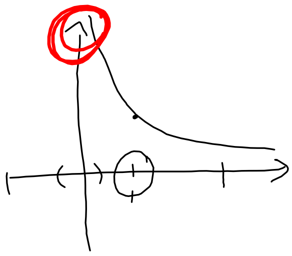
5.1.5. e) $f(x) = \frac{1}{x}$ er kontinuerlig i 1.



Gitt $\varepsilon > 0$, betrakt

$$|f(x) - f(1)| = \left|\frac{1}{x} - 1\right|$$
$$= \left|\frac{1-x}{x}\right|$$
$$= \frac{|x-1|}{|x|}$$

$h = x - 1$

$$= \frac{|h|}{|h+1|}$$
$$= \frac{1}{|h+1|} \cdot |h|$$

Velg $\delta_1 = \frac{1}{2}$, for $|h| < \frac{1}{2}$ så er $\frac{3}{2} > |h+1| > \frac{1}{2}$

$$< \frac{1}{\frac{1}{2}} \cdot |h| = 2|h|$$

Velg $\delta_2 = \frac{\varepsilon}{2}$, for $|h| < \frac{\varepsilon}{2}$ så er

$$< 2 \cdot \frac{\varepsilon}{2} = \varepsilon.$$

Velg $\delta = \min\{\delta_1, \delta_2\}$ og da er $|f(x) - f(1)| < \varepsilon$, og f er kontinuerlig i 1.

5.4.2 b) Skal vise at $\lim_{x \to 3} x^2 = 9$ v/ definisjonen. $a^2 - b^2 = (a-b)(a+b)$

Gitt $\varepsilon > 0$, se på $|x^2 - 9| = |(x-3)(x+3)|$

$= |x-3| \cdot |x+3|$

$= |h| \cdot |h+6|$ $\quad h = x-3$, $x = h+3$

Velg $\delta_1 = 1$, så for $|h| < 1$ har vi at $|h+6| < 7$

$< 7 \cdot |h|$

Velg $\delta_2 = \frac{\varepsilon}{7}$, så for $|h| < \frac{\varepsilon}{7}$ er $\quad < 7 \cdot \frac{\varepsilon}{7} = \varepsilon$.

Velg $\delta = \min\{1, \frac{\varepsilon}{7}\}$, så for $x \in \mathbb{R}$ og $|x-3| < \delta$

så er $|f(x) - 9| < \varepsilon$ og $\lim_{x \to 3} x^2 = 9$.

OBSERVASJON 5.4.7. VIKTIG

En funksjon $f:[a,b] \to \mathbb{R}$ er kontinuerlig i et indre punkt $c \in (a,b)$ $\Longleftrightarrow$ $\lim_{x \to c} f(x) = f(c)$.

Samme for endepunktene, men da med ensidige grenser.

$\lim_{x \to a^+} f(x) = f(a)$ eller $\lim_{x \to b^-} f(x) = f(b)$.

5.4.4. c) Vis at $f(x) = \begin{cases} \frac{1}{x} & \text{for } 0 < x \leq 6 \\ \frac{\sqrt{x+3}-3}{x-6} & \text{for } x > 6 \end{cases}$ er kontinuerlig i $x = 6$.

Nok å vise at $\underbrace{\lim_{x \to 6} f(x)} = f(6) = \frac{1}{6}$

$$\lim_{x \to 6^+} \frac{\sqrt{x+3}-3}{x-6} = \lim_{x \to 6^+} \frac{(\sqrt{x+3}-3)(\sqrt{x+3}+3)}{(x-6)(\sqrt{x+3}+3)}$$

$$= \lim_{x \to 6^+} \frac{x+3-9}{(x-6)(\sqrt{x+3}+3)}$$

$$= \lim_{x \to 6^+} \frac{\cancel{(x-6)}}{\cancel{(x-6)}(\sqrt{x+3}+3)}$$

$$= \frac{1}{(\sqrt{9}+3)}$$

$$= \underline{\underline{\frac{1}{6}}}$$