# Plenum 6/9 - 2017

3.1 f)

$$\frac{4+3i}{2+i} = \frac{(4+3i)(2-i)}{(2+i)(2-i)} = \frac{8-4i+6i-3\overset{-1}{i^2}}{2^2-\underbrace{i^2}_{-1}} = \frac{8+2i+3}{4+1}$$

$$= \frac{11+2i}{5} = \frac{11}{5}+\frac{2}{5}i$$

3f)

$$\overline{(2-3i)+i\,\frac{4+5i}{1-i}}$$

to strategier / forenkle uttrykket under streken så mye som mulig

\ Bruke regneregler for konjugasjon til å forenkle

$\overline{z+w}=\bar{z}+\bar{w}$, $\overline{z-w}=\bar{z}-\bar{w}$

$\overline{zw}=\bar{z}\,\bar{w}$ $\overline{\left(\frac{z}{w}\right)}=\frac{\bar{z}}{\bar{w}}$

$$= \overline{2-3i}+\overline{\left(i\,\frac{4+5i}{1-i}\right)}$$

$$= 2+3i+\bar{i}\cdot\overline{\left(\frac{4+5i}{1-i}\right)}$$

$$= 2+3i+(-i)\,\frac{4-5i}{1+i} = 2+3i+(-i)\,\frac{(4-5i)(1-i)}{(1+i)(1-i)} = 2+3i+(-i)\,\frac{4-4i-5i+\overset{-5}{5i^2}}{\underbrace{1^2-i^2}_{2}}$$

$$= 2+3i+(-i)\,\frac{-1-9i}{2} = 2+3i+\frac{i+\overset{-9}{9i^2}}{2} = 2+3i+\frac{i}{2}-\frac{9}{2}$$

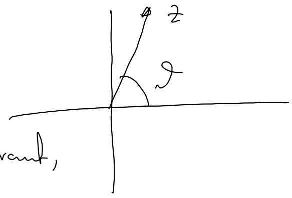
$$= -\frac{5}{2}+\frac{7}{2}i$$

# Seksjon 3.2

**Oppgave 3e)** $z = 1 + i\sqrt{3}$ Finn modulus og argument

$r = \sqrt{a^2 + b^2} = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = \underline{\underline{2}}$

$\sin\vartheta = \frac{b}{r} = \underline{\underline{\frac{\sqrt{3}}{2}}}$

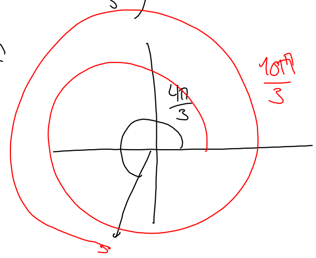



Siden $z$ ligger i første kvadrant, betyr dette at $\underline{\underline{\vartheta = \frac{\pi}{3}}}$

**6b)** $r = 2$, $\vartheta = \frac{10\pi}{3}$ Hva er $a$ og $b$?

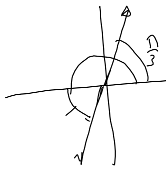
$z = r(\cos\vartheta + i\sin\vartheta) = 2\left(\cos\frac{10\pi}{3} + i\sin\frac{10\pi}{3}\right)$

Mellomregning: $\frac{10\pi}{3} = \frac{6\pi}{3} + \frac{4\pi}{3} = 2\pi + \frac{4\pi}{3}$




$z = 2\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

$= 2\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = \underline{\underline{-1 - i\sqrt{3}}}$

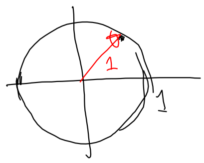



$\frac{4\pi}{3} = \pi + \frac{1}{3}\pi$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

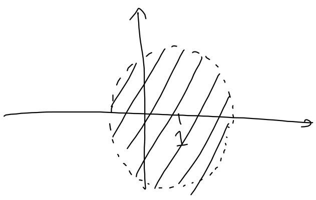
$\cos\frac{\pi}{3} = \frac{1}{2}$

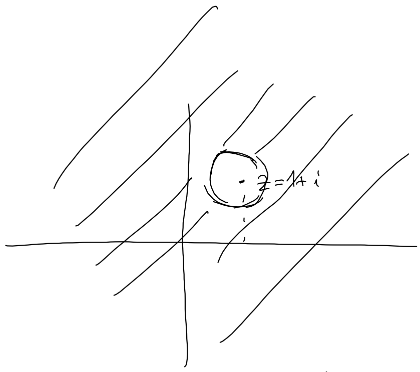
**Oppgave 10** a) $\{z : |z| = 1\}$




b) $\{z : \underbrace{|z-1|}_{\text{avstanden mellom } z \text{ og } 1} < 2\}$

$|z - w| =$ avstanden mellom $z$ og $w$

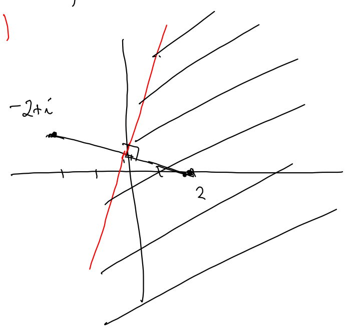



c) $\{z \mid \underbrace{|z-(i+1)|}_{\text{avstanden fra } z \text{ til } i+1} \geq 1/2\}$

$|z-(i-2)| = |z - \underline{(-2+i)}|$

d) $\{z : \underbrace{|z-2|}_{\text{avstanden fra } z \text{ til } \underline{2}} \leq \underbrace{|\overbrace{z-i+2}|}_{\text{avstanden fra } z \text{ til } \underline{-2+i}}\}$



**13** $z = 1 + i\sqrt{3}$, $w = 1 + i$

a) $zw = (1+i\sqrt{3})(1+i) = 1 + i + i\sqrt{3} + i^2\sqrt{3} = (1-\sqrt{3}) + i(1+\sqrt{3})$

$-\sqrt{3}$

$\frac{z}{w} = \frac{1+i\sqrt{3}}{1+i} = \frac{(1+i\sqrt{3})(1-i)}{(1+i)(1-i)} = \frac{1 - i + i\sqrt{3} - i^2\sqrt{3}}{\underbrace{1^2 - i^2}_{+1}} = \frac{1+\sqrt{3}+i(\sqrt{3}-1)}{2}$

$+\sqrt{3}$

b) Finn polarformen til $z$ og $w$.

$z$: $r_1 = \sqrt{a^2+b^2} = \sqrt{1^2+\sqrt{3}^2} = 2$

$\sin\theta_1 = \frac{b}{r} = \frac{\sqrt{3}}{2}$, som gir $\theta_1 = \frac{\pi}{3}$ siden $z$ ligger i første kvadrant.

$w$: $r_2 = \sqrt{1^2+1^2} = \sqrt{2}$

$\sin\theta_2 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$, som gir $\theta_2 = \frac{\pi}{4}$ — " — $w$ — " —

Polarformen til $\frac{z}{w}$: Modulus $\frac{r_1}{r_2} = \frac{2}{\sqrt{2}} = \frac{\sqrt{2}\cdot\sqrt{2}}{\sqrt{2}} = \sqrt{2}$

Argument: $\theta_1 - \theta_2 = \frac{\pi}{3} - \frac{\pi}{4} = \frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{12}$

c) To uttrykk for det samme

$\frac{z}{w} = \frac{1+\sqrt{3}}{2} + i\,\frac{\sqrt{3}-1}{2}$

$\frac{z}{w} = r(\cos\theta + i\sin\theta) = \sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right) = \sqrt{2}\cos\frac{\pi}{12} + i\,\sqrt{2}\sin\frac{\pi}{12}$

Altså: $\sqrt{2}\cos\frac{\pi}{12} = \frac{1+\sqrt{3}}{2} \Rightarrow \cos\frac{\pi}{12} = \frac{1+\sqrt{3}}{2\sqrt{2}} = \frac{(1+\sqrt{3})\sqrt{2}}{2\sqrt{2}\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{4}$

$\sqrt{2}\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2} \Rightarrow \sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{(\sqrt{3}-1)\sqrt{2}}{2\sqrt{2}\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}$

$\sin(u-v) = \sin u\cos v - \cos u\sin v$

$\sin 15^\circ = \sin(45^\circ - 30^\circ) = \sin 45^\circ\cos 30 - \cos 45\sin 30^\circ$

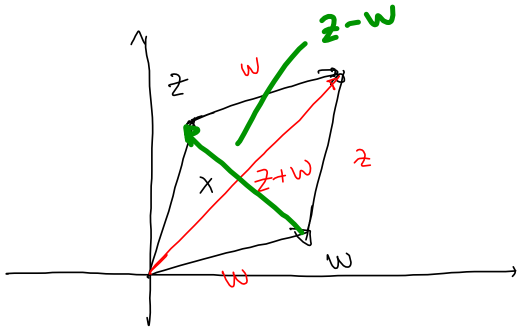
15 Vis at $|z+w|^2 + |z-w|^2 = 2|z|^2 + 2|w|^2$

(Husk at $|z|^2 = z \cdot \bar{z}$).

$|z+w|^2 + |z-w|^2 = (z+w)\overline{(z+w)} + (z-w)\overline{(z-w)} = (z+w)(\bar{z}+\bar{w}) + (z-w)(\bar{z}-\bar{w})$

$= z\bar{z} + \cancel{z\bar{w}} + \cancel{w\bar{z}} + w\bar{w} + z\bar{z} - \cancel{z\bar{w}} - \cancel{w\bar{z}} + w\bar{w} = 2z\bar{z} + 2w\bar{w}$

$= 2|z|^2 + 2|w|^2$




I et parallellogram er summen av kvadratet av diagonalen lik summen av kvadratet av sidene.

Seksjon 3.3:

Oppgave 6: Bruk De Moivres formel til å utlede formlene for $\cos 2u$ og $\sin 2u$.

De Moivre: $(\cos u + i \sin u)^n = \cos nu + i \sin nu$

$n = 2$: $(\cos u + i \sin u)^2 = \cos 2u + i \sin 2u$

$$\begin{aligned} &\| \\ &\cos^2 u + 2i \cos u \sin u - \sin^2 u \\ &\| \\ &(\cos^2 u - \sin^2 u) + i\, 2 \sin u \cos u \end{aligned}$$

Dermed:

$$\cos 2u = \overbrace{\cos^2 u}^{1-\sin^2 u} - \underbrace{\sin^2 u}_{1-\cos^2 u} = 1 - 2\sin^2 u = 2\cos^2 u - 1$$

$$\sin 2u = 2 \sin u \cos u$$

Bonus Oppgave 3.2, nr 11a)

$\{z : \underbrace{2\text{Re}(z) < |z|^2}\}$  $z = x + iy$  $|z| = \sqrt{x^2+y^2}$

$2x < x^2 + y^2 \Rightarrow 0 < x^2 - 2x + y^2$

$= \underbrace{x^2 - 2x + 1}_{(x-1)^2} - 1 + y^2$

$1 < (x-1)^2 + y^2$

$(x-1)^2 + y^2 = 1^2$