Sek. 5.4

c) $f(x) = \begin{cases} \frac{1}{x}, & 0 \le x \le 6 \\ \frac{\sqrt{x+3}-3}{x-6}, & x > 6 \end{cases}$ i plkt. 6 ?

Ensidige grenser:

$$\lim_{x \to 6^-} f(x) = \lim_{x \to 6^-} \frac{1}{x} = \frac{1}{6}$$

$$\lim_{x \to 6^+} f(x) = \lim_{x \to 6^+} \frac{\overbrace{\sqrt{x+3}-3}^{\to 0}}{\underbrace{x-6}_{\to 0}}$$

$$= \lim_{x \to 6^+} \frac{(\sqrt{x+3}-3)(\sqrt{x+3}+3)}{(x-6)(\sqrt{x+3}+3)}$$

$$= \lim_{x \to 6^+} \frac{x+3-9}{(x-6)(\sqrt{x+3}+3)} = \lim_{x \to 6^+} \frac{\cancel{x-6}}{\cancel{(x-6)}(\sqrt{x+3}+3)}$$

$$= \lim_{x \to 6^+} \frac{1}{\sqrt{x+3}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$$

$\boxed{a^2 + b^2 = c^2}$

Så $\lim_{x \to 6^-} f(x) = \lim_{x \to 6^+} f(x) = \frac{1}{6}$, så $\lim_{x \to 6} f(x)$ eksisterer og funk. er kont. i plkt 6 (se OBS. 5.4.7)

9) a) Vis fra fig: $\frac{1}{2}\sin x < \frac{x}{2} < \frac{1}{2}\tan x$

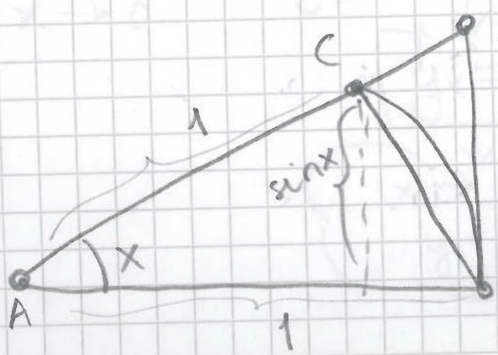
(sml $Ar(\Delta ABC)$, $Ar(\bigcirc ABC)$ og $Ar(\Delta ABD)$):

Areal sirkel: πr^2

Her: $r = 1$

$\Rightarrow$ Areal $= \pi$

$\Rightarrow$ x i radian $\Rightarrow$ $Ar(\bigcirc ABC) = \frac{x}{2}$



tan x

pga. adjacent = 1

$\tan = \frac{opp}{adj} = \frac{opp}{1}$

$Ar(\Delta ABC) = \frac{1 \cdot \sin x}{2}$ $= \frac{\sin x}{2}$

(gr. linje · høyde / 2)

$Ar(\bigcirc ABC) = \frac{x}{2}$

Sek. 5.4

$$Ar(\Delta ABD) = \frac{1 \cdot \tan x}{2} = \frac{\tan x}{2}$$

$$\frac{\text{gr. linje} \cdot \text{høyde}}{2}$$

Ser at $Ar(\Delta ABC) < Ar(\bigcirc ABC) < Ar(\Delta ABD)$

$$\Downarrow$$

$$\frac{\sin x}{2} < \frac{x}{2} < \frac{\tan x}{2}$$

b)

$$\frac{\sin x}{2} < \frac{x}{2}$$

$$\Downarrow \quad (x > 0)$$

$$\frac{\sin x}{x} < \frac{2}{2} = 1$$

$$\Downarrow$$

$$\boxed{\frac{\sin x}{x} < 1}$$

og:

$$\frac{x}{2} < \frac{\tan x}{2}$$

$$\Downarrow$$

$$x < \frac{\sin x}{\cos x}$$

$$\Downarrow \quad (x \in (0, \tfrac{\pi}{2}) \rightarrow \cos x > 0$$

$$x \cos x < \sin x$$

$$\Downarrow \quad (x > 0)$$

$$\boxed{\cos x < \frac{\sin x}{x}}$$

Dermed er

$$\cos x < \frac{\sin x}{x} < 1.$$