4.3 1, 3abd, 4, 11, 13, 14, 15

1a $\lim\limits_{n\to\infty} \dfrac{8n^4+2n}{3n^4-7}$

$$\frac{8n^4+2n}{3n^4-7} = \frac{8+2\frac{n}{n^4}}{3-7\frac{1}{n^4}} = \frac{8+\frac{2}{n^3}}{3-\frac{7}{n^4}}$$

$$\lim_{n\to\infty} \frac{8+\frac{2}{n^3}}{3-\frac{7}{n^4}} = \frac{8}{3} \quad \text{så} \quad \lim_{n\to\infty} \frac{8n^4+2n}{3n^4-7} = \underline{\frac{8}{3}}$$

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1e $\lim_{n\to\infty} \frac{n^5 + 2\sin n}{e^{-n} + 6n^5}$

$$\lim_{n\to\infty} \frac{n^5 + 2\sin n}{e^{-n} + 6n^5} = \lim_{n\to\infty} \frac{1 + \frac{2}{n^5}\sin n}{\frac{1}{n^5}\cdot e^{-n} + 6} = \underline{\underline{\frac{1}{6}}}$$

$\underline{3a}$ $\lim_{n\to\infty} \left(\sqrt{n+2} - \sqrt{n}\right) =$

$$\lim_{n\to\infty} \frac{\left(\sqrt{n+2} - \sqrt{n}\right)\left(\sqrt{n+2} + \sqrt{n}\right)}{\left(\sqrt{n+2} + \sqrt{n}\right)} = \lim_{n\to\infty} \frac{n+2-n}{\sqrt{n+2} + \sqrt{n}} = \lim_{n\to\infty} \frac{2}{\sqrt{n+2} + \sqrt{n}} = 0$$

3d $\lim_{n\to\infty}\left(\sqrt{1+e^{-2n}} - e^{-n}\right)$

$\lim_{n\to\infty} \sqrt{1+e^{-2n}} = 1$ og $\lim_{n\to\infty} e^{-n} = 0$

så $\lim_{n\to\infty}\left(\sqrt{1+e^{-2n}} - e^{-n}\right) = 1$

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4 $\lim_{n\to\infty} a_n = L$ dersom det for hver $\varepsilon > 0$

finns en N slik at $|a_n - L| < \varepsilon$ når $n > N$

b) $\lim_{n\to\infty} \frac{2\sin n}{n} = 0$

Gitt $\varepsilon > 0$

$$\left|\frac{2\sin n}{n} - 0\right| = \frac{2\,|\sin n|}{n} \leq \frac{2}{n} < \varepsilon$$

når $n > \frac{2}{\varepsilon}$

Så hvis $N > \frac{2}{\varepsilon}$ og $n > N$ så er $\left|\frac{2\sin n}{n}\right| < \varepsilon$.

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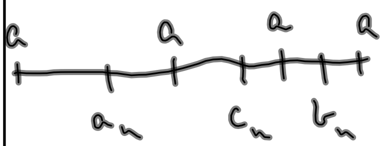
11

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = a$$

$a_n \le c_n \le b_n$ for alle n

$$|c_n - a| \le \max\{|a_n - a|, |b_n - a|\}$$



Gitt $\varepsilon > 0$. La N_a og N_b være slik at $|a_n - a| < \varepsilon$ og $|b_n - a| < \varepsilon$ når $n > N_a$ og $n > N_b$

Velg $N = \max\{N_a, N_b\}$.

13 $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = 0$

$\underline{b}$ $a_n = \frac{1}{n}$ $b_n = \frac{1}{n^2}$

si tr $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} n = \infty$

14 $\lim_{n\to\infty} a_n = \infty = \lim_{n\to\infty} b_n$

$\underline{b}$

$a_n = n$ $b_n = n^2$

si tr $\lim_{n\to\infty} a_n - b_n = \lim_{n\to\infty} n - n^2 = -\infty$

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15 $a_n \geq a_{n+1}$ for alle n

$\{a_n, n=1,2,\ldots\}$ er begrenset

det vil si at det fins en L slik at

$a_n > L$ for alle n.

La $a = \inf\{a_n\}$

La $\varepsilon > 0$, da er $a+\varepsilon$ ikke en nedre grense,

så det fins en N slik at $a_N < a+\varepsilon$.

Hvis nå $n > N$ så er $a+\varepsilon > a_N \geq a_n \geq a$

altså $|a_n - a| < \varepsilon$.

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