\ln|x+1| + \frac{1}{2}\int \frac{2x+2}{x^2+1}\,dx$$

$$= -2\ln|x+1| + \frac{1}{2}\int \frac{2x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx$$

$u = x^2+1$

$du = 2x\,dx$

$$= -2\ln|x+1| + \frac{1}{2}\int \frac{1}{u}\,du + \arctan x$$

$$= -2\ln|x+1| + \frac{1}{2}\ln|x^2+1| + \arctan x + C$$

Se på h 2015 oppg. 11.