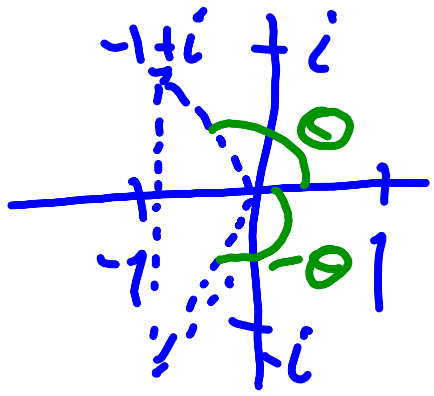
3.4.8 a) Finn alle $z \in \mathbb{C}$ slik at

$$z^3 = -1 + i$$

og tegn de i en figur.



Ukesoppgaver:

3.4: 1bd, 3a, 8, 9a, 11d, 15

3.5: 1ab, 3a, 5, 9

3.3: 10

4.3: 1, 3abd, 4, 11, 13, 14, 15, 18

MEK12: 1, 2, 3, 13

$$r = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$$

$$\cos\theta = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\iff \quad \theta = \frac{3\pi}{4} \text{ eller } \theta = -\frac{3\pi}{4}$$

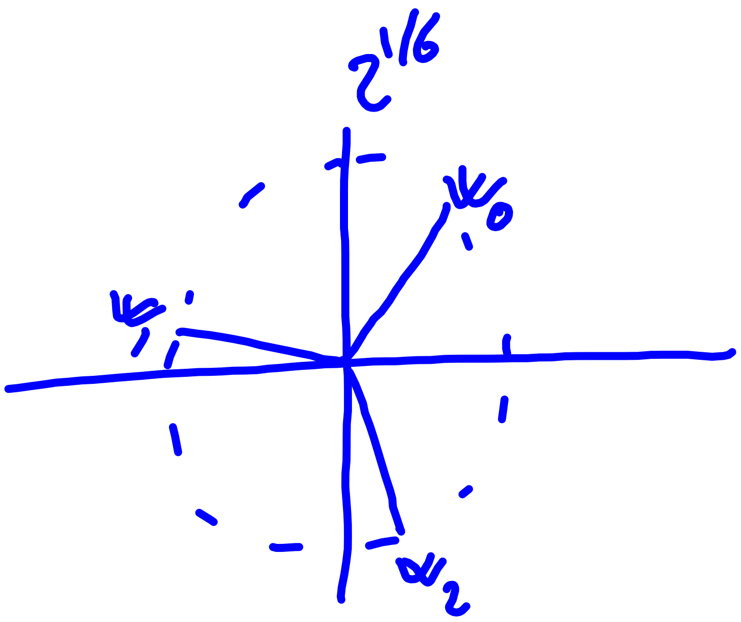
Grafisk $\Rightarrow \quad \theta = \frac{3\pi}{4}$

Ligningen blir da $\quad z^3 = \sqrt{2} \cdot e^{i\frac{3\pi}{4}}$

- $w_0 = \sqrt[3]{\sqrt{2}} \cdot e^{i\frac{3\pi}{4}/3} = (2^{1/2})^{1/3} \cdot e^{i\frac{\pi}{4}} = \underline{2^{1/6} \cdot e^{i\frac{\pi}{4}}}$
- $w_+ = e^{i\frac{2\pi}{3}}$

$$w_1 = w_0 \cdot w_+ = 2^{1/6} e^{i\frac{\pi}{4}} \cdot e^{i\frac{2\pi}{3}} = \underline{2^{1/6} e^{i\frac{11\pi}{12}}}$$

$$w_2 = w_1 \cdot w_+ = 2^{1/6} e^{i\frac{11\pi}{12}} e^{i\frac{2\pi}{3}} = \underline{2^{1/6} e^{i\frac{19\pi}{12}}}$$



b) La $z = z_1 = 2^{1/6} e^{i\frac{11\pi}{12}}$. Vi vil finne en $n \in \mathbb{N}$ s.a.

$$z^n = a,$$

der $a \in \mathbb{R}$.

$$z^n = (2^{1/6})^n \left(e^{i\frac{11\pi}{12}}\right)^n = 2^{n/6} e^{i\frac{11n\pi}{12}}$$

Hvis $\frac{11\pi}{12} \cdot n$ skal være et multiplum av π, så må $\frac{11}{12} \cdot n$ være et heltall.

$$\frac{11}{12} = \frac{11}{2 \cdot 2 \cdot 3}.$$

Altså må vi ha $n = 12$ for å få til dette.

$$z^{12} = 2^{12/6} \cdot e^{i\frac{11\pi}{12} \cdot 12} = 2^2 \cdot e^{i11\pi} = \underline{-4}.$$

3.5.3a) Finn reell og kompleks faktorisering av $z^4 + 2z^2 + 1$.

$$z^4 + 2z^2 + 1 = 0$$

$$(z^2)^2 + 2(z^2) + 1 = 0$$

$w = z^2$, $\quad (z^2+1)^2 = 0 \quad$ (1. kvadratsetn.)

$$\Rightarrow \quad z^2 + 1 = 0$$

$$\Leftrightarrow \quad z^2 = -1$$

$$\Leftrightarrow \quad z = \pm\sqrt{-1} = \pm i$$

Altså har vi at

$$z^4 + 2z^2 + 1 = (z^2+1)^2 = \big((z-i)(z+i)\big)^2$$

$$= \underbrace{(z-i)^2 (z+i)^2}_{\text{Komplekse}}$$

Reelle