# LØSNING AV POLYNOMLIKNINGER MED KOMPLEKSE TALL.

① $z^2 + iz + 2 = 0$

② $z^3 = \dfrac{-8\sqrt{2}}{1+i}$

③ $z^2 + 3z + 7 = 0$

④ $p(z) = z^5 - z^4 + z^3 - z^2 - 12z + 12$

LØS LIKNINGEN $p(z) = 0$

FAKTORISER $p(z)$ I REELLE FAKTORER

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$$z^2 + iz + 2 = 0$$

$$z = \frac{-i \pm \sqrt{i^2 - 4\cdot 2}}{2}$$

$$= \frac{-i \pm \sqrt{-1-8}}{2} = \frac{-i \pm \sqrt{-9}}{2}$$

$$= \frac{-i \pm \sqrt{-1}\cdot\sqrt{9}}{2} = \frac{-i \pm i\cdot 3}{2} = \begin{cases} i \\ -2i \end{cases}$$

LØSNINGER $z = i$ , $z = -2i$

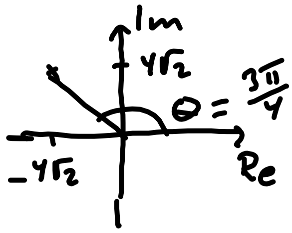
(2)

$$z^3 = \frac{-8\sqrt{2}}{1+i}$$

FINNER REAL OG IMAGINÆR DEL TIL HØYRE SIDE
$(a + b \cdot i)$

H.S.: $\dfrac{-8\sqrt{2}}{1+i} = \dfrac{-8\sqrt{2}(1-i)}{(1+i)(1-i)} = \dfrac{-8\sqrt{2}(1-i)}{1-(-1)} = \dfrac{-8\sqrt{2}}{2}(1-i)$

$= -4\sqrt{2} + 4\sqrt{2} \cdot i$




FINNER SÅ POLARKOORDINATENE TIL H.S. $(r e^{i\theta})$

$r = \sqrt{(4\sqrt{2})^2 + (4\sqrt{2})^2} = \sqrt{32+32} = 8$

$\theta = \frac{3\pi}{4}$

LIKNINGEN KAN SKRIVES: $z^3 = 8 e^{i\frac{3\pi}{4}}$

$$z^3 = 8e^{i\cdot\frac{3\pi}{4}}$$

$$z = 8^{\frac{1}{3}}e^{i\theta_0}\ ,\ 8^{\frac{1}{3}}e^{i\theta_1}\ ,\ 8^{\frac{1}{3}}e^{i\theta_2}$$

der $\theta_0 = \frac{1}{3}\cdot\frac{3\pi}{4} = \frac{\pi}{4}$ $\qquad \theta_1 = \frac{1}{3}\frac{3\pi}{4} + \frac{2\pi}{3} = \frac{\pi}{4} + \frac{2\pi}{3} = \frac{11\pi}{12}$ $\qquad \theta_2 = \frac{1}{3}\frac{3\pi}{4} + \frac{4\pi}{3} = \frac{\pi}{4} + \frac{4\pi}{3} = \frac{19}{12}\pi$

MELLOM REGNING:

$\cos\frac{\pi}{4} = \frac{1}{2}\sqrt{2}$

$\sin\frac{\pi}{4} = \frac{1}{2}\sqrt{2}$

$\cos\frac{11\pi}{12} = \cos\left(\frac{\pi}{4} + \frac{2\pi}{3}\right) = \frac{1}{2}\sqrt{2}\cdot\left(-\frac{1}{2}\right) - \frac{1}{2}\sqrt{2}\left(\frac{1}{2}\sqrt{3}\right) = \frac{1}{4}\left(-\sqrt{2}-\sqrt{6}\right)$ $\qquad \left(\cos\frac{2\pi}{3} = -\frac{1}{2}\right.$

$\sin\frac{11\pi}{12} = \sin\left(\frac{\pi}{4} + \frac{2\pi}{3}\right) = \frac{1}{2}\sqrt{2}\cdot\left(-\frac{1}{2}\right) + \frac{1}{2}\sqrt{2}\cdot\left(\frac{1}{2}\sqrt{3}\right) = \frac{1}{4}\left(-\sqrt{2}+\sqrt{6}\right)$ $\qquad \left.\sin\frac{2\pi}{3} = \frac{1}{2}\sqrt{3}\right)$

$\sin\frac{19\pi}{12} = \sin\left(\frac{\pi}{4} + \frac{4\pi}{3}\right) = \frac{1}{2}\sqrt{2}\cdot\left(-\frac{1}{2}\right) + \frac{1}{2}\sqrt{2}\left(-\frac{1}{2}\sqrt{3}\right) = \frac{1}{4}\left(-\sqrt{2}-\sqrt{6}\right)$

$\cos\frac{19\pi}{12} = \cos\left(\frac{\pi}{4} + \frac{4\pi}{3}\right) = \frac{1}{2}\sqrt{2}\left(-\frac{\sqrt{3}}{2}\right) - \frac{1}{2}\sqrt{2}\left(-\frac{1}{2}\right) = \frac{1}{4}\left(+\sqrt{2}-\sqrt{6}\right)$

LØSNINGER

$$z = 2\left(\frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{2}\,i\right) = \sqrt{2} + \sqrt{2}\cdot i$$

$$z = 2\left(\frac{1}{4}\left(-\sqrt{2}-\sqrt{6}\right) + \frac{1}{4}\left(-\sqrt{2}+\sqrt{6}\right)\cdot i\right) = \frac{1}{2}\left(-\sqrt{2}-\sqrt{6}\right) + \frac{1}{2}\left(-\sqrt{2}+\sqrt{6}\right)\cdot i$$

$$z = \frac{1}{2}\left(\sqrt{2}-\sqrt{6}\right) + \frac{1}{2}\left(-\sqrt{2}-\sqrt{6}\right)\cdot i$$

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$$z^2 + 3z + 7 = 0$$

$$z = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 7}}{2} = \frac{-3 \pm \sqrt{-19}}{2}$$

$$= \frac{-3 \pm \sqrt{-1} \cdot \sqrt{19}}{2} = \frac{-3 \pm i\sqrt{19}}{2}$$

LØSNINGER: $z = -\frac{3}{2} + \frac{\sqrt{19}}{2} \cdot i$

$z = -\frac{3}{2} - \frac{\sqrt{19}}{2} \cdot i$

(4) $p(z) = z^5 - z^4 + z^3 - z^2 - 12z + 12$

LIKNING: $p(z) = 0$

PRØVER OSS FRAM MED HELTALL SOM DELER 12.

F. EKS $z = 1$ : $p(1) = 1^5 - 1^4 + 1^3 - 1^2 - 12 \cdot 1 + 12 = 0!$

DA ER $z = 1$ LØSNING OG $z - 1$ ER FAKTOR I $P(z)$

$P(z) : (z-1)$

$$z^5 - z^4 + z^3 - z^2 - 12z + 12 : (z-1) = z^4 + z^2 - 12$$

$$\begin{array}{l} \underline{z^5 - z^4} \\ 0 \quad z^3 - z^2 - 12z + 12 \\ \quad\;\; \underline{z^3 - z^2} \\ \qquad\quad -12z + 12 \\ \qquad\quad \underline{-12z + 12} \\ \qquad\qquad 0 \end{array}$$

SÅ $p(z) = (z-1)(z^4 + z^2 - 12)$

LØSER $z^4 + z^2 - 12 = 0$

$$(z^2)^2 + z^2 - 12 = 0$$

$$z^2 = \frac{-1 \pm \sqrt{1 - 4 \cdot (-12)}}{2} = \frac{-1 \pm \sqrt{49}}{2} = \frac{-1 \pm 7}{2} = \begin{cases} 3 \\ -4 \end{cases}$$

SÅ $\underline{P(z) = (z-1)(z^2-3)(z^2+4)}$

$= \underline{(z-1)(z-\sqrt{3})(z+\sqrt{3})(z^2+4)}$ ← REEL FAKTOR-ISERING

$(z^2+4) = (z+2i)(z-2i)$

LØSNINGER: $z = 1$, $z = \sqrt{3}$, $z = -\sqrt{3}$, & $z = -2i$, $z = 2i$.