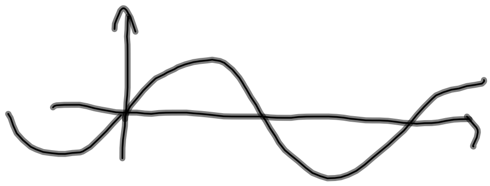
5.1 1abc 3ab 5ab e g 6ab 7 9abc

1 b $f(x) = \ln(x^2 - 4)$ $x^2 - 4 > 0$

så $D_f = (-\infty, -2) \cup (2, \infty)$

3 b $f(x) = \sin x^2$



$V_f = [-1, 1]$

5 $f(x)$ er kontinuerlig i $x=a$ dersom $a \in D_f$ og det for hver $\varepsilon > 0$ finnes en $\delta > 0$ slik at

$$|f(x) - f(a)| < \varepsilon \quad \text{når} \quad |x-a| < \delta$$

e) $f(x) = \frac{1}{x}$ $\quad x = 1$

Gitt $\varepsilon > 0$, skal finne passe δ.

$$\left|\frac{1}{x} - 1\right| = \left|\frac{1-x}{x}\right| = \frac{|x-1|}{|x|} \underset{|x| > \frac{1}{2}}{<} 2|x-1|$$

Velg $\delta < \min\left\{\frac{1}{2}, \frac{1}{2}\varepsilon\right\}$.

Hvis $|x-1| < \delta$ så er $\left|\frac{1}{x} - 1\right| < 2|x-1| < 2\delta < \varepsilon$

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g) $f(x) = \sqrt{x}$ $\qquad x = 4$ $\qquad x > 0$

$\varepsilon > 0 \qquad |\sqrt{x} - \sqrt{4}| = \left| \frac{(\sqrt{x} - 2)(\sqrt{x} + 2)}{\sqrt{x} + 2} \right|$

$$= \frac{|x-4|}{|\sqrt{x}+2|} < \frac{1}{2}|x-4|$$

Velg $\delta = 2\varepsilon$ så vil $|\sqrt{x} - \sqrt{4}| \overset{x>0}{<} \frac{1}{2}|x-4| < \frac{1}{2} \cdot 2\varepsilon = \varepsilon$

når $|x-4| < \delta$

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6 b

$$f(x) = \begin{cases} \cos\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$\cos \frac{1}{\frac{1}{2k\pi}} = \cos 2k\pi = \underline{1} \qquad k = 1, 2, 3 \ldots$$

La $\varepsilon = \frac{1}{2}$, da fins det for hver

$\delta > 0$ en k slik at $0 < \frac{1}{2k\pi} < \delta$, så

$$f\left(\frac{1}{2k\pi}\right) = 1 > \frac{1}{2} \quad \text{selv om}$$

$$0 < \frac{1}{2k\pi} < \delta .$$

Så f er diskontinuerlig i $x = 0$.

d $\quad f(x) = \cos \ln |\sin(e^{x^2})| \qquad x = 0$

x^2 er kontinuerlig i $x=0$

e^u er kontinuerlig i $u=0$

så e^{x^2} — " — $x=0$ $\qquad e^0 = 1$

$\sin u$ — " — i $u=1$

så $\sin(e^{x^2})$ — " — i $x=0$ $\qquad \sin 1 > 0$

$\ln|u|$ — " — i $u = \sin 1$

så $\ln|\sin(e^{x^2})|$ — " — i $x=0$

$\cos u$ — " — i $\ln|\sin 1|$

så $f(x)$ — " — i $x=0$

9a $\quad f(x) = x^3$ ingen diskontinuiteter

b $\quad f(x) = \begin{cases} \sqrt{x} & x > 0 \\ x+1 & x \leq 0 \end{cases}$

eneste mulige diskontinuitet er i $x = 0$

$f(0) = 1$

Velg $\varepsilon = \frac{1}{2}$ da er $\sqrt{x} < \frac{1}{2}$ når $0 < x < \frac{1}{4}$

så $|\sqrt{x} - 1| > \frac{1}{2}$

så f er disk. i $x = 0$

9c $\quad f(x) = \begin{cases} \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

$$\sin \frac{1}{\frac{1}{\frac{\pi}{2} + 2k\pi}} = \sin\left(\frac{\pi}{2} + 2k\pi\right) = 1$$

så med $\varepsilon = \frac{1}{2}$ så fins det for hver

$\delta > 0$ en k slik at $0 < \frac{1}{\frac{\pi}{2} + 2k\pi} < \delta$

mens $f\left(\frac{1}{\frac{\pi}{2} + 2k\pi}\right) = 1 > \frac{1}{2} = \varepsilon$

så f er diskk. i $x = 0$.

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